

## Article

# Advertising Decisions of Platform Supply Chains Considering Network Externalities and Fairness Concerns

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**Abstract:** With the popularization of platform economics, many manufacturers are shifting their operations from offline to online, forming platform supply chains (PSCs), which combine e-commerce with supply chain management. To study the influences of network externalities and fairness concerns on advertising strategies of the platform supply chain (PSC), we construct decentralized decision-making models, with and without fairness concerns. Then, we solve the optimal decisions regarding PSC and use numerical examples to verify the conclusions of the decision models. We further analyze the internal influences of advertising strategies on network externalities in the extended model. We find that the network externalities are beneficial to the PSC system, but the manufacturer's fairness concerns are not beneficial to the PSC. The advertising strategies of the network platform are not affected by network externalities and fairness concerns. In the extended model, the manufacturer can obtain more profits, but the network platform yields less profit than the decentralized model without fairness concerns. Moreover, the more sensitive the network externalities are to the change in advertising strategies, the greater the profits for the PSC members.

**Keywords:** e-commerce; platform supply chain; network externalities; advertising strategy; fairness concerns

**MSC:** 65K10; 90B06; 90C05; 90C31; 90C47; 91A80; 91B50



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## 1. Introduction

The markets in which supply chain firms operate are growing increasingly complicated in today's globalized environment [1]. The e-commerce sector is gaining attention due to the fast expansion of the Internet industry and the supply chain, including the use of dynamic updating mechanisms [2], urban logistics [3], product identification [4], and so on. Currently, a mushrooming number of startups are fundamentally changing their business models, focusing largely on business intelligence and the platform economy [5]. According to statistics, China's online retail sales reached CNY 11,760.1 billion in 2020, an increase of 10.9% over the previous year on a comparable basis. In 2020, due to the impact of the COVID-19 epidemic, online shopping has become the mainstream consumption pattern. Many manufacturers have switched from offline to online, cooperating with network platforms to form platform supply chains (PSC). The PSC is the combination of e-commerce and supply chain management, which is a novel supply chain model [6,7]. In the PSC, the e-commerce platform replaces the offline retailers, which makes the operation of the PSC system present some new characteristics, including network externality.

In the era of the network economy, network externalities that are generated in the sales process have become one of the important factors that increase the revenue of network platforms. The network externalities indicate that an increased total number of consumers buying similar or compatible products can raise the individual utility [8]. When consumers purchase commodities on the platform, consumers not only get utility from purchasing services, but also gain additional utility due to the increased scale of consumers on the platform. Similarly, manufacturers also believe that choosing a larger-scale network platform can bring greater utility to themselves [9]. With consumers increasing and more manufacturers settling in, network externalities will increase the volume of orders for the network platform; therefore, the platform will also obtain more profits, which ultimately leads to a huge gap in profits between the manufacturer and the platform.

With the mature technologies of information processing [10] and machine learning [11], the e-commerce industry tends to form a situation where the winner takes all, which in turn leads to a monopoly on the network platform. As the network platform occupies a dominant position, the platform would intervene in the manufacturer's decisions by using its strong economic power. For example, since 2015, Alibaba (accessed from: [alibabagroup.com](http://alibabagroup.com), accessed on 24 June 2022) has implemented the either-or monopolistic behavior, restricting manufacturers to only trade with them. In 2019, because Galanz (accessed from: [galanz.com](http://galanz.com), accessed on 24 June 2022) intended to join Pinduoduo (accessed from: [pinduoduo.com](http://pinduoduo.com), accessed on 24 June 2022), Tmall (accessed from: [tmall.com](http://tmall.com), accessed on 24 June 2022) suspended the promotion of Galanz (accessed from: [galanz.com](http://galanz.com), accessed on 24 June 2022) products, causing Galanz (accessed from: [galanz.com](http://galanz.com), accessed on 24 June 2022) to suffer a huge profit loss (accessed from: <https://baijiahao.baidu.com/s?id=1649408587476714773&wfr=spider&for=pc>, accessed on 24 June 2022). On 10 April 2021, the State Administration for Market Regulation of China imposed a fine of CNY 18.228 billion on Alibaba ( [alibabagroup.com](http://alibabagroup.com), accessed on 24 June 2022) under the law (accessed from: <https://baijiahao.baidu.com/s?id=1696616953878995740&wfr=spider&for=pc>, accessed on 24 June 2022). It can be seen that the network platform's compulsory behavior regarding manufacturers has a significant impact on the network economy.

The network platforms interfere with manufacturers' decision making using their strong economic power, which triggers the manufacturers' dissatisfaction and fairness concerns. Empirical evidence has found that supply chain members show great concern about fairness [12]. The manufacturers have a strong willingness to care about fairness because of the profit gap among PSC companies. The large network platforms and the great income gap can lead to the manufacturers' stronger dissatisfaction and fairness concerns, which can be reflected in the decision-making process [13]. In the PSC, fairness concerns affect the decision making and profits of the channel's members. This study aims to explore whether fairness concerns are favorable or unfavorable to the operation of companies and the PSC system.

Given the network externalities generated in the sales process, network platforms have adopted some advertising strategies to attract more consumers [14]. For example, large network platforms, such as Tmall (accessed from: [tmall.com](http://tmall.com), accessed on 24 June 2022), often put advertisements in subway stations and bus stations to attract potential consumers and increase the network externalities of the platform. Moreover, some e-commerce platforms also invest a lot of money in advertising in the early stages of their establishment to quickly compete for market share and increase the scale of the platform. For example, Pinduoduo (accessed from: [pinduoduo.com](http://pinduoduo.com), accessed on 24 June 2022) was established less than 5 years ago, but its market value has exceeded USD 100 billion. One of the important reasons for Pinduoduo (accessed from: [pinduoduo.com](http://pinduoduo.com), accessed on 24 June 2022) to counterattack in a short time is its advertising decisions. According to statistics, in 2019, Pinduoduo (accessed from: [pinduoduo.com](http://pinduoduo.com), accessed on 24 June 2022) invested about CNY 27.2 billion in advertising and marketing, while its annual revenue was only CNY 30.1 billion (accessed from: <https://www.163.com/dy/article/F7QRL5G80517MSIB.html>, accessed on 24 June 2022). It can be seen that the advertising decisions of

e-commerce platforms have become one of the important factors affecting product sales and the development of companies.

This paper considers the network externalities and manufacturers' fairness concerns and studies the advertising strategies and product pricing in the PSC. We try to explore the following issues: (1) What are the optimal decisions for the manufacturer and the network platform, with or without fairness concerns? (2) How do network externalities and fairness concerns affect the decision making and profits of companies and the PSC system? (3) How do the network platform's advertising strategies affect network externalities?

We contribute to the existing literature in the following three aspects.

Firstly, we introduce the advertising level of the network platform as the decision-making variable of the platform. Most of the existing research on the PSC considers the sales service of the network platform and does not consider the network platform's advertising service cost [13]. The advertising can increase the scale of the platform and help achieve good development, but the network platform's advertising is limited by the investment. Therefore, the research on the platform's advertising level can guide the company's decisions on advertising investment.

Secondly, we extend the research of the network externalities to the PSC and study the influences of the network externalities on the decision-making of the PSC members. The research of network externalities focused on product pricing in traditional service supply chains and other fields [15]. However, the decision makers and operation model of the PSC are different from those in the traditional service supply chain. Moreover, we explore the internal impacts of advertising strategy on network externalities.

Thirdly, we study the manufacturer's fairness concerns by introducing the fairness concerns coefficient and the fairness reference point. Different from the traditional service supply chain, manufacturers become subordinate companies due to economic strength of the PSC. The utility impact of fairness concerns has been highlighted in previous literature, but studies have not taken into account the features of the PSC. Meanwhile, network externalities will cause producers to become increasingly concerned about fairness. Following the previous research of Nie and Du [16] and Guan et al. [17], we study the influences of subordinate manufacturer's fairness concerns on decisions in the PSC. Additionally, we introduce the fairness reference point for the manufacturer's profit distribution [18], thereby further increasing the practicability of our conclusions.

The rest of the paper is organized as follows. In Section 2, a literature review is briefly provided. Model descriptions and hypotheses for decision models are given in Section 3. Section 4 studies decentralized models and solves optimal decisions. Section 5 conducts a numerical analysis. The extended model is studied in Section 6. Section 7 summarizes the paper, presenting findings and managerial insights.

## 2. Literature Review

Our study involves a literature review of three aspects of the supply chain, namely, advertising strategies, network externalities, and decisions considering fairness concerns.

### 2.1. Advertising Strategies in the Supply Chain

The first aspect is the advertising strategies of the supply chain. Many studies have researched cooperative advertising strategies in the supply chain [19,20]. In detail, for the traditional supply chain, Zhang et al. [21] found that supply chain performance can be improved by other advertising investment information. Xiao et al. [22] researched the optimal order quantity and advertising investment strategy in the supply chain. In the luxury fashion supply chain, Choi et al. [23] studied the advertising budget allocation problem with different brand-level products and found that the best advertising strategy is always the polarization strategy. As for the closed-loop supply chain, from the perspective of marketing strategy, Chen et al. [24] studied the impacts of manufacturers' trade-in discounts and advertising strategies on manufacturers' profits and found that the trade-in policy can bring more profits to manufacturers by saving cost. Hong et al. [25] studied how

local advertising affects product pricing, product recycling, and supply chain members' profits. In the dual-channel supply chain, three different advertising decision models are proposed by Wang et al. [26] to analyze the impact of advertising strategies on corporate decisions. Liu et al. [27] used the Nash game theory to study the optimal strategy of brand advertising and local advertising and found that the advertising strategies of national brand manufacturers remain unchanged in direct sales channels.

Considering fairness concerns, Guan et al. [17] and Li et al. [28] have studied corporate advertising decisions from different perspectives. However, they did not consider the advertising decisions of network platforms in the PSC, nor did they involve the influence of network externalities on advertising decisions.

## 2.2. Network Externalities in the Supply Chain

Our study also involved the research of network externalities. Currently, the research on network externalities mainly focuses on the traditional offline supply chain [15,29]. Zhou and Yuen [30] analyzed the impacts of network externalities, government subsidies, and consumer behavior on the pricing of remanufacturers. They found that if there is no budget constraint, network externalities can increase remanufacturers' profits. Regarding the information product supply chain, Liu et al. [31] researched how network externalities influence the choice of sales channels. They found that the supplier should sell the information product through a dual-channel when the strength of network externalities is weak. Xing [32] and Naskar and Pal [33] focused on the impacts of network externalities on product development and production. Pal [34] analyzed the influences of network externalities using the Cournot game model and the Bertrand game model and found that if there are positive network externalities, the Bertrand model can lead to lower prices and corporate profits. Based on different decision-making models, Zhou and Xu [35] studied the impact of network externalities on the company's product pricing, market share, and profits. They found that in the model of a nonstrategic firm facing nonstrategic consumers, the increase in network externalities is beneficial to the profits of one company, but not to the profits of another company.

The above studies have enriched the theoretical studies of network externalities, but these studies did not involve the decisions of network platforms. The network platform is an economic organization with obvious characteristics of network externalities; therefore, it is necessary to further clarify the impact of network externalities on the operation of the network platform.

## 2.3. Decisions Considering Fairness Concerns

In the third aspect, fairness concerns have recently attracted considerable attention in the supply chain management field. For example, both Zhang et al. [36] and Yoshihara and Matsubayashi [37] found that supply chain efficiency can be affected by retailers' behavior regarding fairness concerns. Zhao [38] studied how retailers' fairness concerns influence corporate decisions and found that retailers' fairness concerns can lead to the differential pricing of extended warranty services. Li et al. [28] examined the manufacturer's fairness concerns in the dual-channel supply chain, and they found that although the manufacturer's fairness concerns decrease the retailer's profit, the retailer would adopt cooperative advertising with the manufacturer. Li et al. [39] found that the retailers' fairness concerns have a greater impact on the stability of the supply chain system. Ma et al. [40] constructed four different recycling models and found that as the disadvantageous inequality coefficient increases, the retailer's marketing level and the manufacturer's recovery rate decrease. Both Li et al. [41] and Du et al. [42] researched how fairness concerns affect green technology investment.

Although Zhen et al. [43] and Wang et al. [44] discussed fairness concerns from different perspectives, they did not consider the influence of network externalities on corporate decisions in the PSC. The fairness concerns in the PSC are caused by the characteristics of the PSC, and network externalities are conducive to the profits of the network platform,

leading to manufacturers' fairness concerns increasing. Therefore, we introduce network externalities into the PSC and analyze the influences of network externalities and fairness concerns in the PSC. The differences between this paper and related literature are shown in Table 1.

**Table 1.** Papers that are most similar to this paper.

| Author                 | Online Sales Channels | Network Externalities Involved | Pricing Strategy | Advertising Strategy Included | Advertising Provider      | Fairness Concerns Considered | Fairness Concerns Regarding Companies |
|------------------------|-----------------------|--------------------------------|------------------|-------------------------------|---------------------------|------------------------------|---------------------------------------|
| Chu and Manchanda [45] | Y                     | Y                              | Y                | N                             | —                         | N                            | —                                     |
| Li [46]                | N                     | Y                              | Y                | Y                             | Manufacturer and retailer | N                            | —                                     |
| Kim et al. [47]        | N                     | Y                              | Y                | N                             | —                         | N                            | —                                     |
| Guan et al. [17]       | N                     | N                              | Y                | Y                             | Retailer                  | Y                            | Manufacturer and retailer             |
| Li et al. [28]         | Y                     | N                              | N                | Y                             | Manufacturer and retailer | Y                            | Manufacturer                          |
| Wang et al. [48]       | N                     | N                              | Y                | N                             | —                         | Y                            | Retailer                              |
| Wang et al. [49]       | Y                     | N                              | Y                | N                             | —                         | Y                            | Manufacturer                          |
| Zhou et al. [50]       | N                     | N                              | N                | Y                             | Manufacturer and retailer | Y                            | Retailer                              |
| Yang et al. [51]       | N                     | N                              | N                | Y                             | Manufacturer and retailer | Y                            | Retailer                              |
| This study             | Y                     | Y                              | Y                | Y                             | Network platform          | Y                            | Manufacturer                          |

Y represents Yes; N represents No; —represents nonexistent.

### 3. Model Descriptions and Assumptions

In the PSC, network externalities will boost the market size of items because customers usually feel that products with substantial sales and a good shop reputation will provide them with more utility, therefore they will pick products with higher sales. As a result, the business will opt for the e-commerce platform's advertising services for established merchants to enhance the number of views and sales.

Consider that the PSC consists of only one manufacturer and one network platform. On the one hand, consumers submit orders for items through the network platform and pay for them, while producers receive equivalent sales and advertising promotion services from the network platform. On the other hand, the manufacturer directly sells products through the network platform and formulates an advertising strategy to help promote products. Correspondingly, the manufacturer pays the commission fee to the network platform when the order is finished. The model structure is shown in Figure 1.

First, the paper constructs a decentralized decision-making model, without fairness concerns for manufacturers, as well as a decentralized decision-making model, with fairness concerns for manufacturers, and solves the optimal decision for each model. Furthermore, numerical analysis is utilized to ensure that each model's result is correct. On this foundation, an expanded model including the influence of advertising tactics on product network externalities is created in the sixth section of the paper, and the internal impact of advertis-

ing strategies on network externalities is analyzed. The model notations are explained, as shown in Table 2.

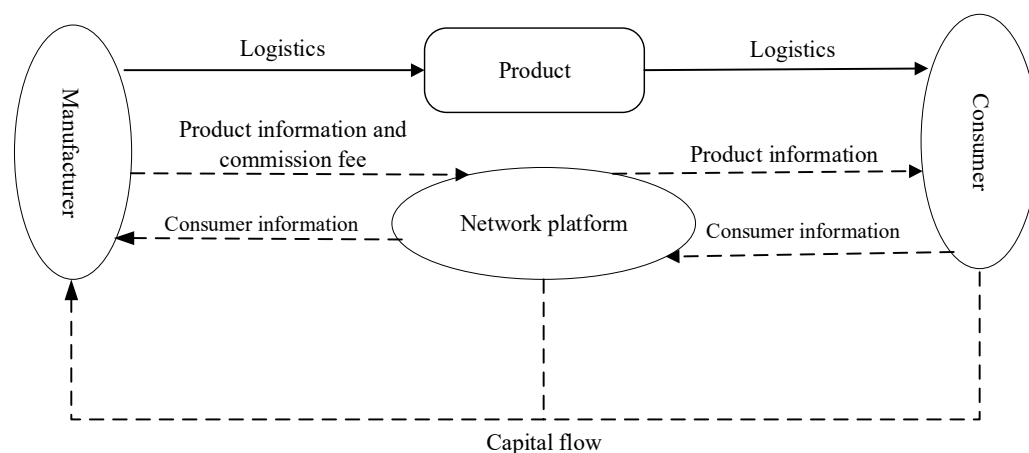


Figure 1. Structure of the PSC.

Table 2. Model's notations and descriptions.

| Notations       | Descriptions                                                                                                                                                                                            |
|-----------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\alpha$        | The potential market scale, $\alpha > 0$                                                                                                                                                                |
| $\beta$         | The elasticity coefficient of the sales price, $\beta > 0$                                                                                                                                              |
| $\gamma$        | The elasticity coefficient of advertising level $\gamma > 0$                                                                                                                                            |
| $k$             | Advertising cost parameters, $k > 0$                                                                                                                                                                    |
| $c$             | Manufacturer's unit production cost                                                                                                                                                                     |
| $\rho$          | The unit commission charged by network platforms, $0 < \rho < p$                                                                                                                                        |
| $q$             | The products market demand                                                                                                                                                                              |
| $\mu$           | The network externality strength coefficient, $0 < \mu < 1$                                                                                                                                             |
| $\theta$        | The fairness concerns coefficient, $0 \leq \theta \leq 1$                                                                                                                                               |
| $\xi$           | The fairness reference point, $\xi > 0$                                                                                                                                                                 |
| $u_0$           | The network externalities' level when the network platform does not adopt an advertising strategy, $0 < u_0 < 1$                                                                                        |
| $l$             | The elasticity coefficient of advertising level to network externalities, $0 < l < 1$                                                                                                                   |
| $p$             | The sale price of the unit product (manufacturer's decision variable)                                                                                                                                   |
| $A$             | The network platform's advertising level (network platform's decision variable)                                                                                                                         |
| $N, M$          | $N$ represents network platform, $M$ represents the manufacturer                                                                                                                                        |
| $E, E\theta, C$ | $E$ represents the decentralized model without manufacturer's fairness concerns, $E\theta$ represents the decentralized model with manufacturer's fairness concerns, $C$ represents the extended model. |
| $\pi_n^i$       | Profit of decision maker $n$ under the model $i$ , $n = M, N$ , $i = E, E\theta, C$                                                                                                                     |
| $\pi^i$         | Profit of PSC system under the model $i$ , $i = E, E\theta, C$                                                                                                                                          |

The model assumptions are as follows:

**Assumption 1.** A specific product sold by a manufacturer is considered in this paper. The network platform is the leading company and the manufacturer is the following company, based on the actual operation of the PSC.

**Assumption 2.** The higher the advertising level provided by the network platform, the higher the advertising cost. According to Zhang et al. [52] and Liu et al. [27], when the network platform provides advertising level  $A$ , the cost function  $C(A)$  is

$$C(A) = kA^2/2. \quad (1)$$

**Assumption 3.** The product's market demand is affected by sales price, network externalities, and advertising level; based on Pokharel and Liang [53] and Yi and Yang [15], it is assumed that

$$q = \alpha(1 + \mu) - \beta p + \gamma A. \quad (2)$$

**Assumption 4.** The potential market scale  $\alpha$  describes the market demand when the price is zero and no advertising service is available. The elasticity coefficients satisfy  $\beta > \gamma > 0$ , which indicates that consumers' preferences for product price are higher than for the advertising level. At the same time, assume that  $2k > \gamma$ , which means that twice of the advertising cost parameters is greater than the elasticity coefficient of the advertising level.

**Assumption 5.** To ensure that the research is useful and the decisions are positive, the parameters meet the conditions that  $\alpha > c\beta$  (the necessity is presented in Appendix A).

According to the model descriptions and assumptions, we can further get:  
The manufacturer's profit function is

$$\pi_M = (p - \rho - c)q. \quad (3)$$

The network platform's profit function is

$$\pi_N = \rho q - kA^2/2 \quad (4)$$

The PSC's profit function is

$$\pi = (p - c)q - kA^2/2. \quad (5)$$

#### 4. Decentralized Models

##### 4.1. Benchmark Model without Fairness Concerns (Model 1)

Under the benchmark model without fairness concerns, both the manufacturer and the network platform make decisions for maximizing their profits. In such a decision scenario, the network platform takes the lead in determining the advertising level  $A$ ; then, the manufacturer determines the sales price  $p$ . The optimal solutions are obtained by the backward induction method.

To derive the manufacturer's specific profit function, substitute Equation (2) into Equation (3):

$$\pi_M = (p - \rho - c)[\alpha(1 + \mu) - \beta p + \gamma A] \quad (6)$$

There is a maximum of  $\pi_M$  due to  $\frac{\partial^2 \pi_M}{\partial p^2} = -2\beta < 0$ . By solving  $\frac{\partial \pi_M}{\partial p} = 0$ , we can obtain that the response function of  $p$  is

$$p(A) = \frac{A\gamma + \alpha(1 + \mu) + \beta(c + \rho)}{2\beta} \quad (7)$$

In Equation (7), the sales price increases with the network externalities, which shows that the network externalities are conducive to increasing the sales price.

Substituting Equation (7) into Equation (4), there is the optimal solution of  $\pi_N$  due to  $\frac{\partial^2 \pi_N}{\partial A^2} = -k < 0$ . Solving  $\frac{\partial \pi_N}{\partial A} = 0$ , we derive the optimal advertising level

$$A^{E*} = \frac{\gamma\rho}{2k}$$

To obtain the manufacturer's optimal selling price, substitute  $A^{E*}$  into Equation (7):

$$p^{E*} = \frac{\alpha(1 + \mu)}{2\beta} + \frac{\gamma^2\rho}{4k\beta} + \frac{c + \rho}{2}$$

Similarly, by substituting  $p^{E*}$  into (2), the market demand can be obtained:

$$q^{E*} = \frac{\alpha(1 + \mu) - \beta(c + \rho)}{2} + \frac{\gamma^2\rho}{4k}$$

Then, to derive the optimal profit of the network platform, manufacturer, and PSC, put  $A^{E*}$  and  $q^{E*}$  into Equations (3)–(5), respectively. We can get:

$$\begin{aligned}\pi_N^{E*} &= \frac{\rho\{4k[\alpha(1+\mu) - \beta(c+\rho)] + \gamma^2\rho\}}{8k} \\ \pi_M^{E*} &= \frac{\{2k[\alpha(1+\mu) - \beta(c+\rho)] + \gamma^2\rho\}^2}{16k^2\beta} \\ \pi^{E*} &= \frac{[\alpha(1+\mu) - c\beta]^2 - \beta^2\rho^2}{4\beta} + \frac{\gamma^2\rho[2\alpha(1+\mu) - \beta(2c+\rho)]}{8k\beta} + \frac{\gamma^4\rho^2}{16k^2\beta}\end{aligned}$$

**Proposition 1.**  $p^{E*}$ ,  $q^{E*}$ ,  $\pi_M^{E*}$ ,  $\pi_N^{E*}$ ,  $\pi^{E*}$  are positively related with  $\mu$  and  $A^{E*}$  is independent of  $\mu$ .

**Proof of Proposition 1.**  $\frac{\partial p^{E*}}{\partial \mu} = \frac{\alpha}{2\beta} > 0$ ,  $\frac{\partial q^{E*}}{\partial \mu} = \frac{\alpha}{2} > 0$ ,  $\frac{\partial \pi_M^{E*}}{\partial \mu} = \frac{\alpha[\alpha(1+\mu) - \beta(c+\rho)]}{2\beta} + \frac{\alpha\gamma^2\rho}{4k\beta} > 0$ ,  $\frac{\partial \pi_N^{E*}}{\partial \mu} = \frac{\alpha\rho}{2} > 0$ ,  $\frac{\partial A^{E*}}{\partial \mu} = 0$ . Since  $\pi^{E*} = \pi_M^{E*} + \pi_N^{E*}$ ,  $\frac{\partial \pi^{E*}}{\partial \mu} > 0$ .  $\square$

It can be seen that the advertising level is not affected by network externalities. The consumers' utility increases as the network externalities increase, which can further increase the product demand and bring more customers to buy products through the platform. At this time, the manufacturer intends to increase the sales price slightly to ensure more profits. Moreover, commission income increases with the increase of product demand, which makes the network platform's profit increase as the network externalities increase. It can be seen that network externalities are beneficial to the operation of the PSC's companies and the whole PSC system.

**Proposition 2.**  $\pi_M^{E*} > \pi_N^{E*}$ .

**Proof of Proposition 2.**  $\pi_M^{E*} - \pi_N^{E*} = \frac{\gamma^2\rho[2\alpha(1+\mu) - \beta(2c+3\rho)]}{8k\beta} + \frac{[\alpha(1+\mu) - \beta(c+2\rho)]^2}{4\beta} + \frac{\gamma^4\rho^2 - 4k^2\beta^2\rho^2}{16k^2\beta} > 0$ .  $\square$

Under the decentralized model without fairness concerns, the following manufacturer's profit is larger than the leading network platform's profit. Proposition 2 is exactly inconsistent with the conclusion that whoever leads the system makes more profit in the traditional supply chain [54]. This is determined by the network platform's characteristics. The network platform is a platform that provides sales services, and it belongs to an industry with economies of scale. Only when the scale continues to increase can the network platform's profit continue to increase. Therefore, in the proposed model, which only considers one manufacturer and one network platform, the network platform obtains less profit than the manufacturer.

Currently, to improve competitiveness and make better use of the economies of scale to increase profits, the network platform chooses to cooperate with major brands in different fields; so far, more than 70,000 brands and more than 50,000 shops have cooperated with Tmall (accessed from: [tmall.com](http://tmall.com), accessed on 24 June 2022) (accessed from: <http://www.wiki.cn/wiki/162673.htm>, accessed on 24 June 2022). With the increase in the network platform's scale, the platform's profits have shown explosive growth.

Proposition 1 and Proposition 2 illustrate that improving network externalities is conducive to improving the platform's competitiveness, which is consistent with real life. In practice, when consumers browse products on the network platform, they not only pay attention to the number of shops on the platform, but also consider the product reviews. The consumers' utility would increase as the number of product reviews increases.

**Proposition 3.**  $p^{E*}, q^{E*}, A^{E*}, \pi_M^{E*}, \pi_N^{E*}, \pi^{E*}$  are positively related with  $\gamma$ .

**Proof of Proposition 3.** The proof process is similar to that in Proposition 1. Please refer to Appendix C for details.  $\square$

When the elasticity coefficient of the advertising level increases, the advertising level is improved to attract more consumers. On the other hand, consumers' sensitivity to sales price decreases due to the increase in sensitivity to the advertising level, which leads to the sales price increase. However, the influence of sales price is smaller than the advertising level, so product demand increases, which ultimately leads to the improvements in the profits of companies and the PSC system. Compared with lesser-known network platforms, consumers generally perceive that network platforms with good advertising services and high reputations are more secure, even if their sales prices are higher than those of lesser-known network platforms.

Proposition 3 also indicates that the network platform's advertising services not only contribute to the platform itself, but also have a positive effect on the operation of manufacturers and the PSC. Therefore, in the network platform's early stage, advertising is one of the most significant strategies. For example, Pinduoduo (accessed from: [pinduoduo.com](http://pinduoduo.com), accessed on 24 June 2022) was established many years after Tmall (accessed from: [tmall.com](http://tmall.com), accessed on 24 June 2022) and JingDong (accessed from: [jd.com](http://jd.com), accessed on 24 June 2022), but it completed a counterattack in just five years and became the second Chinese e-commerce company to exceed USD 100 billion (accessed from: <https://baijiahao.baidu.com/s?id=1670253826812715115&wfr=spider&for=pc>, accessed on 24 June 2022). Its advertising strategy has played an important role in its success. According to relevant statistics, in 2018, Pinduoduo (accessed from: [pinduoduo.com](http://pinduoduo.com), accessed on 24 June 2022) spent about CNY 7 billion on online advertising, while the total cost of offline advertising and brand activities was about CNY 2.2 billion (accessed from: [https://www.sohu.com/a/335591401\\_114819](https://www.sohu.com/a/335591401_114819), accessed on 24 June 2022).

#### 4.2. Decentralized Model with Manufacturer's Fairness Concerns (Model 2)

In the PSC, the network platform occupies the dominant position, and the manufacturer is in a subordinate position, which causes the manufacturer to pay attention to whether the profit distribution is fair. Moreover, the network externalities are beneficial to the platform, which further causes the manufacturer's fairness concerns to be stronger.

Proposition 2 shows that although the network platform is in a dominant position, the network platform can only obtain limited profits. This requires that when describing the manufacturer's fairness concerns, it is necessary to introduce a parameter  $\zeta$  as the fairness reference point, considering the profit gap. The manufacturer uses the network platform's profit as the reference point to judge the fairness of profit distribution. The parameter  $\zeta$  indicates the value when the manufacturer considers that the proportion of its profit to the profit of the network platform is fair. When the manufacturer's profit is  $\zeta$  times higher than the network platform's, the distribution of the profit is fair. The larger the  $\zeta$ , the greater the contribution of the manufacturer in the PSC. Based on the assumptions of Katok et al. [55] and Li and Li [18], the manufacturer's utility is

$$U_M = \pi_M - \theta(\zeta\pi_E - \pi_M) \quad (8)$$

Equation (8),  $0 < \theta < 1$  refers to the coefficient of the manufacturer's fairness concern. The fairness concerns get stronger as  $\theta$  increases. If  $\zeta\pi_E > \pi_M$ , the manufacturer faces disadvantageous inequality; if  $\zeta\pi_E < \pi_M$ , the manufacturer faces advantageous inequality.

In Model 2, the leading network platform and the following manufacturer form a Stackelberg game model. The network platform still takes profit maximization as the decision-making goal, while the manufacturer takes utility maximization as the decision-

making goal. The solving process is similar to that in Model 1, and the optimal decisions are as follows:

$$\begin{aligned}
 A^{E\theta*} &= \frac{\gamma\rho}{2k} \\
 p^{E\theta*} &= \frac{\gamma^2\rho}{4k\beta} + \frac{\alpha(1+\mu)}{2\beta} + \frac{c+\rho}{2} + \frac{\rho\xi\theta}{2(1+\theta)} \\
 q^{E\theta*} &= \frac{\alpha(1+\mu)}{2} + \frac{\gamma^2\rho}{4k} - \beta \left[ \frac{\xi\theta\rho}{2(1+\theta)} + \frac{c+\rho}{2} \right] \\
 \pi_N^{E\theta*} &= \frac{\rho[\alpha(1+\mu) - c\beta]}{2} + \frac{\gamma^2\rho^2}{8k} - \frac{\rho^2\beta[1+\theta(1+\xi)]}{2(1+\theta)} \\
 \pi_M^{E\theta*} &= \frac{\{2k\alpha(1+\theta)(1+\mu) + \gamma^2\rho(1+\theta) - 2k\beta[(c+\rho)(1+\theta) + \xi\theta\rho]\}^2}{16k^2\beta(1+\theta)^2} \\
 &\quad + \frac{\xi\theta\rho[\alpha(1+\mu) - \beta(c+\rho)]}{2(1+\theta)} + \frac{\gamma^2\rho^2\xi\theta}{4k(1+\theta)} - \frac{\xi^2\theta^2\rho^2\beta}{2(1+\theta)^2} \\
 \pi^{E\theta*} &= \frac{\{2k\alpha(1+\theta)(1+\mu) + \gamma^2\rho(1+\theta) - 2k\beta[(c+\rho)(1+\theta) + \xi\theta\rho]\}^2}{16k^2\beta(1+\theta)^2} - \frac{\beta\xi^2\theta^2\rho^2}{2(1+\theta)^2} \\
 &\quad + \frac{\xi\theta\rho[\alpha(1+\mu) - c\beta] - 2\beta\rho^2\xi\theta}{2(1+\theta)} + \frac{\rho[\alpha(1+\mu) - \beta(c+\rho)]}{2} + \frac{\gamma^2\rho^2[1+\theta(1+2\xi)]}{8k(1+\theta)}
 \end{aligned}$$

**Proposition 4.**  $p^{E\theta*} > p^{E*}$ ,  $A^{E\theta*} = A^{E*}$ ;  $p^{E\theta*}$  is positively related with  $\theta$ ;  $\pi_M^{E\theta*}$ ,  $\pi_N^{E\theta*}$ ,  $\pi^{E\theta*}$ ,  $q^{E\theta*}$  are negatively related with  $\theta$ .

**Proof of Proposition 4.**  $\frac{\partial \pi_M^{E\theta*}}{\partial \theta} = -\frac{\beta\theta\xi^2\rho^2}{2(1+\theta)^3} < 0$ ,  $\frac{\partial \pi_N^{E\theta*}}{\partial \theta} = -\frac{\beta\xi\rho^2}{2(1+\theta)^2} < 0$ ,  $\frac{\partial p^{E\theta*}}{\partial \theta} = \frac{\xi\rho}{2(1+\theta)^2} > 0$ ,  $\frac{\partial q^{E\theta*}}{\partial \theta} = -\frac{\beta\xi\rho}{2(1+\theta)^2} < 0$ ,  $\frac{\partial A^{E\theta*}}{\partial \theta} = 0$ ,  $p^{E\theta*} - p^{E*} = \frac{\xi\theta\rho}{2(1+\theta)} > 0$ . Since  $\pi^{E\theta*} = \pi_M^{E\theta*} + \pi_N^{E\theta*}$ ,  $\frac{\partial \pi^{E\theta*}}{\partial \theta} > 0$ .  $\square$

As the coefficient of the manufacturer's fairness concern increases, the manufacturer will increase the sales price to achieve a fair distribution of profits. The network platform relies on the dominant position in the PSC to take the lead in making decisions, and the advertising level remains unchanged as fairness concerns change. The fixed advertising level and the increased sales price reduce the product demand, which ultimately leads to a decrease in profits for the manufacturers, the network platforms, and the PSC. Although the manufacturer's fairness concerns have narrowed the profit gap between the manufacturer and the network platform, it has also reduced the profits of both parties, which is disadvantageous to the operation of the PSC.

**Proposition 5.**  $p^{E\theta*}$  is positively related with  $\xi$ ,  $\pi_M^{E\theta*}$ ,  $\pi_N^{E\theta*}$ ,  $\pi^{E\theta*}$ ,  $q^{E\theta*}$  are negatively related with  $\xi$ ,  $A^{E\theta*}$  is independent of  $\xi$ .

**Proof of Proposition 5.** The proof process is similar to Proposition 1. Please refer to the Appendix C for details.  $\square$

Proposition 5 shows the larger the fairness reference point  $\xi$ , the greater the contribution the manufacturer makes to the PSC, and the manufacturer is more likely to have disadvantageous inequality. When a manufacturer falls into the situation of disadvantageous inequality, he will punish the network platform at the expense of sacrificing his profit by increasing the sales price. The increased sales price makes the product demand

decrease, which ultimately causes the profits of the manufacturer, network platform, and PSC to decrease.

Propositions 4 and 5 demonstrate that the manufacturer's fairness concerns can result in the loss of the overall efficiency of PSC. Therefore, in practice, the network platform should not use dominance over the channels to suppress the manufacturer and should avoid causing the manufacturer to fall into the situation of disadvantageous inequality.

## 5. Numerical Analysis

Numerical analysis is used to validate the conclusions and explore more findings. Following Hsiao and Chen [56], Yi and Yang [57], Zhang and Wang [58], and Wang et al. [49], it is assumed that  $\alpha = 500$ ,  $k = 1$ ,  $\gamma = 2$ ,  $\rho = 8$ ,  $\beta = 5$ ,  $c = 5$ ,  $\theta = 0.5$ . Further,  $\mu$  and  $\xi$  are independent variables, and we assume that  $\mu \in [0, 1]$ ,  $\xi \in [0, 20]$ . Thus, the changes in decision variables with  $\mu$  and  $\xi$  under decentralized models are shown in Figures 2–6.

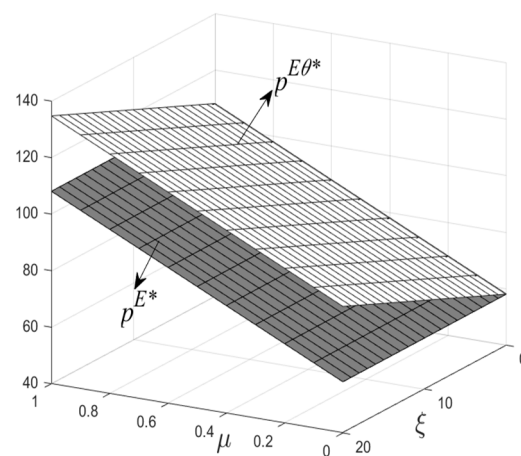


Figure 2. Changes in sales price.

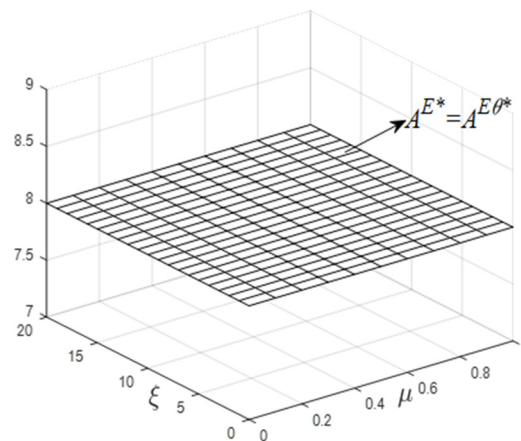


Figure 3. Changes in advertising level.

Propositions 1 and 5 are verified in Figures 2–6. Moreover, the numerical analysis indicates that compared with the benchmark model, the sales price is higher, and the manufacturer, the network platform, and the PSC get less profit in the model with manufacturer's fairness concerns.

Let  $\mu = 0.08$ ,  $\theta = 0.5$ ,  $\xi = 8$ , and  $\rho \in [0, 25]$  be the independent variables. The changes in decisions with  $\rho$  are shown in Figures 7–11.

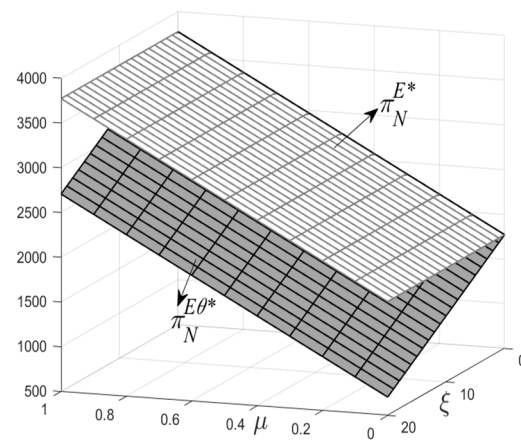


Figure 4. Changes in network platform's profit.

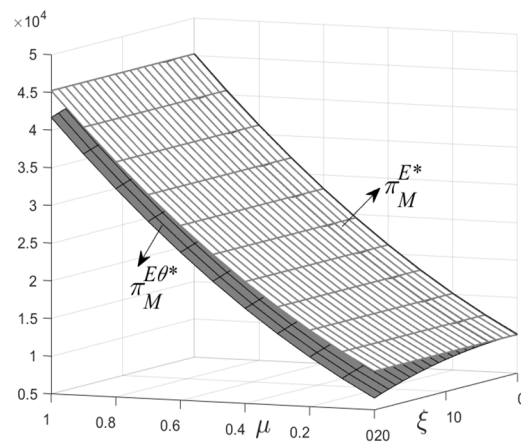


Figure 5. Changes in manufacturer's profit.

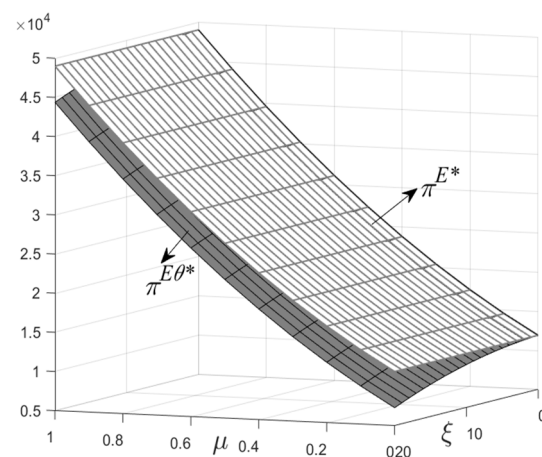


Figure 6. Changes in PSC's profit.

It can be seen from Figures 7–11:

The sales price and the advertising level all increase with the increase in the commission. Moreover, the sales price with fairness concerns is more sensitive to the commissions than that without fairness concerns. This is because the manufacturer is more likely to fall into an inequality situation as the commission increases, thus paying attention to the income gap between himself and the platform. Therefore, the manufacturer with fairness concerns will eagerly increase profit by increasing sales price. For the network platform,

as commissions increase, the network platform can receive more funds to improve the advertising level.

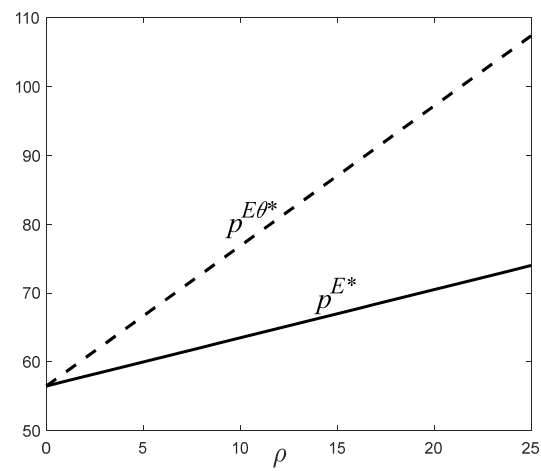


Figure 7. Changes in sales price.

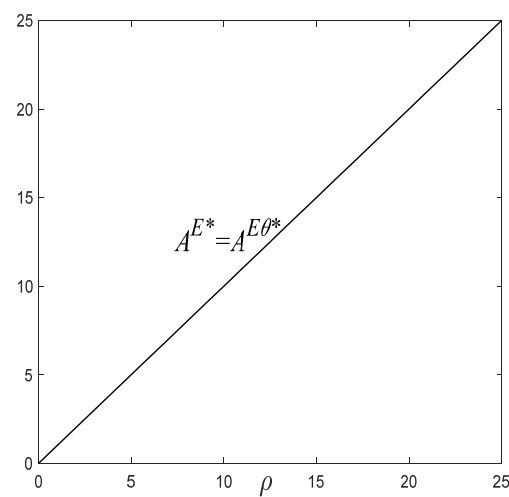


Figure 8. Changes in advertising level.

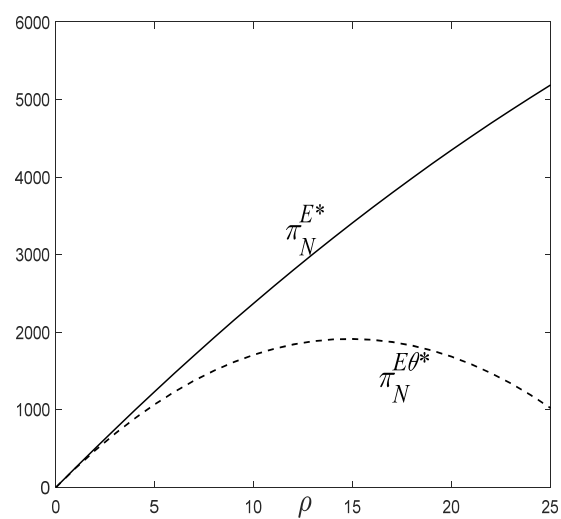


Figure 9. Changes in the network platform's profit.

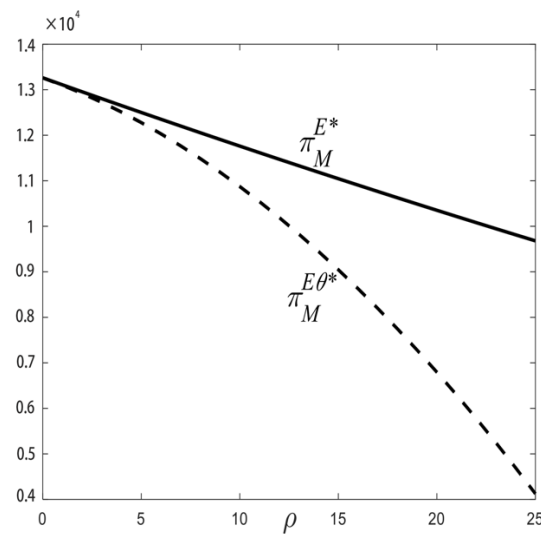


Figure 10. Changes in the manufacturer's profit.

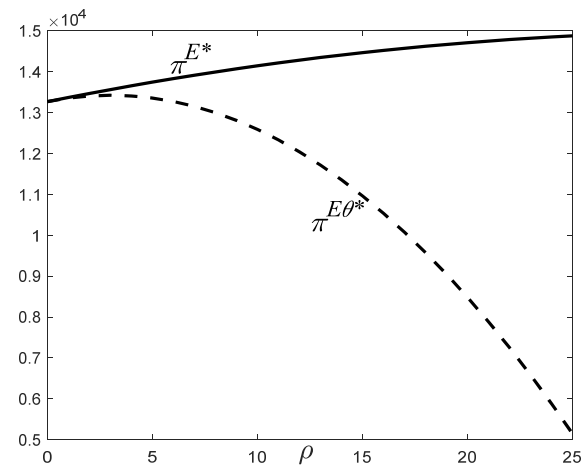


Figure 11. Changes in the PSC's profit.

In the benchmark model, as the commission increases, the manufacturer's profit decreases due to the increased manufacturer's sales cost, but the improved commission revenue helps improve the network platform's profit. The PSC's profit increases, since the increase in the network platform's profit is greater than the decrease in the manufacturer's profit.

The manufacturer's profit still decreases as the commission increases in the decentralized model with the manufacturer's fairness concerns. However, both the network platform's profit and PSC's profit increase first and then decrease. This is because when the commission is at a high level, the sales price increases too much, resulting in a significant decrease in product demand. For the network platform, the profit loss incurred by decreasing the product demands surpasses the profit increased by increasing the commission. As a result, both the network platform's and the PSC's profits tend to decrease.

## 6. Extended Model (Model 3)

It is shown that the advertising strategy positively affects product sales (Proposition 3), so the platform's advertising also has an impact on the strength of the network externalities. Assume that the network externalities strength coefficient  $\mu$  is an increasing function of the advertising level  $A$  to analyze the internal impact of advertising decisions on network externalities, that is:

$$\mu = lA + u_0 \quad (9)$$

Equation (9),  $l(0 < l < 1)$  denotes the elasticity coefficient of the advertising level to the network externalities, and  $u_0(0 < u_0 < 1)$  denotes the network externalities' level when the network platform does not adopt an advertising strategy.

The function of product demand is:

$$q = \alpha(1 + lA + u_0) - \beta p + \gamma A = \alpha(1 + u_0) + (\alpha l + \gamma)A - \beta p \quad (10)$$

Both the manufacturer and the network platform make decisions for maximizing their profits under the extended model. A Stackelberg game model is formed with the leading network platform and the following manufacturer. The optimal solutions are as follows. The solving process is similar to that of Model 1.

$$\begin{aligned} A^{C*} &= \frac{\rho(l\alpha + \gamma)}{2k} \\ p^{C*} &= \frac{\alpha(1 + u_0) + \beta(c + \rho)}{2\beta} + \frac{(l\alpha + \gamma)^2 \rho}{4k\beta} \\ q^{C*} &= \frac{\rho(l\alpha + \gamma)^2}{4k} + \frac{\alpha(1 + u_0) - \beta(c + \rho)}{2} \\ \pi_N^{C*} &= \frac{\rho^2(l\alpha + \gamma)^2}{8k} + \frac{\rho[\alpha(1 + u_0) - \beta(c + \rho)]}{2} \\ \pi_M^{C*} &= \frac{\left\{ \rho(l\alpha + \gamma)^2 + 2k[\alpha(1 + u_0) - \beta(c + \rho)] \right\}^2}{16k^2\beta} \\ \pi^{C*} &= \frac{\rho^2(l\alpha + \gamma)^2 + 4\rho k[\alpha(1 + u_0) - \beta(c + \rho)]}{8k} + \frac{\left\{ \rho(l\alpha + \gamma)^2 + 2k[\alpha(1 + u_0) - \beta(c + \rho)] \right\}^2}{16k^2\beta} \end{aligned}$$

**Proposition 6.**  $p^{C*}, A^{C*}, q^{C*}, \pi_N^{C*}, \pi_M^{C*}, \pi^{C*}$  are positively related with  $l$ .

**Proof of Proposition 6.** The proof process is similar to that of Proposition 1. Please refer to Appendix D for details.  $\square$

Proposition 6 shows that in the extended model, as the network externalities are more sensitive to the advertising level, the network platform will increase the advertising level, and thereby increase network externalities. From Proposition 1, it can be seen that with the increase in network externalities, the product demand, sales price, and manufacturer's profit all increase. For the network platform, although improving the advertising level has increased the network platform's operating cost, the network platform's profit eventually increases because of the significant increase in the product demand.

**Proposition 7.**  $p^{C*}, A^{C*}, q^{C*}, \pi_N^{C*}, \pi_M^{C*}, \pi^{C*}$  are positively related with  $\gamma$ .

**Proof of Proposition 7.** The proof process is similar to that in Proposition 1. Please refer to the Appendix E for details.  $\square$

As consumers are more sensitive to the advertising level, the network platform will strive to increase its advertising level to attract more consumers, thereby increasing the product demand and the scale of the platform. Meanwhile, the manufacturer will slightly increase the sales price. Therefore, the profits of companies and PSC are all improved because of the increase in product demand.

Propositions 3, 6, and 7 illustrate the fact that promoting the network platform's advertising level is not only beneficial to the platform's development, but also contributes to the development of the manufacturer and the PSC.

**Proposition 8.**  $p^{C*}, A^{C*}, q^{C*}, \pi_N^{C*}, \pi_M^{C*}, \pi^{C*}$  are positively related with  $\rho$ .

**Proof of Proposition 8.** The proof process is similar to that in Proposition 1. Please refer to Appendix F for details.  $\square$

Similar to the conclusions illustrated in Figures 7–9 and 11, Proposition 8 shows that the commission has positive effects on the sales price, advertising level, product demand, and the profits of the network platform and the PSC. Different from the conclusion in Figure 10, the manufacturer's profit increases as the commission increases in the extended model. This is mainly because considering the impact of advertising level on network externalities, the network platform would improve the advertising level as the commission increases in the extended model. For the manufacturer, the product demand significantly increases because of the improved advertising level, increasing the manufacturer's profit.

**Conclusion 1.** Sales price satisfies  $p^{C*} > p^{E*}$ , advertising level satisfies  $A^{C*} > A^{E*}$ , product demand satisfies  $q^{C*} > q^{E*}$ , network platform's profit satisfies  $\pi_N^{E*} > \pi_N^{C*}$ , manufacturer's profit satisfies  $\pi_M^{C*} > \pi_M^{E*}$ , and PSC's profit satisfies  $\pi^{C*} > \pi^{E*}$ .

**Proof of Conclusion 1.**

$$p^{C*} - p^{E*} = \frac{\alpha[l\rho(l\alpha + 2\gamma) - 2k(\mu - u_0)]}{4k\beta} \quad (11)$$

$$A^{C*} = \frac{\rho(l\alpha + \gamma)}{2k} \quad (12)$$

From Equation (8) we have

$$\mu - u_0 = lA^{C*} \quad (13)$$

Substituting Equations (12) and (13) into Equation (11), we drive  $p^{C*} - p^{E*} = \frac{\alpha[l\rho(l\alpha + 2\gamma) - l\rho(l\alpha + \gamma)]}{4k\beta} = \frac{\alpha l\rho\gamma}{4k\beta} > 0$ .

Similarly,  $A^{C*} - A^{E*} = \frac{l\alpha\rho}{2k} > 0$ ,  $q^{C*} - q^{E*} = \frac{\alpha l\rho\gamma}{4k} > 0$ ,  $\pi_N^{E*} - \pi_N^{C*} > 0$ ,  $\pi_M^{C*} - \pi_M^{E*} > 0$ ,  $\pi^{C*} - \pi^{E*} > 0$ .  $\square$

Conclusion 1 indicates that the network platform and the manufacturer would set a higher advertising level and a higher sales price in the extended model. This is mainly because the advertising strategy has a positive impact on the network externalities; thus, the network platform would improve the advertising level to increase the network externalities. Meanwhile, the manufacturer would slightly increase the sales price to obtain more sales revenue. Moreover, the influence of the advertising level is greater than the sales price, which is advantageous to the product demand in the extended model. The manufacturer and the PSC obtain more profits in the extended model, which is because of the higher product demand. However, for the network platform, the substantial increase in advertising level has caused the platform's profit in the extended model to be lower than that in the benchmark model.

## 7. Conclusions

Benefiting from the rapid development of the Internet, the PSC has gradually matured, and has been valued by many companies. We construct a PSC to analyze the impacts of network externalities and fairness concerns on advertising decisions. We first solve and analyze the optimal decisions under decentralized models, without and with manufacturer's fairness concerns. Then, we study the impacts of fairness concerns on system operation

by comparing equilibrium solutions. On this basis, we extend the model to analyze the internal influence of advertising strategy on network externalities. The findings are as follows:

- (i). Increasing product network externalities is conducive to improving the profits of companies and the PSC. Therefore, the manufacturer should cooperate with the network platform to improve the network externalities, thereby obtaining more profits and ensuring the long-term operation of the PSC.
- (ii). Increasing consumer preference for advertising level is beneficial to the profits of both the companies and the PSC. Therefore, increasing the advertising level of the network platform not only helps improve profit, but also benefits the PSC system.
- (iii). The manufacturer's fairness concerns prompt the manufacturer to increase the sales price, which not only causes the manufacturer's profit to decrease, but also causes the profit loss of the network platform and the PSC. Moreover, the advertising level of the network platform is not affected by fairness concerns. Therefore, the network platform should avoid the unfairness to the manufacturer.
- (iv). In the extended model, where the advertising strategy is the internal influencing factor of the network externalities, the manufacturer, network platform, and PSC can profit more when the network externalities become more sensitive to the advertising level. Moreover, the manufacturer and the PSC obtain more profits in the extended model, while the network platform's profit is higher in the benchmark model.

The following managerial insights can be drawn:

First, as the leading company in the PSC, network platforms should improve the advertising level. Moreover, the platform should not only strengthen brand promotion and enhance brand awareness, but also standardize operation activities. Reliable advertising is an essential precondition for maximizing profit, which requires platforms to strictly check the shops' qualifications and product quality and improve their after-sales service. It is necessary to create a safe and secure online shopping environment and ensure the long-term operation of its platform.

Second, regarding companies in the PSC, manufacturers should cooperate with network platforms to increase the network externalities and improve the maximization of system efficiency. For example, manufacturers can meet the different needs of consumers and cultivate customer loyalty by improving the products' quality and providing related products, thereby enhancing network externalities.

Third, the cooperation mechanism of the e-commerce industry should be standardized and the online transactions should be fair. Network platforms tend to infringe on the manufacturers' profits, causing the cooperation to fail. Therefore, market regulators should strengthen the publicity and implementation of antitrust laws to prevent the monopoly behaviors of the network platforms. Moreover, relevant regulatory authorities need to strengthen the supervision of platform operations to avoid network platforms from interfering in manufacturers' decisions, such as by forcing manufacturers to participate in shopping festivals.

The network platform's advertising strategy is discussed in this paper, but the manufacturer also constructs advertising strategies. Based on this reality, how to balance the manufacturer's and the network platform's advertising strategies should be explored in future research. The methods for determining advertising investment should also be further explored.

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## Appendix A

**Proof of Assumption 5.** When there is no advertising influence, the market demand for regular commodities should be larger than zero,  $q = \alpha - \beta p > 0$ , can be obtained, and after the item is moved,  $\alpha > \beta p$  can be obtained. Moreover, the product's selling price is larger than or equal to the product's cost, i.e.,  $\alpha > \beta p \geq c\beta$ .  $\square$

## Appendix B

**Proof of Proposition 3.**

$$\frac{\partial p^{E*}}{\partial \gamma} = \frac{\gamma \rho}{2k\beta} > 0, \frac{\partial q^{E*}}{\partial \gamma} = \frac{\gamma \rho}{2k} > 0, \frac{\partial A^{E*}}{\partial \gamma} = \frac{\rho}{2k} > 0, \frac{\partial \pi_N^{E*}}{\partial \gamma} = \frac{\gamma \rho^2}{4k} > 0, \\ \frac{\partial \pi_M^{E*}}{\partial \gamma} = \frac{\gamma^3 \rho^2}{4k^2 \beta} + \frac{\gamma \rho [\alpha(1+\mu) - \beta(c+\rho)]}{2k\beta} > 0. \text{ Since } \pi^{E*} = \pi_M^{E*} + \pi_N^{E*}, \frac{\partial \pi^{E*}}{\partial \gamma} > 0.$$

$\square$

## Appendix C

**Proof of Proposition 5.**

$$\frac{\partial \pi_M^{E\theta*}}{\partial \xi} = -\frac{\beta \xi \theta^2 \rho^2}{2(1+\theta)^2} < 0, \frac{\partial \pi_N^{E\theta*}}{\partial \xi} = -\frac{\beta \theta \rho^2}{2(1+\theta)} < 0, \frac{\partial p^{E\theta*}}{\partial \xi} = \frac{\theta \rho}{2(1+\theta)} > 0, \frac{\partial q^{E\theta*}}{\partial \xi} = -\frac{\beta \theta \rho}{2(1+\theta)} < 0, \frac{\partial A^{E\theta*}}{\partial \xi} = 0. \text{ Since } \\ \pi^{E\theta*} = \pi_M^{E\theta*} + \pi_N^{E\theta*}, \frac{\partial \pi^{E\theta*}}{\partial \xi} > 0.$$

$\square$

## Appendix D

**Proof of Proposition 6.**

$$\frac{\partial p^{C*}}{\partial l} = \frac{\alpha \rho(l\alpha + \gamma)}{2k\beta} > 0, \frac{\partial A^{C*}}{\partial l} = \frac{\alpha \rho}{2k} > 0, \frac{\partial q^{C*}}{\partial l} = \frac{\alpha \rho(l\alpha + \gamma)}{2k} > 0, \frac{\partial \pi_N^{C*}}{\partial l} = \frac{\alpha \rho^2(l\alpha + \gamma)}{4k} > 0, \\ \frac{\partial \pi_M^{C*}}{\partial l} = \frac{\alpha \rho(l\alpha + \gamma) \{ \rho(l\alpha + \gamma)^2 + 2k[\alpha(1+u_0) - \beta(c+\rho)] \}}{4k^2 \beta} > 0. \text{ Since } \pi^{C*} = \pi_M^{C*} + \pi_N^{C*}, \frac{\partial \pi^{C*}}{\partial l} > 0.$$

$\square$

## Appendix E

**Proof of Proposition 7.**

$$\frac{\partial p^{C*}}{\partial \gamma} = \frac{\rho(l\alpha + \gamma)}{2k\beta} > 0, \frac{\partial A^{C*}}{\partial \gamma} = \frac{\rho}{2k} > 0, \frac{\partial q^{C*}}{\partial \gamma} = \frac{\rho(l\alpha + \gamma)}{2k} > 0, \frac{\partial \pi_N^{C*}}{\partial \gamma} = \frac{\rho^2(l\alpha + \gamma)}{4k} > 0, \\ \frac{\partial \pi_M^{C*}}{\partial \gamma} = \frac{\rho(l\alpha + \gamma) \{ \rho(l\alpha + \gamma)^2 + 2k[\alpha(1+u_0) - \beta(c+\rho)] \}}{4k^2 \beta} > 0. \text{ Since } \pi^{C*} = \pi_M^{C*} + \pi_N^{C*}, \frac{\partial \pi^{C*}}{\partial \gamma} > 0.$$

$\square$

## Appendix F

### Proof of Proposition 8.

$$\frac{\partial p^{C*}}{\partial \rho} = \frac{(l\alpha + \gamma)^2}{4k\beta} + \frac{1}{2} > 0, \quad \frac{\partial A^{C*}}{\partial \rho} = \frac{l\alpha + \gamma}{2k} > 0, \quad \frac{\partial q^{C*}}{\partial \rho} = \frac{(l\alpha + \gamma)^2}{4k} - \frac{\beta}{2} > 0, \quad \frac{\partial \pi_N^{C*}}{\partial \rho} = \frac{\rho(l\alpha + \gamma)^2}{4k} + \frac{\alpha(1+u_0) - \beta(c+2\rho)}{2} > 0,$$

$$\frac{\partial \pi_M^{C*}}{\partial \rho} = \frac{[(l\alpha + \gamma)^2 - 2k\beta] \{ \rho(l\alpha + \gamma)^2 + 2k[\alpha(1+u_0) - \beta(c+\rho)] \}}{8k^2\beta} > 0. \text{ Since } \pi^{C*} = \pi_M^{C*} + \pi_N^{C*}, \quad \frac{\partial \pi^{C*}}{\partial \rho} > 0.$$

□

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