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Equilibrium in a Bargaining Game of Two Sellers and Two Buyers

Jiawei Li [1,*], Tianxiang Cui [1] and Graham Kendall [2]

[1] School of Computer Science, University of Nottingham Ningbo China, Ningbo 315100, China; tianxiang.cui@nottingham.edu.cn
[2] School of Computer Science, University of Nottingham UK, Nottingham NG7 2RD, UK; graham.kendall@nottingham.ac.uk
* Correspondence: jiawei.li@nottingham.edu.cn

Abstract: The uniqueness of equilibrium in bargaining games with three or more players is a problem preventing bargaining theory from general real world applications. We study the uniqueness of bargaining equilibrium in a bargaining game of two sellers and two buyers, which has instances in real-world markets. Each seller (or buyer) wants to reach an agreement with a buyer (or seller) on the division of a pie in the bargaining game. A seller and a buyer will receive their agreed divisions if they can reach an agreement. Otherwise, they receive nothing. The bargaining game includes a finite number of rounds. In each round, a player can propose an offer or accept an offer. Each player has a constant discounting factor. Under the assumption of complete information, we prove that the equilibrium of this bargaining game is the same division of two pies. The ratio of division as a function of the discount factors of all players is also deduced. The result can be extended to a bargaining game of n-sellers and n-buyers, which reveals the relevance of bargaining equilibrium to the general equilibrium of a market.

Keywords: bargaining; Nash bargaining equilibrium; bargaining game of two sellers and two buyers

MSC: 91A10

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1. Introduction

Bargaining games concern situations in which the players bargain over the division of certain goods. Game theorists have developed both axiomatic approaches and sequential approaches for two-player bargaining games. With an axiomatic approach, Nash proved that there was a unique equilibrium for two-player bargaining games, called the Nash bargaining equilibrium, under a series of axiomatic assumptions [1]. Rubinstein [2] developed a sequential strategic approach in which two players took turns making alternating offers. In the case where each player has a constant discounting factor (δ_1 and δ_2), the equilibrium is proven to be $(1-\delta_2)/(1-\delta_1\delta_2)$. Binmore et al. [3] discussed the relationship between these two approaches. Stâhl [4] discussed a finite version of Rubinstein's method. Backus et al. [5] conducted experiments on sequential online bargaining and concluded that behavioral norms played important roles in making bargaining successful.

Nash bargaining equilibrium could be widely used in solving decision-making problems if it could be extended to a bargaining game with more than two players. Researchers have found that the uniqueness of bargaining equilibrium does not hold in the bargaining games where multiple players bargain over how to share a pie, even under the condition of a common discounting factor [6–8]. Baron and Ferejohn [9] presented a sequential model of multiple players bargaining in a cake-dividing game with a simple majority rule. They concluded that the output of bargaining depends on the bargaining settings, that is, the open and close rules and the sequence of offer making. Recent work on $n \geq 3$-person bargaining games also includes [10–19]. A recent survey of the theoretical literature on legislative and multilateral bargaining is [20].

It has been revealed that the uniqueness of a bargaining equilibrium can be achieved by introducing some extra constraints to the model of bargaining, that is, the exit opportunity [21–23], delay effect [24], asymmetric players [25], and incomplete information assumption [26,27]. Collard-Wexler et al. [28] studied bargaining among multiple upstream and downstream firms, in which the upstream and downstream firms would bargain in pairs. Each pair's bargaining solution was arbitrarily close to the Nash bargaining equilibrium. This research shows that the uniqueness of an equilibrium can be achieved by assigning the bargainers different roles. Eraslan [25] introduced some monotonicity properties of the equilibrium payoffs in a proof of the uniqueness of an equilibrium. The bargainers choose to join coalitions in which they receive identical payoffs, so they will choose the coalition that maximises individual payoffs. Kawamori [29] showed that all players receive the same payoff if sufficiently patient players with linear preferences have similar recognition probabilities. Kalandrakis [30] established a link between the cost of coalitions of any two players, which provided an alternative proof of the uniqueness of the equilibrium. Montero [31] showed that the payoff of a player could be irrespective of the discount factor, and a patient player might be worse off in a multiplayer bargaining game.

In this article, we study the multiple-player bargaining games by assuming that every player has a specific role in bargaining: either a seller or a buyer. A bargain can be implemented only between different roles, say a buyer and a seller. This is a reasonable assumption since it coincides with the situations in real-world markets.

We study the equilibrium of a bargaining game of two sellers and two buyers. In the game, two buyers and two sellers bargain on how to share two pies. In every round of bargaining, each bargainer can offer a price (division of a pie) or accept a price offered by another player of a different role. A pair of seller and buyer will share a pie if they reach an agreement. Each player has a constant discounting factor. The sellers and buyers who have made agreements divide their pies, and other players receive nothing. We prove that this bargaining game has a perfect equilibrium in which four players reach the same division of two pies.

The contributions of this paper are threefold. First, we prove the uniqueness of the bargaining equilibrium in the bargaining game of two buyers and two sellers. It is the first time that the uniqueness of equilibrium in such a game that reflects multiple players' bargaining behaviours in a real-world market is proven. Second, we deduce the price in equilibrium is

$$p = \frac{2 - \delta_{b1} - \delta_{b2}}{2 - \delta_{s1} - \delta_{s2}},$$

where the players have constant discounting factors (δ_{si} and δ_{bi}, $i = 1, 2$, for the sellers and buyers, respectively). Third, the analysis method can be extended to a bargaining game of n sellers and n buyers. The outcome of the bargaining is a market clearance price, which provide a microscopic explanation of the concept of general equilibrium in market.

The remainder of this paper is arranged as follows. In Section 2, the preliminary knowledge of the sequential approach is introduced. The advantage of moving first can be eliminated by introducing a bidding stage in which two players bid for the right to make the first offer. We also prove the consistency between the axiomatic approach and the sequential strategic approach. In Section 3, the perfect equilibrium of a bargaining game of two sellers and two buyers is deduced. Then, we provide the conclusions in Section 4.

2. Two-Player Bargaining Game

According to [2], a two-player bargaining game is described as below:

Two players, 1 and 2, are bargaining on the division of a pie. The pie will be divided only after the players reach an agreement. Each player, in turn offers a partition and his opponent may agree to the offer or reject it. Acceptance of the offer ends the bargaining.

After rejection, the rejecting player then has to make a counter offer and so on. If no agreement is achieved, both players keep their status quo (no gain no loss).

Let X be the set of possible agreements, D be the status quo (no agreement), and x_1 and x_2 the partitions of the pie that 1 and 2 receive, respectively. The players' preferences are defined on the set of ordered pairs of the type (x,t) , where t is a non-negative integer and denotes the time when the agreement is reached, $0 \leq x \leq 1$, $x_1 = x$, and $x_2 = 1 - x$. Let $\succsim_i$ denote player i's preference ordering over $X \cup \{D\}$. There are the following assumptions:

A-1. Disagreement is the worst outcome: for every $(x,t) \in X \times T$, we have $(x,t) \succsim_i D$.

A-2. 'Pie' is desirable: $(x,t) \succsim_i (y,t)$ iff $x_i \geq y_i$.

A-3. 'Time' is valuable: for every $x \in X$, $t_1 < t_2$, if $(x,t_2) \succ_i (d,0)$, then $(x,t_1) \succ_i (x,t_2)$.

A-4. Stationarity: for every $x, y \in X$, $\Delta > 0$, if $(x,t_1) \succsim_i (y,t_1+\Delta)$, then $(x,t_2) \succsim_i (y,t_2+\Delta)$.

A-5. Continuity: if $(x,t_1) \succsim_i (y,t_2)$, there always exists a small positive value $\epsilon \to 0$ such that $(x+\epsilon,t_1) \succsim_i (y,t_2)$.

A-6. Increasing loss to delay: for any $c_1, c_2 > 0$, if $(x+c_1,t) \sim_i (x,0)$, $(y+c_2,t) \sim_i (y,0)$ and $x_i > y_i$ then $c_1 \geq c_2$.

The players have constant discounting factors: each player has a number $0 \leq \delta_i \leq 1$ such that $(x,t_1) \succsim_i (y,t_2)$ iff $x_i\delta_i^{t_1} \geq y_i\delta_i^{t_2}$. Under these assumptions, Rubinstein [2] has proven the following Proposition 1.

Proposition 1. *(a) There exists a unique perfect equilibrium of this bargaining game (b). If at least one of the δ_i is less than 1 and at least one of them is positive, the bargaining solution is $(x,0)$, where $x = (1-\delta_2)/(1-\delta_1\delta_2)$.*

Notice that player 1 is supposed to begin the bargaining. If player 2 began, the solution would be $x = (1-\delta_1)/(1-\delta_1\delta_2)$. The player who makes the first offer has an advantage in bargaining and receives a larger partition of the pie than what would be received if another player had made the first offer.

Binmore et al. [3] gave a procedure to eliminate the advantage of moving first, as follows: Let the time delay between successive periods be Δ, and represent the discount factor as δ^Δ. Then, in the limit $\Delta \to 0$, it is irrelevant who makes the opening demand.

$$\lim_{\Delta\to 0} x^*(\Delta) = \lim_{\Delta\to 0} y^*(\Delta) = x_N^{TP}(\succsim_1,\succsim_2)$$

where $x^*(\Delta)$ and $y^*(\Delta)$ denote the pair of agreements, and $x_N^{TP}(\succsim_1,\succsim_2)$ the Nash bargaining equilibrium.

We introduce another procedure to eliminate the advantage of moving first as follows: Two players bid for the right to make the first offer before bargaining for the division of the pie. Player 1 offers a bid w ($0 \leq w \leq 1$) to player 2 to exchange the right of moving first. If player 2 accepts the bid, she receives w division of the pie, and player 1 begins the bargaining to divide the rest $1-w$. If player 2 refuses the bid, she wins the right to make the first offer, and player 1 receives w. We then have the following Proposition 2.

Proposition 2. *In the case where two players bid for the right of moving first, the bargaining equilibrium is $(x,0)$, where $x = (1-\delta_2)/(2-\delta_1-\delta_2)$.*

Proof. According to Proposition 1, if player 2 accepts player 1's bid of exchanging w for the right of moving first, the two players will receive x_1 and x_2, respectively, where

$$x_1 = \frac{1-\delta_2}{1-\delta_1\delta_2}(1-w)$$

and $x_2 = 1 - x_1$. On the other hand, if player 2 refuses player 1's bid, two players will receive x_1^* and x_2^*, where

$$x_1^* = w + \frac{\delta_1(1-\delta_2)}{1-\delta_1\delta_2}(1-w)$$

and $x_2^* = 1 - x_1^*$.

It is obvious that there should be $x_1 = x_1^*$. Then, we have

$$w = \frac{(1-\delta_1)(1-\delta_2)}{2-\delta_1-\delta_2}$$

$$x_1 = \frac{1-\delta_2}{2-\delta_1-\delta_2}$$

$$x_2 = \frac{1-\delta_1}{2-\delta_1-\delta_2}$$

□

Note that $x = 1/2$ when $\delta_1 = \delta_2$, and it is irrelevant who makes the first offer. If it is player 2 who offers the bid, the result will remain the same. Therefore, the players are indifferent between whether to make an offer or to accept the opponent's offer. If we regard player 1 as the seller and player 2 the buyer, $p = x_1/x_2 = (1-\delta_2)/(1-\delta_1)$ can be treated as the price of exchange. Specifically, two players share the pie equally when $p = 1$; player 2 receives the whole pie when $p = 0$, and player 1 receives the whole pie when $p = \infty$. Proposition 2 also shows the consistency between the strategic bargaining approach and the Nash bargaining solution. Let u_1 and u_2 be the von Neumann–Morgenstern utilities of two players. If we define

$$p_N = \frac{1-\delta_2}{1-\delta_1} = \text{arg max } (u_1(p)u_2(p)) \tag{1}$$

the outcome of Proposition 2 is just the Nash bargaining solution.

Given that the players are equal in bargaining, we assume that the players accept the price in (1) without the extra biding process in the following sections.

3. Bargaining Problem of Two Sellers and Two Buyers

Consider a market where there are only two goods, A and B. Participants in this market negotiate to exchange A for B or to exchange B for A. Without loss of generality, let us assume that those who want to exchange A for B are the sellers, and those who want to exchange B for A are the buyers. The price p is the exchange ratio of the amount of B to A, and it is the only factor that every player is concerned with in the bargaining. The utility function of each player is linearly relevant to the price. The bargaining game is described as below.

Two sellers and two buyers bargain over the divisions of two pies (each pie is of a size of 1). A seller (buyer) has to reach an agreement with a buyer (seller) on the division of a pie. Each player has a constant discounting factor (δ_{si} and δ_{bi} for the sellers and buyers, respectively, $i = 1, 2$). The bargaining game contains a number of rounds in which the sellers and the buyers, in turn, offer divisions to their opponents. In the first round, two sellers, in turn, choose a buyer and offer a division, and the chosen buyer may agree the offer "*Y*" or reject it "*N*". Acceptance of the offer leads to an agreement on sharing the pie, and both players involved in the agreement quit the bargaining. If an offer is rejected, the rejected player remains in the bargaining, and so does the rejecting player unless he has accepted an offer from another seller. In the second round, the buyers remaining in the bargaining choose their opponents and make offers. This process will continue for m rounds if no agreements are reached. We assume rationality and complete information in the game, that is, all players are payoff maximised, and the discount factors of all players are common knowledge.

The sellers are given the advantage of making their offers first. As shown in Section 2, this advantage can be eliminated so that it is irrelevant who makes the first offer. It is possible that two or more players choose the same opponent to make their offers in a single round. Any player in this game will either reach an agreement with an opponent or keep the *status quo* (no gain, no loss). The price here denotes the ratio of the seller's share of a pie to the buyer's. Therefore, each seller prefers a higher price, and each buyer prefers a lower price.

Let F be the set of all pure strategies of the players who offer the divisions, and G be the set of all pure strategies of the players who have to respond to an offer. The result of this bargaining can be expressed by the quad (x_1, t_1, x_2, t_2), where $x_1, x_2 \in X$ denote the divisions of two pies that two pairs of players agree with, respectively.

For the outcome (x_1, t_1, x_2, t_2) to be a perfect equilibrium division, the following conditions should be satisfied:

(P-1) There is no $x' > x_1$ such that $(x', t_1) \succ_{b_2} (x_2, t_2)$.
(P-2) There is no $x' < x_1$ such that $(x', t_1) \succ_{s_2} (x_2, t_2)$.
(P-3) There is no $x' > x_2$ such that $(x', t_2) \succ_{b_1} (x_1, t_1)$.
(P-4) There is no $x' < x_2$ such that $(x', t_2) \succ_{s_1} (x_1, t_1)$.

Note that (P-1) and (P-2) ensure that it is not better for s_1 and b_1 to bargain with b_2 or s_2. Similarly, (P-3) and (P-4) ensure that it is not better for s_2 and b_2 to bargain with b_1 or s_1. We then have the following Proposition 3:

Proposition 3. *The bargaining game of two sellers and two buyers has an equilibrium—the same division of both pies.*

Proof. The bargaining game has at least one equilibrium according to Nash (1951). Let the quad (x_1, t_1, x_2, t_2) be an equilibrium satisfying (P-1) to (P-4).

We first prove that if two players reach an agreement, the agreement will be made in the first round. Assume that a seller and a buyer reach an agreement at time t ($t \neq 0$). Obviously, both sides receive a higher payoff if they make the same agreement at time $t = 0$, given that they have positive discounting factors.

Second, we prove that the divisions of both pies must be the same if the players reach two agreements. Assume that the division ratios are different in two agreements. Without loss of generality, suppose that the seller s_1 and the buyer b_1 reach the agreement of division $(x_1, 0)$, and s_2 and b_2 reach the agreement of division $(x_2, 0)$ and $x_1 > x_2$. The seller s_2 and buyer b_1 can be better off by making an agreement with any x' ($x_1 > x' > x_2$). The division of s_2 increases from x_2 to x', and the division of b_1 increases from $1 - x_1$ to $1 - x'$. Thus, there must be $x_1 = x_2$.

Third, we prove the uniqueness of equilibrium. Assume that there are two quads $(x_1, 0, x_1, 0)$ and $(x_2, 0, x_2, 0)$, and $x_1 \neq x_2$. It is a contradiction that both quads are equilibria of the game. □

With the same division, it is irrelevant for each player to choose who to bargain with. Then, the bargaining turns out to be symmetric: the bargaining between one pair of seller and buyer mirrors the bargaining between another pair. According to Proposition 1, there is a unique perfect equilibrium to the bargaining problem of two players. We now deduce that the following.

(1) If the sellers first make offers, each seller receives x and each buyer receives y.

$$x = \frac{2(2 - \delta_{b1} - \delta_{b2})}{4 - (\delta_{b1} + \delta_{b2})(\delta_{s1} + \delta_{s2})}$$

$$y = \frac{(\delta_{b1} + \delta_{b2})(2 - \delta_{s1} - \delta_{s2})}{4 - (\delta_{b1} + \delta_{b2})(\delta_{s1} + \delta_{s2})}$$

(2) If the sellers and buyers bid for the right to make first offers, each seller receives x^* and each buyer receives y^*.

$$x^* = \frac{2-\delta_{b1}-\delta_{b2}}{4-\delta_{b1}-\delta_{b2}-\delta_{s1}-\delta_{s2}}$$

$$y^* = \frac{2-\delta_{s1}-\delta_{s2}}{4-\delta_{b1}-\delta_{b2}-\delta_{s1}-\delta_{s2}}$$

The price is

$$p = \frac{x^*}{y^*} = \frac{2-\delta_{b1}-\delta_{b2}}{2-\delta_{s1}-\delta_{s2}}$$

Following Rubinstein [2,32], we define a group of functions v_i $(i = s_1, s_2, b_1, b_2)$ as follows:

$$v_i(x,t) = \begin{cases} y, & \exists y \in X \text{ such that } (y,0) \sim_i (x,t) \\ 0, & \forall y \in X \text{ there is } (y,0) \succ_i (x,t) \end{cases}$$

This means that for any (x,t), either there is $y \in X$ such that player i ($i \in \{b_1, b_2, s_1, s_2\}$) is indifferent between (x,t) and $(y,0)$, or every $(y,0)$ is preferred by i to (x,t). In order for two pairs of sellers and buyers to reach the same division, there should be $v_{s_1} = v_{s_2}$ and $v_{b_1} = v_{b_2}$. This means that two sellers (buyers) are equal in the bargaining regardless of the values of their discount factors.

In order for two sellers (buyers) to form the same bargaining strategy, there should be

$$v_{s_1}(x,t) = v_{s_2}(x,t) = \frac{1}{2}(\delta_{s_1}^t + \delta_{s_2}^t)x$$

$$v_{b_1}(x,t) = v_{b_2}(x,t) = \frac{1}{2}(\delta_{b_1}^t + \delta_{b_2}^t)x$$

The intersection of $y_{s_1} = v_1(x_{s_1}, 1)$ and $x_{b_1} = v_1(y_{b_1}, 1)$ reflects the division (x^*, y^*). This can be expressed as Figure 1.

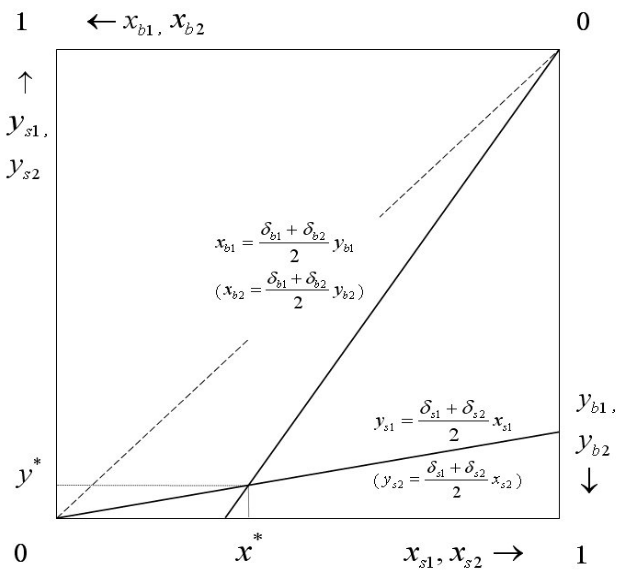

Figure 1. Perfect equilibrium (x^*, y^*) for the bargaining game of two sellers and two buyers.

From Figure 1, we have

$$x^* = \frac{2(2-\delta_{b1}-\delta_{b2})}{4-(\delta_{b1}+\delta_{b2})(\delta_{s1}+\delta_{s2})}$$

$$y^* = \frac{(\delta_{b1} + \delta_{b2})(2 - \delta_{s1} - \delta_{s2})}{4 - (\delta_{b1} + \delta_{b2})(\delta_{s1} + \delta_{s2})}$$

When the sellers and buyers bid for the right to make the first offer, the advantage of first offer can be eliminated. The process is that the sellers first offer a bid w ($0 \le w \le 1$) to the buyers to exchange the right of first offer. If the buyers accept the bid, each buyer receives w division of a pie, and the sellers begin the bargaining to divide the rest of pies. If the buyers refuse the bid, they win the right to make an offer first, and each seller receives w.

If the buyers accept the bid, each seller will receive

$$x_1 = \frac{2(2 - \delta_{b1} - \delta_{b2})}{4 - (\delta_{s1} + \delta s2)(\delta_{b1} + \delta_{b2})}(1 - w)$$

If the buyers refuse the bid, each seller will receive

$$x_2 = \frac{(\delta_{s1} + \delta_{s2})(2 - \delta_{b1} - \delta_{b2})}{4 - (\delta_{s1} + \delta s2)(\delta_{b1} + \delta_{b2})}(1 - w) + w$$

Obviously, there should be $x_1 = x_2$. Then, we have

$$x_1 = \frac{2 - \delta_{b1} - \delta_{b2}}{4 - \delta_{b1} - \delta_{b2} - \delta_{s1} - \delta_{s2}}$$

Each buyer receives

$$y_1 = 1 - x_1 = \frac{2 - \delta_{s1} - \delta_{s2}}{4 - \delta_{b1} - \delta_{b2} - \delta_{s1} - \delta_{s2}}$$

Hence,

$$p = x_1/y_1 = \frac{2 - \delta_{b1} - \delta_{b2}}{2 - \delta_{s1} - \delta_{s2}}.$$

According to Proposition 3, a 'patient' player receives an equal division to an 'impatient' player in the bargaining. This counter-intuitive result can be explained as follows. Because every player would like to choose the impatient player to be their bargaining opponent, the impatient player could increase their share by threatening to change their bargaining opponent. Similarly, the patient player had to lower their share because of their opponent's threat of changing their bargaining opponent. Consequently, a division will be reached so that the sellers (buyers) receive equal division no matter how patient or impatient they are.

Figure 2 shows the relationship between two-player bargaining and two pairs of players bargaining. If the players bargain with each other independently, the solution to two pairs of players bargaining will be either A, C or B, D. Because of the interaction between two pairs of players, A, C and B, D converge to (x^*, y^*).

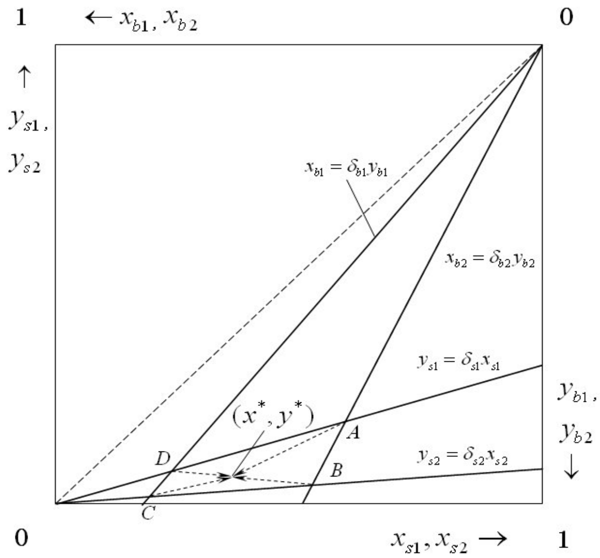

Figure 2. Relationship between two-player bargaining and two pairs of players bargaining.

4. Conclusions

The bargaining game of two sellers and two buyers is analysed by using a sequential approach. We have proven that this game has an unique equilibrium in which all players agree on a price. The price is exactly the Nash bargaining equilibrium between the coalition of sellers and the coalition of buyers. The sellers and buyers will equally share two pies when the sellers and buyers have equal aggregated discount factors, or $\delta_{b1} + \delta_{b2} = \delta_{s1} + \delta_{s2}$. When the buyers are patient, ($\delta_{b1} \to 1$ and $\delta_{b2} \to 1$), and when the sellers are not, the price tends to zero. The buyers' coalition obtains a bargaining payoffs monopoly in this extreme situation. The price tends to infinity ($p \to \infty$) when the sellers are patient ($\delta_{s1} \to 1$ and $\delta_{s2} \to 1$) and the buyers are not.

The uniqueness of equilibrium in this game can also be proven via cooperative game theory. Among the possible coalitions of players, the coalition of all sellers and the coalition of all buyers are the cheapest winning coalitions for two types of players. This coincides with the current bargaining theory in the literature.

This is the first time that the uniqueness of equilibrium in a bargaining game of two sellers and two buyers has been proven, which can be further extended to a bargaining problem of n sellers and n buyers with complete information, that is, the price is computed by

$$p = \frac{n - \delta_{b1} - \delta_{b2} - \cdots - \delta_{bn}}{n - \delta_{s1} - \delta_{s2} - \cdots - \delta_{sn}}$$

The price is the Nash bargaining equilibrium between the coalition of sellers and the coalition of buyers. This result is non-trivial in understanding the essence of market price and equilibrium state in real world markets. We will give the proof of this in another paper.

The numbers of sellers and buyers are not necessarily equal as long as the amount of goods being bargained are equal. This provides an explanation of general equilibrium in a two-goods market by showing that the market clearance price is actually determined by the bargaining between sellers and buyers.

In a bargaining game of incomplete or asymmetric information, different prices are possible, taking into consideration the cost of acquiring information. The analysis in this study may still be non-trivial when the bargaining game is played repeatedly or in an evolutionary context. To study the bargaining equilibrium in an evolutionary dynamic using evolutionary game theory will be a topic of our future research.

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