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Medical Diagnosis and Pattern Recognition Based on Generalized Dice Similarity Measures for Managing Intuitionistic Hesitant Fuzzy Information

Majed Albaity [1,*] and Tahir Mahmood [2,*]

1 Department of Mathematics, Faculty of Science, King Abdulaziz University, P.O. Box 80348, Jeddah 22254, Saudi Arabia
2 Department of Mathematics and Statistics, International Islamic University, Islamabad 44000, Pakistan
* Correspondence: malbaity@kau.edu.sa (M.A.); tahirbakhat@yahoo.com or tahirbakhat@iiu.edu.pk (T.M.)

Abstract: Pattern recognition is the computerized identification of shapes, designs, and reliabilities in information. It has applications in information compression, machine learning, statistical information analysis, signal processing, image analysis, information retrieval, bioinformatics, and computer graphics. Similarly, a medical diagnosis is a procedure to illustrate or identify diseases or disorders, which would account for a person's symptoms and signs. Moreover, to illustrate the relationship between any two pieces of intuitionistic hesitant fuzzy (IHF) information, the theory of generalized dice similarity (GDS) measures played an important and valuable role in the field of genuine life dilemmas. The main influence of GDS measures is that we can easily obtain a lot of measures by using different values of parameters, which is the main part of every measure, called DGS measures. The major influence of this theory is to utilize the well-known and valuable theory of dice similarity measures (DSMs) (four different types of DSMs) under the assumption of the IHF set (IHFS), because the IHFS covers the membership grade (MG) and non-membership grade (NMG) in the form of a finite subset of [0, 1], with the rule that the sum of the supremum of the duplet is limited to [0, 1]. Furthermore, we pioneered the main theory of generalized DSMs (GDSMs) computed based on IHFS, called the IHF dice similarity measure, IHF weighted dice similarity measure, IHF GDS measure, and IHF weighted GDS measure, and computed their special cases with the help of parameters. Additionally, to evaluate the proficiency and capability of pioneered measures, we analyzed two different types of applications based on constructed measures, called medical diagnosis and pattern recognition problems, to determine the supremacy and consistency of the presented approaches. Finally, based on practical application, we enhanced the worth of the evaluated measures with the help of a comparative analysis of proposed and existing measures.

Keywords: intuitionistic hesitant fuzzy sets; generalized dice similarity measures; medical diagnosis; pattern recognition; artificial intelligence

MSC: 03B52; 68T27; 68T37; 94D05; 03E72

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1. Introduction

The decision-making procedure covers four main stages: intelligence, design, choice, and implementation. The principle of the decision-making technique begins with the intelligence stage. In this stage, the intellectual determines reality and identifies and explains the troubles. However, before 1965, no one had utilized or studied the decision-making troubles in the environment of the fuzzy set (FS) theory. For this, the well-known idea of FS was initiated by Zadeh [1] by modifying the technique of crisp set into FS, which covers the MG belonging to [0, 1]. FS has received considerable attention from the distinct intellectual, and certain applications have been carried out by different scholars. For example, Aydin [2] proposed the fuzzy multicriteria decision-making technique by using the Fermatean fuzzy sets, John [3] discussed the certain application of the type-2 FSs, Mandel and John [4] explored the type-2 fuzzy sets made simple, and Mahmood [5] initiated the idea of a bipolar soft set, discussed operational laws, and applied it in decision-making problems.

FS has received attention from the distinct intellectual, and certain applications have been carried out by different scholars. However, if an intellectual faces information in the shape of {0.8,0.9,0.7}, then the principle of FS has been neglected. For this, the well-known idea of hesitant FS (HFS) was initiated by Torra [6] by modifying the technique of FS into HFS, which covers the MG, whose supremum value is belonging to [0, 1]. HFS is a modified version of FS and has received attention from the distinct intellectual; certain applications have been performed by different scholars. For example, Meng and Chen [7] developed the correlation measures for HFSs, Li et al. [8] investigated the distance and similarity measures for HFSs, Su et al. [9] proposed certain measures based on dual HFSs, and Wei et al. [10] investigated the entropy and certain types of measures based on HFSs.

If a piece of intellectual faced information in the shape of "yes" or "no", then the principle of FS has been neglected. For this, the well-known idea of intuitionistic FS (IFS) was initiated by Atanassov [11] by modifying the technique of FS into IFS, which covers the MG and NMG, whose sum is belonging to [0, 1]. IFS is a modified version of FS and has received attention from the distinct intellectual; certain applications have been carried out by different scholars. For example, Ye [12] initiated the certain cosine measure by using IFSs, Rani and Garg [13] developed the distance measures by using complex IFSs, Liang and Shi [14] also explored certain measures based on IFSs, Xu and Chen [15] examined the distance and similarity measures for IFSs, Xu [16] proposed the intuitionistic fuzzy similarity measures, Garg and Rani [17] presented the correlation among any number of complex IFSs, Zeshui [18] utilized certain measures for interval-valued IFSs, Wei et al. [19] investigated the entropy and similarity measures for interval-valued IFSS, and Wang and Xin [20] proposed the distance measures for IFSs.

It was demonstrated that the prevailing information computed based on FSs, HFSs, and IFSs has a variety of applications in many different fields, for instance, computer science, economics and finance, engineering sciences, and road signals. However, it is also clear that they have many limitations and restrictions. For instance, we know that IFS has managed only with two-dimensional information in a singleton set, and each dimension of information can express only one value, but what if someone provided two-dimensional information in the shape of singleton sets, and each dimension of information could represent more than one value? In such a situation, experts noticed that the theory of IFS was not able to proceed with the above information accurately. For this, the well-known idea of an intuitionistic hesitant fuzzy set (IHFS) was initiated by Beg and Rashid [21] by modifying the technique of IFS into IHFS, which covers the MG and NMG in the form of a finite subset of [0, 1], whose sum of the supremum of the duplet is belonging to [0, 1]. IHFS is a modified version of IFS and HFS to cope with complicated and unreliable information in genuine life troubles, and it has gotten massive attraction from the distinct intellectual. Certain applications have been carried out by different scholars. For example, Peng et al. [22] initiated the cross-entropy measures by using the IHFSs, and Zhai et al. [23] examined probabilistic interval-valued IHFSs.

In statistics and related theories, a similarity function, i.e., similarity metric or similarity measure, is a real-valued function that computes the similarity among two terms. Even though no single idea of similarity exists, generally such measures are, in a particular sense, the inverse of distance metrics. Cosine similarity, Tangent similarity, hamming similarity, Euclidean similarity, dice similarity, and generalized dice similarity measures are the commonly employed types of similarity measures for real-valued vectors, used in data retravel to score the similarity of documents in the vector space model. In machine learning, common kernel mappings such as the Radial based function kernel can be observed as similarity measures. In all these measures, we noticed that the GDS measures are massively valuable and effective, as they are more generalized than the prevailing studied measures.

Furthermore, GDS measures are a very significant part of the decision-making technique to determine the closeness between any number of attributes. A certain application has been performed by different scholars. By using different values of the parameter, we can easily obtain the prevailing measures of cosine similarity, tangent similarity, hamming similarity, Euclidean similarity, dice similarity. However, the principle of dice and GDS measures are not implemented in the environment of IHFSs. The main goal of this study is to utilize the principle of GDS measures in the environment of IHFS to improve the quality of the research. We propose this theory, due to the following reasons:

1. How do we find the relation between two objects?
2. How do we propose new types of measures based on IHF information?
3. How do we find our required result?

To handle the above questions, we aim to illustrate the following investigations, which are briefly explained in the form of certain points below:

1. To diagnose certain dice similarity measures based on IHF information.
2. To evaluate different types of GDS measures based on IHF information.
3. To investigate many cases of the investigated measures in order to improve the worth of the evaluated measures.
4. To utilize two different types of applications, called medical diagnosis and pattern recognition, based on pioneered measures.
5. To describe the sensitive analysis, advantages, and geomatical expressions of the evaluated theories to determine the partibility of the investigated measures.

The main contribution of this study is constructed as follows: In Section 2, we briefly recall the idea of IFSs, HFSs, and IHFSs. The main idea of dice similarity measure (DSM) is also revised. In Section 3, we propose certain types of DSM measures based on IHFSs. In Section 4, we explore the IHF GDS measure and IHF-weighted GDS measure. Based on the investigated measures, certain special cases are also evaluated. In Section 5, we utilize two different types of applications, called medical diagnosis and pattern recognition, based on pioneered measures and discuss their comparative analysis. The conclusion of this study is discussed in Section 6.

2. Preliminaries

The theory of IFSs, HFSs, IHFSs, and DSMs are the parts of this section. Further, the mathematical term X, represented as a universal set with MG "$\mathfrak{M}_I$" and NMG "$\mathfrak{N}_I$".

Definition 1 ([11]). *An IFS I is investigated by:*

$$I = \{(\mathfrak{X}, \mathfrak{M}_I(\mathfrak{X}), \mathfrak{N}_I(\mathfrak{X})) : \mathfrak{X} \in X\}$$

with a rule: $0 \leq \mathfrak{M}_I(\mathfrak{X}) + \mathfrak{N}_I(\mathfrak{X}) \leq 1$. *Moreover, the hesitancy degree is shown by:* $d_I(\mathfrak{X}) = 1 - (\mathfrak{M}_I(\mathfrak{X}) + \mathfrak{N}_I(\mathfrak{X}))$. *During this study, the IFN is elaborated by* $I = (\mathfrak{M}, \mathfrak{N})$.

Definition 2 ([6]). *A HFS I is investigated by:*

$$I = \{(\mathfrak{X}, \mathfrak{M}_I(\mathfrak{X})) : \mathfrak{X} \in X\}$$

where $\mathfrak{M}_I = \{\mathfrak{M}_1, \mathfrak{M}_2, \ldots, \mathfrak{M}_n\}$ *with a rule:* $0 \leq \sup(\mathfrak{M}_I) \leq 1$.

Definition 3 ([21]). *An IHFS Ξ is investigated by:*

$$\Xi = \{(\mathfrak{X}, \mathfrak{M}_\Xi(\mathfrak{X}), \mathfrak{N}_\Xi(\mathfrak{X})) : \mathfrak{X} \in X\}$$

where $\mathfrak{M}_\Xi(\mathfrak{X})$ *and* $\mathfrak{N}_\Xi(\mathfrak{X})$ *are expressed the hesitant fuzzy numbers (HFNs), with a rule:* $0 \leq \mathfrak{MAX}(\mathfrak{M}_\Xi(\mathfrak{X})) + max(\mathfrak{N}_\Xi(\mathfrak{X})) \leq 1$. *Moreover, the refusal grade is initiated by:* $\pi_\Xi(\mathfrak{X}) =$

$1-(\mathfrak{MAX}(\mathfrak{M}_{\Xi}(\mathfrak{X}))+max(\mathfrak{N}_{\Xi}(\mathfrak{X})))$. *The intuitionistic hesitant fuzzy number is expressed by:* $\Xi=\left(\mathfrak{M}^{j}_{\Xi},\mathfrak{N}^{j}_{\Xi}\right)$.

Definition 4 ([24]). *For any two-positive vector X and Y, the DSM is initiated by:*

$$D(X,Y)=\frac{2X.Y}{\|X\|_2^2+\|Y\|_2^2}=\frac{2\sum_{j=1}^{l}\mathfrak{X}_j y_j}{\sum_{j=1}^{l}\mathfrak{X}_j^2+\sum_{j=1}^{l}y_j^2}$$

where $X.Y=\sum_{j=1}^{l}\mathfrak{X}_j y_j$ is expressed as the inner product and $\|X\|_2=\sqrt{\sum_{j=1}^{l}\mathfrak{X}_j^2}$ and $\|Y\|_2=\sqrt{\sum_{j=1}^{l}y_j^2}$ is expressed in the Euclidean or L_2 norms of X and Y.

3. DSM for IHFSs

To illustrate the relationship between any two pieces of IHF information, the theory of DSMs played an important and valuable role in the field of genuine life dilemmas. The main influence of GDS measures is that we can easily obtain many measures by using different values of parameters, which is the main part of every measure, called DGS measures. In this study, we chose one of the most flexible and genuine principles, called the IHFS, which covers the MG and NMG in the form of a finite subset of [0, 1], with the rule that the sum of the supremum of the duplet is limited to [0, 1] and GDS measures are to develop the four sorts of IHF dice similarity measure and IHF weighted dice similarity measure. Based on the investigated measures, certain special cases were also evaluated.

Definition 5. *By using any two IHFNS Ξ and Ξ', a DSM $D^1{}_{P\Xi F}(\Xi,\Xi')$ is investigated by:*

$$\begin{aligned}&D^1{}_{P\Xi F}(\Xi,\Xi')\\&=\frac{1}{M}\sum_{i=1}^{M}\frac{2\left(\frac{1}{\mathfrak{M}}\sum_{j=1}^{l}\mathfrak{M}^{j}_{\Xi}(\mathfrak{X}_i)\mathfrak{M}^{j}_{\Xi'}(\mathfrak{X}_i)+\frac{1}{\mathfrak{N}}\sum_{j=1}^{l}\mathfrak{N}^{j}_{\Xi}(\mathfrak{X}_i)\mathfrak{N}^{j}_{\Xi'}(\mathfrak{X}_i)\right)}{\left(\frac{1}{L_{\mathfrak{M}_{\Xi}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{M}^{j}_{\Xi}(\mathfrak{X}_i)\right)^2+\frac{1}{L_{\mathfrak{N}_{\Xi}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{N}^{j}_{\Xi}(\mathfrak{X}_i)\right)^2+\frac{1}{L_{\mathfrak{M}_{\Xi'}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{M}^{j}_{\Xi'}(\mathfrak{X}_i)\right)^2+\frac{1}{L_{\mathfrak{N}_{\Xi'}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{N}^{j}_{\Xi'}(\mathfrak{X}_i)\right)^2\right)}\end{aligned}$$

which holds the necessary rules:

1. $0\le D^1{}_{P\Xi F}(\Xi,\Xi')\le 1$
2. $D^1{}_{P\Xi F}(\Xi,\Xi')=D^1{}_{P\Xi F}(\Xi',\Xi)$
3. $D^1{}_{P\Xi F}(\Xi,\Xi')=1\Leftrightarrow\Xi=\Xi'$

Using some conditions, we can easily obtain further particular cases from the above theory; for instance, to put $\mathfrak{N}^{j}_{\Xi}(\mathfrak{X}_i)=\mathfrak{N}^{j}_{\Xi'}(\mathfrak{X}_i)=0$ in $D^1{}_{P\Xi F}(\Xi,\Xi')$, then $D^1{}_{P\Xi F}(\Xi,\Xi')$ will change for HFSs. Furthermore, to put $\mathfrak{M}^{j}_{\Xi}(\mathfrak{X}_i),\mathfrak{M}^{j}_{\Xi'}(\mathfrak{X}_i)$ and $\mathfrak{N}^{j}_{\Xi}(\mathfrak{X}_i),\mathfrak{N}^{j}_{\Xi'}(\mathfrak{X}_i)$ as a singleton set, then $D^1{}_{P\Xi F}(\Xi,\Xi')$ will change for IFSs, meaning the theory diagnosed in this study is massively powerful and dominant compared to others.

Definition 6. *By using any two IHFNS Ξ and Ξ', a WDSM $WD^1{}_{P\Xi F}(\Xi,\Xi')$ is investigated by:*

$$\begin{aligned}&WD^1{}_{P\Xi F}(\Xi,\Xi')\\&=\sum_{i=1}^{M}w_i\frac{2\left(\frac{1}{\mathfrak{M}}\sum_{j=1}^{l}\mathfrak{M}^{j}_{\Xi}(\mathfrak{X}_i)\mathfrak{M}^{j}_{\Xi'}(\mathfrak{X}_i)+\frac{1}{\mathfrak{N}}\sum_{j=1}^{l}\mathfrak{N}^{j}_{\Xi}(\mathfrak{X}_i)\mathfrak{N}^{j}_{\Xi'}(\mathfrak{X}_i)\right)}{\left(\frac{1}{L_{\mathfrak{M}_{\Xi}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{M}^{j}_{\Xi}(\mathfrak{X}_i)\right)^2+\frac{1}{L_{\mathfrak{N}_{\Xi}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{N}^{j}_{\Xi}(\mathfrak{X}_i)\right)^2+\frac{1}{L_{\mathfrak{M}_{\Xi'}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{M}^{j}_{\Xi'}(\mathfrak{X}_i)\right)^2+\frac{1}{L_{\mathfrak{N}_{\Xi'}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{N}^{j}_{\Xi'}(\mathfrak{X}_i)\right)^2\right)}\end{aligned}$$

which holds the necessary rules of Definition 5.

Using some conditions, we can easily obtain further particular cases from the above theory, for instance, to put $\mathfrak{N}^{j}_{\Xi}(\mathfrak{X}_i)=\mathfrak{N}^{j}_{\Xi'}(\mathfrak{X}_i)=0$ in $WD^1{}_{P\Xi F}(\Xi,\Xi')$, $WD^1{}_{P\Xi F}(\Xi,\Xi')$ will change for HFSs. Furthermore, to put $\mathfrak{M}^{j}_{\Xi}(\mathfrak{X}_i),\mathfrak{M}^{j}_{\Xi'}(\mathfrak{X}_i)$ and $\mathfrak{N}^{j}_{\Xi}(\mathfrak{X}_i),\mathfrak{N}^{j}_{\Xi'}(\mathfrak{X}_i)$ as a singleton set, $WD^1{}_{P\Xi F}(\Xi,\Xi')$ will change for IFSs, meaning the theory diagnosed in this manuscript is massively powerful and dominant as compared to others. For $w = \left(\frac{1}{M}, \frac{1}{M}, \ldots, \frac{1}{M}\right)^T$, the WDSM is converted for DSM based on IHFS such that $WD^1{}_{P\Xi F}(\Xi, \Xi') = D^1{}_{P\Xi F}(\Xi, \Xi')$.

Definition 7. *By using any two IHFNS Ξ and Ξ', a DSM $D^2{}_{P\Xi F}(\Xi, \Xi')$ is investigated by:*

$$D^2{}_{P\Xi F}(\Xi, \Xi') = \frac{1}{M}\sum_{i=1}^{M} \frac{2\left(\frac{1}{\mathfrak{M}}\sum_{j=1}^{l}\mathfrak{M}^j_{\Xi}(\mathfrak{X}_i)\mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i) + \frac{1}{\mathfrak{N}}\sum_{j=1}^{l}\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i)\mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i) + \frac{1}{\pi}\sum_{j=1}^{l}\pi^j_{\Xi}(\mathfrak{X}_i)\pi^j_{\Xi'}(\mathfrak{X}_i)\right)}{\left(\begin{array}{c}\frac{1}{L_{\mathfrak{M}_{\Xi}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{M}^j_{\Xi}(\mathfrak{X}_i)\right)^2 + \frac{1}{L_{\mathfrak{N}_{\Xi}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i)\right)^2 + \frac{1}{L_{\pi_{\Xi}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\pi^j_{\Xi}(\mathfrak{X}_i)\right)^2 + \\ \frac{1}{L_{\mathfrak{M}_{\Xi'}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i)\right)^2 + \frac{1}{L_{\mathfrak{N}_{\Xi'}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)\right)^2 + \frac{1}{L_{\pi_{\Xi'}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\pi^j_{\Xi'}(\mathfrak{X}_i)\right)^2\end{array}\right)}$$

which holds the necessary rules of Definition 5.

Using some conditions, we can easily obtain a lot of further particular cases from the above theory; for instance, to put $\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i) = \mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i) = 0$ in $D^2{}_{P\Xi F}(\Xi, \Xi')$, then $D^2{}_{P\Xi F}(\Xi, \Xi')$ will change for HFSs. Furthermore, to put $\mathfrak{M}^j_{\Xi}(\mathfrak{X}_i), \mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i)$ and $\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i)$, $\mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)$ as a singleton set, $D^2{}_{P\Xi F}(\Xi, \Xi')$ will change for IFSs, meaning the theory diagnosed in this study is massively powerful and dominant compared to others.

Definition 8. *By using any two IHFNS Ξ and Ξ', a WDSM $WD^2{}_{P\Xi F}(\Xi, \Xi')$ is investigated by:*

$$WD^2{}_{P\Xi F}(\Xi, \Xi') = \sum_{i=1}^{M} w_i \frac{2\left(\frac{1}{\mathfrak{M}}\sum_{j=1}^{l}\mathfrak{M}^j_{\Xi}(\mathfrak{X}_i)\mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i) + \frac{1}{\mathfrak{N}}\sum_{j=1}^{l}\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i)\mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i) + \frac{1}{\pi}\sum_{j=1}^{l}\pi^j_{\Xi}(\mathfrak{X}_i)\pi^j_{\Xi'}(\mathfrak{X}_i)\right)}{\left(\begin{array}{c}\frac{1}{L_{\mathfrak{M}_{\Xi}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{M}^j_{\Xi}(\mathfrak{X}_i)\right)^2 + \frac{1}{L_{\mathfrak{N}_{\Xi}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i)\right)^2 + \frac{1}{L_{\pi_{\Xi}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\pi^j_{\Xi}(\mathfrak{X}_i)\right)^2 + \\ \frac{1}{L_{\mathfrak{M}_{\Xi'}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i)\right)^2 + \frac{1}{L_{\mathfrak{N}_{\Xi'}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)\right)^2 + \frac{1}{L_{\pi_{\Xi'}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\pi^j_{\Xi'}(\mathfrak{X}_i)\right)^2\end{array}\right)}$$

which holds the necessary rules of Definition 5.

Using some conditions, we can easily obtain a lot of further particular cases from the above theory; for instance, to put $\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i) = \mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i) = 0$ in $WD^2{}_{P\Xi F}(\Xi, \Xi')$, $WD^2{}_{P\Xi F}(\Xi, \Xi')$ will change for HFSs. Furthermore, to put $\mathfrak{M}^j_{\Xi}(\mathfrak{X}_i), \mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i)$ and $\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i), \mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)$ as a singleton set, $WD^2{}_{P\Xi F}(\Xi, \Xi')$ will change for IFSs, meaning that the theory diagnosed in this study is massively powerful and dominant compared to others. For $w = \left(\frac{1}{M}, \frac{1}{M}, \ldots, \frac{1}{M}\right)^T$ then $WD^2{}_{P\Xi F}(\Xi, \Xi') = D^2{}_{P\Xi F}(\Xi, \Xi')$.

Definition 9. *By using any two IHFNS Ξ and Ξ', a DSM $D^3{}_{P\Xi F}(\Xi, \Xi')$ is investigated by:*

$$D^3{}_{P\Xi F}(\Xi, \Xi') = \frac{\sum_{i=1}^{M} 2\left(\frac{1}{\mathfrak{M}}\sum_{j=1}^{l}\mathfrak{M}^j_{\Xi}(\mathfrak{X}_i)\mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i) + \frac{1}{\mathfrak{N}}\sum_{j=1}^{l}\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i)\mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)\right)}{\begin{array}{c}\sum_{i=1}^{M}\left(\frac{1}{L_{\mathfrak{M}_{\Xi}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{M}^j_{\Xi}(\mathfrak{X}_i)\right)^2 + \frac{1}{L_{\mathfrak{N}_{\Xi}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i)\right)^2\right) + \\ \sum_{i=1}^{M}\left(\frac{1}{L_{\mathfrak{M}_{\Xi'}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i)\right)^2 + \frac{1}{L_{\mathfrak{N}_{\Xi'}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)\right)^2\right)\end{array}}$$

which holds the necessary rules of Definition 5.

Using some conditions, we can easily obtain further particular cases from the above theory; for instance, to put $\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i) = \mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i) = 0$ in $D^3{}_{P\Xi F}(\Xi, \Xi')$, $D^3{}_{P\Xi F}(\Xi, \Xi')$ will change for HFSs. Furthermore, to put $\mathfrak{M}^j_{\Xi}(\mathfrak{X}_i), \mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i)$ and $\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i), \mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)$ as a singleton set, $D^3{}_{P\Xi F}(\Xi, \Xi')$ will change for IFSs, meaning that the theory diagnosed in this study is massively powerful and dominant compared to others.

Definition 10. *By using any two IHFNS Ξ and Ξ', a WDSM $WD^3{}_{P\Xi F}(\Xi,\Xi')$ is investigated by:*

$$WD^3{}_{P\Xi F}(\Xi,\Xi')=\frac{\sum_{i=1}^{M}2w_i^2\left(\frac{1}{\mathfrak{M}}\sum_{j=1}^{l}\mathfrak{M}^j_{\Xi}(\mathfrak{X}_i)\mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i)+\frac{1}{\mathfrak{N}}\sum_{j=1}^{l}\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i)\mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)\right)}{\begin{array}{c}\sum_{i=1}^{M}w_i^2\left(\frac{1}{L_{\mathfrak{M}_{\Xi}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{M}^j_{\Xi}(\mathfrak{X}_i)\right)^2+\frac{1}{L_{\mathfrak{N}_{\Xi}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i)\right)^2\right)+\\ \sum_{i=1}^{M}w_i^2\left(\frac{1}{L_{\mathfrak{M}_{\Xi'}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i)\right)^2+\frac{1}{L_{\mathfrak{N}_{\Xi'}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)\right)^2\right)\end{array}}$$

which holds the necessary rules of Definition 5.

Using some conditions, we can easily obtain further particular cases from the above theory; for instance, to put $\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i)=\mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)=0$ in $WD^3{}_{P\Xi F}(\Xi,\Xi')$, $WD^3{}_{P\Xi F}(\Xi,\Xi')$ will change for HFSs. Furthermore, to put $\mathfrak{M}^j_{\Xi}(\mathfrak{X}_i),\mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i)$ and $\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i),\mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)$ as a singleton set, $WD^3{}_{P\Xi F}(\Xi,\Xi')$ will change for IFSs, meaning that the theory diagnosed in this study is massively powerful and dominant compared to others. For $w=\left(\frac{1}{M},\frac{1}{M},\ldots,\frac{1}{M}\right)^T$ then $WD^3{}_{P\Xi F}(\Xi,\Xi')=D^3{}_{P\Xi F}(\Xi,\Xi')$.

Definition 11. *By using any two IHFNS Ξ and Ξ', a DSM $D^4{}_{P\Xi F}(\Xi,\Xi')$ is investigated by:*

$$D^4{}_{P\Xi F}(\Xi,\Xi')=\frac{2\sum_{i=1}^{M}\left(\frac{1}{\mathfrak{M}}\sum_{j=1}^{l}\mathfrak{M}^j_{\Xi}(\mathfrak{X}_i)\mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i)+\frac{1}{\mathfrak{N}}\sum_{j=1}^{l}\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i)\mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)+\frac{1}{\pi}\sum_{j=1}^{l}\pi^j_{\Xi}(\mathfrak{X}_i)\pi^j_{\Xi'}(\mathfrak{X}_i)\right)}{\begin{array}{c}\sum_{i=1}^{M}\left(\frac{1}{L_{\mathfrak{M}_{\Xi}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{M}^j_{\Xi}(\mathfrak{X}_i)\right)^2+\frac{1}{L_{\mathfrak{N}_{\Xi}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i)\right)^2+\frac{1}{L_{\pi_{\Xi}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\pi^j_{\Xi}(\mathfrak{X}_i)\right)^2\right)+\\ \sum_{i=1}^{M}\left(\frac{1}{L_{\mathfrak{M}_{\Xi'}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i)\right)^2+\frac{1}{L_{\mathfrak{N}_{\Xi'}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)\right)^2+\frac{1}{L_{\pi_{\Xi'}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\pi^j_{\Xi'}(\mathfrak{X}_i)\right)^2\right)\end{array}}$$

which holds the necessary rules of Definition 5.

Using some conditions, we can easily obtain further particular cases from the above theory; for instance, to put $\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i)=\mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)=0$ in $D^4{}_{P\Xi F}(\Xi,\Xi')$, $D^4{}_{P\Xi F}(\Xi,\Xi')$ will change for HFSs. Furthermore, to put $\mathfrak{M}^j_{\Xi}(\mathfrak{X}_i),\mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i)$ and $\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i),\mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)$ as a singleton set, $D^4{}_{P\Xi F}(\Xi,\Xi')$ will change for IFSs, meaning the theory diagnosed in this study is massively powerful and dominant compared to others.

Definition 12. *By using any two IHFNS Ξ and Ξ', a WDSM $WD^4{}_{P\Xi F}(\Xi,\Xi')$ is investigated by:*

$$WD^4{}_{P\Xi F}(\Xi,\Xi')=\frac{2\sum_{i=1}^{M}w_i^2\left(\frac{1}{\mathfrak{M}}\sum_{j=1}^{l}\mathfrak{M}^j_{\Xi}(\mathfrak{X}_i)\mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i)+\frac{1}{\mathfrak{N}}\sum_{j=1}^{l}\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i)\mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)+\frac{1}{\pi}\sum_{j=1}^{l}\pi^j_{\Xi}(\mathfrak{X}_i)\pi^j_{\Xi'}(\mathfrak{X}_i)\right)}{\begin{array}{c}\sum_{i=1}^{M}w_i^2\left(\frac{1}{L_{\mathfrak{M}_{\Xi}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{M}^j_{\Xi}(\mathfrak{X}_i)\right)^2+\frac{1}{L_{\mathfrak{N}_{\Xi}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i)\right)^2+\frac{1}{L_{\pi_{\Xi}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\pi^j_{\Xi}(\mathfrak{X}_i)\right)^2\right)+\\ \sum_{i=1}^{M}w_i^2\left(\frac{1}{L_{\mathfrak{M}_{\Xi'}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i)\right)^2+\frac{1}{L_{\mathfrak{N}_{\Xi'}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)\right)^2+\frac{1}{L_{\pi_{\Xi'}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\pi^j_{\Xi'}(\mathfrak{X}_i)\right)^2\right)\end{array}}$$

which holds the necessary rules of Definition 5.

Using some conditions, we can easily obtain further particular cases from the above theory; for instance, to put $\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i)=\mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)=0$ in $WD^4{}_{P\Xi F}(\Xi,\Xi')$, $WD^4{}_{P\Xi F}(\Xi,\Xi')$ will change for HFSs. Furthermore, to put $\mathfrak{M}^j_{\Xi}(\mathfrak{X}_i),\mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i)$ and $\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i),\mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)$ as a singleton set, $WD^4{}_{P\Xi F}(\Xi,\Xi')$ will change for IFSs, meaning the theory diagnosed in this manuscript is massively powerful and dominant compared to others. For $w=\left(\frac{1}{M},\frac{1}{M},\ldots,\frac{1}{M}\right)^T$, $WD^4{}_{P\Xi F}(\Xi,\Xi')=D^4{}_{P\Xi F}(\Xi,\Xi')$.

4. GDSM for IHFSs

To illustrate the relationship between any two pieces of IHF information, the theory of GDS measures played an important and valuable role in the field of genuine life dilemmas. The main influence of GDS measures is that we can easily obtain a large number of measures by using different values of parameters, which is the main part of every measure, called DGS measures. In this study, we chose one of the most flexible and genuine principles, called the IHFS, which covers the MG and NMG in the form of a finite subset of [0, 1], with the rule that the sum of the supremum of the duplet is limited to [0, 1], GDS measures are to develop the four sorts of IHF GDS measure, and IHF weighted GDS measure. Based on the investigated measures, certain special cases are also evaluated, with $0 \leq \rho \leq 1$.

Definition 13. *By using any two IHFNS Ξ and Ξ', a GDSM $GD^1{}_{P\Xi F}(\Xi,\Xi')$ is investigated by:*

$$GD^1{}_{P\Xi F}(\Xi,\Xi') = \frac{1}{M}\sum_{i=1}^{M}\frac{\left(\frac{1}{\mathfrak{M}}\sum_{j=1}^{l}\mathfrak{M}^j_{\Xi}(\mathfrak{x}_i)\mathfrak{M}^j_{\Xi'}(\mathfrak{x}_i)+\frac{1}{\mathfrak{N}}\sum_{j=1}^{l}\mathfrak{N}^j_{\Xi}(\mathfrak{x}_i)\mathfrak{N}^j_{\Xi'}(\mathfrak{x}_i)\right)}{\left(\begin{array}{c}\gamma\left(\frac{1}{L_{\mathfrak{M}_{\Xi}(\mathfrak{x})}}\sum_{j=1}^{l}\left(\mathfrak{M}^j_{\Xi}(\mathfrak{x}_i)\right)^2+\frac{1}{L_{\mathfrak{N}_{\Xi}(\mathfrak{x})}}\sum_{j=1}^{l}\left(\mathfrak{N}^j_{\Xi}(\mathfrak{x}_i)\right)^2\right)+\\(1-\gamma)\left(\frac{1}{L_{\mathfrak{M}_{\Xi'}(\mathfrak{x})}}\sum_{j=1}^{l}\left(\mathfrak{M}^j_{\Xi'}(\mathfrak{x}_i)\right)^2+\frac{1}{L_{\mathfrak{N}_{\Xi'}(\mathfrak{x})}}\sum_{j=1}^{l}\left(\mathfrak{N}^j_{\Xi'}(\mathfrak{x}_i)\right)^2\right)\end{array}\right)}$$

which holds the necessary rules of Definition 5.

Using some conditions, we can easily obtain further particular cases from the above theory; for instance, to put $\mathfrak{N}^j_{\Xi}(\mathfrak{x}_i) = \mathfrak{N}^j_{\Xi'}(\mathfrak{x}_i) = 0$ in $GD^1{}_{P\Xi F}(\Xi,\Xi')$, $GD^1{}_{P\Xi F}(\Xi,\Xi')$ will change for HFSs. Furthermore, to put $\mathfrak{M}^j_{\Xi}(\mathfrak{x}_i), \mathfrak{M}^j_{\Xi'}(\mathfrak{x}_i)$ and $\mathfrak{N}^j_{\Xi}(\mathfrak{x}_i), \mathfrak{N}^j_{\Xi'}(\mathfrak{x}_i)$ as a singleton set, $GD^1{}_{P\Xi F}(\Xi,\Xi')$ will change for IFSs, meaning the theory diagnosed in this study is massively powerful and dominant compared to others.

Definition 14. *By using any two IHFNS Ξ and Ξ', a WGDSM $WGD^1{}_{P\Xi F}(\Xi,\Xi')$ is investigated by:*

$$WGD^1{}_{P\Xi F}(\Xi,\Xi') = \sum_{i=1}^{M}w_i\frac{\left(\frac{1}{\mathfrak{M}}\sum_{j=1}^{l}\mathfrak{M}^j_{\Xi}(\mathfrak{x}_i)\mathfrak{M}^j_{\Xi'}(\mathfrak{x}_i)+\frac{1}{\mathfrak{N}}\sum_{j=1}^{l}\mathfrak{N}^j_{\Xi}(\mathfrak{x}_i)\mathfrak{N}^j_{\Xi'}(\mathfrak{x}_i)\right)}{\left(\begin{array}{c}\gamma\left(\frac{1}{L_{\mathfrak{M}_{\Xi}(\mathfrak{x})}}\sum_{j=1}^{l}\left(\mathfrak{M}^j_{\Xi}(\mathfrak{x}_i)\right)^2+\frac{1}{L_{\mathfrak{N}_{\Xi}(\mathfrak{x})}}\sum_{j=1}^{l}\left(\mathfrak{N}^j_{\Xi}(\mathfrak{x}_i)\right)^2\right)+\\(1-\gamma)\left(\frac{1}{L_{\mathfrak{M}_{\Xi'}(\mathfrak{x})}}\sum_{j=1}^{l}\left(\mathfrak{M}^j_{\Xi'}(\mathfrak{x}_i)\right)^2+\frac{1}{L_{\mathfrak{N}_{\Xi'}(\mathfrak{x})}}\sum_{j=1}^{l}\left(\mathfrak{N}^j_{\Xi'}(\mathfrak{x}_i)\right)^2\right)\end{array}\right)}$$

which holds the necessary rules of Definition 5.

Using some conditions, we can easily obtain further particular cases from the above theory; for instance, to put $\mathfrak{N}^j_{\Xi}(\mathfrak{x}_i) = \mathfrak{N}^j_{\Xi'}(\mathfrak{x}_i) = 0$ in $WGD^1{}_{P\Xi F}(\Xi,\Xi')$, $WGD^1{}_{P\Xi F}(\Xi,\Xi')$ will change for HFSs. Furthermore, to put $\mathfrak{M}^j_{\Xi}(\mathfrak{x}_i), \mathfrak{M}^j_{\Xi'}(\mathfrak{x}_i)$ and $\mathfrak{N}^j_{\Xi}(\mathfrak{x}_i), \mathfrak{N}^j_{\Xi'}(\mathfrak{x}_i)$ as a singleton set, $WGD^1{}_{P\Xi F}(\Xi,\Xi')$ will change for IFSs, meaning the theory diagnosed in this study is massively powerful and dominant compared to others. For $w = \left(\frac{1}{M}, \frac{1}{M}, \ldots, \frac{1}{M}\right)^T$, $WGD^1{}_{P\Xi F}(\Xi,\Xi') = GD^1{}_{P\Xi F}(\Xi,\Xi')$.

Definition 15. *By using any two IHFNS Ξ and Ξ', a GDSM $GD^2{}_{P\Xi F}(\Xi,\Xi')$ is investigated by:*

$$GD^2{}_{P\Xi F}(\Xi,\Xi') = \frac{1}{M}\sum_{i=1}^{M}\frac{\left(\frac{1}{\mathfrak{M}}\sum_{j=1}^{l}\mathfrak{M}^j_{\Xi}(\mathfrak{x}_i)\mathfrak{M}^j_{\Xi'}(\mathfrak{x}_i)+\frac{1}{\mathfrak{N}}\sum_{j=1}^{l}\mathfrak{N}^j_{\Xi}(\mathfrak{x}_i)\mathfrak{N}^j_{\Xi'}(\mathfrak{x}_i)+\frac{1}{\pi}\sum_{j=1}^{l}\pi^j_{\Xi}(\mathfrak{x}_i)\pi^j_{\Xi'}(\mathfrak{x}_i)\right)}{\left(\begin{array}{c}\gamma\left(\frac{1}{L_{\mathfrak{M}_{\Xi}(\mathfrak{x})}}\sum_{j=1}^{l}\left(\mathfrak{M}^j_{\Xi}(\mathfrak{x}_i)\right)^2+\frac{1}{L_{\mathfrak{N}_{\Xi}(\mathfrak{x})}}\sum_{j=1}^{l}\left(\mathfrak{N}^j_{\Xi}(\mathfrak{x}_i)\right)^2+\frac{1}{L_{\pi_{\Xi}(\mathfrak{x})}}\sum_{j=1}^{l}\left(\pi^j_{\Xi}(\mathfrak{x}_i)\right)^2\right)+\\(1-\gamma)\left(\frac{1}{L_{\mathfrak{M}_{\Xi'}(\mathfrak{x})}}\sum_{j=1}^{l}\left(\mathfrak{M}^j_{\Xi'}(\mathfrak{x}_i)\right)^2+\frac{1}{L_{\mathfrak{N}_{\Xi'}(\mathfrak{x})}}\sum_{j=1}^{l}\left(\mathfrak{N}^j_{\Xi'}(\mathfrak{x}_i)\right)^2+\frac{1}{L_{\pi_{\Xi'}(\mathfrak{x})}}\sum_{j=1}^{l}\left(\pi^j_{\Xi'}(\mathfrak{x}_i)\right)^2\right)\end{array}\right)}$$

which holds the necessary rules of Definition 5.

Using some conditions, we can easily obtain further particular cases from the above theory; for instance, to put $\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i) = \mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i) = 0$ in $GD^2{}_{P\Xi F}(\Xi, \Xi')$, $GD^2{}_{P\Xi F}(\Xi, \Xi')$ will change for HFSs. Furthermore, to put $\mathfrak{M}^j_{\Xi}(\mathfrak{X}_i), \mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i)$ and $\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i), \mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)$ as a singleton set, $GD^2{}_{P\Xi F}(\Xi, \Xi')$ will change for IFSs, meaning the theory diagnosed in this study is massively powerful and dominant compared to others.

Definition 16. *By using any two IHFNS Ξ and Ξ', a WGDSM $WGD^2{}_{P\Xi F}(\Xi, \Xi')$ is investigated by:*

$$WGD^2{}_{P\Xi F}(\Xi, \Xi') = \sum_{i=1}^{M} w_i \frac{2\left(\frac{1}{\mathfrak{M}} \sum_{j=1}^{l} \mathfrak{M}^j_{\Xi}(\mathfrak{X}_i)\mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i) + \frac{1}{\mathfrak{N}} \sum_{j=1}^{l} \mathfrak{N}^j_{\Xi}(\mathfrak{X}_i)\mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i) + \frac{1}{\pi} \sum_{j=1}^{l} \pi^j_{\Xi}(\mathfrak{X}_i)\pi^j_{\Xi'}(\mathfrak{X}_i)\right)}{\left(\begin{array}{c} \gamma\left(\frac{1}{L_{\mathfrak{M}_{\Xi}(\mathfrak{X})}} \sum_{j=1}^{l} \left(\mathfrak{M}^j_{\Xi}(\mathfrak{X}_i)\right)^2 + \frac{1}{L_{\mathfrak{N}_{\Xi}(\mathfrak{X})}} \sum_{j=1}^{l} \left(\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i)\right)^2 + \frac{1}{L_{\pi_{\Xi}(\mathfrak{X})}} \sum_{j=1}^{l} \left(\pi^j_{\Xi}(\mathfrak{X}_i)\right)^2\right) + \\ (1-\gamma)\left(\frac{1}{L_{\mathfrak{M}_{\Xi'}(\mathfrak{X})}} \sum_{j=1}^{l} \left(\mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i)\right)^2 + \frac{1}{L_{\mathfrak{N}_{\Xi'}(\mathfrak{X})}} \sum_{j=1}^{l} \left(\mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)\right)^2 + \frac{1}{L_{\pi_{\Xi'}(\mathfrak{X})}} \sum_{j=1}^{l} \left(\pi^j_{\Xi'}(\mathfrak{X}_i)\right)^2\right) \end{array}\right)}$$

which holds the necessary rules of Definition 5.

Using some conditions, we can easily obtain further particular cases from the above theory; for instance, to put $\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i) = \mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i) = 0$ in $WGD^2{}_{P\Xi F}(\Xi, \Xi')$, $WGD^2{}_{P\Xi F}(\Xi, \Xi')$ will change for HFSs. Furthermore, to put $\mathfrak{M}^j_{\Xi}(\mathfrak{X}_i), \mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i)$ and $\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i), \mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)$ as a singleton set, $WGD^2{}_{P\Xi F}(\Xi, \Xi')$ will change for IFSs, meaning the theory diagnosed in this study is massively powerful and dominant compared to others. For $w = \left(\frac{1}{M}, \frac{1}{M}, \ldots, \frac{1}{M}\right)^T$, $WD^2{}_{P\Xi F}(\Xi, \Xi') = D^2{}_{P\Xi F}(\Xi, \Xi')$.

Definition 17. *By using any two IHFNS Ξ and Ξ', a GDSM $GD^3{}_{P\Xi F}(\Xi, \Xi')$ is investigated by:*

$$GD^3{}_{P\Xi F}(\Xi, \Xi') = \frac{\sum_{i=1}^{M}\left(\frac{1}{\mathfrak{M}} \sum_{j=1}^{l} \mathfrak{M}^j_{\Xi}(\mathfrak{X}_i)\mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i) + \frac{1}{\mathfrak{N}} \sum_{j=1}^{l} \mathfrak{N}^j_{\Xi}(\mathfrak{X}_i)\mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)\right)}{\begin{array}{c} \gamma \sum_{i=1}^{M}\left(\frac{1}{L_{\mathfrak{M}_{\Xi}(\mathfrak{X})}} \sum_{j=1}^{l} \left(\mathfrak{M}^j_{\Xi}(\mathfrak{X}_i)\right)^2 + \frac{1}{L_{\mathfrak{N}_{\Xi}(\mathfrak{X})}} \sum_{j=1}^{l} \left(\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i)\right)^2\right) + \\ (1-\gamma) \sum_{i=1}^{M}\left(\frac{1}{L_{\mathfrak{M}_{\Xi'}(\mathfrak{X})}} \sum_{j=1}^{l} \left(\mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i)\right)^2 + \frac{1}{L_{\mathfrak{N}_{\Xi'}(\mathfrak{X})}} \sum_{j=1}^{l} \left(\mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)\right)^2\right) \end{array}}$$

which holds the necessary rules of Definition 5.

Using some conditions, we can easily obtain further particular cases from the above theory; for instance, to put $\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i) = \mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i) = 0$ in $GD^3{}_{P\Xi F}(\Xi, \Xi')$, $GD^3{}_{P\Xi F}(\Xi, \Xi')$ will change for HFSs. Furthermore, to put $\mathfrak{M}^j_{\Xi}(\mathfrak{X}_i), \mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i)$ and $\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i), \mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)$ as a singleton set, $GD^3{}_{P\Xi F}(\Xi, \Xi')$ will change for IFSs, meaning the theory diagnosed in this study is massively powerful and dominant compared to others.

Definition 18. *By using any two IHFNS Ξ and Ξ', a WGDSM $WGD^3{}_{P\Xi F}(\Xi, \Xi')$ is investigated by:*

$$WGD^3{}_{P\Xi F}(\Xi, \Xi') = \frac{\sum_{i=1}^{M} w_i^2\left(\frac{1}{\mathfrak{M}} \sum_{j=1}^{l} \mathfrak{M}^j_{\Xi}(\mathfrak{X}_i)\mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i) + \frac{1}{\mathfrak{N}} \sum_{j=1}^{l} \mathfrak{N}^j_{\Xi}(\mathfrak{X}_i)\mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)\right)}{\begin{array}{c} \gamma \sum_{i=1}^{M} w_i^2\left(\frac{1}{L_{\mathfrak{M}_{\Xi}(\mathfrak{X})}} \sum_{j=1}^{l} \left(\mathfrak{M}^j_{\Xi}(\mathfrak{X}_i)\right)^2 + \frac{1}{L_{\mathfrak{N}_{\Xi}(\mathfrak{X})}} \sum_{j=1}^{l} \left(\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i)\right)^2\right) + \\ (1-\gamma) \sum_{i=1}^{M} w_i^2\left(\frac{1}{L_{\mathfrak{M}_{\Xi'}(\mathfrak{X})}} \sum_{j=1}^{l} \left(\mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i)\right)^2 + \frac{1}{L_{\mathfrak{N}_{\Xi'}(\mathfrak{X})}} \sum_{j=1}^{l} \left(\mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)\right)^2\right) \end{array}}$$

which holds the necessary rules of Definition 5.

Using some conditions, we can easily obtain further particular cases from the above theory; for instance, to put $\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i) = \mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i) = 0$ in $WGD^3{}_{P\Xi F}(\Xi, \Xi')$, $WGD^3{}_{P\Xi F}(\Xi, \Xi')$ will change for HFSs. Furthermore, to put $\mathfrak{M}^j_{\Xi}(\mathfrak{X}_i), \mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i)$ and $\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i), \mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)$ as a singleton set, $WGD^3{}_{P\Xi F}(\Xi, \Xi')$ will change for IFSs, meaning the theory diagnosed in this study is massively powerful and dominant compared to others. For $w = \left(\frac{1}{M}, \frac{1}{M}, \ldots, \frac{1}{M}\right)^T$, $WGD^3{}_{P\Xi F}(\Xi, \Xi') = GD^3{}_{P\Xi F}(\Xi, \Xi')$.

Definition 19. *By using any two IHFNS Ξ and Ξ', a GDSM $GD^4{}_{P\Xi F}(\Xi, \Xi')$ is investigated by:*

$$GD^4{}_{P\Xi F}(\Xi, \Xi') = \frac{\sum_{i=1}^{M}\left(\frac{1}{\mathfrak{M}}\sum_{j=1}^{l}\mathfrak{M}^j_{\Xi}(\mathfrak{X}_i)\mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i) + \frac{1}{\mathfrak{N}}\sum_{j=1}^{l}\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i)\mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i) + \frac{1}{\pi}\sum_{j=1}^{l}\pi^j_{\Xi}(\mathfrak{X}_i)\pi^j_{\Xi'}(\mathfrak{X}_i)\right)}{\begin{array}{c}\gamma\sum_{i=1}^{M}\left(\frac{1}{L_{\mathfrak{M}_{\Xi}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{M}^j_{\Xi}(\mathfrak{X}_i)\right)^2 + \frac{1}{L_{\mathfrak{N}_{\Xi}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i)\right)^2 + \frac{1}{L_{\pi_{\Xi}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\pi^j_{\Xi}(\mathfrak{X}_i)\right)^2\right) + \\ (1-\gamma)\sum_{i=1}^{M}\left(\frac{1}{L_{\mathfrak{M}_{\Xi'}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i)\right)^2 + \frac{1}{L_{\mathfrak{N}_{\Xi'}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)\right)^2 + \frac{1}{L_{\pi_{\Xi'}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\pi^j_{\Xi'}(\mathfrak{X}_i)\right)^2\right)\end{array}}$$

which holds the necessary rules of Definition 5.

Using some conditions, we can easily obtain further particular cases from the above theory; for instance, to put $\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i) = \mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i) = 0$ in $GD^4{}_{P\Xi F}(\Xi, \Xi')$, $GD^4{}_{P\Xi F}(\Xi, \Xi')$ will change for HFSs. Furthermore, to put $\mathfrak{M}^j_{\Xi}(\mathfrak{X}_i), \mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i)$ and $\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i), \mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)$ as a singleton set, $GD^4{}_{P\Xi F}(\Xi, \Xi')$ will change for IFSs, meaning the theory diagnosed in this study is massively powerful and dominant compared to others.

Definition 20. *By using any two IHFNS Ξ and Ξ', a WGDSM $WGD^4{}_{P\Xi F}(\Xi, \Xi')$ is investigated by:*

$$WGD^4{}_{P\Xi F}(\Xi, \Xi') = \frac{\sum_{i=1}^{M} w_i^2\left(\frac{1}{\mathfrak{M}}\sum_{j=1}^{l}\mathfrak{M}^j_{\Xi}(\mathfrak{X}_i)\mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i) + \frac{1}{\mathfrak{N}}\sum_{j=1}^{l}\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i)\mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i) + \frac{1}{\pi}\sum_{j=1}^{l}\pi^j_{\Xi}(\mathfrak{X}_i)\pi^j_{\Xi'}(\mathfrak{X}_i)\right)}{\begin{array}{c}\gamma\sum_{i=1}^{M} w_i^2\left(\frac{1}{L_{\mathfrak{M}_{\Xi}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{M}^j_{\Xi}(\mathfrak{X}_i)\right)^2 + \frac{1}{L_{\mathfrak{N}_{\Xi}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i)\right)^2 + \frac{1}{L_{\pi_{\Xi}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\pi^j_{\Xi}(\mathfrak{X}_i)\right)^2\right) + \\ (1-\gamma)\sum_{i=1}^{M} w_i^2\left(\frac{1}{L_{\mathfrak{M}_{\Xi'}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i)\right)^2 + \frac{1}{L_{\mathfrak{N}_{\Xi'}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)\right)^2 + \frac{1}{L_{\pi_{\Xi'}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\pi^j_{\Xi'}(\mathfrak{X}_i)\right)^2\right)\end{array}}$$

which holds the necessary rules of Definition 5.

Using some conditions, we can easily obtain further particular cases from the above theory; for instance, to put $\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i) = \mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i) = 0$ in $WGD^4{}_{P\Xi F}(\Xi, \Xi')$, $WGD^4{}_{P\Xi F}(\Xi, \Xi')$ will change for HFSs. Furthermore, to put $\mathfrak{M}^j_{\Xi}(\mathfrak{X}_i), \mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i)$ and $\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i), \mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)$ as a singleton set, $WGD^4{}_{P\Xi F}(\Xi, \Xi')$ will change for IFSs, meaning the theory diagnosed in this study is massively powerful and dominant compared to others. For $w = \left(\frac{1}{M}, \frac{1}{M}, \ldots, \frac{1}{M}\right)^T$, $WGD^4{}_{P\Xi F}(\Xi, \Xi') = GD^4{}_{P\Xi F}(\Xi, \Xi')$.

By using the investigated measures, we discussed certain special cases of the DSM, WDSM, GDSM, and WGDSM.

For $\gamma = 0$, in $GD^1{}_{P\Xi F}(\Xi, \Xi')$, we obtained

$$GD^1{}_{P\Xi F}(\Xi, \Xi') = \frac{1}{M}\sum_{i=1}^{M}\frac{\left(\frac{1}{\mathfrak{M}}\sum_{j=1}^{l}\mathfrak{M}^j_{\Xi}(\mathfrak{X}_i)\mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i) + \frac{1}{\mathfrak{N}}\sum_{j=1}^{l}\mathfrak{N}^j_{\Xi}(\mathfrak{X}_i)\mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)\right)}{\left(\frac{1}{L_{\mathfrak{M}_{\Xi'}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{M}^j_{\Xi'}(\mathfrak{X}_i)\right)^2 + \frac{1}{L_{\mathfrak{N}_{\Xi'}(\mathfrak{X})}}\sum_{j=1}^{l}\left(\mathfrak{N}^j_{\Xi'}(\mathfrak{X}_i)\right)^2\right)}$$

Similarly, for $\gamma = 0.5$,

$$GD^1{}_{P\Xi F}(\Xi,\Xi')=\frac{1}{M}\sum_{i=1}^{M}\frac{2\left(\frac{1}{\dot{\mathfrak{M}}}\sum_{j=1}^{l}\mathfrak{M}^j_{\Xi}(\mathfrak{x}_i)\mathfrak{M}^j_{\Xi'}(\mathfrak{x}_i)+\frac{1}{\dot{\mathfrak{A}}}\sum_{j=1}^{l}\mathfrak{A}^j_{\Xi}(\mathfrak{x}_i)\mathfrak{A}^j_{\Xi'}(\mathfrak{x}_i)+\frac{1}{\dot{\mathfrak{N}}}\sum_{j=1}^{l}\mathfrak{N}^j_{\Xi}(\mathfrak{x}_i)\mathfrak{N}^j_{\Xi'}(\mathfrak{x}_i)\right)}{\left(\begin{array}{c}\left(\frac{1}{L_{\mathfrak{M}_{\Xi}(\mathfrak{x})}}\sum_{j=1}^{l}\left(\mathfrak{M}^j_{\Xi}(\mathfrak{x}_i)\right)^2+\frac{1}{L_{\mathfrak{A}_{\Xi}(\mathfrak{x})}}\sum_{j=1}^{l}\left(\mathfrak{A}^j_{\Xi}(\mathfrak{x}_i)\right)^2+\frac{1}{L_{\mathfrak{N}_{\Xi}(\mathfrak{x})}}\sum_{j=1}^{l}\left(\mathfrak{N}^j_{\Xi}(\mathfrak{x}_i)\right)^2\right)+\\ \left(\frac{1}{L_{\mathfrak{M}_{\Xi'}(\mathfrak{x})}}\sum_{j=1}^{l}\left(\mathfrak{M}^j_{\Xi'}(\mathfrak{x}_i)\right)^2+\frac{1}{L_{\mathfrak{A}_{\Xi'}(\mathfrak{x})}}\sum_{j=1}^{l}\left(\mathfrak{A}^j_{\Xi'}(\mathfrak{x}_i)\right)^2+\frac{1}{L_{\mathfrak{N}_{\Xi'}(\mathfrak{x})}}\sum_{j=1}^{l}\left(\mathfrak{N}^j_{\Xi'}(\mathfrak{x}_i)\right)^2\right)\end{array}\right)}=D^1{}_{P\Xi F}(\Xi,\Xi')$$

For $\gamma = 0.5$, in $GD^1{}_{P\Xi F}(\Xi,\Xi')$, we obtained

$$GD^1{}_{P\Xi F}(\Xi,\Xi')=\frac{1}{M}\sum_{i=1}^{M}\frac{\left(\frac{1}{\dot{\mathfrak{M}}}\sum_{j=1}^{l}\mathfrak{M}^j_{\Xi}(\mathfrak{x}_i)\mathfrak{M}^j_{\Xi'}(\mathfrak{x}_i)+\frac{1}{\dot{\mathfrak{N}}}\sum_{j=1}^{l}\mathfrak{N}^j_{\Xi}(\mathfrak{x}_i)\mathfrak{N}^j_{\Xi'}(\mathfrak{x}_i)\right)}{\left(\frac{1}{L_{\mathfrak{M}_{\Xi}(\mathfrak{x})}}\sum_{j=1}^{l}\left(\mathfrak{M}^j_{\Xi}(\mathfrak{x}_i)\right)^2+\frac{1}{L_{\mathfrak{N}_{\Xi}(\mathfrak{x})}}\sum_{j=1}^{l}\left(\mathfrak{N}^j_{\Xi}(\mathfrak{x}_i)\right)^2\right)}$$

For $\gamma = 0$ and 0.5, $GD^2{}_{P\Xi F}(\Xi,\Xi')$, $GD^3{}_{P\Xi F}(\Xi,\Xi')$, and $GD^4{}_{P\Xi F}(\Xi,\Xi')$ are similar.

5. Decision-Making Processes

Pattern recognition is the computerized identification of shapes, designs, and reliabilities in information. It has applications in information compression, machine learning, statistical information analysis, signal processing, image analysis, information retrieval, bioinformatics, and computer graphics. Similarly, a medical diagnosis is a procedure to illustrate or identify diseases or disorders, which would account for a person's symptoms and signs. The decision-making procedure covers four main stages: intelligence, design, choice, and implementation. The principle of decision-making technique begins with the intelligence stage. In this stage, the intellectual determines reality and identifies and explains the troubles. The main influence of this theory is to explore the main idea of medical diagnosis and pattern recognition under the consideration of IHF information. The main importance and briefing explanation about every application is available below. These applications are taken from Ref. [17]. By using the proposed measures, the applications of medical diagnosis and pattern recognition are discussed below.

5.1. Medical Diagnosis

Certain sorts of diseases have distinct symptoms and different affection; the medical diagnosis procedure is determined by the distinct symptoms of the required diseases of the intellectual which is safer from them. The diseases are expressed using the symbols $\Xi_1, \Xi_2, \ldots, \Xi_n$, and their symptoms are expressed by the values of universal sets. Using the proposed measures, the numerical example is discussed below.

Example 1. *For any set of diseases whose expressions are in the form of* $\Xi = \left\{\begin{array}{c}\Xi_1(Typhoid), \Xi_2(Flu), \Xi_3(Heart\ Probelms), \\ \Xi_4(Pneumonia), \Xi_5(coronavirus)\end{array}\right\}$ *and their symptoms whose expressions are in the form of* $X = \left\{\begin{array}{c}Fever,\ Cough,\ Heart\ pain, \\ Loss\ of\ appetite, Short\ of\ breath\end{array}\right\}$. *The symptoms of the distinct diseases are discussed below in the form of unknown diseases:*

$$\Xi_1 = \left\{\begin{array}{c}(\{0.1, 0.2\}, \{0.2, 0.3, 0.4\}), \\ (\{0.11, 0.21\}, \{0.21, 0.31, 0.41\}), \\ (\{0.12, 0.22\}, \{0.22, 0.32, 0.42\}), \\ (\{0.13, 0.23\}, \{0.23, 0.33, 0.43\}), \\ (\{0.14, 0.24\}, \{0.24, 0.34, 0.44\})\end{array}\right\}, \Xi_2 = \left\{\begin{array}{c}(\{0.2, 0.3\}, \{0.1, 0.3, 0.2\}), \\ (\{0.21, 0.31\}, \{0.11, 0.31, 0.21\}), \\ (\{0.22, 0.32\}, \{0.12, 0.32, 0.22\}), \\ (\{0.23, 0.33\}, \{0.13, 0.33, 0.23\}), \\ (\{0.24, 0.34\}, \{0.14, 0.34, 0.24\})\end{array}\right\},$$

$$\Xi_3 = \left\{\begin{array}{c}(\{0.3, 0.1\}, \{0.5, 0.2, 0.1\}), \\ (\{0.31, 0.11\}, \{0.51, 0.21, 0.11\}), \\ (\{0.32, 0.12\}, \{0.52, 0.22, 0.12\}), \\ (\{0.33, 0.13\}, \{0.53, 0.23, 0.13\}), \\ (\{0.34, 0.14\}, \{0.54, 0.24, 0.14\})\end{array}\right\}, \Xi_4 = \left\{\begin{array}{c}(\{0.1, 0.1\}, \{0.2, 0.2, 0.4\}), \\ (\{0.11, 0.11\}, \{0.21, 0.21, 0.41\}), \\ (\{0.12, 0.12\}, \{0.22, 0.22, 0.42\}), \\ (\{0.13, 0.13\}, \{0.23, 0.23, 0.43\}), \\ (\{0.14, 0.14\}, \{0.24, 0.24, 0.44\})\end{array}\right\}, \Xi_5 =$$

$\left\{\begin{array}{c}(\{0.3, 0.5\}, \{0.1, 0.2, 0.3\}), \\ (\{0.31, 0.51\}, \{0.11, 0.21, 0.31\}), \\ (\{0.32, 0.52\}, \{0.12, 0.22, 0.32\}), \\ (\{0.33, 0.53\}, \{0.13, 0.23, 0.33\}), \\ (\{0.34, 0.54\}, \{0.14, 0.24, 0.34\})\end{array}\right\}$. *For this, we choose the known diseases* $\Xi' = \left\{\begin{array}{c}(\{1, 1\}, \{0.0, 0.0.0.0\}), \\ (\{1, 1\}, \{0.0, 0.0.0.0\}), (\{1, 1\}, \{0.0, 0.0.0.0\}), \\ (\{1, 1\}, \{0.0, 0.0.0.0\}), (\{1, 1\}, \{0.0, 0.0.0.0\})\end{array}\right\}$. *Then, by using the* $GD^1{}_{P\Xi F}(\Xi, \Xi')$, $WGD^1{}_{P\Xi F}(\Xi, \Xi')$, $GD^2{}_{P\Xi F}(\Xi, \Xi')$, *and* $WGD^2{}_{P\Xi F}(\Xi, \Xi')$, *the examined measures are discussed in the form of Table 1 by using the weight vector 0.2, 0.3, 0.2, 0.2, and 0.1. For this, we chose the value of* $\gamma = 1$, *then*

Table 1. Expressions of the measured values by using different measures.

Methods	Values
$\boldsymbol{GD^1{}_{P\Xi F}(\Xi, \Xi')}$	0.5056, 0.8248, 0.542, 0.4772, 0.7232
$\boldsymbol{WGD^1{}_{P\Xi F}(\Xi, \Xi')}$	0.0595, 0.1036, 0.0640, 0.0553, 0.0866
$\boldsymbol{GD^2{}_{P\Xi F}(\Xi, \Xi')}$	0.4649, 0.7866, 0.4981, 0.4396, 0.6635
$\boldsymbol{WGD^2{}_{P\Xi F}(\Xi, \Xi')}$	0.0473, 0.0821, 0.0509, 0.0441, 0.0687

Further, information computed in Table 2 is constructed based on the information available in Table 1.

Table 2. Contained ranking analysis of the information in Table 1.

Methods	Values
$\boldsymbol{GD^1{}_{P\Xi F}(\Xi, \Xi')}$	$\Xi_{\cdot 2} \geq \Xi_{\cdot 5} \geq \Xi_{\cdot 3} \geq \Xi_{\cdot 1} \geq \Xi_{\cdot 4}$
$\boldsymbol{WGD^1{}_{P\Xi F}(\Xi, \Xi')}$	$\Xi_{\cdot 2} \geq \Xi_{\cdot 5} \geq \Xi_{\cdot 3} \geq \Xi_{\cdot 1} \geq \Xi_{\cdot 4}$
$\boldsymbol{GD^2{}_{P\Xi F}(\Xi, \Xi')}$	$\Xi_{\cdot 2} \geq \Xi_{\cdot 5} \geq \Xi_{\cdot 3} \geq \Xi_{\cdot 1} \geq \Xi_{\cdot 4}$
$\boldsymbol{WGD^2{}_{P\Xi F}(\Xi, \Xi')}$	$\Xi_{\cdot 2} \geq \Xi_{\cdot 5} \geq \Xi_{\cdot 3} \geq \Xi_{\cdot 1} \geq \Xi_{\cdot 4}$

From Table 2, all sorts of measures are provided with the same ranking results. the best optimal is Ξ_2. Additionally, by using distinct types of measures based on IFSs and IHFSs, the comparative analysis of the elaborated measures with certain prevailing measures are discussed in the form of Table 3. The information related to prevailing measures is as follows: Ye [12] initiated certain cosine measures based on IFSs, Beg and Rashid [21] proposed certain measures based on IHFSs, and Peng et al. [22] proposed the cross-entropy measures based on IHFSs. By using the information in Section 5.1, the comparative analysis is discussed in the form of Table 3.

Table 3. Contained comparative information.

Methods	Values	Ranking Results
Ye [12]	*Cannot be Calculated*	*Cannot be Calculated*
Beg and Rashid [21]	0.0484, 0.1025, 0.0530, 0.0442, 0.0755	$\Xi_{\cdot 2} \geq \Xi_{\cdot 5} \geq \Xi_{\cdot 3} \geq \Xi_{\cdot 1} \geq \Xi_{\cdot 4}$
Peng et al. [22]	0.3538, 0.6755, 0.3870, 0.3285, 0.5524	$\Xi_{\cdot 2} \geq \Xi_{\cdot 5} \geq \Xi_{\cdot 3} \geq \Xi_{\cdot 1} \geq \Xi_{\cdot 4}$
$\boldsymbol{GD^1{}_{P\Xi F}(\Xi, \Xi')}$	0.5056, 0.8248, 0.542, 0.4772, 0.7232	$\Xi_{\cdot 2} \geq \Xi_{\cdot 5} \geq \Xi_{\cdot 3} \geq \Xi_{\cdot 1} \geq \Xi_{\cdot 4}$
$\boldsymbol{WGD^1{}_{P\Xi F}(\Xi, \Xi')}$	0.0595, 0.1036, 0.0640, 0.0553, 0.0866	$\Xi_{\cdot 2} \geq \Xi_{\cdot 5} \geq \Xi_{\cdot 3} \geq \Xi_{\cdot 1} \geq \Xi_{\cdot 4}$
$\boldsymbol{GD^2{}_{P\Xi F}(\Xi, \Xi')}$	0.4649, 0.7866, 0.4981, 0.4396, 0.6635	$\Xi_{\cdot 2} \geq \Xi_{\cdot 5} \geq \Xi_{\cdot 3} \geq \Xi_{\cdot 1} \geq \Xi_{\cdot 4}$
$\boldsymbol{WGD^2{}_{P\Xi F}(\Xi, \Xi')}$	0.0473, 0.0821, 0.0509, 0.0441, 0.0687	$\Xi_{\cdot 2} \geq \Xi_{\cdot 5} \geq \Xi_{\cdot 3} \geq \Xi_{\cdot 1} \geq \Xi_{\cdot 4}$

From Table 2, all sorts of measures are provided with the same ranking results. The best optimal is Ξ_2.

5.2. Pattern Recognition

By using the elaborated measures, we aimed to use a practical application called pattern recognition and try to evaluate it by using pioneered information.

Example 2. *Without any complication or difficulty, the construction of any building is very complicated. For this, a decision-maker collects the information for different places and resolves it using the elaborated measures; then a very safe decision can be made. For this, we chose the different types of building material, the information associated with which is discussed below.*

$$\Xi_1 = \left\{\begin{array}{c} (\{0.1, 0.2\}, \{0.1, 0.2, 0.3\}), \\ (\{0.11, 0.21\}, \{0.11, 0.21, 0.31\}), \\ (\{0.12, 0.22\}, \{0.12, 0.22, 0.32\}), \\ (\{0.13, 0.23\}, \{0.13, 0.23, 0.33\}), \\ (\{0.14, 0.24\}, \{0.14, 0.24, 0.34\}) \end{array}\right\}, \Xi_2 = \left\{\begin{array}{c} (\{0.2, 0.3\}, \{0.2, 0.3, 0.4\}), \\ (\{0.21, 0.31\}, \{0.21, 0.31, 0.41\}), \\ (\{0.22, 0.32\}, \{0.22, 0.32, 0.42\}), \\ (\{0.23, 0.33\}, \{0.23, 0.33, 0.43\}), \\ (\{0.24, 0.34\}, \{0.24, 0.34, 0.44\}) \end{array}\right\},$$

$$\Xi_3 = \left\{\begin{array}{c} (\{0.1, 0.3\}, \{0.2, 0.1, 0.1\}), \\ (\{0.11, 0.31\}, \{0.21, 0.11, 0.11\}), \\ (\{0.12, 0.32\}, \{0.22, 0.12, 0.12\}), \\ (\{0.13, 0.33\}, \{0.23, 0.13, 0.13\}), \\ (\{0.14, 0.34\}, \{0.24, 0.14, 0.14\}) \end{array}\right\}, \Xi_4 = \left\{\begin{array}{c} (\{0.1, 0.2\}, \{0.3, 0.2, 0.4\}), \\ (\{0.11, 0.21\}, \{0.31, 0.21, 0.41\}), \\ (\{0.12, 0.22\}, \{0.32, 0.22, 0.42\}), \\ (\{0.13, 0.23\}, \{0.33, 0.23, 0.43\}), \\ (\{0.14, 0.24\}, \{0.34, 0.24, 0.44\}) \end{array}\right\},$$

$$\Xi_5 = \left\{\begin{array}{c} (\{0.4, 0.5\}, \{0.1, 0.1, 0.1\}), \\ (\{0.41, 0.51\}, \{0.11, 0.11, 0.11\}), \\ (\{0.42, 0.52\}, \{0.12, 0.12, 0.12\}), \\ (\{0.43, 0.53\}, \{0.13, 0.13, 0.13\}), \\ (\{0.44, 0.54\}, \{0.14, 0.14, 0.14\}) \end{array}\right\}$$

For this, we choose the known diseases, which are expressed below:

$$\Xi' = \left\{\begin{array}{c} (\{1, 1\}, \{0.0, 0.0.0.0\}), \\ (\{1, 1\}, \{0.0, 0.0.0.0\}), (\{1, 1\}, \{0.0, 0.0.0.0\}), \\ (\{1, 1\}, \{0.0, 0.0.0.0\}), (\{1, 1\}, \{0.0, 0.0.0.0\}) \end{array}\right\}$$

Then, by using the $GD^1{}_{P\Xi F}(\Xi, \Xi')$, $WGD^1{}_{P\Xi F}(\Xi, \Xi')$, $GD^2{}_{P\Xi F}(\Xi, \Xi')$, and $WGD^2{}_{P\Xi F}(\Xi, \Xi')$, the examined measures are discussed in the form of Table 4 by using the weight vector 0.2, 0.3, 0.2, 0.2, and 0.1. For this, we chose the value of $\gamma = 1$.

Table 4. Expressions of the measured values using different measures.

Methods	Values
$\boldsymbol{GD^1{}_{P\Xi F}(\Xi, \Xi')}$	0.8166, 0.5859, 0.8271, 0.5057, 0.8219
$\boldsymbol{WGD^1{}_{P\Xi F}(\Xi, \Xi')}$	0.098, 0.0728, 0.1313, 0.0596, 0.0984
$\boldsymbol{GD^2{}_{P\Xi F}(\Xi, \Xi')}$	0.749, 0.5589, 0.9916, 0.4649, 0.7514
$\boldsymbol{WGD^2{}_{P\Xi F}(\Xi, \Xi')}$	0.0777, 0.0578, 0.104, 0.0474, 0.078

Further, the information computed in Table 5 was constructed based on the information available in Table 4.

Table 5. Contained ranking analysis.

Methods	Values
$GD^1{}_{P\Xi F}(\Xi,\Xi')$	$\Xi_{\cdot 5} \geq \Xi_{\cdot 3} \geq \Xi_{\cdot 1} \geq \Xi_{\cdot 2} \geq \Xi_{\cdot 4}$
$WGD^1{}_{P\Xi F}(\Xi,\Xi')$	$\Xi_{\cdot 3} \geq \Xi_{\cdot 5} \geq \Xi_{\cdot 1} \geq \Xi_{\cdot 2} \geq \Xi_{\cdot 4}$
$GD^2{}_{P\Xi F}(\Xi,\Xi')$	$\Xi_{\cdot 5} \geq \Xi_{\cdot 3} \geq \Xi_{\cdot 1} \geq \Xi_{\cdot 2} \geq \Xi_{\cdot 4}$
$WGD^2{}_{P\Xi F}(\Xi,\Xi')$	$\Xi_{\cdot 5} \geq \Xi_{\cdot 3} \geq \Xi_{\cdot 1} \geq \Xi_{\cdot 2} \geq \Xi_{\cdot 4}$

From Table 5, all sorts of measures are provided with the different ranking results. the best optimal is Ξ_5 and Ξ_3. Additionally, by using distinct types of measures based on IFSs and IHFSs, the comparative analysis of the elaborated measures with certain prevailing measures are discussed in the form of Table 6. The information related to prevailing measures is as follows: Ye [12] initiated certain cosine measures based on IFSs, Beg and Rashid [21] proposed certain measures based on IHFSs, and Peng et al. [22] proposed the cross-entropy measures based on IHFSs. By using the information from Example 1, the comparative analysis is discussed in the form of Table 6.

Table 6. Contained comparative analysis.

Methods	Values	Ranking Results
Ye [12]	*Cannot be Calculated*	*Cannot be Calculated*
Beg and Rashid [21]	0.638, 0.4478, 0.8805, 0.3538, 0.6403	$\Xi_5 \geq \Xi_3 \geq \Xi_1 \geq \Xi_2 \geq \Xi_4$
Peng et al. [22]	0.0666, 0.0467, 0.103, 0.0363, 0.067	$\Xi_5 \geq \Xi_3 \geq \Xi_1 \geq \Xi_2 \geq \Xi_4$
$GD^1{}_{P\Xi F}(\Xi,\Xi')$	0.8166, 0.5859, 0.8271, 0.5057, 0.8219	$\Xi_5 \geq \Xi_3 \geq \Xi_1 \geq \Xi_2 \geq \Xi_4$
$WGD^1{}_{P\Xi F}(\Xi,\Xi')$	0.098, 0.0728, 0.1313, 0.0596, 0.0984	$\Xi_3 \geq \Xi_5 \geq \Xi_1 \geq \Xi_2 \geq \Xi_4$
$GD^2{}_{P\Xi F}(\Xi,\Xi')$	0.749, 0.5589, 0.9916, 0.4649, 0.7514	$\Xi_5 \geq \Xi_3 \geq \Xi_1 \geq \Xi_2 \geq \Xi_4$
$WGD^2{}_{P\Xi F}(\Xi,\Xi')$	0.0777, 0.0578, 0.104, 0.0474, 0.078	$\Xi_5 \geq \Xi_3 \geq \Xi_1 \geq \Xi_2 \geq \Xi_4$

From Table 2, all sorts of measures are provided with the different ranking results. the best optimal is Ξ_5 and Ξ_3. In the future, we will utilize different types of operators, methods, and measures in the environment of picture hesitant fuzzy sets and neutrosophic hesitant fuzzy sets [24–31] to improve the quality of the proposed works. Therefore, the elaborated measures based on IHFS are more powerful and more fixable than the prevailing ideas [23–31].

6. Conclusions

The main and major features of this analysis are described below:

1. We pioneered the main theory of DSM based on IHFS and evaluated their particular cases.
2. We established the GDS measures in the environment of IHFSs and discussed IHFDSM, IHFWDSM, IHFGDSM, and IHFWGDSM.
3. Based on the investigated measures, certain special cases were also evaluated. Furthermore, by using the discovered measures, medical diagnosis and pattern recognition problems were determined.
4. Finally, we determined the supremacy of the explored work and the sensitivity analysis and advantages of the explored measures. Their geometrical expressions are also discussed.

Our recent work focused on the prevailing information computed based on complex q-rung orthopair FSs [32], spherical FSs (SFSs) [33], Aczel-Alsina operational laws [34], different types of measures [35,36], Aczel-Alsina aggregation operators [37], Maclaurin operators [38], Complex SFSs [39,40], linguistic group decision-making techniques [41], and unbalanced linguistic information [42], and we aim to employ it in the field of computer science, road signals, software engineering, and decision-making.

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