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Bipartite Synchronization of Fractional-Order Memristor-Based Coupled Delayed Neural Networks with Pinning Control

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Abstract: This paper investigates the bipartite synchronization of memristor-based fractional-order coupled delayed neural networks with structurally balanced and unbalanced concepts. The main result is established for the proposed model using pinning control, fractional-order Jensen's inequality, and the linear matrix inequality. Further, new sufficient conditions are derived using the Lyapunov–Krasovskii functional with delay-dependent criteria. Finally, numerical simulations are provided including two numerical examples to show the effectiveness of the theoretical results.

Keywords: bipartite synchronization; Filippov sense; memristor; structurally balanced; Caputo derivative; pinning control

MSC: 34D06; 93D20; 93D15; 37M99

1. Introduction

Fractional differential calculus (FDC) has a vital mechanism compared to ordinary differential calculus. Thus, it is practically used to model several problems in engineering and sciences. The main reason for the success of FDC applications is that they produce fractional-order models that are often more accurate than the integer-order ones. Indeed, it has been witnessed that all fractional operators consider the entire history of the process, thus helping to model the non-local and distributed effects often encountered in the natural and technical phenomena. Fractional calculus is therefore an excellent tool for describing the memory and hereditary properties of various materials and processes, including various applications of real-life engineering problems, such as the diffusion process, heat transfer, mechanics, electric circuits, medicine and a wide range of fields. For more applications, we refer the reader to [1–4] and their references therein.

Synchronization signifies multiple engaging collective behaviors obtained in natural and artificial systems. It is successfully used in pattern recognition, object detection, image processing, etc. [5,6]. There are many examples, such as local synchronization [7], global synchronization [8], multi-quasi synchronization, projective synchronization [9], and bipartite synchronization [10–12]. All nodes which converge to the value and are equal in every modulus, but not in sign, are called bipartite consensus [13]. This consensus which is applied to the synchronization is said to be bipartite synchronization, and this type of synchronization has attracted the attention of more researchers [10,11,14–16].

In 1971, Leon O. Chua postulated that the memristor is a new circuit element. Indeed, it offers non-volatile memory potential applications stored in a simple device structure. In 2008, Chua received great attention for the experimental observation in (Hewlett–Packard) HP labs. The two-terminal variable resistance of the memristor is known as memristance [17,18]. A neural network emulates the human brain by placing the memristor instead of the resistor. It is known as a memristor neural network (MNN). The memristor holds many advantages, such as lower power, high density, and good scalability. It finds the key to engineering problems, such as power circuits, static function, and impacting machines. Moreover, several types of neural network have been studied in the past, a few among them being the Hopfield neural network [19], biological neural network [6], cellular neural network [20], stochastic neural network [21], complex-valued neural network [22], Chebyshev neural network [23], BAM neural network [24,25], delayed neural network [26–29] and coupled delayed neural network [2,11]. Nowadays, many authors are studying the fractional-order memristor-based neural network (FMNN), but their solutions are still unveiled. The fractional-order memristor coupled delayed neural network was studied in [2,30], while the complex-valued memristor-based neural network was discussed in [9,14,17,31]. The network models are efficiently employed to solve several problems in image processing, cryptography and secure communication [14,18,32]. Different kind of controllers are used within the ordinary and fractional order control systems, for example, pinning control [11,31,33], adaptive control [31], event-triggered control [34], and impulsive control [20,24,35,36]. However, the pinning control strategy is mostly used to reduce large-scale network costs. Moreover, the Laplace transform has been used as a tool to study the qualitative behavior of the fractional-order differential system. Recently, the linear matrix inequality (LMI) was used to study the stability behavior of linear and non-linear fractional differential systems [37].

Motivated by the foregoing results, a fractional-order coupled delayed neural network (FCDNN) with delay-dependent criteria is considered in this paper, with the pinning control, fractional-order Jensen's inequality, and the linear matrix inequality. Furthermore, new sufficient conditions are derived using the Lyapunov–Krasovskii functional (LKF). The main contributions of this paper are described as follows:

(a) Constructing a new LKF and examining effects on the stability by considering time delay information which effectively improves the stability condition.
(b) Deducing delay-dependent synchronization criteria for the FCDNN under both structurally balanced and unbalanced networks.
(c) The pinning controller is used, which is cost efficient, and is found effective compared to the other controllers.
(d) Obtaining the boundedness of LKF via fractional-order Jensen's inequality.

The objective of this paper is to provide a different approach that will yield less restrictive and more efficient delay dependent stability conditions. Based on the above discussion, this paper's main outline is derived as follows: In Section 2, we discuss essential preliminaries, definitions and lemmas, and Section 3 explains the problem formulation of memristor based FCDNN. Sections 4 and 5 study memristor-based FCDNN with the bipartite leader and leaderless synchronization through structurally balanced and unbalanced concepts. In Section 6, the applications of the theory are proved by numerical examples.

Notations: The sign$(\cdot)$ represents the signum or sign function. w_p denotes gauge transformation, $\otimes$ denotes the Kronecker product [38], l_{pj} denotes the Laplacian matrix, L^s and L^u are the Laplacian of signed and unsigned matrices, respectively, and $\bar{co}$ means convexity closure. A symmetric matrix Π is denoted as $\Pi = \begin{pmatrix} a & b \\ * & d \end{pmatrix}$, $*$ represents the transpose of b and $\Gamma(*)$ is the gamma function defined by $\Gamma(q) = \int_0^\infty e^{-t} t^{q-1} dt$, where the real part of q is positive.

2. Preliminaries

This part provides some essential preliminaries that are necessary to address the important results.

Lemma 1 ([39]). *The matrix A is known as M–matrix if it satisfies the following conditions:*

(i) *All eigenvalues of A are non-negative.*
(ii) *All off-diagonal elements of A are non-positive.*

Definition 1 ([11,13]). *The signed network $\mathcal{G}^{SN}$ is structurally balanced if the bipartition of the vertices v_1, v_2, $v_1 \cup v_2 = V, v_1 \cap v_2 = \emptyset$, V is the set of nodes of $\mathcal{G}^{SN}$, the adjacency matrix a_{pj} is positive if for all $v_p, v_j \in v_s, s \in \{1,2\}$, and a_{pj} is negative if for all $v_p \in v_s, v_j \in v_t$, $s \neq t$, where $s, t \in \{1,2\}$. Otherwise it is said to be structurally unbalanced.*

Definition 2 ([13]). *If a memristor-based FCDNN of $\mathcal{G}^{SN}$ is structurally balanced with gauge transformation then, $Wa^s_{pj}W = a^u_{pj} = |a^s_{pj}|$, where $W = diag(w_1, w_2 ... w_N)$, $w_p = \{-1,1\}$, and a^s_{pj}, a^u_{pj} represent the adjacency matrices of signed and unsigned graphs.*

Lemma 2 ([11,12]). *If $D = diag(d_1 ... d_N), d_p \in \{-1,1\}, p \in \{1,2,...N\}$ is pinning feedback gains, the Laplacian $L^s = D - A^s$ and $L^u = D - A^u$ of the signed network $\mathcal{G}^{SN}$ such that $l^u_{pj} = Wl^s_{pj}W = -|a^s_{pj}|$ for $p \neq j$ and $l^u_{pp} = \sum_{k=1,k\neq p}^{N} |a^s_{pj}|$, where A^s and A^u are adjacency matrices of signed and unsigned graphs, respectively, then the H-matrix is defined as*

$$H = (h_{pj}) = L^u + D. \tag{1}$$

Lemma 3 ([40]). *(Jensen's inequality) Let $\varrho(t) \in \mathbb{R}^n$ be an integral function, $q \in (0,1]$, R be positive definite $n \times n$ matrix, then the fractional-order integral inequality is defined as follows:*

$${}_{t_0}I_t^q(\varrho^T(t)R\varrho(t)) \geq \frac{\Gamma(q+1)}{(t-t_0)^q}({}_{t_0}I_t^q\varrho(t))^T R({}_{t_0}I_t^q\varrho(t)). \tag{2}$$

Lemma 4 ([41]). *If the following LMI $\begin{pmatrix} Q & T \\ T^T & Q_1 \end{pmatrix} > 0$ holds, then either one of the following conditions will exist,*

(i) $Q > 0$, $Q_1 - T^T Q^{-1} T > 0$,
(ii) $Q_1 > 0$, $Q - TQ_1^{-1}T^T > 0$.

Definition 3 ([1]). *The fractional-order integration of the function $f(t)$ is defined as follows:*

$${}_{t_0}I_t^q f(t) = \frac{1}{\Gamma(q)} \int_{t_0}^{t} \frac{f(\tau)d\tau}{(t-\tau)^{1-q}}, \tag{3}$$

where $q \in (0,1)$.

Definition 4 ([1]). *Caputo's fractional-order derivative of $f(t)$ is defined as*

$${}_{t_0}^{C}D_t^q f(t) = \frac{1}{\Gamma(n-q)} \int_{t_0}^{t} \frac{f^{(n)}(s)}{(t-s)^{q-n+1}} ds,$$

where $q > 0$, $n \in Z^+$ satisfying $(n-1) < q < n$. Particularly, if $0 < q < 1$, then

$${}_{t_0}^{C}D_t^q f(t) = \frac{1}{\Gamma(1-q)} \int_{t_0}^{t} \frac{f'(s)}{(t-s)^q} ds. \tag{4}$$

3. Problem Formulation

Consider the memristor-based FCDNN of signed network $\mathcal{G}^{SN}$ expressed by the following conditions:

$$\begin{aligned} {}_{t_0}^{C}D_t^q(x_p(t)) &= -Cx_p(t) + A(x_p(t))f(x_p(t)) + B(x_p(t))g(x_p(t-\tau(t))) \\ &\quad -\sigma\sum_{j=1}^{N}|a_{pj}^s|(x_p(t) - sign(a_{pj}^s)x_j(t)) + u_p, \end{aligned} \tag{5}$$

where $x_p(t), (t \geq 0)$ denote the state variable of p^{th} neuron (capacitor's voltage), $p = \{1, 2, ...N\}$, $C = diag(c_1, c_2, ...c_n)$ is a diagonal matrix of the neurons, $A(x_p(t)) = [a_{gj}(x_{pj}(t))]_{n\times n}$ and $B(x_p(t)) = [b_{gj}(x_{pj}(t))]_{n\times n}$ represent the connective weighted memristor matrices, $f(x_p(t)) = (f_1(x_{p1}(t)), ...f_n(x_{pn}(t)))^T$ and $g(x_p(t-\tau(t))) = (g_1(x_{p1}(t-\tau(t))), ...g_n(x_{pn}(t-\tau(t))))^T$ are bounded feedback functions without and with delay, respectively, the coupling strength of the node is denoted as σ, $\tau(t)$ is called bounded and differentiable node delay $0 \leq \tau(t) < \bar{\tau}, 0 \leq {}_{t_0}^{C}D_t^q\tau(t) \leq \mu < 1$. Using the concept of a signed network, some justification for the coupling terms is needed to be given. A coupling adjacency matrix is symmetric if $a_{pj} = a_{jp}$. However, the coupling term is not necessarily symmetric. The signed network $\mathcal{G}^{SN}$ (5) is strongly connected if $a_{pj} > 0$ is positive. Further, it has a direct link between j to p. Then a coupling term is $a_{pj}^s(x_p(t) - x_j(t))$; otherwise -$a_{pj}^s(x_p(t) + x_j(t))$. The initial condition of $\mathcal{G}^{SN}$ is represented as $x_p(t) = \phi_p(t), t \in [-r, 0]$, where $r = \sup_{t\geq 0}\tau(t)$. $\phi_p(t)$ belongs to the bounded feedback functions on $[-r, 0]$. The network of leader node (5) is defined as

$${}_{t_0}^{C}D_t^q(s_p(t)) = -Cs_p(t) + A(s_p(t))f(s_p(t)) + B(s_p(t))g(s_p(t-\tau(t))), \tag{6}$$

where $a_{gj}(s_{pj}(t))$ and $b_{gj}(s_{pj}(t))$ represent the connective weights of memristor matrices.

Remark 1. *In memristor-based FCDNN (6), the memristor connective weights of $a_{gj}(s_{pj}(t))$, $b_{gj}(s_{pj}(t))$ are discontinuous, and the solution for differential Equation (6) cannot be found directly. Filippov developed a solution of the discontinuous right-hand side of an integer-order differential equation. Based on the definition, the discontinuous right-hand side of a differential equation has the same values as a certain inclusion.*

Definition 5 ([17])**.** *The set-valued map $F : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ is defined as*

$$F(t, x) = \cap_{\delta>0} \cap_{\mu(M)=0} \bar{co}[f(t, \mathbb{B}(x, \delta))/M],$$

where $\mathbb{B}(x, \delta)$ is a ball with x as the center and radius δ, $\bar{co}$ is the convex closure, and $\mu(M)$ denotes the Lebesgue measure. $x(t)$, a vector valued function defined on $I \subset \mathbb{R}$, forms a solution of the Filippov system if it is absolutely continuous on $[t_1, t_2]$ of I and also satisfies the differential inclusion $\frac{dx}{dt} \in F(t, x)$, $t \in I$. Let $A(x_p(t))$ and $B(x_p(t))$ be memristor connection weights of a signed network, which are defined as

$$a_{gj}(x_{pj}(t)) = \begin{cases} \hat{a}_{gj}, |x_{pj}(t)| > T_j, \\ \check{a}_{gj}, |x_{pj}(t)| < T_j, \end{cases} \quad b_{gj}(x_{pj}(t)) = \begin{cases} \hat{b}_{gj}, |x_{pj}(t)| > T_j, \\ \check{b}_{gj}, |x_{pj}(t)| < T_j, \end{cases}$$

$A(s_p(t))$ and $B(s_p(t))$ are memristor connection weights of the leader node of the network, which are defined as

$$a_{gj}(s_{pj}(t)) = \begin{cases} \hat{a}_{gj}, |s_{pj}(t)| > T_j, \\ \check{a}_{gj}, |s_{pj}(t)| < T_j, \end{cases} \quad b_{gj}(s_{pj}(t)) = \begin{cases} \hat{b}_{gj}, |s_{pj}(t)| > T_j, \\ \check{b}_{gj}, |s_{pj}(t)| < T_j, \end{cases}$$

$A(\pm T_j) = \hat{a}_{gj}$ (or) $\check{a}_{gj}$, $B(\pm T_j) = \hat{b}_{gj}$ (or) $\check{b}_{gj}$, where switching jumps $T_j > 0$ and weights $\hat{a}_{gj}$, $\check{a}_{gj}$, $\hat{b}_{gj}$, $\check{b}_{gj}$, for $g, j = \{1, 2, ...n\}$, are constant.

Assumption 1. (A_1) *The neuron activation functions f_j, and g_j are odd and bounded, $f_j(\pm T_j) = 0$. Let F and G be Lipschitz constants and the neuron activation functions satisfy the Lipschitz condition as follows:*

$$\begin{aligned} |f_j(x) - f_j(y)| &\leq F_j|x-y|, \\ |g_j(x) - g_j(y)| &\leq G_j|x-y|, \end{aligned} \tag{7}$$

where $F_j > 0, G_j > 0$, $j = \{1, 2, ...n\}$, $x, y \in \mathbb{R}$.

Lemma 5 ([27])**.** *Under the assumption $f(\pm T_p) = g(\pm T_p) = 0$, then the following holds:*

$$|co[a_{gj}x_{pj}(t)]f(x_p(t)) - co[a_{gj}s_{pj}(t)]f(s_p(t))| \leq \mathcal{A}F_p|x_p(t) - s_p(t)|,$$
$$|co[b_{gj}x_{pj}(t)]g(x_p(t)) - co[b_{gj}s_{pj}(t)]g(s_p(t))| \leq \mathcal{B}G_p|x_p(t) - s_p(t)|,$$

where $F_p > 0, G_p > 0$, $p = \{1, 2, ...N\}$, $\mathcal{A} = max\{|\hat{a}_{gj}|, |\check{a}_{gj}|\}, \mathcal{B} = max\{|\hat{b}_{gj}|, |\check{b}_{gj}|\}$.

Remark 2. *Let us consider the memristor of FCDNN of leader node (6). The set valued map is defined as follows:*

$$co[(a_{gj}(s_{pj}(t)))] = \begin{cases} \hat{a}_{gj}, |s_{pj}(t)| > T_j, \\ \bar{co}\{\hat{a}_{gj}, \check{a}_{pj}\}, |s_{pj}(t)| = T_j, \\ \check{a}_{gj}|s_{pj}(t)| < T_j. \end{cases}$$

If the network of leader node $s_p(t)$ is absolutely continuous on $[0, T)$, then the Filippov solution of (6) on $[0, T)$ is defined as

$${}_{t_0}^{C}D_t^q(s_p(t)) \in -Cs_p(t) + co[a_{gj}(s_{pj}(t))]f(s_p(t)) + co[b_{gj}(s_{pj}(t))]g(s_p(t - \tau(t))),$$

for $t \geq 0$, where $0 < q < 1$, or there exist $\mathcal{A} \in co[a_{gj}(s_{pj}(t))]$, $\mathcal{B} \in co[b_{gj}(s_{pj}(t))]$ such that

$${}_{t_0}^{C}D_t^q(s_p(t)) = -Cs_p(t) + \mathcal{A}f(s_p(t)) + \mathcal{B}g(s_p(t - \tau(t))). \tag{8}$$

Applying the theories of the set valued map and fractional-order differential inclusion in (6), the Filippov sense of fractional order differential inclusion is written as (8). Additionally, the Filippov sense in the memristor-based FCDNN (5) of the signed network is written as

$${}_{t_0}^{C}D_t^q(x_p(t)) = -Cx_p(t) + \mathcal{A}f(x_p(t)) + \mathcal{B}g(x_p(t - \tau(t))) - \sigma\sum_{j=1}^{N} l_{pj}^u x_j(t) + u_p, \tag{9}$$

where $\mathcal{A} \in co[a_{gj}(x_{pj}(t))]$, $\mathcal{B} \in co[b_{gj}(x_{pj}(t))]$.

Definition 6 ([11,12])**.** *Memristor-based FCDNN is said to be a bipartite leader synchronization if $\lim_{t\to\infty}(x_p(t) - w_p s_p(t)) = 0$ and is said to be bipartite leaderless synchronization if $\lim_{t\to\infty}(x_p(t) - w_p x_j(t)) = 0$.*

Now, we are defining the pinning controller $u_p(t) = -\sigma d_p(x_p(t) - w_p s_p(t))$, where d_p is known as the pinning feedback gain. If the vertices are pinned, then d_p is positive, otherwise d_p is zero. If $w_p = 1$, $p \in v_1$ and if $w_p = -1$, $p \in v_2$. Further, if we consider $w_p = I_N$, the bipartite synchronization changes to the standard leader-following synchronization.

Remark 3. *If the network of leader node is rooted, then it contains a spanning tree. Hence the network of leader node (8) has a directed path to each node of the signed network (9). Moreover, if the Laplacian matrix of a leaderless node has all its eigenvalues to be positive, except at least one zero value, then the signed graph has a spanning tree [11–13,42].*

4. Bipartite Leader Synchronization of Memristor-Based FCDNN

In this section, we discuss the bipartite leader synchronization of the memristor-based FCDNN of leader node (8) and signed network (9). The error system is described as $e_p(t) = \bar{x}_p(t) - s_p(t)$, where $\bar{x}_p(t) = w_p x_p(t)$, $w_p^2 = 1$,

$$ {}_{t_0}^{C}D_t^q(e_p(t)) = -Ce_p(t) + \mathcal{A}F_p e_p(t) + \mathcal{B}G_p e_p(t-\tau(t)) - \sigma \sum_{j=1}^{N} l_{pj}^u e_j(t) - \sigma d_p e_p(t). $$

If $\sum_{k=1,k\neq q}^{N} l_{pk}^s \neq 0$ when $a_{pk}^s < 0$ holds, then the row sum of the matrix, denoted by L^s, may or may not be zero. It is different from the unsigned Laplacian matrix; by the Lemma 2, we obtain

$$ {}_{t_0}^{C}D_t^q(e_p(t)) = -Ce_p(t) + \mathcal{A}F_p e_p(t) + \mathcal{B}G_p e_p(t-\tau(t)) - \sigma \sum_{j=1}^{N} h_{pj} e_j(t). \tag{10} $$

Theorem 1. *Assume the hypothesis (A_1), and consider the positive definite matrices $P, P_1, P_2 \in \mathbb{R}^{n\times n}$, a positive real number $\mu < 1$, and a symmetric matrix Q_1, if the following LMI condition holds:*

$$ \Pi = \begin{pmatrix} \mathring{a} & P\mathcal{B}G & 0 & 0 \\ * & -(1-\mu)P_1 & 0 & 0 \\ * & * & -\frac{\bar{\tau}^q}{\Gamma(q+1)}(Q_1 P_2) & 0 \\ * & * & * & -\frac{\Gamma(q+1)}{\bar{\tau}^q}P_2 \end{pmatrix} \leq 0; \tag{11} $$

where $\mathring{a} = -PC - C^T P + P_1 + 2P\mathcal{A}F - 2\Psi P$ satisfies $0 < \Psi < \sigma \min_{1\leq p\leq N} Re(\lambda_p)$, λ_p is an eigenvalue of matrix H, and $\Psi > 0$ is a positive number, then the memristor-based FCDNN of the error system (10) is the bipartite leader, synchronized.

Proof. Take the Lyapunov–Krasovskii candidate:

$$ \begin{aligned} V(t) &= \sum_{k=1}^{3} V_k(t), \\ V_1(t) &= \sum_{p=1}^{N} \xi_p e_p^T(t) P e_p(t), \\ V_2(t) &= \sum_{p=1}^{N} \xi_p \int_{t-\tau(t)}^{t} e_p^T(s) P_1 e_p(s) ds, \\ V_3(t) &= \sum_{p=1}^{N} \xi_p \int_{-\bar{\tau}}^{0} (-\theta)^{q-1} \int_{t+\theta}^{t} ({}_{t-\bar{\tau}}^{C}D_t^q e_p(s))^T P_2 ({}_{t-\bar{\tau}}^{C}D_t^q e_p(s)) ds d\theta, \end{aligned} $$

Now, using the Caputo fractional-order derivative,

$$
\begin{aligned}
{}_{t_0}^{C}D_t^q V_1(t) &\leq 2\sum_{p=1}^{N}\xi_p e_p^T(t)P\{-Ce_p(t)+\mathcal{A}F_p e_p(t)+\mathcal{B}G_p e_p(t-\tau(t))-\sigma\sum_{j=1}^{N}h_{pj}e_j(t)\} \\
{}_{t_0}^{C}D_t^q V_2(t) &= \sum_{p=1}^{N}\xi_p\{e_p^T(t)P_1 e_p(t)-(1-{}_{t_0}^{C}D_t^q\tau(t))e_p^T(t-\tau(t))P_1 e_p(t-\tau(t))\} \\
{}_{t_0}^{C}D_t^q V_3(t) &\leq \sum_{p=1}^{N}\xi_p\{\frac{\bar{\tau}^q}{\Gamma(q+1)}({}_{t-\bar{\tau}}^{C}D_t^q e_p(t))^T P_2({}_{t-\bar{\tau}}^{C}D_t^q e_p(t)) \\
&\quad -\frac{1}{\Gamma(q)}\int_{\bar{\tau}}^{0}(-\theta)^{q-1}({}_{t+\theta-\bar{\tau}}^{C}D_{t+\theta}^q e_p(t+\theta))^T P_2({}_{t+\theta-\bar{\tau}}^{C}D_{t+\theta}^q e_p(t+\theta)), \\
&= \sum_{p=1}^{N}\xi_p\{\frac{\bar{\tau}^q}{\Gamma(q+1)}({}_{t-\bar{\tau}}^{C}D_t^q e_p(t))^T P_2({}_{t-\bar{\tau}}^{C}D_t^q e_p(t))\} \\
&\quad -\int_{t-\bar{\tau}}^{t}\frac{1}{(t-\theta)^{1-q}\Gamma(q)}({}_{\theta-\bar{\tau}}^{C}D_\theta^q e_p(\theta))^T P_2({}_{\theta-\bar{\tau}}^{C}D_\theta^q e_p(\theta))d\theta.
\end{aligned}
$$

Now, consider

$$
\begin{aligned}
&\int_{t-\bar{\tau}}^{t}\frac{1}{(t-\theta)^{1-q}\Gamma(q)}({}_{\theta-\bar{\tau}}^{C}D_\theta^q e_p(\theta))^T P_2({}_{\theta-\bar{\tau}}^{C}D_\theta^q e_p(\theta))d\theta \\
&\quad \geq \frac{\Gamma(q+1)}{\bar{\tau}^q}(\frac{1}{\Gamma(q)}\int_{t-\bar{\tau}}^{t}(t-\theta)^{q-1}({}_{\theta-\bar{\tau}}^{C}D_\theta^q e_p(\theta)d\theta))^T P_2 \\
&\qquad \times(\frac{1}{\Gamma(q)}\int_{t-\bar{\tau}}^{t}(t-\theta)^{q-1}({}_{\theta-\bar{\tau}}^{C}D_\theta^q e_p(\theta)d\theta)) \\
&\quad = \frac{\Gamma(q+1)}{\bar{\tau}^q}(\frac{1}{\Gamma(q)}\int_{t-\bar{\tau}}^{t}(t-\theta)^{q-1}({}_{\theta-\bar{\tau}}^{C}D_\theta^q e_p(\theta)-y_p(\theta)d\theta)^T P_2 \\
&\qquad \times(\frac{1}{\Gamma(q)}\int_{t-\bar{\tau}}^{t}(t-\theta)^{q-1}({}_{\theta-\bar{\tau}}^{C}D_\theta^q e_p(\theta)-y_p(\theta))d\theta) \\
&\quad = \frac{\Gamma(q+1)}{\bar{\tau}^q}(e_p(t)-e_p(t-\bar{\tau})-{}_{t-\bar{\tau}}I_t^q y_p(t))^T P_2 \\
&\qquad \times(e_p(t)-e_p(t-\bar{\tau})-{}_{t-\bar{\tau}}I_t^q y_p(t)),
\end{aligned}
$$

where $y_p(\theta)=\frac{1}{\Gamma(1-q)}\int_{t-\bar{\tau}}^{\theta-\bar{\tau}}(u-\theta)^{-q}\dot{e}_p(u)du$, $e(t)=(e_1(t),e_2(t),...e_N(t))^T$, $e(t-\tau(t))=(e_1(t-\tau(t)),e_2(t-\tau(t)),...e_N(t-\tau(t)))^T$.

$$
\begin{aligned}
{}^{C}_{t_0}D^q_t V(t) \leq\ & 2\sum_{p=1}^{N}\xi_p e_p^T(t)P\{-Ce_p(t)+\mathcal{A}F_p e_p(t)+\mathcal{B}G_p e_p(t-\tau(t))-\sigma\sum_{j=1}^{N}h_{pj}e_j(t)\} \\
+\ & \sum_{p=1}^{N}\xi_p\{e_p^T(t)P_1 e_p(t)-(1-{}^{C}_{t_0}D^q_t\tau(t))e_p^T(t-\tau(t))P_1 e_p(t-\tau(t))\} \\
+\ & \sum_{p=1}^{n}\xi_p\{\frac{\bar{\tau}^q}{\Gamma(q+1)}({}^{C}_{t-\bar{\tau}}D^q_t e_p(t))^T P_2({}^{C}_{t-\bar{\tau}}D^q_t e_p(t)) \\
-\ & \frac{\Gamma(q+1)}{\bar{\tau}^q}(e_p(t)-e_p(t-\bar{\tau})-{}_{t-\bar{\tau}}I^q_t y_p(t))^T P_2(e_p(t)-e_p(t-\bar{\tau})-{}_{t-\bar{\tau}}I^q_t y_p(t)) \\
\leq\ & -2e^T(t)(\Xi\otimes PC)e(t)+2e^T(t)(\Xi\otimes P\mathcal{A}F)e(t)+2e^T(t)(\Xi\otimes P\mathcal{B}G)e(t-\tau(t)) \\
-\ & 2e^T(t)[\sigma(\Xi H)\otimes P]e(t)+2\Psi e^T(t)(\Xi\otimes P)e(t)-2\Psi e^T(t)(\Xi\otimes P)e(t) \\
+\ & e^T(t)(\Xi\otimes P_1)e(t)-e^T(t-\tau(t))(1-\mu)(\Xi\otimes P_1)e(t-\tau(t)) \\
-\ & \frac{\bar{\tau}^q}{\Gamma(q+1)}\Omega_1^T(t)(\Xi\otimes Q_1P_2)\Omega_1(t)-\frac{\Gamma(q+1)}{\bar{\tau}^q}\Omega_2^T(t)(\Xi\otimes P_2)\Omega_2(t) \\
\leq\ & z_p^T(t)\Pi z_p(t)-2e^T(t)([\Xi(\sigma H-\Psi I_N)]\otimes P)e(t)-\frac{\bar{\tau}^q}{\Gamma(q+1)}\Omega_1^T(t)(\Xi\otimes(T^TQ^{-1}T)P_2)\Omega_1(t), \qquad (12)
\end{aligned}
$$

here $z_p(t) = \text{col}\ (e_p(t), e_p(t-\tau(t)), \Omega_1(t), \Omega_2(t)), y(\theta) = (y_1(\theta), y_2(\theta), ...y_N(\theta))^T$, $\Omega_1(t) = {}^{C}_{t-\bar{\tau}}D^q_t e_p(t)$, $\Omega_2(t) = e_p(t) - e_p(t-\bar{\tau}) - {}_{t-\bar{\tau}}I^q_t y_p(t)$, by Lemmas 2 and 4, H matrix is a non-singular M-matrix, also $\sigma H - \Psi I_N$ is a non-singular M-matrix such that $[\Xi(\sigma H - \Psi I_N)] > 0$, where $\Xi = diag(\xi_1, \xi_2, ...\xi_N)$. Hence, by using the LMI (11), the error system is globally asymptotically stable. Therefore, the desired result is achieved. □

5. Bipartite Leaderless Synchronization of Memristor Based FCDNN

In this section, we discuss the bipartite leaderless synchronization (self-synchronization) of the memristor-based FCDNN of signed networks (9). Here, the external force is not required for leaderless consensus. Hence, the pinning value is zero. Let $\bar{x}_p(t) = w_p x_p(t)$, which also gives $x_p = w_p\bar{x}_p(t)$. Then, from (9), it follows that

$$
\begin{aligned}
{}^{C}_{t_0}D^q_t(\bar{x}_p(t)) &= -C\bar{x}_p(t)+\mathcal{A}w_p f(w_p\bar{x}_j(t))+\mathcal{B}w_p g(w_p\bar{x}_p(t-\tau(t)))-\sigma\sum_{j=1}^{N}l^u_{pj}\bar{x}_j(t), \\
{}^{C}_{t_0}D^q_t(\bar{x}_p(t)) &= -C\bar{x}_p(t)+\mathcal{A}f(\bar{x}_p(t))+\mathcal{B}g(\bar{x}_p(t-\tau(t)))-\sigma\sum_{j=1}^{N}l^u_{pj}\bar{x}_j(t). \qquad (13)
\end{aligned}
$$

Here, $e_p(t) = \bar{x}_p(t) - \bar{x}_r(t)$, $L^u_{pj} = l^u_{pj} - l^u_{rj}$. Recall that $\sum_{j=1}^{N} l^u_{pj} = 0$.

$$
{}^{C}_{t_0}D^q_t(e_p(t)) = -Ce_p(t)+\mathcal{A}F_p e_p(t)+\mathcal{B}G_p e_p(t-\tau(t))-\sigma\sum_{j=1}^{N}L^u_{pj}e_j(t). \qquad (14)
$$

Theorem 2. *Under Assumption (A_1), if $\mu < 1$ is a positive real number, $P, P_1, P_2 \in \mathbb{R}^{n\times n} > 0$ are positive definite matrices, Q_1 is a symmetric matrix, and the following LMI condition holds:*

$$
\Pi_1 = \begin{pmatrix} \mathring{a}_1 & P\mathcal{B}G & 0 & 0 \\ * & -(1-\mu)P_1 & 0 & 0 \\ * & * & -\frac{\bar{\tau}^q}{\Gamma(q+1)}(Q_1P_2) & 0 \\ * & * & * & -\frac{\Gamma(q+1)}{\bar{\tau}^q}P_2 \end{pmatrix} \leq 0; \qquad (15)
$$

where $\mathring{a}_1 = -PC - C^TP + P_1 + 2P\mathcal{A}F$, then the memristor-based FCDNN of error system (14) is bipartite leaderless, synchronized.

The proof is similar to Theorem 1.

6. Numerical Examples

For the sake of confirmation and validation, two numerical examples are discussed in this section.

Example 1. *Consider the leader node of the network* (8) *with the memristor values as*

$$\mathcal{A} = \begin{pmatrix} -0.40 & -0.32 \\ -1.30 & 0.70 \end{pmatrix}, \mathcal{B} = \begin{pmatrix} -0.82 & -0.50 \\ -1.35 & 0.64 \end{pmatrix} \text{ and } C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix};$$

and $f(x_p(t)) = g(x_p(t-\tau(t))) = (tanh(x_{p1}(t)), tanh(x_{p2}(t)))^T$, $p = \{1,2,3...8\}$. *The arbitrary trajectories of the leader node of network* (8) *are shown in Figure 1. Consider the memristor of FCDNN with the following parameters. The memristor connective weights are taken from [27] as*

$$a_{11}(x_1) = \begin{cases} 2, |x_1(t)| < 1, \\ -2, |x_1(t)| > 1, \end{cases} \quad a_{12}(x_1) = \begin{cases} -1, |x_1(t)| < 1, \\ 1, |x_1(t)| > 1, \end{cases} \quad a_{13}(x_1) = \begin{cases} 0.5, |x_1(t)| < 1, \\ -0.5, |x_1(t)| > 1, \end{cases}$$

$$a_{21}(x_2) = \begin{cases} 1.5, |x_2(t)| < 1, \\ -1.5, |x_2(t)| > 1, \end{cases} \quad a_{22}(x_2) = \begin{cases} 1, |x_2(t)| < 1, \\ -1, |x_2(t)| > 1, \end{cases} \quad a_{23}(x_2) = \begin{cases} 2, |x_2(t)| < 1, \\ -2, |x_2(t)| > 1, \end{cases}$$

$$a_{31}(x_3) = \begin{cases} 1.5, |x_3(t)| < 1, \\ -1.5, |x_3(t)| > 1, \end{cases} \quad a_{32}(x_3) = \begin{cases} 2, |x_3(t)| < 1, \\ -2, |x_3(t)| > 1, \end{cases} \quad a_{33}(x_3) = \begin{cases} 1, |x_3(t)| < 1, \\ -1, |x_3(t)| > 1, \end{cases}$$

$$b_{11}(x_1) = \begin{cases} 1, |x_1(t)| < 1, \\ -1, |x_1(t)| > 1, \end{cases} \quad b_{12}(x_1) = \begin{cases} 0.5, |x_1(t)| < 1, \\ -0.5, |x_1(t)| > 1, \end{cases} \quad b_{13}(x_1) = \begin{cases} 1.5, |x_1(t)| < 1, \\ -1.5, |x_1(t)| > 1, \end{cases}$$

$$b_{21}(x_2) = \begin{cases} 2, |x_2(t)| < 1, \\ -2, |x_2(t)| > 1, \end{cases} \quad b_{22}(x_2) = \begin{cases} 1.5, |x_2(t)| < 1, \\ -1.5, |x_2(t)| > 1, \end{cases} \quad b_{23}(x_2) = \begin{cases} 1, |x_2(t)| < 1, \\ -1, |x_2(t)| > 1, \end{cases}$$

$$b_{31}(x_3) = \begin{cases} 2, |x_3(t)| < 1, \\ -2, |x_3(t)| > 1, \end{cases} \quad b_{32}(x_3) = \begin{cases} 3, |x_3(t)| < 1, \\ -3, |x_3(t)| > 1, \end{cases} \quad b_{33}(x_3) = \begin{cases} 1.5, |x_3(t)| < 1, \\ -1.5, |x_3(t)| > 1. \end{cases}$$

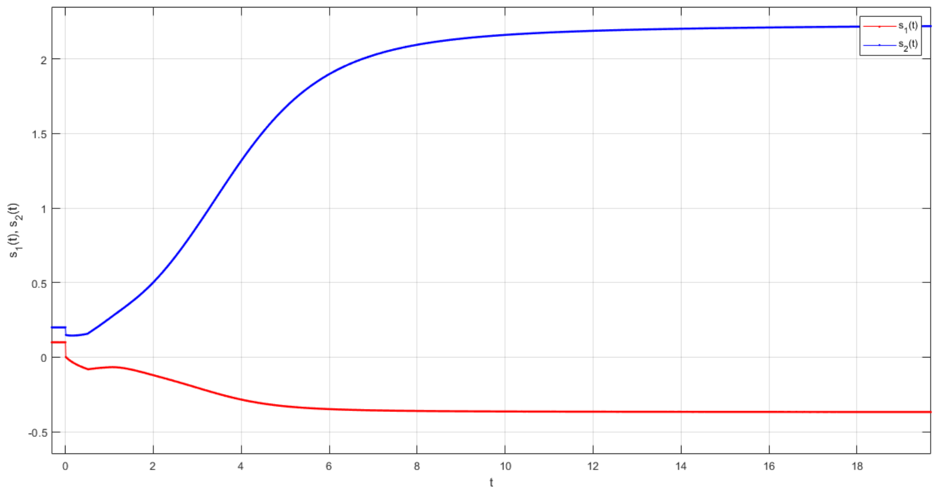

Figure 1. Orbit trajectory of leader node.

Further, consider the coupling strength $\sigma = 30$ *and* $q = 0.9$, *by the bipartite consensus* $v_1 = \{1,2,3,5\}$, $v_2 = \{4,6,7,8\}$, $w = \{1,1,1,-1,1,-1,-1,-1\}$, $d_4 = 30$, *with the activation function* $f(x_p(t)) = g(x_p(t-\tau(t))) = tanh(x_{pj}(t))$ *which satisfies* $F = G = 1$. *Hence from Lemma 2, we obtain* $\lambda_p = 0.5134$. *Let* $\tau(t) = log(1+e^t)$, *then* ${}_{t_0}^{C}D_t^q\tau(t) = \lim_{w\to 0} \frac{1}{w^q}\sum_{r=0}^{\infty}(-1)^r \frac{\Gamma(q+1)log(1+e^{t-rw})}{\Gamma(r+1)\Gamma(1-r+q)} \leq \mu = 0.5 < 1$. *Hence all the hypotheses of Theorem 1 are satisfied and with the support of the MATLAB LMI toolbox, Inequality* (11) *is validated with feasible positive definite matrices of the form,* $P = \begin{pmatrix} 0.1386 & -0.0033 & 0.0048 \\ -0.0033 & 0.1186 & 0.0081 \\ 0.0048 & 0.0081 & 0.1105 \end{pmatrix}$;

$P_1 = \begin{pmatrix} 1.3577 & -0.0550 & -0.0668 \\ -0.0550 & 1.2878 & -0.1290 \\ -0.0668 & -0.1290 & 1.2170 \end{pmatrix}$; $P_2 = \begin{pmatrix} 1.1279 & 0 & 0 \\ 0 & 1.1279 & 0 \\ 0 & 0 & 1.1279 \end{pmatrix}$. *Figure 2 represents directed signed graph with eight vertices, and Figure 3 shows the bipartite leader synchronization of equation (10). Further, using inequality (11), we obtain* $0 \leq \bar{\tau} \leq 1.1057$.

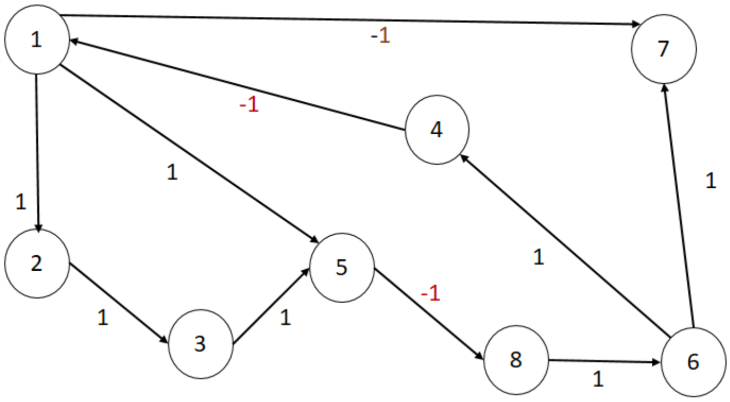

Figure 2. Structurally balanced directed signed graph of eight vertices.

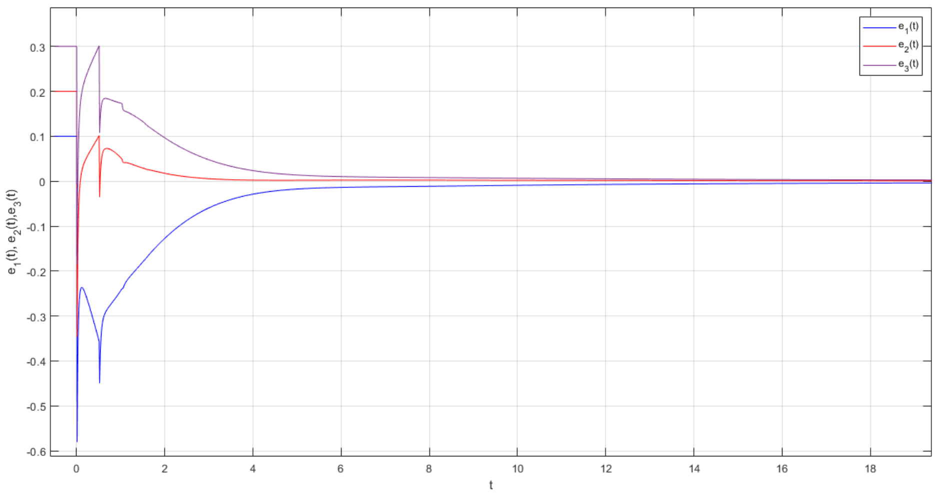

Figure 3. Bipartite leader synchronization of signed graph with eight vertices.

Example 2. *Consider equation (14) with* $C = \begin{pmatrix} 10 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 20 \end{pmatrix}$, *and the memristor values, activation functions and* μ *are taken as same as in example* 1. *By the bipartite consensus* $v_1 = \{1,2,3,5\}$, $v_2 = \{4,6,7,8\}$, $w = \{1,1,1,-1,1,-1,-1,-1\}$, *Hence all the hypotheses of Theorem* 2 *are satisfied, and with the support of MATLAB LMI toolbox, inequality (15) is validated with feasible positive definite matrices as follows:*

$$P = \begin{pmatrix} 0.1568 & 0.0076 & 0.0146 \\ 0.0076 & 0.1135 & 0.0062 \\ 0.0146 & 0.0062 & 0.0992 \end{pmatrix}; P_1 = \begin{pmatrix} 1.2781 & 0.0566 & -0.0490 \\ 0.0566 & 1.0928 & -0.1944 \\ -0.0490 & -0.1944 & 0.8368 \end{pmatrix};$$

$P_2 = \begin{pmatrix} 0.8435 & 0 & 0 \\ 0 & 0.8435 & 0 \\ 0 & 0 & 0.8435 \end{pmatrix}$. *Error system (14) achieves the bipartite leaderless synchronization as shown in Figure 4. Further using inequality (15), we obtain* $0 \leq \bar{\tau} \leq 0.7724$.

Remark 4. *The memristor-based FCDNN of signed network equation (5) shows that signed graph of eight vertices is structurally balanced. If the signed network has at least one negative cycle, it* *is structurally unbalanced. For Equation* (10)*, the matrix values are the same as in Example* 1 *with* $\lambda_p = 0.6048$*. Hence by Theorem* 1*, we derive the structurally unbalanced bipartite leader synchronization. Figures* 5 *and* 6 *show the structurally unbalanced directed signed graph with eight vertices and its bipartite leader synchronization, respectively.*

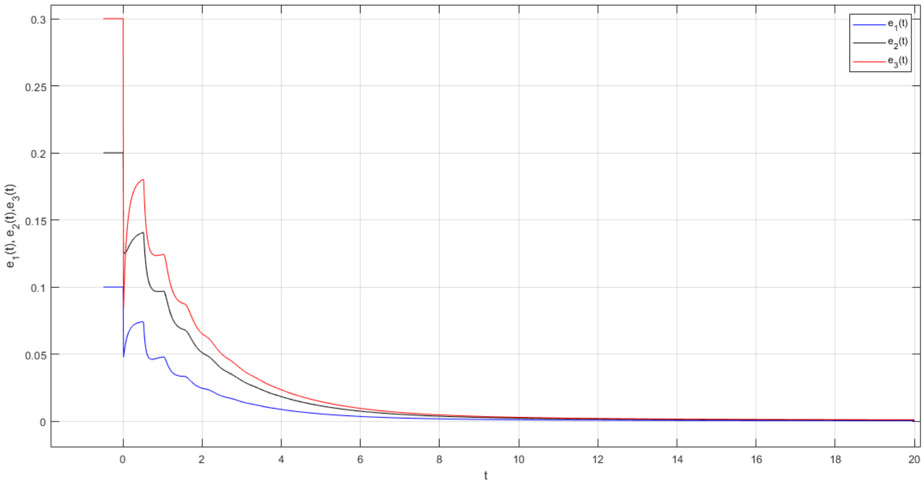

Figure 4. Bipartite leaderless synchronization of signed graph with eight vertices.

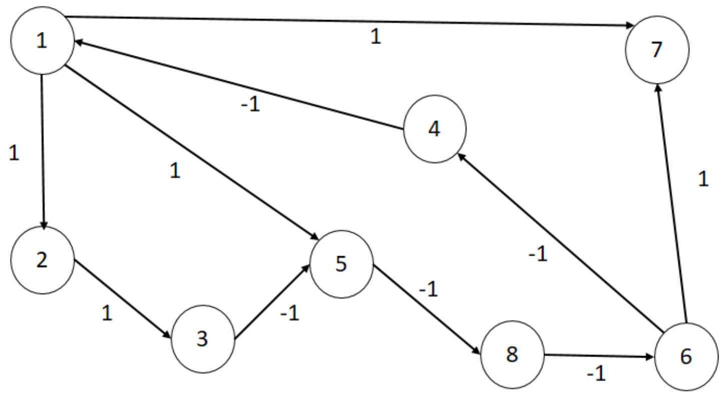

Figure 5. Structurally unbalanced directed signed graph.

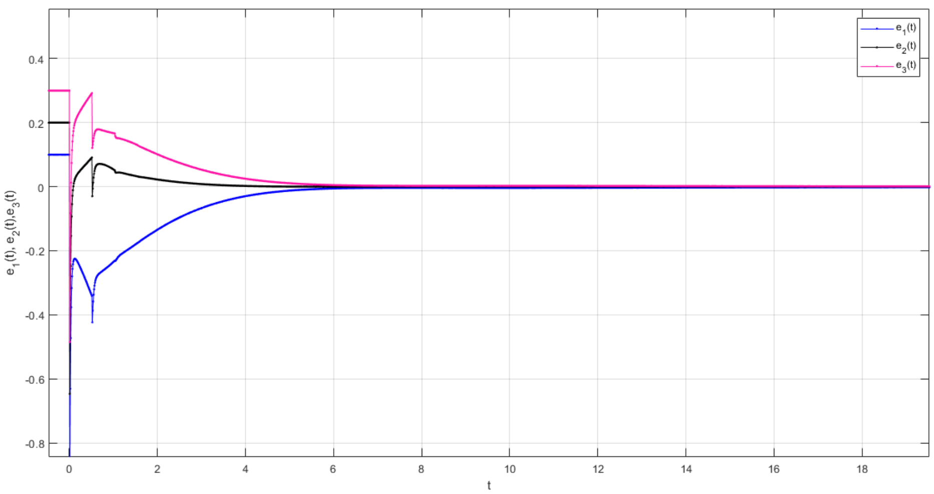

Figure 6. Structurally unbalanced bipartite leader synchronization.

7. Conclusions

In this paper, we studied the memristor-based FCDNN under pinning control. The main results are proved under less restrictive conditions on the bipartite leader and leaderless synchronization of the signed network through structurally balanced and unbalanced concepts. In future, the FCDNN of Equation (10) with non-differentiable delay could be discussed within the fractional settings.

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