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Editorial

Preface to the Special Issue on "Fuzzy Natural Logic in IFSA-EUSFLAT 2021"

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In [1], we gave reasons to focus on one of the possible future directions of fuzzy logic towards the concept of fuzzy natural logic (FNL). The concept of FNL continues the program of fuzzy logic in a broader sense (FLb-logic) introduced in [2]. This theory was established as a formal logic aiming at modeling natural human reasoning, which necessarily proceeds in natural language. The goal is to make FNL a mathematical theory being extension of the mathematical fuzzy logic. Its paradigm extends the classical concept of natural logic suggested by Lakoff in [3]. According to him, natural logic is a collection of terms and rules that come with natural language and that allows us to reason and argue in it. Its goals can be characterized as follows:

- To express all concepts capable of being expressed in natural language,
- To characterize all the valid inferences that can be made in natural language,
- To mesh with adequate linguistic descriptions of all natural languages.

It is essential to employ meaning-postulates that do not vary from language to language. In other words, all natural languages reflect ability of human mind to reason that is common to all of us, and thus, its principles are independent on the use of the concrete natural language. The concept of natural logic has been further developed by several other authors (cf. [4,5] and others).

We argue that fuzzy set theory has the potential to be a good tool for modeling of linguistic semantics because it provides a reasonable mathematical model of the vagueness phenomenon. This is important because, as argued by many authors (cf., e.g., [6]), vagueness is an unavoidable feature of natural language semantics. The role of fuzzy sets in modeling of linguistic semantics has been discussed already by L. A. Zadeh in many of his papers since the very beginning (cf., e.g., [7–9]). Interesting is his concept of precisiated natural language [10]. Its main idea is to develop a "reasonable working formalization of the semantics of natural language without pretensions to capture it in detail and fineness." The goal is to provide an acceptable and applicable technical solution, i.e., to relax some of the requirements of thorough linguistic analysis and, in line with the paradigm of Zadeh's precisiated natural language, to focus on smaller parts of natural language and try to capture only their essential properties.

Therefore, following the definition of natural logic, we can define fuzzy natural logic as a system of theories of mathematical fuzzy logic enabling us to model terms and rules that come with natural language together with their inherent vagueness and allowing us to reason and argue using tools developed in it. A necessary constituent of FNL is a mathematical model of semantics of a specific part of natural language independent of a concrete language.

The following are the main sources for the development of FNL:

- Results of classical linguistics.
- Logical analysis of concepts and semantics of natural language Transparent Intensional Logic (P. Tichý [11], P. Materna [12]).
- Montague grammar [13].

- Mathematical fuzzy logic, especially the higher order one called *Fuzzy Type Theory* (FTT) [14].

  The current constituents of FNL are:

- Theory of evaluative linguistic expressions (small, very small, medium, large, etc.).
- Theory of fuzzy and intermediate quantifiers (most, a lot of, few, many, etc.) and generalized Aristotle's syllogisms.
- Theory of fuzzy/linguistic IF-THEN rules and logical inference (Perception-based Logical Deduction).

FNL is expected to contribute to the development of methods for construction of models of systems and processes on the basis of expert knowledge expressed in genuine natural language and to develop special algorithms making computer to "understand" natural language and suggest a corresponding behavior. Let us remark that FNL has already many interesting applications (cf. [15]).

There are five contributions in this Special Issue devoted to extended versions of the papers presented in the conference "The 19th World Congress of the International Fuzzy Systems Association and the 12th Conference of the European Society for Fuzzy Logic and Technology jointly with the AGOP, IJCRS, and FQAS conferences" that took place in Bratislava (Slovakia) from September 19 to September 24, 2021. These contributions use various parts and concepts of FNL mentioned above and apply it to a wide range of problems, theoretical as well as application-oriented.

A very important building block of FNL is the theory of evaluative linguistic expressions. In [16], this theory is developed in an exciting direction from the perspective of theoretical linguistics. The range of evaluative linguistic expressions is considerably broadened to also contain verbs ("love" in "I love you very much"), proper names ("Einstein" in "Mark is an Einstein"), etc. Essential for this extension is the Fuzzy Property Grammar—the topic of [17]. In this contribution, the Fuzzy Property Grammar permits to describe linguistic complexity of a natural language and linguistic universality (presence of a grammatical characteristics in all or most natural languages) as vague concepts. It allows, among other things, to better understand similarities and differences between natural languages.

One of the important directions of FNL development is the study of intermediate quantifiers and generalized syllogisms. In [18], the authors continue this research program by studying syllogisms whose constituents (quantified expressions) can contain negated terms, such as "most people who do not drink alcohol have healthy livers." The validity of certain forms of these syllogisms (related to the so-called graded Peterson's cube of opposition) is proved syntactically. Examples of syllogisms on finite models are also elaborated.

Paper [19] presents a more applied facet of FNL. It proposes a model for exploring and extracting knowledge of auction frauds using IF-THEN rules (a crucial component of FNL). An innovative fuzzy neural network model based on or-neurons using a t-conorm as their underlying operation is presented in detail and compared with several state-of-art neuro-fuzzy models. The proposed model shows its superiority by achieving more than 98% accuracy with fewer fuzzy rules and greater assertiveness than other models.

In [20], the authors study the so-called preimage problem in the context of F-transform: how it is possible to describe the class of all functions mapped onto the same result of direct F-transform. Note that F-transform is an important technique necessary in various kinds of applications of FNL. The relationship between objects is determined by closeness (a weaker concept than metric). The preimage problem is formulated using the language of matrix calculus. The authors show that its solutions can be given in three different ways (using a weighted arithmetic mean, any right inverse of the closeness matrix or any element of a certain affine subspace). The study of this problem contributes to better understanding of ill-posed problems frequent in machine learning.

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