mathematics

Article

An Optimized Double-Nested Anti-Missile Force Deployment Based on the Deep Kuhn–Munkres Algorithm

Wen Sun, Zeyang Cao, Gang Wang, Yafei Song * and Xiangke Guo

Air Defense and Antimissile School, Air Force Engineering University, Xi'an 710051, China
* Correspondence: yafei_song@163.com

Abstract: In view of a complex multi-factor interaction relationship and high uncertainty of a battlefield environment in the anti-missile troop deployment, this paper analyzes the relationships between the defending stronghold, weapon system, incoming target, and ballistic missile. In addition, a double nested optimization architecture is designed by combining deep learning hierarchy concept and hierarchical dimensionality reduction processing. Moreover, a deployment model based on the double nested optimization architecture is constructed with the interception arc length as an optimization goal and based on the basic deployment model, kill zone model, and cover zone model. Further, by combining the target full coverage adjustment criterion and depth-first search, a deep Kuhn–Munkres algorithm is proposed. The model is validated by simulations of typical scenes. The results verify the rationality and feasibility of the proposed model, high adaptability of the proposed algorithm. The research of this paper has important enlightenment and reference function for solving the force deployment optimization problems in uncertain battlefield environment.

Keywords: anti-missile; force deployment; double nested optimization; deep Kuhn–Munkres algorithm

MSC: 68Q25; 90C29

check for updates

Citation: Sun, W.; Cao, Z.; Wang, G.; Song, Y.; Guo, X. An Optimized Double-Nested Anti-Missile Force Deployment Based on the Deep Kuhn–Munkres Algorithm. *Mathematics* **2022**, *10*, 4627. https://doi.org/10.3390/math10234627

Academic Editors: Fuyuan Xiao and David Greiner

Received: 8 October 2022
Accepted: 1 December 2022
Published: 6 December 2022

1. Introduction

The process of anti-missile troop deployment is mainly to judge the current conditions, predict the trajectory and other related information based on the enemy's situation, and deploy an appropriate weapon system at the right location so that it can effectively deal with the high uncertainty of a battlefield environment and maximize its operational effectiveness [1,2]. Anti-missile troop deployment is crucial for system-level joint anti-missile operations [3,4], the core part of pre-war mission planning [5], and an important prerequisite for the successful interception of ballistic missiles. This is conducive to grasping the initiative of a war and influencing or even deciding the victory or defeat of the war. Therefore, the research on anti-missile troop deployment has important practical value and military significance.

At present, the existing research on troop deployment has mostly focused on air defense operations, and relatively little research has been conducted on anti-missile troop deployment. However, there is a significant difference between anti-missile operations and air defense operations [6]. Namely, anti-missile operations are characterized by fast target flight speed, strong penetration performance, short combat time, and difficulty in interception and destruction [7,8], making an air defense troop deployment plan difficult to adapt to anti-missile operations. In addition, the current research on anti-missile troop deployment lacks analysis of multiple elements and is less capable of handling variable and uncertain battlefield environments. In terms of troop deployment models, based on the relevant theories of operations-related research, previous studies proposed various model-construction methods based on different focuses. The military troop deployment models have been mainly focused on the field of air defense. Zhao and Li proposed a calculation model for the penetration probability of air defense weapons based on the queuing theory and established a fan-shaped deployment optimization model for multi-type air defense weapons [9]. Xu analyzed the matrix countermeasures under the pure strategies of both sides participating in a battle, and established an optimization model of the air defense deployment [10]. Wan et al. analyzed the deployment strategy, principle, and form of the multi-platform coordinated air defense and established a deployment model based on a hybrid deployment strategy [11]. Yan et al. proposed using a dynamic programming algorithm to obtain the route with minimum risk of attack and incorporated the obtained result into the fitness function of a genetic algorithm to obtain an optimization model of the air defense deployment based on a dynamic programming genetic algorithm [12]. However, most of the intelligent optimization algorithms introduced in the previous research are not specific for the deployment problem. The recently proposed algorithms for deployment problem solving have been mainly based on the genetic algorithm [13–17], particle swarm algorithm, improved Memetic algorithm, and Hungarian algorithm. However, there has been little systematic research on the troop deployment of anti-missile, and the obtained solutions have low universality. Liu et al. analyzed the deployment area selection from the perspective of firing favorability for mid-range anti-missiles [18]. Yu et al. analyzed the advantages and disadvantages of deployment solutions from the perspective of result evaluation for late-range anti-missiles [19]. The existing research lacks the analysis of the complex interaction of the elements, and the ability to deal with the uncertain battlefield environment is insufficient.

This paper provides a comprehensive analysis of the anti-missile troop deployment problem, focusing on the complexity of the interaction between various elements in anti-missile operations and considering the high uncertainty of battlefield environmental conditions, such as attack direction and scale. This study comprehensively considers the relationship between defended strongholds, preselected positions, weapon systems, and incoming ballistic missiles and uses the concept of deep learning hierarchy as a guide [20] to design a double nested optimization architecture. In this architecture, the first-layer optimization provides an optimal configuration of weapon systems and ballistic missiles, whereas the second-layer optimization preselects positions and weapon systems for the optimal configuration obtained by the first-layer optimization. Based on this optimization architecture, an anti-missile troop deployment optimization model based on double nesting is constructed with the interception arc length as an optimization goal, reducing problem complexity. Following the idea of optimization-problem solving in operation-related research [21,22], a deep Kuhn–Munkres (KM) algorithm is used to solve the anti-missile troop deployment problem by adopting the depth-first search algorithm according to the full-coverage adjustment criterion of the incoming target. This provides a solution with better adaptability and higher satisfaction and achieves more desirable results.

2. Problem Analysis

2.1. Anti-Missile Troop Deployment Analysis

Anti-missile warfare is a countdown operation based on preplanning [23], and the general operational time is ten to twenty minutes. The predictability of a ballistic missile flight trajectory provides the basis for pre-war deployment plan, while uncertainties, such as the size, number, and direction of a target when it arrives, increase the difficulty of generating a pre-war deployment plan [24].

The main reasons for the troop deployment before war in anti-missile operations are as follows:

1. Anti-missile force maneuver deployment is time-consuming, and the time planned for anti-missile operations is extremely short, making deployment adjustments based on a real-time battlefield situation extremely difficult;
2. Based on the available data, such as information on enemy situation and judgment, launch and drop points, and trajectory prediction, the best points are found in advance

for targeted deployment to handle possible attack situations of enemy targets, reducing the impact of uncertainty and helping to improve the interception probability;

3. The depth of the kill and cover zones and route shortcuts differ among deployment locations, and early deployment at the best point is conducive to the operational effectiveness of a weapon system;
4. The contradictory relationship between limited anti-missile troop resources and the cost-efficiency ratio of intercepting incoming missiles.

The problem of anti-missile troop deployment mainly relates to the defending strongholds, weapons systems, preselected positions, and incoming ballistic missile exhibitions. Among them, defending strongholds belong to the targets for weapons systems to defend and protect. They are fixed in place and have different levels of importance according to the geographical location and comprehensive factors, including political, economic, cultural, and military factors. Weapon systems mainly refer to different types of firepower units with various combat capabilities that intercept ballistic missiles and defend defending strongholds, involving indicators such as kill and cover zones. The preselected positions mainly denote deployment positions that can be selected for the weapon systems filtered based on the information on terrain cover, traffic, communication, and other related factors combined with possible enemy incoming attacks. Further, incoming missiles are generally ballistic missiles launched by the enemy to attack the defending strongholds, and related data include the number, type, incoming direction, and range of missiles [25]. Therefore, the anti-missile troop deployment represents a complex multi-constraint optimization problem that integrates deterministic and uncertain factors, combines qualitative analysis and quantitative calculations, and employs scientific methods to realize the optimal matching of weapon systems and preselected positions to achieve the interception of incoming missiles and stronghold defense.

2.2. Current Difficulties in Multi-Constraint Optimization Problem Solving

The deployment of anti-missile forces involves many elements, and it is necessary to fully consider the impact of uncertain battlefield environment to solve the following difficulties.

1. Complex multi-factor and multi-constraint interaction relationships

The anti-missile troop deployment involves different factors, such as defending strongholds, weapon systems, preselected positions, and incoming missiles, each of which has a number of sub-factors with complex correlations and mutual constraints [26,27]. Therefore, sorting out the relationships between the factors to achieve the best solution under multiple constraints represents a complex non-equilibrium problem.

2. Certain deployment plan to deal with the uncertain battlefield environment

Ballistic missile attacks have a high degree of uncertainty, which is mainly reflected in the uncertainty of the direction and scale of an attack. The uncertainty of the incoming direction mainly refers to the problem that a specific direction and the number of directions cannot be determined. The uncertainty of the scale mainly refers to the difficulty in determining an appropriate scale among small, medium, and large scales. The missile range and type are also uncertain. However, the deployment plan is relatively deterministic, and there is almost no time to adjust the deployment plan in a real-time battlefield environment, so the plan has to have a high degree of generalization. Therefore, the conflicting relationship between certainty and uncertainty of missile attack and deployment makes the solution complex.

2.3. Solution Ideas Analysis

1. Solution ideas

Through comprehensive analysis of each factor and their complex correlations, an abstract military deployment problem is transformed into a concrete mathematical problem, which represents a complex multi-constraint optimization matching problem [28], and an efficient and reasonable mathematical model is constructed to reduce problem complexity. Scientific methods and optimization algorithms are employed to solve the considered problem and generate a highly satisfactory solution to handle the uncertainty of the deployment problem.

2. Specific implementation

A simplified mathematical expression of the troop deployment problem can be defined as follows. Assume that m weapon systems are deployed in n preselected positions, each of which can deploy at most one weapon system to achieve the overall optimum of the interception arc length w.

Particularly, an anti-missile troop deployment plan is created according to the following steps: element analysis, architecture definition, model construction, algorithm implementation, simulation, and verification. The attributes of each element are analyzed, and the defending strongholds are set as a constant element; then, the weapon system, incoming target, and preselected position are used as changing elements to integrate the elements organically. The hierarchical concept of deep learning is adopted to design a double nested optimization architecture to simplify problem complexity. Further, the basic deployment model, kill zone model, and cover zone model are combined to construct a mathematical model based on the double-nested optimization architecture with the interception arc length as an optimization criterion. Furthermore, the optimal matching problem in operation-related research is considered [29], and the non-equilibrium problem is transformed into an equilibrium problem [30]. Finally, incorporating the target full-coverage adjustment step and combining the KM algorithm and depth-first search to develop the deep KM for solving the problem can improve solution satisfaction, enhance the ability to handle uncertainty, and realize the scientific optimization of the considered problem.

3. Proposed Model Design

3.1. Basic Deployment Model

An anti-missile troop deployment model mainly takes the overall optimal interception arc length as an optimization goal. In this study, F represents the objective function, and w_{ij} denotes the interception arc length of a weapon system on the incoming ballistic target. The interception arc length is the arc section part of the incoming target ballistic curve within the kill zone of a weapon system [31]. The longer the interception arc length is, the higher the interception success probability in anti-missile operations will be. The interceptor can encounter the target on this arc section, whose starting point is the earliest encounter point (t_{ls}), and the ending point is the latest encounter point (t_{le}). The time difference between the earliest and latest encounter points is calculated as a measure of the intercept arc length, and the calculation formula is $w = t_{le} - t_{ls}$.

When preselected positions are selected for deployment, each position can deploy at most one weapon system, and according to the 0–1 integer planning idea [32], the decision variable is denoted by x_{ij}. Namely, for any position, one is taken for a deployed weapon system, whereas zero is taken for a not-deployed system. The basic constraint is that each weapon system can be deployed in only one position, and each position can deploy at most one weapon system. The basic deployment model can be expressed by Equation (1).

$$
\begin{gathered}
F = \mathbf{max}\sum_{i=1}^{m}\sum_{j=1}^{n} w_{ij}x_{ij} \\
s.t.\begin{cases}
\sum\limits_{j=1}^{n} x_{ij} = 1, \quad i = 1,2,\ldots,m \\
\sum\limits_{i=1}^{m} x_{ij} \leq 1, \quad j = 1,2,\ldots,n \\
x_{ij} \in \{0,1\}, i = 1,2,\ldots,m \quad j = 1,2,\ldots,n \\
n \geq m
\end{cases}
\end{gathered} \tag{1}
$$

where m denotes the number of weapon systems, n denotes the number of preselected positions, i denotes a weapon system's number, and j denotes a preselected position's number, w_{ij} represents the interception arc length calculated after the weapon system i is deployed in the position j.

3.2. Kill and Cover Zone Models

1. Kill zone model

The kill zone is a spatially closed three-dimensional area where the kill probability of a weapon system does not fall below a certain value [33], and it mainly includes the high bound, low bound, far bound, and near bound, as well as high near bound and side bound [34]. A weapon system is located at the origin point O of the coordinate system. The S-axis is related to the incoming target, and it is parallel but points opposite to the horizontal projection of the incoming target velocity vector; the H-axis is perpendicular to the horizontal plane (upward is positive). Lastly, the P-axis forms a right-handed Cartesian coordinate system with the S-axes and H-axes [35]. This coordinate system can be mathematically modeled, the kill zone model can be expressed by Equations (2)–(8). The kill zone is shown in Figure 1.

$$Far\ bound\ ABCD: \ S^2 + P^2 + H^2 = D_{\max}^2, H_{\min} \le H \le H_{\max}, \frac{|P|}{S} \le \tan\gamma \tag{2}$$

$$Near\ bound\ KLMN: \ S^2 + P^2 + H^2 = D_{\min}^2, H_{\min} \le H \le \sqrt{D_{\min}\sin\theta} \tag{3}$$

$$High\ bound\ ABFE: \ H = H_{\max} \tag{4}$$

$$lower\ bound\ CDMN: \ H = H_{\min} \tag{5}$$

$$high\ near\ bound\ EFLK: \ \frac{H}{S} = \tan\theta \tag{6}$$

$$side\ bound\ one\ BCNLF: \ \frac{P}{S} = \tan\gamma \tag{7}$$

$$side\ bound\ two\ ADMKE: \ -\frac{P}{S} = \tan\gamma \tag{8}$$

where H is the flight height of the target, P is the shortcut of the target's route, S is the horizontal projection of the target distance, γ is the target azimuth, and θ is the target height angle.

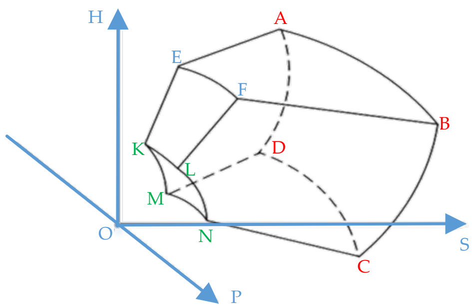


Figure 1. The kill zone.

2. Cover zone model

The cover zone mainly refers to the projected area of a weapon system's kill zone on the ground according to the incoming direction of a ballistic missile; the projected area is capable of covering the defended target [36]. The cover zone model can be expressed by Equation (9).

$$B(d,f) = \begin{cases} \left(d * \sin f + \frac{h_a}{\tan q}\right)^2 + (d * \cos f)^2 - s_a^2 = 0 \\ \left(d * \sin f + \frac{h_b}{\tan q}\right)^2 + (d * \cos f)^2 - s_b^2 = 0 \\ d * \sin f + \frac{f_{AB}(d * \sin f)}{\tan q} = 0 \end{cases} \tag{9}$$

where d is the distance between the deployment point and the boundary of the cover zone; f is the angle between the line connecting the boundary point of the cover zone and the origin and horizontal direction; q is the angle between the average velocity of a ballistic missile in the kill zone and the ground plane; s_a and s_b are the heading distances of the incoming ballistic missile when it flies to points A and B, respectively; h_a and h_b are the heights of the incoming ballistic missile when it flies to points A and B, respectively; f_{AB} is the curve distance between points A and B.

3.3. Optimized Double Nested Architecture-Based Deployment Model

For the problems involving a large number of factors, high degree of complexity, and uncertainty in weapon types, preselected positions, incoming targets, and defending strongholds in force deployment, this study adopts the concept of deep learning hierarchy to represent high-level features as low-level features and defines complex concepts using simple concepts [37,38]. In this study, a double nested architecture is used for hierarchical dimensionality reduction, which can reduce problem complexity and helps to obtain deployment solutions faster under the conditions of covering all incoming ballistic missiles and defending strongholds. The double nested optimization architecture is shown in Figure 2.

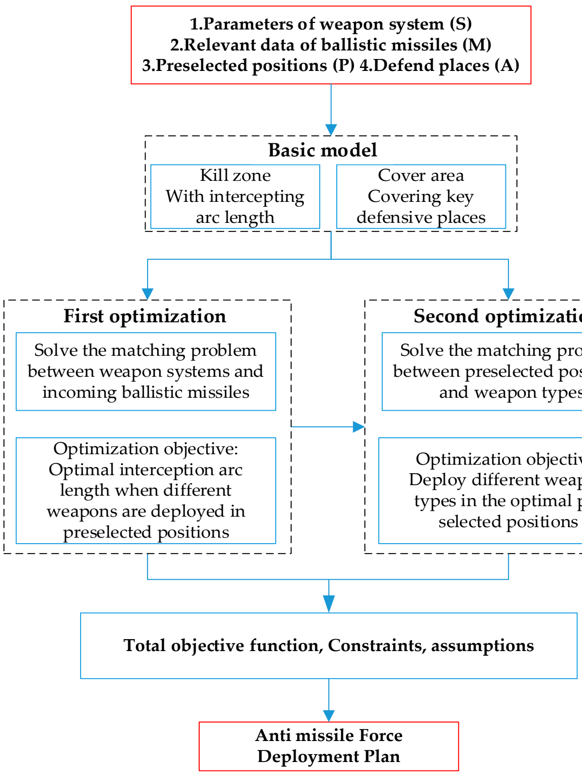


Figure 2. Structure of a double nested optimization architecture.

The double nested optimization architecture can simplify the multi-dimensional force deployment problem into two two-dimensional problems. It is convenient for rapid optimization of the deployment plan. Based on the known input information of weapon system, ballistic missiles, preselected positions, defense places, and the basic models of kill zone and cover zone, the weapon system and ballistic missiles are optimized for the first-layer optimization; then, the formed intercept arc length matrix is used as input to the second-layer optimization, the optimal matching between preselected positions and weapon system can be realized; finally, this architecture can get the deployment plan.

A weapon system is represented by $S = \{s_1, s_2, \ldots, s_t\}$, the description of the weapon system is given by $s_i = \{T_i, R_i, B_i\}$, where T_i denotes the weapon system type, R_i is the kill zone, and B_i represents the cover zone. It should be noted that the same type of weapon system has the same interception capability. Further, the incoming ballistic missile is represented as $M = \{m_1, m_2, \ldots, m_{n'}\}$, and its description is given by $m_i = \{b_{itm}, t_{itm}, d_{itm}\}$, where b_{itm}, t_{itm}, and d_{itm} denote the trajectory position information over time in the East-North-Up (ENU) coordinate system. The preselected position is expressed by $P = \{p_1, p_2, \ldots, p_{m''}\}$, and its description is given by $z_i = \{b_{ip}, t_{ip}, d_{ip}\}$, where b_{ip}, t_{ip}, and d_{ip} denote the position information in the ENU coordinate system. The defending strongholds are represented as $A = \{a_1, a_2, \ldots, a_S\}$, and their description is given by $a_i = \{L_i, b_{ia}, t_{ia}, d_{ia}\}$, where L_i denotes the stronghold level, and b_{ia}, t_{ia}, and d_{ia} denote the location information in the ENU coordinate system.

The first-layer optimization mainly solves the relationship between weapon system S and ballistic missile M. For a certain stronghold, under the constraint conditions, the same types of weapon systems are combined to obtain an optimal interception arc length w of different weapon types. Meanwhile, the second-layer optimization mainly solves the relationship between the preselected position P and weapon system S. Based on the optimal interception arc length obtained by the first-layer optimization part, the best position is selected from the preselected positions under the compliance constraint so that all weapon systems can be deployed. The schematic diagram of the hierarchical process is shown in Figure 3.

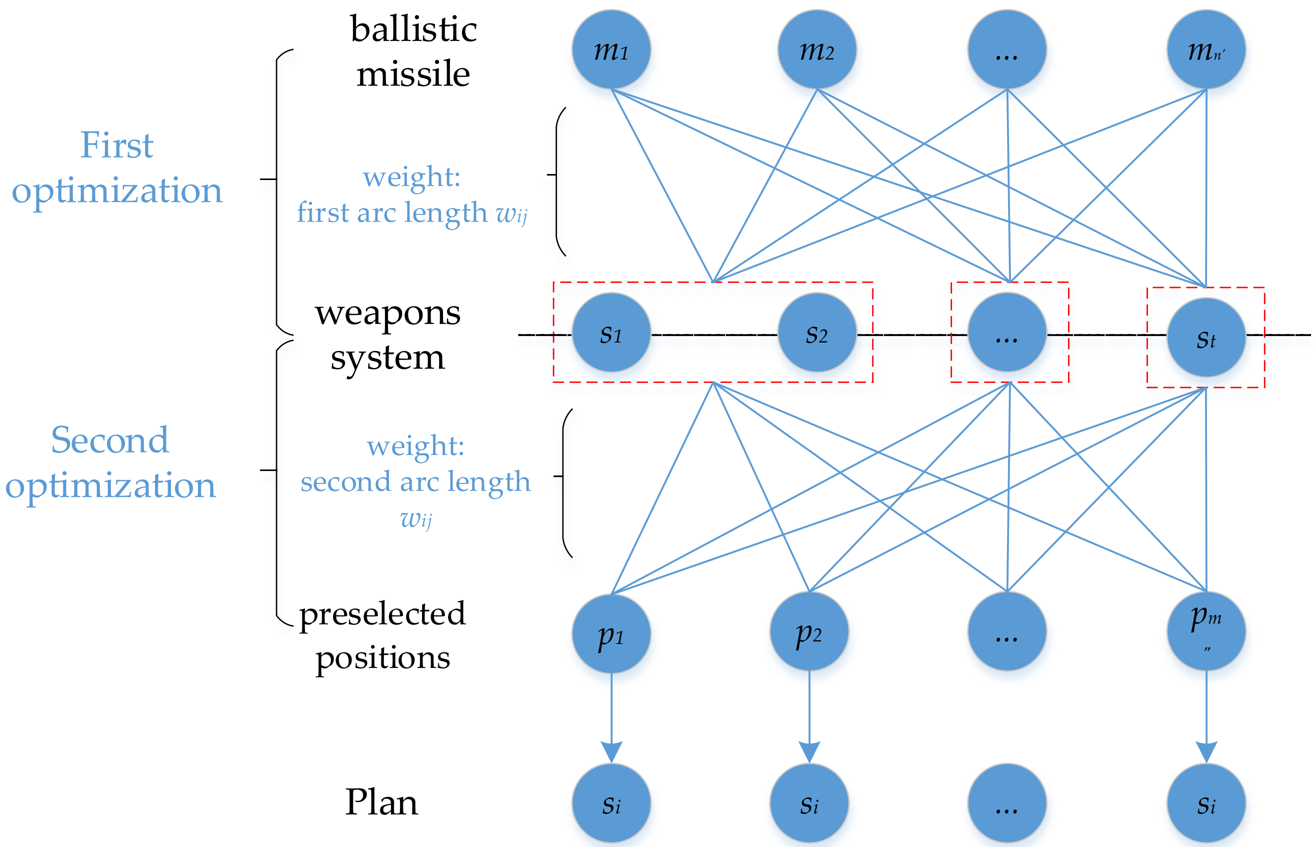


Figure 3. Diagram of hierarchical processing.

The objective function F defines the overall optimal interception arc length based on the double nested optimization architecture. The corresponding mathematical model can be expressed by Equation (10).

$$F = \max \sum_{i''=1}^{m''} \sum_{j''=1}^{n''} \max(\max w_{1j'} + \sum_{i'=2}^{m'} \sum_{j'=1}^{n'} w_{i'j'} {x'}_{i'j'}) {x''}_{i''j''}$$
$$s.t.\begin{cases} \sum_{j'=1}^{n'} {x'}_{i'j'} \le 1, \quad i' = 1,2,\ldots,m' \\ \sum_{j''=1}^{n''} {x''}_{i''j''} \le 1, \quad i'' = 1,2,\ldots,m'' \\ \sum_{i=1}^{m'} {x'}_{i'j'} \le 1, \quad j' = 1,2,\ldots,n' \\ \sum_{i''=1}^{m''} {x''}_{i''j''} = {y''}_{j''}, \quad j'' = 1,2,\ldots,n'' \\ {x'}_{i'j'} \in \{0,1\}, i' = 2,\ldots,m' j' = 1,2,\ldots,n' \\ {x''}_{i''j''} \in \{0,1\}, i'' = 1,2,\ldots,m'' j'' = 1,2,\ldots,n'' \\ \sum_{j''=1}^{n''} {y''}_{j''} = t, {y''}_1 = t_a, {y''}_2 = t_b \ldots \\ w \in \{a,b,\ldots\} \\ m' \le n' \\ m'' \ge n'' \end{cases} \tag{10}$$

where ${x'}_{i'j'}$ with a value of one indicates that the incoming target j' has the interception condition; otherwise, ${x'}_{i'j'}$ has a value of zero; ${x''}_{i''j''}$ with a value of one means that weapon j'' is deployed in stronghold i''; otherwise, ${x''}_{i''j''}$ has a value of zero.

The maximum single-interception arc length of the incoming ballistic missile is a mandatory value. The meaning of each symbol is given in Table 1.

Table 1. List of parameters.

Parameter	Description
i'	Number of interceptions of a certain type of weapon
j'	Incoming ballistic missile number
m'	Maximum number of interceptions of a weapon
n'	Total number of incoming ballistic missiles
w	Interception arc length
I''	Preselected position number
j''	Weapon number; A corresponds to one, B corresponds to two, ...
m''	Total number of preselected positions
n''	Total number of weapons
A, B ...	Label of weapon A, B ...
$a, b \ldots$	Interception arc length of weapons A, B, ...
t	Total number of weapons
$t_a, t_{b.} \ldots$	Total number of weapons A, B, ... corresponding to ${y_1}''$, ${y_2}''$, ...

For the anti-missile troop deployment problem, the main assumptions and constraints are as follows:

- Assumptions
  1. The cover zone can cover defending strongholds when a weapon system is deployed in the preselected position;
  2. The shelter angle of the preselected position meets the interception demand of a target;
  3. When the incoming target enters the kill zone of a fire unit, it will be intercepted with a certain probability;

4 The traffic and communication conditions of the preselected position meet the deployment demand.

- Constraints

In addition to satisfying the conditions based on the basic matching problem, the following constraints should be satisfied.

1 All incoming ballistic missiles must be covered, and after the calculation is completed, the $x'_{i'j'} = 1$ is selected to take the value of j' corresponding to max $w_{1j'}$, and all incoming ballistic missile numbers must be covered;
2 A weapon system's cover zone covers all defending strongholds;
3 Each position deploys at most one set of weapon systems;
4 A weapon system does not produce resistance to only one ballistic missile but can resist multiple incoming missiles on the premise that the response time of the weapon system meets the requirements. In addition, considering the capabilities of combatant commanders and operators and following balancing guidelines and force requirements, the number of ballistic missiles that can be intercepted by a weapon system is limited to two.

3.4. Interception Arc Length Matrix

The anti-missile troop deployment represents a non-equilibrium problem, and the obtained arc-length matrix needs to be transformed into a balanced matrix for further processing by a method of adding edges to complement zeros and a method of transforming one set into m sets [39,40]. The initial matrix of the arc length relationship of all weapon systems and incoming ballistic missiles is denoted by M; one type of weapon system in a preselected position can intercept at most m' ballistic missiles, where $m' \leq n'$. The interception arc length matrix is transformed by changing one weapon system set into m' sets, and the remaining $(n' - m')$ rows are assigned zeros to obtain the n'th order interception arc-length matrix W. All weapon types are calculated, and the remaining $(m''\, n' - m'\, n'')$ rows are assigned zeros to obtain $m''\, n'$ order interception arc length matrix Q. After determining the best value of each weapon type in a certain position according to the constraints, each of the determined values is assigned to each row, $m'' \geq n''$, and the remaining $(m'' - n'')$ columns are assigned zeros; the m'' order intercept arc length matrix R is formed by all weapon types in all positions.

$$M_{t\times n'} = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1n'-1} & w_{1n'} \\ w_{21} & w_{22} & \cdots & w_{2n'-1} & w_{2n'} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ w_{t-11} & w_{t-12} & \cdots & w_{t-1n'-1} & w_{t-1n'} \\ w_{t1} & w_{t2} & \cdots & w_{tn'-1} & w_{tn'} \end{bmatrix} \tag{11}$$

$$W_{n'\times n'} = \begin{bmatrix} w'_{11} & w'_{12} & \cdots & w'_{n'-1} & w'_{1n'} \\ w'_{21} & w'_{22} & \cdots & w'_{2n'-1} & w'_{2n'} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ w'_{m'-11} & w'_{m'-12} & \cdots & w'_{m'-1n'-1} & w'_{m'-1n'} \\ w'_{m'1} & w'_{m'2} & \cdots & w'_{m'n'-1} & w'_{m'n'} \\ & & 0_{(n'-m')\times n'} & & \end{bmatrix} \tag{12}$$

$$Q_{m''n' \times m''n'} = \begin{bmatrix} a_{11} \ldots a_{1n'} & \ldots & a_{11} \ldots a_{1n'} \\ \ldots & \ldots & \ldots \\ a_{m'1} \ldots a_{m'n'} & \ldots & a_{m'1} \ldots a_{m'n'} \\ & \ldots & \\ w_{11} \ldots w_{1n'} & \ldots & w_{11} \ldots a_{1n'} \\ \ldots & \ldots & \ldots \\ w_{m'1} \ldots w_{m'n'} & \ldots & w_{m'1} \ldots w_{m'n'} \\ & 0_{(m''n' - m'n'') \times m''n'} & \end{bmatrix} \quad (13)$$

$$R_{m'' \times m''} = \begin{bmatrix} A_{11(\max)} & \ldots & W_{2n''(\max)} & \\ A_{21(\max)} & \ldots & W_{2n''(\max)} & \\ \ldots & \ldots & \ldots & 0_{m'' \times (m'' - n'')} \\ A_{m''-11(\max)} & \ldots & W_{m''-1n''(\max)} & \\ A_{m''1(\max)} & \ldots & W_{m''n''(\max)} & \end{bmatrix} \quad (14)$$

4. Algorithm Implementation

The anti-missile deployment problem represents a complex multi-constraint optimization matching problem. Following the idea of operations-related research and based on the established double nested optimization model, a depth-first search method is integrated, and each possible branch path can be explored as deep as it can go [41]. The KM algorithm is a typical algorithm for solving weighted-matching and assignment problems [42], which is conducive to obtaining a solution with a high degree of matching [43]. Depth-first search can traverse all feasible solutions while performing a backtracking operation, which is conducive to the overall solution search and adjustment. Therefore, the deep KM algorithm can obtain more optimal solutions with higher satisfaction and effectively handle uncertainty in the anti-missile deployment problem.

4.1. Algorithm Core Ideas

The core idea of the proposed deep KM algorithm is to transform the problem of finding the maximum right matching into the problem of finding the perfect matching by assigning a vertex with a label [44], converting weights into feasible vertex labels, and deeply traversing all feasible solutions to obtain the most satisfactory matching. When the most satisfactory matching cannot be found, the total number of feasible edges is increased by modifying the feasible vertex labels to determine the matching of each point satisfying the constraints so that the final and best matchings are achieved.

4.2. Algorithm Design

The algorithm flow diagram is shown in Figure 4, the specific algorithm steps are as follows:

Step 1: Input the required parameters: the number of weapon types and related data; the number of incoming ballistic missiles and related data; the number of preselected positions and their locations; and the number of defending strongholds and their locations;

Step 2: Construct the bipartite maps of ballistic missiles and weapon systems and of weapon systems and preselected positions;

Step 3: Initialize the feasible vertex label; assign the left vertex to the maximum weight of the edge connected to it and the right vertex to zero;

Step 4: Perform the depth matching process by performing the following steps:

Step 4.1: According to the criterion of selecting the maximum value and the determination criterion of whether the second ballistic missile can be intercepted, obtain the arc length matrix and perform the normalization process, including adding the edge and filling with zeros and converting one set into m sets, to transform the matrix to the n-order standard matrix;

Step 4.2: Process the matrix for finding zeros; subtract the maximum of a row from each element in that row, and repeat the same procedure for columns;

Step 4.3: Process the matrix for depth marking zeros, starting from the first row (column) of the first zero, circle zero, and cross out the row and column where it is located; cross out the new matrix in the same way in turn until all zero elements in the matrix are marked; if the initial row (column) has multiple zeros, proceed sequentially from the second zero to obtain multiple marked matrices;

Step 5: Determine whether the matching is complete; check whether the number of zeros in the matrix is equal to the number of matrix dimensions; if so, proceed to Step 6; otherwise, proceed to Step 7;

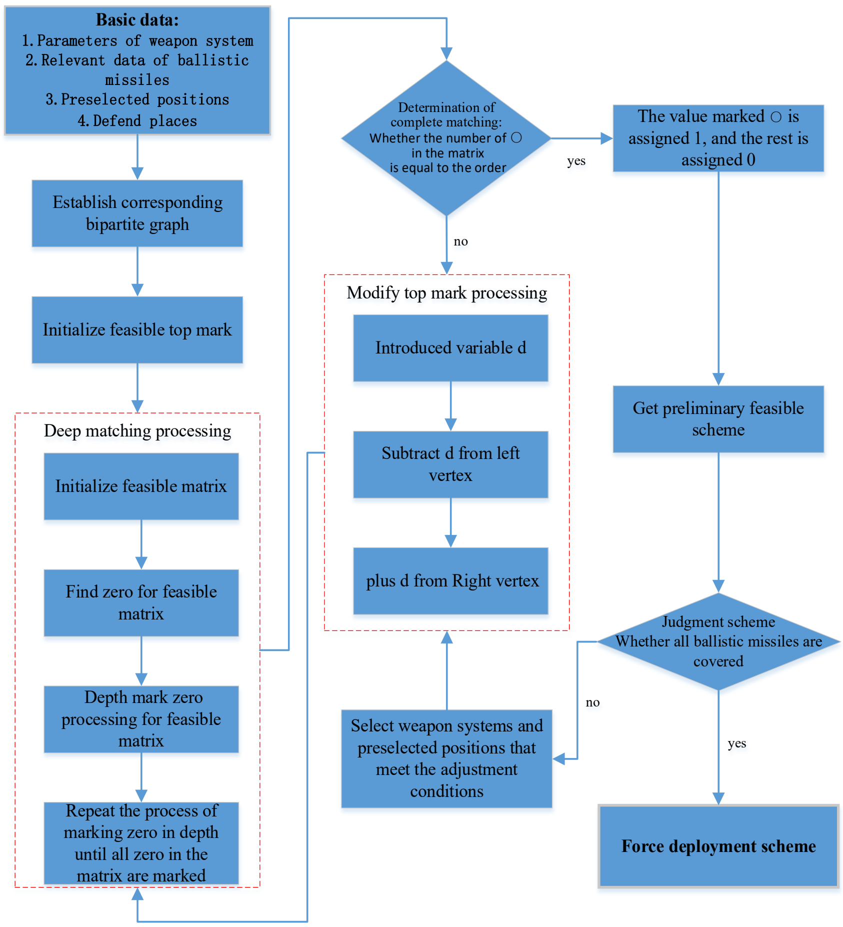


Figure 4. Diagram of the algorithm design.

Step 6: Assign the marked position to one and the remaining positions to zeros to obtain a set of feasible solutions; then, proceed to Step 8;

Step 7: Modify the vertex label; introduce variable d; subtract d from the vertex label of the left endpoint and add d to the vertex label of the right endpoint; return to Step 4;

Step 8: Determine whether the scenario covers all ballistic missiles; if so, proceed to Step 9; otherwise, proceed to Step 10;

Step 9: Output the resulting troop deployment plan;

Step 10: According to the ballistic missile full-coverage adjustment criteria, screen out the weapons systems and preselected positions that meet the adjustment conditions and return to Step 7.

4.3. *Ballistic Missile Full-Coverage Adjustment Criteria*

The incoming ballistic missile's number is determined based on the preliminarily obtained results, and if not all ballistic missiles are covered, an adjustment needs to be performed according to the adjustment criteria by conducting the following steps:

Step 1: Among the weapon systems of each type, identify the weapon systems that have the capability to intercept an incoming ballistic missile two or more times and their deployed positions. A weapon system that can intercept the missile more times has a higher adjustment priority. A weapon system with a smaller arc length also has a higher adjustment priority. For instance, type-A weapon systems deployed in positions 1 and 5 both meet the interception conditions for incoming ballistic missile 1. Next, the arc lengths of positions 1 and 5 are compared, and the one that has smaller arc lengths is first adjusted;

Step 2: Determine whether the selected weapon models and their deployment positions have interception arc lengths for incoming ballistic missiles for which the weapons do not meet interception conditions; proceed to Step 3 if they have interception arc lengths; otherwise, proceed to Step 4;

Step 3: After screening out the weapon models with interception capability and their deployment locations, perform deployment adjustments;

Step 4: Adjust the interception arc lengths for the incoming ballistic missile given in Step 1 from smallest to largest to determine whether a weapon that does not satisfy interception conditions for the ballistic missile has the interception capability. If so, return to Step 3; however, if not even after adjusting the interception arc length to the maximum arc length, proceed to Step 5;

Step 5: Adjust the preselected positions and re-execute the automatic troop deployment optimization operation.

5. Simulation Verification

5.1. Scene Setting

In the simulation scene, there were three defending strongholds denoted by D1–D3. Six preselected positions denoted by Z1–Z6 were determined based on the possible direction of the enemy. Further, both A and B weapon types had two deployed sets, and there were a total of six attack ballistic missiles, with lot numbers of 001–006.

The variable $d = 0.1$, the initial matrix $R_0 = [0_{6\times 6}]$, the number of operations $T = 100$, number of interceptions of a certain type of weapon $i' \in \{0,1,2\}$, the incoming ballistic missile number $j' \in \{1,2,3,4,5,6\}$, 1~6 represents ballistic missile 001~006, the maximum number of interceptions of a weapon $m' = 2$, the total number of incoming ballistic missiles $n'= 6$, the preselected position number $i'' \in \{1,2,3,4,5,6\}$, 1~6 represents preselected position Z1~Z6, the weapon number $j'' \in \{1,2\}$, 1 represents weapon type A, 2 represents weapon type B, the total number of preselected positions $m'' = 6$, the total number of weapons $n'' = 2$, the total number of weapons $t = 4$, the total solution time is not more than 30 min, it depends on the solution scale. It was assumed that the coverage requirements of the defending strongholds were met after the deployment of weapon in the preselected positions, and each weapon system was intercepting maximum of two ballistic missiles. After the weapon system deployment, it was required to cover all incoming ballistic missiles and defending strongholds. The coordinates of the defending strongholds and preselected positions in the ENU coordinate system are given in Table 2. The weapon system capability parameters are given in Table 3. The schematic diagram of the weapon system capability is shown in Figure 5. Finally, the ballistic trajectory simulation data, positions of the defending strongholds, and preselected positions are shown in Figures 6 and 7.

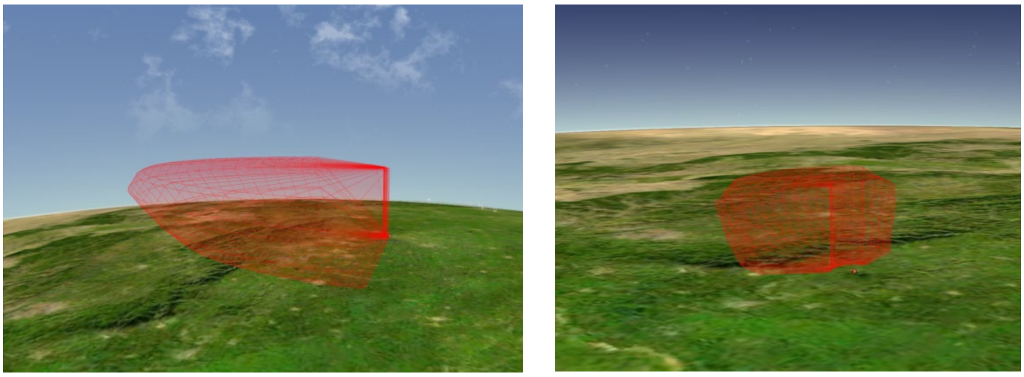
Table 2. Coordinates of the defend places and preselected positions.

North-Up-East	DS 1	DS 2	DS 3	PP 1	PP 2	PP 3	PP 4	PP 5	PP 6
North	−13,979	−4184	−13,924	−14,690	−12,289	−5730	−4786	−13,865	−14,300
Up	−288	−303	−221	−300	−290	−268	−269	−209	−230
East	59,784	62,374	51,588	59,453	61,029	62,069	61,388	50,987	49,398

Note: DS stands for a defending stronghold, and PP means a preselected position.

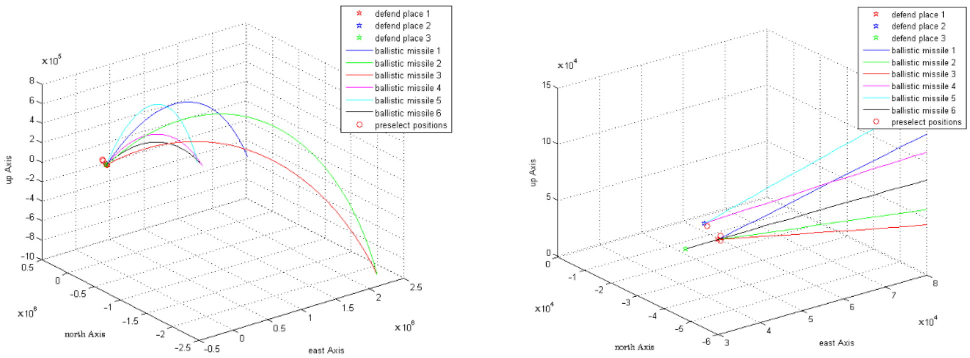
Table 3. Weapon system combat capability parameters.

Weapon Type	High Bound (km)	Lower Bound (km)	Far Bound (km)	Near Bound (km)	Sector Range
A	100	30	180	60	−50°–+50°
B	60	20	80	30	−45°–+45°



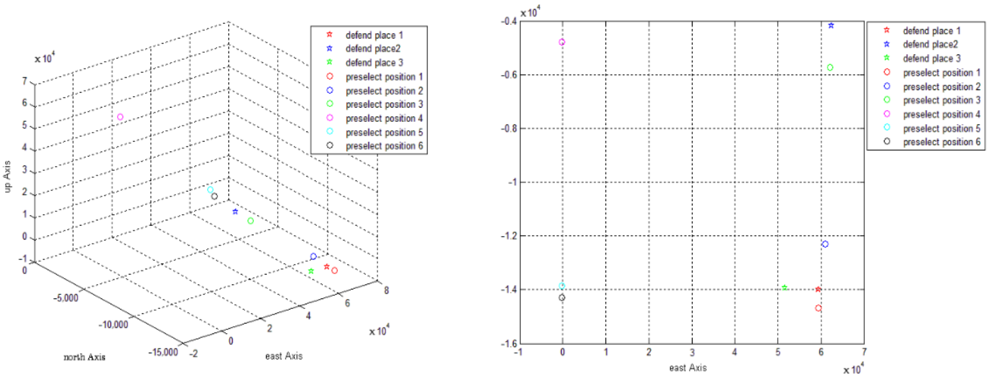
(**a**) (**b**)

Figure 5. Diagram of the combat capability of weapons. (**a**) The combat capability of weapon A; (**b**) the combat capability of weapon B.



(**a**) (**b**)

Figure 6. The ballistic trajectory of the incoming missile. (**a**) Global view, (**b**) detail view.



(**a**) (**b**)

Figure 7. Locations of defend places and preselected positions. (**a**) 3D (three-dimensional) deployment point, (**b**) 2D plane deployment point.

5.2. Deployment Plan Analysis

Based on the weapon system operational performance and trajectory information of the incoming ballistic missile, the scene setting has assumed that a weapon system can intercept at most two ballistic missiles, so the number of targets that a weapon system can intercept can be 0, 1, and 2. The specific number of interceptions is determined according to the solution results. And it has set up six preselected positions, each of which is numbered Z1~Z6 and set a total of six incoming ballistic missiles in the scene settings, whose numbers are 001–006. At the same time, it analyzed six preselected positions and deploy two types of weapon systems, A and B. The interception arc length matrix for six ballistic missiles is Q_{Z1}–Q_{Z6}.

After processing, the sixth-order interception arc length matrix R was obtained with two sets of different weapon types A and B in six different preselected positions. The first column indicated the A-type weapon system, while the second column indicated the B-type weapon system. The first and second columns in R were copied once, so the final calculation results in each row and column had at most one "1" while the remaining columns were zeros. the final deployment matrix R' was obtained by performing a series of processing as follows:

$$Q_{Z1}=\begin{bmatrix} 3.4 & 3.1 & 3.9 & 1.9 & 1.8 & 2.3 \\ 3.4 & 3.1 & 3.9 & 1.9 & 1.8 & 2.3 \\ 3.1 & 3.0 & 3.4 & 1.7 & 1.5 & 1.8 \\ 3.1 & 3.0 & 3.4 & 1.7 & 1.5 & 1.8 \\ \multicolumn{6}{c}{\mathbf{0}_{2\times6}} \end{bmatrix};\ Q_{Z2}=\begin{bmatrix} 3.6 & 3.3 & 4.1 & 2.0 & 1.9 & 2.4 \\ 3.6 & 3.3 & 4.1 & 2.0 & 1.9 & 2.4 \\ 3.3 & 2.6 & 3.7 & 1.9 & 1.6 & 1.9 \\ 3.3 & 2.6 & 3.7 & 1.9 & 1.6 & 1.9 \\ \multicolumn{6}{c}{\mathbf{0}_{2\times6}} \end{bmatrix}\ Q_{Z3}=\begin{bmatrix} 1.5 & 1.3 & 1.9 & 5.6 & 4.8 & 2.1 \\ 1.5 & 1.3 & 1.9 & 5.6 & 4.8 & 2.1 \\ 1.3 & 1.2 & 1.7 & 5.4 & 4.7 & 2.0 \\ 1.3 & 1.2 & 1.7 & 5.4 & 4.7 & 2.0 \\ \multicolumn{6}{c}{\mathbf{0}_{2\times6}} \end{bmatrix}$$

$$Q_{Z4}=\begin{bmatrix} 1.4 & 1.3 & 1.8 & 5.3 & 4.7 & 2.0 \\ 1.4 & 1.3 & 1.8 & 5.3 & 4.7 & 2.0 \\ 1.2 & 1.1 & 1.6 & 5.1 & 4.5 & 1.9 \\ 1.2 & 1.1 & 1.6 & 5.1 & 4.5 & 1.9 \\ \multicolumn{6}{c}{\mathbf{0}_{2\times6}} \end{bmatrix}\ Q_{Z5}=\begin{bmatrix} 2.5 & 2.0 & 2.7 & 1.9 & 1.8 & 5.7 \\ 2.5 & 2.0 & 2.7 & 1.9 & 1.8 & 5.7 \\ 2.3 & 1.8 & 2.6 & 1.8 & 1.7 & 5.5 \\ 2.3 & 1.8 & 2.6 & 1.8 & 1.7 & 5.5 \\ \multicolumn{6}{c}{\mathbf{0}_{2\times6}} \end{bmatrix}\ Q_{Z6}=\begin{bmatrix} 2.4 & 2.0 & 2.6 & 1.8 & 1.6 & 5.5 \\ 2.4 & 2.0 & 2.6 & 1.8 & 1.6 & 5.5 \\ 2.1 & 1.7 & 2.3 & 1.7 & 1.5 & 5.4 \\ 2.1 & 1.7 & 2.3 & 1.7 & 1.5 & 5.4 \\ \multicolumn{6}{c}{\mathbf{0}_{2\times6}} \end{bmatrix}$$

$$R=\begin{bmatrix} 7.3 & 6.4 & 7.3 & 6.4 & \\ 7.7 & 7.0 & 7.7 & 7.0 & \\ 10.4 & 10.1 & 10.4 & 10.1 & \mathbf{0}_{6\times2} \\ 10 & 9.6 & 10 & 9.6 & \\ 5.7 & 5.5 & 5.7 & 5.5 & \\ 5.5 & 5.4 & 5.5 & 5.4 & \end{bmatrix} \Rightarrow R'=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Matrix R' is obtained after matrix R performs optimization according to the constraints and 0–1 integer programming. One means the deployment of weapon system, 0 means no weapon system is deployed. According to the deployment requirements, it is necessary to find the best deployment scheme from the optimization results of R, so as to optimize the overall interception arc length as much as possible. The obtained result should meet the requirement that each row and column can only have one 1.

The numerical values in the matrix Q_{Z1}–Q_{Z6} represent the interception arc length formed for the incoming ballistic missiles 001~006 when deploying two different weapons, A and B. The interception arc length has been defined before. It can be calculated according to the parameters of the kill zone of the weapon system and the flight path data of the ballistic missile.

In the matrix R', the row represents position and column represents the weapon type, Lines 1–6 in matrix R' represent positions 1–6 respectively, the odd column represents weapon type A, even columns represent weapon type B. The second value in the first row of the matrix R' is 1, which means that the weapon system is deployed in the position $Z1$, and the second value corresponds to the second column, which is the weapon type B. We get the conclusion that the weapon type B is deployed in the preselected position 1. After returning to our analysis of matrix R, we know that it is mainly used to intercept the second batch of 002 targets and the third batch of 003 targets. According to the analysis results of the plan matrix R', the following conclusions could be made:

- B-type weapon deployment in position 1 mainly intercepted target batches 002 and 003;
- A-type weapon deployment in position 2 mainly intercepted target batches 001 and 003;
- B-type weapon deployment in position 3 mainly intercepted target batches 004 and 005;
- A-type weapon deployment in position 5 mainly intercepted target batch 006.

The deployment results indicated that the double-fold coverage of defending stronghold D1 and one-fold coverage of defending strongholds D2 and D3 were achieved. Thus, all incoming ballistic missiles were covered, which verified the reliability, feasibility, and effectiveness of the proposed algorithm.

6. Conclusions

This paper conducts a systematic analysis of the anti-missile troop deployment, develops a model based on a double-nested optimization architecture, and proposes a deep KM algorithm. The proposed algorithm is verified by simulations.

In terms of system cognition, this study clarifies the importance and necessity of troop deployment, along with its uncertainties and complexities, which can lay a solid foundation to solve deployment problems. In terms of complexity, the relationship between key factors is determined, and problem complexity is reduced by decreasing the modeling dimension. In terms of uncertainty, this study is beneficial to solution optimization, which can increase problem solving satisfaction and can enhance efficiency in handling uncertainties on the battlefield. The research presented in the paper provides valuable reference to solving complex multi-constraint optimization problems under uncertainties and has important theoretical guidance and practical application to anti-missile warfare and anti-missile troop deployment.

Further work could include further research regarding the proposed algorithm solving efficiency improvement.

Author Contributions: Conceptualization, W.S. and Z.C.; methodology, W.S. and G.W.; software, X.G.; validation, W.S., Y.S. and G.W.; formal analysis, W.S. and Y.S.; investigation, Z.C.; resources, Y.S.; data curation, W.S.; writing—original draft preparation, W.S.; writing—review and editing, W.S. and Y.S.; visualization, X.G.; supervision, Z.C.; project administration, G.W.; funding acquisition, Y.S. All authors have read and agreed to the published version of the manuscript.

Funding: This work was funded by the National Natural Science Foundation of China (No. 61703426) and the National Natural Science Foundation of Shanxi Province (2021JM-226).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Zheng, B.; Zhao, J.; Zheng, Q. Review of Distribution of Troops Related Concepts. *Fire Control Command Control* **2017**, *42*, 5–8.
2. Zhao, G. Operational mission planning: Issues and review. *J. Command Control* **2017**, *3*, 265–272.
3. Liu, Y.; Zhang, Z.; Wang, Y.; Fan, X. Overview of missile defense development in Russia. *Aerosp. China* **2022**, *6*, 28–31.
4. Wang, W.; Li, B. Development Status and Characteristics of U.S. Military Ballistic Missile Defense System. *J. CAEIT* **2021**, *16*, 805–812.
5. Zhang, J.; Wang, G.; Song, Y.; Zhao, F.; Wang, S. Multiagent Task Planning Based on Distributed Resource Scheduling under Command and Control Structure. *Math. Probl. Eng.* **2019**, *2019*, 4259649. [CrossRef]
6. Army. Air and Missile Defense. *Anonymous* **2021**, *71*, 118–120.
7. La, B.; David, J. From Mutually Assured Destruction to Mutually Assured Stability: A Constructivist Rationale for Expanding Missile Defense. Ph.D. Thesis, Seoul National University, Seoul, Republic of Korea, 2019.

8. David, D. Ballistic Missile Defense. In *The Post–Cold War Era*; Taylor and Francis: Abingdon, UK, 2021.
9. Zhao, P.; Li, J. An Optimization Model for Air Defense Troops Deployment Based on Queuing Theory. *Fire Control Command Control* **2017**, *42*, 38–42.
10. Xu, J. Force Deployment Method for Land–Based Air Defense Based on Decision Theory. *Command Inf. Syst. Technol.* **2017**, *8*, 99–104.
11. Wan, J.; Wang, P.; Tang, J.; Zhang, D.; Li, X. Research on Hybrid Deployment Strategy and Model for Key Position Air-defense Based on Multi-weapon Platforms. *J. Syst. Simul.* **2021**, *33*, 1754–1765.
12. Yan, P.; Liu, S.; Wang, J. Optimization model of air defense disposition based on dynamic programming and genetic algorithm. *Syst. Eng. Electron.* **2018**, *40*, 2249–2255.
13. Pan, Y.; Wu, K.; Dong, S.; Jiang, H.; Lei, J. Application of regional air defense deployment based on genetic algorithm. *J. Command Control* **2020**, *6*, 271–277.
14. Kong, X.; Li, D. Region Air-Defense Deployment Decision Based on Particle Swarm Optimization. *Comput. Digit. Eng.* **2016**, *44*, 2320–2324.
15. Chen, X.; Wen, T.; Liu, S. Optimization for Regional Air Defense Deployment Based on Memetic Algorithms. *J. Proj. Rockets Missiles Guid.* **2018**, *38*, 14–18.
16. Gong, X.; Li, H.; Yang, X. A Task Planning Simulation Method Based on Hungarian Algorithm. *J. Phys. Conf. Ser.* **2022**, *2219*, 012041. [CrossRef]
17. Zhang, S.; Huang, Y. Analysis of operational resources deployment based on intelligent optimization algorithm. *Jiangsu Sci. Technol. Inf.* **2020**, *1*, 57–60.
18. Liu, Y.; Zhang, Z.; Wang, L. Deployment and Optimization of Midcourse Anti-missile Firepower Unit. *Fire Control Command Control* **2018**, *43*, 157–161.
19. Yu, W.; Shi, Z.; Zhou, F.; Li, J. Research on Optimization of Terminal Anti-missile Equipment Resource Allocation. *Mod. Def. Technol.* **2022**, *50*, 25–31.
20. Li, Y.; Han, W.; Wang, Y. Deep Reinforcement Learning with Application to Air Confrontation Intelligent Decision-Making of Manned/Unmanned Aerial Vehicle Cooperative System. *IEEE Access* **2020**, *8*, 67887–67898. [CrossRef]
21. Debabrata, S.; Islam, S.K.H.; Naveen, C.; Mohammad, H. *Data Analytics, Computational Statistics, and Operations Research for Engineers: Methodologies and Applications*; CRC Press: Boca Raton, FL, USA, 2021.
22. Gayialis, S.P.; Kechagias, E.P.; Konstantakopoulos, G.D. A city logistics system for freight transportation: Integrating information technology and operational research. *Oper. Res.* **2022**, *22*, 5953–5982. [CrossRef]
23. John, B. Ballistic Missile Defense: Threats and Challenges. *Arms Control Today* **2022**, *52*, 36.
24. Kohri, K.; Amann, D. Ballistic missile defense WALEXs: Collaborative examination of requirements. *Johns Hopkins APL Tech. Dig.* **2000**, *21*, 238–243.
25. Xing, Q.; Fan, H. *Missile Defense Mission Planning Technology*; Science Press: Beijing, China, 2019.
26. Wang, F.; Yang, Z.; Guo, Z. Research on Intelligent Force Deployment Based on 3D Real Map. *Ship Electron. Eng.* **2021**, *41*, 107–111.
27. Wang, X.; Wang, G.A. Study of the deployment and re-deployment of the US military force and the enlightenment they give us. *Traffic Eng. Technol. Natl. Def.* **2020**, *18*, 1–6.
28. Lötter, D.P. *Modeling Weapon Assignment as a Multi Objective Problem*; Stellenbosch University: Stellenbosch, Western Cape, South Africa, 2012.
29. Nguyen, V.D.; Pham, B.T.; Do, P.T. Efficient Algorithms for Maximum Induced Matching Problem in Permutation and Trapezoid Graphs. *Fundam. Inform.* **2021**, *182*, 257–283. [CrossRef]
30. Kamiyama, N. The envy-free matching problem with pairwise preferences. *Inf. Process. Lett.* **2021**, *172*, 106158. [CrossRef]
31. Sun, H.; Xing, C.; Ma, Q. Simulation and Analysis on Interception Capability of Anti-missile Weapon System. *Ship Electron. Eng.* **2022**, *42*, 92–96.
32. Busetto, F.; Codognato, G.; Tonin, S. Simple majority rule and integer programming. *Math. Soc. Sci.* **2021**, *113*, 160–163. [CrossRef]
33. Liu, Y.; Zhang, Z.; Chai, H. Intercepting Probability of Anti-missile Firepower in Space Killing Area. *Comput. Meas. Control* **2018**, *26*, 132–136.
34. Wang, J.; Zhao, J.; Li, J.; Shao, L. Research on numerical model of ground-to-air missile kill zone. *Syst. Eng. Theory Pract.* **2014**, *34*, 3260–3267.
35. Yu, X.; Du, X. Fitting algorithm design for destruction zone of air defense. *Comput. Eng. Des.* **2012**, *33*, 3470–3474.
36. Wang, J.; Liu, R.; Xue, W. Research on Defended Footprint Modeling. *Mod. Def. Technol.* **2014**, *42*, 14–21.
37. Yu, W.; Yu, H.; Du, J.; Zhang, M.; Wang, D. A deep learning algorithm for joint direct tracking and classification of manoeuvring sources. *IET Radar Sonar Navig.* **2022**, *16*, 1198–1211. [CrossRef]
38. Uday, S.S.; Andreas, S. *Machine and Deep Learning Algorithms and Applications*; Morgan & Claypool Publishers: San Rafael, CA, USA, 2021.
39. Shen, X.; Chen, L. Configuration Transformation Planning Method of Unmanned Aerial Vehicle Cluster Based on Improved Hungarian Algorithm. *J. Phys. Conf. Ser.* **2022**, *2283*, 012001. [CrossRef]
40. Zheng, X.; Ma, C. An intelligent target detection method of UAV swarms based on improved KM algorithm. *Chin. J. Aeronaut.* **2021**, *34*, 539–553. [CrossRef]

41. Baswana, S.; Gupta, S.; Tulsyan, A. Fault Tolerant Depth First Search in Undirected Graphs: Simple Yet Efficient. *Algorithmica* **2022**, *84*, 2028–2049. [CrossRef]
42. Yan, Y.; Chen, X.; Nan, H.; Guan, J. Modern inversion workflow of the multimodal surface wave dispersion curves: Staging strategy and Pattern search with embedded Kuhn–Munkres algorithm. *Geophsical J. Int.* **2022**, *231*, 47–71. [CrossRef]
43. Zhu, H.; Zhou, M. Efficient Role Transfer Based on Kuhn–Munkres Algorithm. *IEEE Trans. Syst. Man Cybern. Part A Syst. Hum.* **2012**, *42*, 491–496. [CrossRef]
44. Amir, M.; Mostafa, H. Minimum-Cost Drone–Nest Matching through the Kuhn–Munkres Algorithm in Smart Cities: Energy Management and Efficiency Enhancement. *Aerospace* **2019**, *6*, 125.