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Dynamic ILC for Linear Repetitive Processes Based on Different Relative Degrees

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Abstract: The current research on iterative learning control focuses on the condition where the system relative degree is equal to 1, while the condition where the system relative degree is equal to 0 or greater than 1 is not considered. Therefore, this paper studies the monotonic convergence of the corresponding dynamic iterative learning controller systematically for discrete linear repetitive processes with different relative degrees. First, a 2D discrete Roesser model of the iterative learning control system is presented by means of 2D systems theory. Then, the monotonic convergence condition of the controlled system is analyzed according to the stability theory of linear repetitive process. Furthermore, the sufficient conditions of the controller existence are given in linear matrix inequality format under different relative degrees, which guarantees the system dynamic performance. Finally, through comparison with static controllers under different relative degrees, the simulation results show that the designed schemes are effective and feasible.

Keywords: linear repetitive process; relative degrees; dynamic iterative learning control; monotonic convergence; linear matrix inequality

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1. Introduction

Iterative learning control (hereafter abbreviated as ILC) is suitable for executing repetitive operation tasks in finite time. The strategy is to use the previous control experience and output error to repeatedly adjust the present system input, and then the system output is able to track the desired trajectory [1,2]. There are different application scenarios in the field of ILC. The research in [3,4] studied state-observer-based ILC algorithms and PD-type ILC algorithms, respectively, in systems with arbitrary relative degrees. The uncertain system survey in [5,6] is also an important domain for ILC application. Additionally, monotonic convergence and controller design have always been hot issues [7–9].

Linear repetitive processes under ILC are usually regarded as two-dimensional systems on the time and batch axes. Research on ILC system-design methods and convergence performance on the basis of 2D system theory has become an important direction for the development of ILC technology [10,11]. Based on the theory of 2D systems, the work in [12,13] discussed a comprehensive predictive ILC strategy to ensure the fast convergence performance of the learning process under model error and uncertain disturbance. In [14,15], a high-order ILC controller was proposed according to the stability theory of linear repetitive processes, which optimized the monotonic convergence and robust performance of the system.

Based on discrete linear repetitive processes of actuator failure, the authors of [16,17] proposed a comprehensive design method of an integrated iterative learning fault-tolerant controller with state feedback and gave sufficient conditions for a closed-loop system to remain stable in the case of failure using Lyapunov stability theory. In [18,19], a P-type ILC algorithm was proposed for high relative linear multivariable discrete systems with iterative initial error.

These existing design methods use static state feedback and feedforward information to improve the system performance. However, in some cases, the static iterative learning controller may be highly conservative, and it is difficult to meet certain performance requirements of the system. In this case, dynamic ILC control law is a better choice method. In terms of the error convergence between channels, relevant applications showed that the dynamic ILC outperforms the static controller. In [20,21], a dynamic ILC controller is proposed to make the system tracking error converge rapidly along the iterative direction.

The authors of [22,23] used 2D system theory to design the controller, but the stability analysis and applicable cases are not completely given. In [24,25], a dynamic filter is designed; however, it is difficult to control the system stability for uncertain systems. For the system with an uncertain ILC model, a dynamic filter was designed to make the system error converge in [26,27], while both of these papers did not consider the system fault.

The research in [28] introduced an ILC method on the basis of the iterative moving-average operator, which is applied in nonlinear dynamic systems with random variable test lengths. The work in [29,30] designed a dynamic ILC law based on the repetitive process setting method for three-dimensional cranes and space manipulator individually, while the research on ILC in systems with a relative degree greater than 1 is relatively rare, the above research only considers the ILC with a relative degree of 1 and does not consider the situation where the system relative degree is equal to 0 or greater than 1.

In view of this, this paper studies the system monotonic convergence under the dynamic ILC algorithm for discrete linear repetitive processes with different degrees of relativity. Combined with 2D system theory, a 2D discrete Roesser model of an ILC system is established. The stability theory of linear repetitive processes is used to analyze the system monotone convergence condition. The sufficient condition for the controller existence is given in linear matrix inequality (hereafter abbreviated as LMI) format, it guarantees the dynamic performance.

In this paper, for matrices X, X^T and $X^{\perp}$ are the transpositions and orthogonal complements, respectively; $X > 0$ and $X < 0$ mean that X is positive definite and negative definite separately; I and 0 represent a unit and zero matrix, respectively; symbol "$\forall$" represents the element transposition in a symmetrical position; $sym(X) = (X + X^T)$ represents the *Hermitian* part of the matrix X; and $diag\{\cdot\}$ represents a diagonal matrix.

2. Problem Description

We consider below the state space model with iteration as follows:

$$\begin{cases} x(t+1,k) = Ax(t,k) + Bu(t,k) \\ y(t,k) = Cx(t,k) + +Du(t,k) \end{cases} \tag{1}$$

where the iteration period t satisfies $0 \le t \le T$; system iteration times $k \ge 0$; $x(t,k) \in R^n$, $u(t,k) \in R^m$, and $y(t,k) \in R^l$ are the system state, input and output at time t and batch k; boundary condition $x(0,k) = x_0$ represents the system initial status at batch k; A, B, C and D are the corresponding system matrices of proper dimension.

Suppose that system (1) is stable and the relative degree $r \ge 0$, which has the following characteristics [31]:

(1) if $D \neq 0$ and the row is full of rank, then $r = 0$;

(2) $r = 1$ if the system meets the following conditions:

(a) for all $i < r - 1$, there are $D = 0$ and $CA^iB \neq 0$;

(b) $CA^{r-1}B \neq 0$ and the row is full of rank.

In ILC process of system (1), we make its desired trajectory $y_d(t)$ and define the tracking error as follows:

$$e(t,k) = y_d(t) - y(t,k). \tag{2}$$

For a given state space model (1), considering the different relative degrees of the system, this paper designs appropriate control inputs $\{u(t,k), 0 \le t \le T\}$ to make the tracking error $e(t,k)$ monotonic convergent along the batch direction. At the same time, the system stability along the time axis is also guaranteed.

3. Iterative Learning Control for Linear Repetitive Processes

For batch $k \geq 0$, state space model [32] of discrete linear repetitive processes running on finite periods $0 \leq t \leq T$ is as follows:

$$\begin{cases} x(t+1,k+1) = A_d x(t,k+1) + B_d u(t,k+1) + B_{d0} y(t,k) \\ y(t,k+1) = C_d x(t,k+1) + D_d u(t,k+1) + D_{d0} y(t,k) \end{cases} \tag{3}$$

where $x(t,k) \in R^n$, $u(t,k) \in R^{m,}$, $y(t,k) \in R^l$ represent system state, input, output at time t and batch k, respectively. Set boundary conditions $x(0,k+1)$ and $y(t,0)$ as known vector functions, the stability condition of linear repetitive processes (3) satisfies the below lemmas.

Lemma 1 ([32]). *The linear repetitive process described in* (3) *is stable if and only if the following inequality is established:*

(i) $\rho(D_{d0}) < 1$*, its modulus is strictly less than 1 to ensure convergence along the batch.*

(ii) $\rho(D_{d0}) < 1$*, its modulus is strictly less than 1 to ensure convergence along time.*

(iii) All the eigenvalues modulus of transfer function $G(z) = C_d * (zI - A_d)^{-1} * B_{d0} + D_{d0}$ *are strictly less than 1 on a unit circle of complex plane* $|z| = 1$*, thereby, ensuring convergence along time and the batch.*

Lemma 2 ([33]). *Given a matrix with proper dimensions* A, B *and* M*, there is* $\det\left(e^{j\omega} I - A\right) \neq 0$ *for* $\omega \in R$*, and the below conditions are equivalent:*

(1) There exists $\omega \in R$ *to hold the below inequality:*

$$\begin{bmatrix} \left(e^{j\omega} I - A\right)^{-1} B \\ I \end{bmatrix}^T M \begin{bmatrix} \left(e^{j\omega} I - A\right)^{-1} B \\ I \end{bmatrix} \leq 0. \tag{4}$$

(2) There exists Hermitian array $P = P^T$ *to hold the below inequality:*

$$M + \begin{bmatrix} A^T PA - P & A^T PB \\ B^T PA & B^T PB \end{bmatrix} \leq 0. \tag{5}$$

To make system (3) convergent, we propose Theorem 1.

Theorem 1. *If matrices* $P_1 > 0$, $P_2 > 0$ *exist, the following LMI is established:*

$$\begin{bmatrix} A_d^T P_1 A_d + C_d^T P_2 C_d - P_1 & A_d^T P_1 B_{d0} + C_d^T P_2 D_{d0} \\ * & B_{d0}^T P_1 B_{d0} + D_{d0}^T P_2 D_{d0} - P_2 \end{bmatrix} < 0. \tag{6}$$

Then, the linear repetitive process (3) *is stable.*

Proof of Theorem 1. Equation (6) can be written as

$$\begin{bmatrix} A_d & B_{d0} \\ C_d & D_{d0} \end{bmatrix}^T \begin{bmatrix} P_1 & 0 \\ 0 & -P_1 \end{bmatrix} \begin{bmatrix} A_d & B_{d0} \\ C_d & D_{d0} \end{bmatrix} + \Theta < 0 \tag{7}$$

where $\Theta = \begin{bmatrix} C_d & D_{d0} \\ 0 & I \end{bmatrix}^T \begin{bmatrix} P_2 & 0 \\ 0 & -P_2 \end{bmatrix} \begin{bmatrix} C_d & D_{d0} \\ 0 & I \end{bmatrix}$. From Lemma 2, we can obtain

$$\left[\left(\begin{matrix} \left(e^{j\omega} I - A_d\right)^{-1} B_{d0} \\ I \end{matrix} \right]^T \Theta \begin{bmatrix} \left(e^{j\omega} I - A_d\right)^{-1} B_{d0}, \\ I \end{bmatrix} < 0, \tag{8}$$

which is

$$\begin{bmatrix} G(e^{j\omega}) \\ I \end{bmatrix}^T \begin{bmatrix} P_2 & 0 \\ 0 & -P_2 \end{bmatrix} \begin{bmatrix} G(e^{j\omega}) \\ I \end{bmatrix} < 0. \tag{9}$$

The $G(e^{j\omega})$ is the frequency response matrix of transfer function matrix $G(z)$ in Lemma 1, and then we can obtain

$$G\left(e^{j\omega}\right)^T P_2 G\left(e^{j\omega}\right) - P_2 < 0. \tag{10}$$

When $P_2 > 0$, for $\omega \in R$, there is $\rho(G(e^{j\omega})) < 1$, which satisfies condition (iii) of Lemma 1. In addition, from (6), we find

$$\begin{cases} A_d^T P_1 A_d + C_d^T P_2 C_d - P_1 < 0 \\ B_{d0}^T P_1 B_{d0} + D_{d0}^T P_2 D_{d0} - P_2 < 0 \end{cases}. \tag{11}$$

As $P_1 > 0, P_2 > 0$, the below inequalities $C_d^T P_2 C_d \geq 0, B_{d0}^T P_1 B_{d0} \geq 0$ hold, and then the following inequality is also true

$$\begin{cases} A_d^T P_1 A_d - P_1 < 0 \\ D_{d0}^T P_2 D_{d0} - P_2 < 0 \end{cases} \tag{12}$$

where $\rho(A_d) < 1, \rho(D_{d0}) < 1$, condition (ii) and (i) of Lemma 1 are also satisfied, respectively. □

4. Design of a Dynamic Iterative Learning Control System

To obtain the form of (6) to ensure system (3) is stable, we propose Theorems 2 and 3 for dynamic ILC algorithms in systems with different relative degrees.

Lemma 3 (The Projection Theorem [34]). *Let Ψ, Λ and Σ be real matrices of appropriate dimension where $\Psi = \Psi^T$, and there is a matrix W that holds*

$$\Psi + \Lambda^T W \sum + \sum^T W^T \Lambda < 0 \tag{13}$$

if and only if the following inequalities hold:

$$\begin{cases} \Lambda^{\perp T} \Psi \Lambda^{\perp} < 0 \\ \sum^{\perp T} \Psi \sum^{\perp} < 0 \end{cases}. \tag{14}$$

Lemma 4 ([35]). *Suppose that W, L and V are given matrices of appropriate dimension. W and V are positive definite, and then $L^T V L - W < 0$ is equivalent to*

$$\begin{bmatrix} -W & L^T \\ L & -V^{-1} \end{bmatrix} < 0 \tag{15}$$

or

$$\begin{bmatrix} -V^{-1} & L \\ L^T & -W \end{bmatrix} < 0. \tag{16}$$

For system (1), the below ILC law is defined as:

$$u(t,k) = u(t,k-1) + \Delta u(t,k) \tag{17}$$

where $u(t,0)$ is the initial value of iterative control, and $\Delta u(t,k)$ is the iteration learning update law, which is used to update the control system input signal. We simultaneously define

$$\delta_k(f(t,k)) = f(t,k) - f(t,k-1) \tag{18}$$

where $\delta_k(f(t,k))$ represents the difference of $f(t,k)$ along batch axis.

4.1. Zero Relativity ($r = 0$)

Considering the relativity of system (1), there is $D \neq 0$. The state equation of a dynamic iterative learning controller is as follows:

$$x_c(t+1,k) = A_c x_c(t,k) + B_c \delta_k(x(t,k)) + D_c e(t,k-1) \tag{19}$$

where $x_c(t,k)$ is the dynamic ILC state vector. The below dynamic ILC update law is adopted as follows:

$$\Delta u(t,k) = K_1 \delta_k(x(t,k)) + K_2 x_c(t,k) + K_3 e(t,k-1) \tag{20}$$

where the corresponding static ILC update law is $\Delta u(t,k) = L_1 \delta_k(x(t,k)) + L_2 e(t,k-1)$. In order to rewrite the dynamic ILC system into a discrete linear repetitive process, define

$$\mathrm{X}(t,k) = \begin{bmatrix} \delta_k(x(t,k)) \\ x_c(t,k) \end{bmatrix}. \tag{21}$$

Then, a 2D dynamic model is obtained as follows:

$$\begin{cases} \mathrm{X}(t+1,k) = A_1 \mathrm{X}(t,k) + B_1 e(t,k-1) \\ e(t,k) = C_1 \mathrm{X}(t,k) + D_1 e(t,k-1) \end{cases} \tag{22}$$

where $A_1 = \begin{bmatrix} A+BK_1 & BK_2 \\ B_c & A_c \end{bmatrix}$, $B_1 = \begin{bmatrix} BK_3 \\ D_c \end{bmatrix}$, $C_1 = \begin{bmatrix} -C-DK_1 & -DK_2 \end{bmatrix}$, $D_1 = I - DK_3$.

Theorem 2. *Considering the 2D closed loop system (22) whose relativity $r = 0$, if positive definite symmetric matrix $Y_{11} > 0, Y_{13} > 0, Y_2 > 0$ are existing, well-dimensioned matrices W_{11}, W_{12} and W_2 and matrices $N_1, N_2, N_3, N_{A_c}, N_{B_c}$ and N_{D_c} make the following linear matrix inequalities hold*

$$\begin{bmatrix} -Y_{11} & -Y_{12} & 0 & AW_{11}+BN_1 & BN_2 & BN_3 \\ * & -Y_{13} & 0 & N_{Bc} & N_{Ac} & N_{Dc} \\ * & * & -Y_2 & -CW_{11}-DN_1 & -DN_2 & W_2 - DN_3 \\ * & * & * & Y_{11}-W_{11}-W_{11}^T & Y_{12} & 0 \\ * & * & * & * & Y_{13}-W_{12}-W_{12}^T & 0 \\ * & * & * & * & * & Y_2 - W_2 - W_2^T \end{bmatrix} < 0. \tag{23}$$

Then, the 2D linear repetitive process (22) under dynamic ILC is stable along both the time and batch axes, the system tracking error has monotonic convergence, and the system controller matrices are

$$\begin{cases} A_c = N_{Ac} W_{12}^{-1}, B_c = N_{Bc} W_{11}^{-1}, D_c = N_{Dc} W_2^{-1} \\ K_1 = N_1 W_{11}^{-1}, K_2 = N_2 W_{12}^{-1}, K_3 = N_3 W_2^{-1} \end{cases}. \tag{24}$$

Proof of Theorem 2. The inequality (23) can be rewritten to be

$$\Phi_1 + sym\left\{ \begin{bmatrix} \Pi_1 \\ -I \end{bmatrix} W \Lambda_1 \right\} < 0 \tag{25}$$

where $\Phi_1 = \begin{bmatrix} -Y_1 & 0 & 0 & 0 \\ * & -Y_2 & 0 & 0 \\ * & * & Y_1 & 0 \\ * & * & * & Y_2 \end{bmatrix}$, $Y_1 = \begin{bmatrix} Y_{11} & Y_{12} \\ * & Y_{13} \end{bmatrix}$, $\Pi_1 = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}$, $\Lambda_1 = \begin{bmatrix} 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}$, $W = \mathrm{diag}\{W_1, W_2\}$, $W_1 = \mathrm{diag}\{W_{11}, W_{12}\}$ and take

$\Lambda_1^{\perp} = \begin{bmatrix} I & I & 0 & 0 \end{bmatrix}^T$. If inequality (25) is true, according to Lemma 3, the following linear matrix inequalities are also true

$$\begin{cases} \begin{bmatrix} I & \Pi_1 \end{bmatrix} \Phi_1 \begin{bmatrix} I \\ \Pi_1^T \end{bmatrix} < 0 \\ \left(\Lambda_1^{\perp}\right)^T \Phi_1 \Lambda_1^{\perp} < 0 \end{cases}. \tag{26}$$

In Theorem 2, according to the matrix $Y_{11} > 0, Y_{13} > 0$, we can conclude $Y_1 = \begin{bmatrix} Y_{11} & Y_{12} \\ * & Y_{13} \end{bmatrix} > 0$ holds. Then, the inequality $\left(\Lambda_1^{\perp}\right)^T \Phi_1 \Lambda_1^{\perp} =$ $\begin{bmatrix} I & I & 0 & 0 \end{bmatrix} \begin{bmatrix} -Y_1 & 0 & 0 & 0 \\ * & -Y_2 & 0 & 0 \\ * & * & Y_1 & 0 \\ * & * & * & Y_2 \end{bmatrix} \begin{bmatrix} I \\ I \\ 0 \\ 0 \end{bmatrix} = -Y_1 - Y_2 < 0$ clearly holds.

According to inequality $\begin{bmatrix} I & \Pi_1 \end{bmatrix} \Phi_1 \begin{bmatrix} I \\ \Pi_1^T \end{bmatrix} < 0$, we obtain

$$\begin{aligned} &\begin{bmatrix} I & 0 & A_1 & B_1 \\ 0 & I & C_1 & D_1 \end{bmatrix} \begin{bmatrix} -Y_1 & 0 & 0 & 0 \\ * & -Y_2 & 0 & 0 \\ * & * & Y_1 & 0 \\ * & * & * & Y_2 \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & I \\ A_1^T & C_1^T \\ B_1^T & D_1^T \end{bmatrix} = \\ &\begin{bmatrix} -Y_1 & 0 \\ * & -Y_2 \end{bmatrix} + \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} Y_1 & 0 \\ * & Y_2 \end{bmatrix} \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}^T < 0. \end{aligned} \tag{27}$$

According to Lemma 4, we find

$$\begin{bmatrix} -Y_1 & 0 & A_1 & B_1 \\ * & -Y_2 & C_1 & D_1 \\ * & * & -Y_1^{-1} & 0 \\ * & * & * & -Y_2^{-1} \end{bmatrix} < 0 \tag{28}$$

where we take $P_1 = Y_1^{-1}, P_2 = Y_2^{-1}$. Using Lemma 4 again, we can obtain the inequality in the form of (6); thus, the condition is met, and (23) holds. □

4.2. Higher Order Relativity ($r \geqslant 1$)

Considering that system (1) has high-order relativity $r \geqslant 1$, the system model satisfies $D = 0, CA^{r-1}B \neq 0, CA^iB = 0 (i = 0, \ldots, r-2)$. In order to compensate for higher order relativity, the state equation and dynamic iterative learning update law are designed as follows:

$$x_c(t+1,k) = A_c x_c(t,k) + B_c \delta_k(x(t,k)) + D_c e(t+r,k-1) \tag{29}$$

$$\Delta u(t,k) = K_1 \delta_k(x(t,k)) + K_2 x_c(t,k) + K_3 e(t+r,k-1). \tag{30}$$

The static iterative learning update law for high relativity is $\Delta u(t,k) = L_1 \delta_k(x(t,k)) + L_2 e(t+r,k-1)$, and thus we can find

$$\delta_k(x(t+1,k)) = (A + BK_1)\delta_k(x(t,k)) + BK_2 x_c(t,k) + BK_3 e(t+r,k-1) \tag{31}$$

and then

$$\begin{aligned} e(t+r,k) &= e(t+r,k-1) - C\delta_k(x(t+r,k)) \\ &= e(t+r,k-1) - C(A+BK_1)\delta_k(x(t+r-1,k)) - CBK_2 x_c(t+r-1,k) - CBK_3 e(t+2r-1,k-1) \\ &= e(t+r,k-1) - CA[(A+BK_1)\delta_k(x(t+r-2,k)) + BK_2 x_c(t+r-2,k) + BK_3 e(t+2r-2,k-1)] \\ &= e(t+r,k-1) - CA^2\delta_k(x(t+r-2,k)) = \cdots = e(t+r,k-1) - CA^{r-1}\delta_k(x(t+1,k)) \\ &= e(t+r,k-1) - CA^{r-1}[(A+BK_1)\delta_k(x(t,k)) + BK_2 x_c(t,k) + BK_3 e(t+r,k-1)] \\ &= -CA^{r-1}(A+BK_1)\delta_k(x(t,k)) - CA^{r-1}BK_2 x_c(t,k) + (I - CA^{r-1}BK_3\big)e(t+r,k-1). \end{aligned} \tag{32}$$

The corresponding 2D dynamic mode is

$$\begin{cases} X(t+1,k) = A_2X(t,k) + B_2e(t+r,k-1) \\ e(t+r,k) = C_2X(t,k) + D_2e(t+r,k-1) \end{cases} \tag{33}$$

where $X(t,k) = \begin{bmatrix} \delta_k(x(t,k)) \\ x_c(t,k) \end{bmatrix}$, $A_2 = \begin{bmatrix} A+BK_1 & BK_2 \\ B_c & A_c \end{bmatrix}$, $B_2 = \begin{bmatrix} BK_3 \\ D_c \end{bmatrix}$, $D_2 = I - CA^{r-1}BK_3$, $C_2 = \begin{bmatrix} -CA^{r-1}(A+BK_1) & -CA^{r-1}BK_2 \end{bmatrix}$.

Theorem 3. *Considering the 2D closed loop system (22) whose relativity $r \geq 1$, if positive definite symmetric matrices $Y_{11} > 0, Y_{13} > 0, Y_2 > 0$ are existing, well-dimensioned matrices W_{11}, W_{13} and W_2 and matrices $N_1, N_2, N_3, N_{A_c}, N_{B_c}$ and N_{D_c} make the following linear matrix inequalities hold*

$$\begin{bmatrix} -Y_{11} & -Y_{12} & 0 & AW_{11}+BN_1 & BN_2 & BN_3 \\ * & -Y_{13} & 0 & N_{Bc} & N_{Ac} & N_{Dc} \\ * & * & -Y_2 & -CA^rW_{11}-CA^{r-1}BN_1 & -CA^{r-1}BN_2 & W_2-CA^{r-1}BN_3 \\ * & * & * & Y_{11}-W_{11}-W_{11}^T & Y_{12} & 0 \\ * & * & * & * & Y_{13}-W_{12}-W_{12}^T & 0 \\ * & * & * & * & * & Y_2-W_2-W_2^T \end{bmatrix} < 0. \tag{34}$$

Then, the 2D linear repetitive process (22) under dynamic ILC is stable along both the time and batch axes, the system tracking error has monotonic convergence, and the controller matrix of the system is

$$\begin{cases} A_c = N_{Ac}W_{12}^{-1}, B_c = N_{Bc}W_{11}^{-1}, D_c = N_{Dc}W_2^{-1} \\ K_1 = N_1W_{11}^{-1}, K_2 = N_2W_{12}^{-1}, K_3 = N_3W_2^{-1} \end{cases}. \tag{35}$$

Proof of Theorem 3. The proving process is same with Theorem 2 and, thus, is omitted here. □

There are two main steps to implement the dynamic ILC for linear repetitive process: step 1 is to use the LMI toolbox to obtain the dynamic ILC law factors $K1$, $K2$ and $K3$ shown in Algorithm 1, and step 2 is to iterate the input according to the dynamic learning control law along both the batch and time axes as demonstrated in Algorithm 2.

Algorithm 1 Linear repetitive process dynamic ILC law calculation

Input: System array A, B, C, D, relative degree r, select dynamic or static ILC law.
Output: Dynamic ILC law factor $K1$, $K2$ and $K3$ in (20).

1: **switch** relative degree **r**.
2: **r = 0**.
3: Substitute known matrices A, B, C, D and r into (23) of Theorem 2 to obtain matrices $N_1, N_2, N_3, N_{A_c}, N_{B_c}, N_{D_c}, W_{11}, W_{12}$ and W_2.
4: Obtain dynamic ILC law $K1$, $K2$ and $K3$ according to (24) of Theorem 2.
5: **r >= 1**.
6: Substitute known matrices A, B, C, D and r into (34) of Theorem 3 to obtain matrices $N_1, N_2, N_3, N_{A_c}, N_{B_c}, N_{D_c}, W_{11}, W_{12}$ and W_2.
7: Obtain dynamic ILC law $K1$, $K2$ and $K3$ according to (35) of Theorem 3.
8: **end.**
9: **return** $K1$, $K2$ and $K3$.

Algorithm 2 Dynamic ILC for linear repetitive process algorithm

Input: System array A, B, C and D; relative degree r; simulation duration T; simulation time step Ts; initial input signal u_0; target trajectory $y_d(t)$; and dynamic ILC law factor $K1$, $K2$ and $K3$ from Algorithm 1

Output: System input u^* for each iteration, actual system output y^*

1: **initialization:** Iteration times $k = 0$, simulation time $t = 0$, $\|e_0\| > \epsilon$
2: Send the initial input signal u_0, calculate the state vector X_{c0} according to (19), obtain the output y_0 and corresponding error e_0
3: Record input u_0, state vector X_{c0} and initial error e_0
4: **while not** $\|e_k\| < \epsilon$ **do**
5: Set $k = k + 1$ for next iteration.
6: **while not** $t > T$ **do**
7: Set $t = t + Ts$ to perform the next calculation.
8: Obtain and record state vector $X_c(t, k)$ according to (19)
9: Update the input signal $u(t, k)$ by $u(t, k) = u(t, k-1) + \Delta u(t, k-1)$, $\Delta u(t, k-1)$ is obtained from (20).
10: Send and the input signal $u(t, k)$ to the simulation model and measure the output trajectory $y(t, k)$.
11: Obtain and record state vector $X(t, k)$ according to (22)
12: Record the input signal $u(t, k)$ and output trajectory error $e(t, k)$ according to (2).
13: **end while**
14: **end while**
15: **return** $u^* = u(t, k)$ and $y^* = y(t, k)$

5. Simulation Results

To verify the proposed dynamic ILC algorithm effectiveness, simulations were performed in MATLAB with the LMI toolbox for the systems with different relative degrees. For reasons of evaluating the system tracking error, the root mean square error is introduced as follows:

$$RMS(k) = \sqrt{\frac{1}{T}\sum_{t=1}^{T} e^2(k, t)}. \tag{36}$$

5.1. Condition 1: Relativity $r = 0$

Considering the system relativity $r = 0$, the coefficient of its state equation is $A = \begin{bmatrix} 1 & 0.2 \\ 0.3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0.1 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 0.5 & 0.2 \end{bmatrix}$, $D = 1$. Suppose that each iteration length of the system is $T = 2s$, and the expected trajectory is as follows:

$$y_d(t) = \sin(\pi t) + 0.25\sin(2\pi t). \tag{37}$$

Using LMI (23) and (24) in Theorem 2, we can obtain

$$\begin{cases} A_c = -0.5491, B_c = \begin{bmatrix} -0.0144 & 0.1329 \end{bmatrix}, D_c = -0.1704 \\ K_1 = \begin{bmatrix} -0.4874 & -0.3796 \end{bmatrix}, K_2 = -0.2790, K_3 = 0.5392 \end{cases}. \tag{38}$$

To further explore the advantages of dynamic ILC, we consider the corresponding static ILC under the same conditions, and the system law is

$$u(t, k) = u(t, k-1) + L_1\delta_k(x(t, k)) + L_2 e(t, k-1). \tag{39}$$

Using the same LMI solution, parameters of static ILC can be obtained as

$$\begin{cases} L_1 = \begin{bmatrix} -0.4913 & -0.3271 \end{bmatrix} \\ L_2 = 0.3687 \end{cases}. \tag{40}$$

The simulation results are shown in Figures 1 and 2.

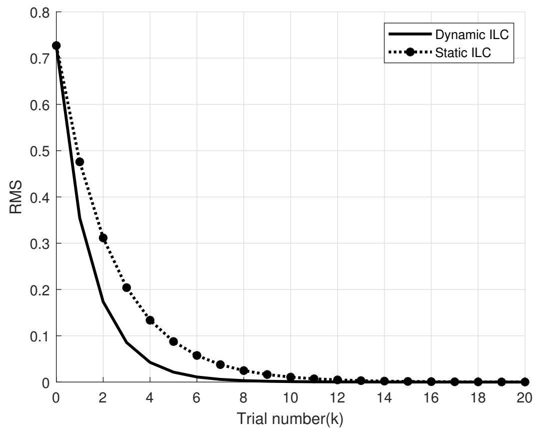

Figure 1. The mean square error trajectory at relative degree $r = 0$.

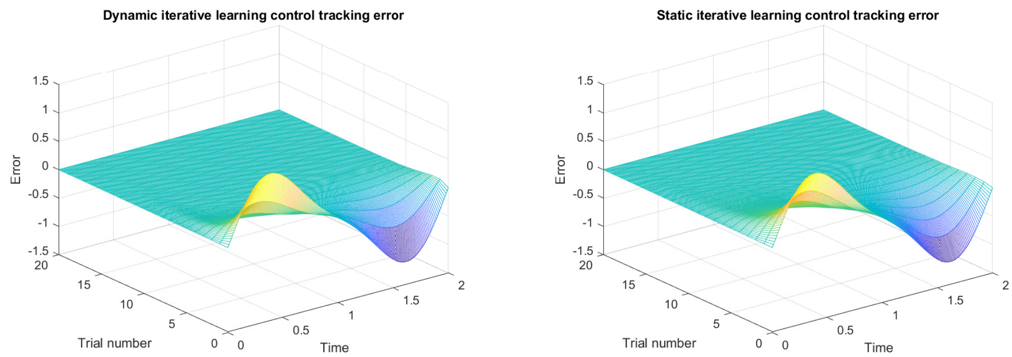

Figure 2. The tracking error curve of ILC at relative degree $r = 0$.

5.2. Condition 2: Relativity $r = 1$

Considering the system relativity $r = 1$, taking the injection speed control of the injection molding process in [35] as the simulation objective, the constant matrix of its state equation is given as follows: $A = \begin{bmatrix} 1.582 & -0.5916 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $C = \begin{bmatrix} 1.69 & 1.419 \end{bmatrix}$. Define the expected trajectory as

$$y_d(t) = \begin{cases} \sin(0.01\pi t), 0 \le t < 200 & \\ 1 & , 200 \le t < 300 \\ 4 - 0.01t & , 300 \le t < 400 \end{cases}. \tag{41}$$

Using LMI (34) and (35) in Theorem 3, we can obtain

$$\begin{cases} A_c = -0.5884, B_c = [-0.2074 - 0.0032], D_c = -0.4093 \\ K_1 = \begin{bmatrix} -2.2461 & -0.5908 \end{bmatrix}, K_2 = -0.0752, K_3 = 0.3912 \end{cases}. \tag{42}$$

The static ILC law under the same conditions is

$$u(t,k) = u(t,k-1) + L_1\delta_k(x(t,k)) + L_2 e(t+1,k-1). \tag{43}$$

The static ILC controller parameters are obtained using the same LMI solution method as follows:

$$\begin{cases} L_1 = \begin{bmatrix} -2.2058 & -0.5915 \end{bmatrix} \\ L_2 = 0.3185 \end{cases}. \tag{44}$$

The simulation results are shown in Figures 3 and 4.

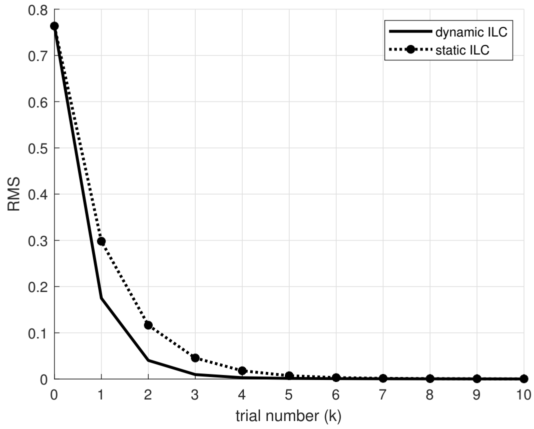

Figure 3. The mean square error trajectory at relative degree $r = 0$.

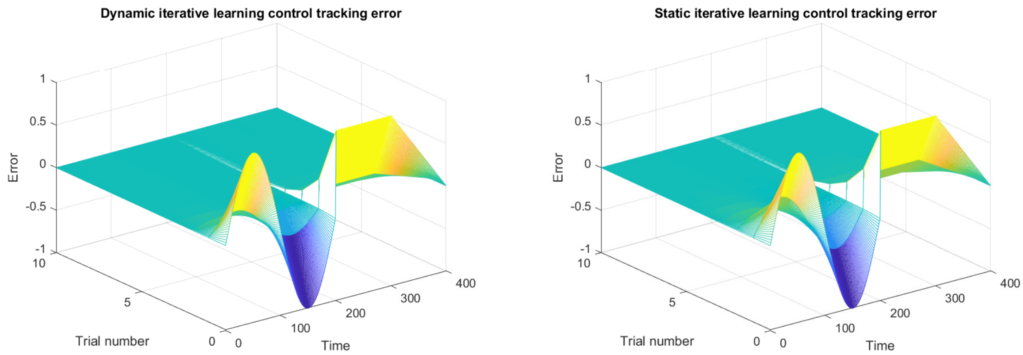

Figure 4. The tracking error curve of ILC at relative degree $r = 1$.

5.3. Condition 3: Relativity $r = 2$

Considering the system relativity $r = 2$, the selected state space model parameters are as follows: $A = \begin{bmatrix} 2 & -0.25 \\ 1 & -0.35 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 2 & 0 \end{bmatrix}$. Suppose that each iteration length of the system is $T = 2s$ and the sampling time $T_s = 0.01s$, then the expected trajectory is as follows:

$$y_d(t) = t^2(1 - 0.5t). \tag{45}$$

The controller parameters can be obtained as

$$\begin{cases} A_c = -0.5880, B_c = [-0.1931 - 0.1367], D_c = -0.4132 \\ K_1 = \begin{bmatrix} -3.5641 & -1.5628 \end{bmatrix}, K_2 = -0.0509, K_3 = 0.3217 \end{cases}. \tag{46}$$

The static ILC law under the same conditions is

$$u(t,k) = u(t,k-1) + L_1\delta_k(x(t,k)) + L_2 e(t+2,k-1). \tag{47}$$

The parameters of the static ILC controller are obtained

$$\begin{cases} L_1 = \begin{bmatrix} -3.5690 & -1.5604 \end{bmatrix} \\ L_2 = 0.2709 \end{cases}. \tag{48}$$

The simulation results are shown in Figures 5 and 6.

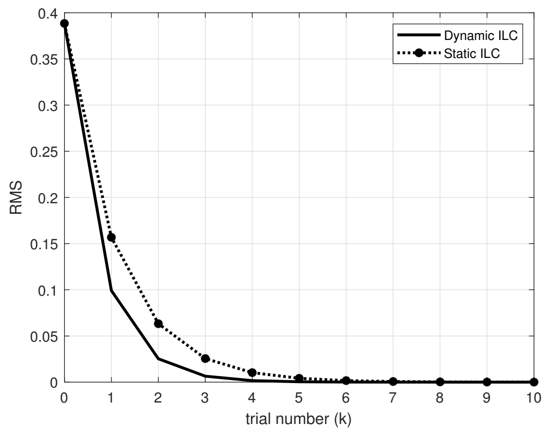

Figure 5. The mean square error trajectory at relative degree $r = 2$.

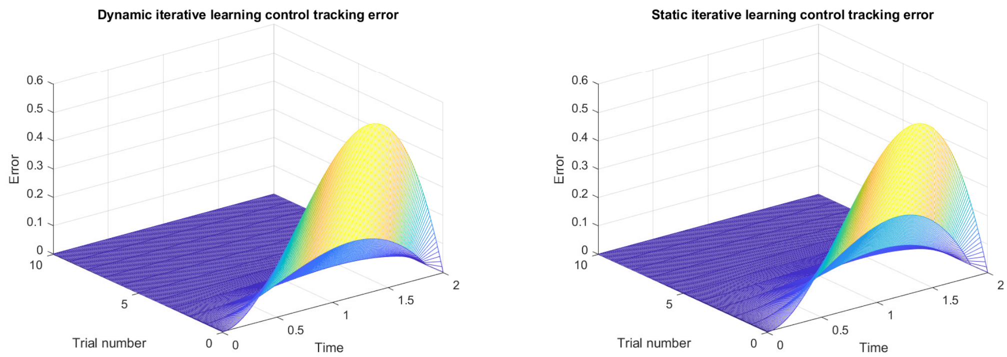

Figure 6. The tracking error curve of ILC at relative degree $r = 2$.

The simulation results show that the actual system output gradually tracks the desired trajectory along an iterative axis, and the system remains stable under the dynamic ILC. Under the same conditions, compared with the static iterative learning controller, the control system using a dynamic iterative learning controller can achieve a better convergence effect.

6. Conclusions and Future Work

The monotonic convergence problem of dynamic ILC systems was studied in this paper in discrete linear repetitive processes with different relative degrees. First, the 2D discrete Roesser model of ILC system was established based on 2D system theory; secondly, the stability theory of linear repetitive processes was used to analyze the system monotone convergence, and sufficient conditions for the controller existence were given to simultaneously guarantee the dynamic performance; finally, the static controller and dynamic controller were simulated under different conditions, and the comparison showed the effectiveness of the dynamic ILC design schemes.

However, we know that the system matrix A, B, C and D has difficulty maintaining accuracy for actual objects, and the system always has disturbances. For certain conditions, the dynamic iterative learning controller proposed in this paper is not suitable. Additionally, the ILC initial state issue studied in [36–38] is also critical for dynamic ILC algorithms, and thus how to eliminate the impacts of parameter uncertainty, disturbances and the initial state issue is one of our future jobs. Our subsequent research will also consider the actuator fault diagnosis and the reaction method, which is a category of fault-tolerant control.

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