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Optimization of the Cognitive Processes in a Virtual Classroom: A Multi-objective Integer Linear Programming Approach

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Abstract: A fundamental problem in the design of a classroom is to identify what characteristics it should have in order to optimize learning. This is a complex problem because learning is a construct related to several cognitive processes. The aim of this study is to maximize learning, represented by the processes of attention, memory, and preference, depending on six classroom parameters: height, width, color hue, color saturation, color temperature, and illuminance. Multi-objective integer linear programming with three objective functions and 56 binary variables was used to solve this optimization problem. Virtual reality tools were used to gather the data; novel software was used to create variations of virtual classrooms for a sample of 112 students. Using an interactive method, more than 4700 integer linear programming problems were optimally solved to obtain 13 efficient solutions to the multi-objective problem, which allowed the decision maker to analyze all the information and make a final choice. The results showed that achieving the best cognitive processing performance involves using different classroom configurations. The use of a multi-objective interactive approach is interesting because in human behavioral studies, it is important to consider the judgement of an expert in order to make decisions.

Keywords: optimization; multi-objective integer linear programming; classroom design; cognitive learning processes

MSC: 90C90; 90C29



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1. Introduction

Multi-objective linear programming (MOLP) is a mathematical model in which two or more conflicting linear objective functions, which dependent on variables subject to certain linear constraints, are optimized simultaneously [1–4]. MOLP has several variations; in particular, if all variables must be integers, the model is known as the multi-objective integer linear programming (MOILP), which will be the fundamental tool for the development of the study presented here.

Given the MOILP problem $Maximize\{Cx : Ax \ge b, x \ge 0 \text{ and integer}\}$, a feasible solution x' is efficient if there is no other feasible solution x such that $Cx' \le Cx$ with at least one strict inequality. In this case, the objective vector Cx' is called non-dominated. It is highly unlikely that a MOILP problem will have an optimal solution, and therefore, solving a MOILP problem generally entails identifying the set of its efficient solutions.

There are several algorithms to solve a MOILP problem. The very recent survey by Halffmann et al. [5] presents all the exact algorithms known up to that date but also gives references on approximate algorithms. To solve a MOILP problem, scalarization methods, which, as their name indicates, turn such a problem into solving single-objective

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integer linear programming problems, play a crucial role in finding all or a subset of the non-dominated objective vectors. Note that a single-objective integer linear programming problem is known in the literature as an integer linear programming (ILP) problem [6–10].

In this work, the scalarization known as the weighted sum is used [11]. This basically consists of solving ILP problems where the objective functions are convex combinations of the objective functions in the original MOILP problem, with the same restrictions ($Ax \ge b$, $x \ge 0$ and integer) plus $Cx \ge g$, where g is a row vector of objective bounds. Each optimal solution to an ILP problem with the above conditions is an efficient solution to the MOILP problem.

Therefore, a method that generates the whole set, or a large subset, of non-dominated objective vectors may require an excessive amount of computational resources, which may make it inappropriate for dealing with large problems. Another approach is to use the so-called interactive methods [3], which basically consist of phases of human intervention alternated with phases of computation. Human intervention is carried out by a decision maker (DM), which is a domain expert who can provide preferences toward solutions and select the most preferred one for implementation. It is essential that, on interactive methods, the computational effort is not too high in the computation phase and that the questions asked to the DM are simple and understandable. In these procedures, the DM makes the final choices.

In this work, an interactive method based on a MOILP weighted-sum scalarization is used to examine how classroom design affects cognitive processes. The objective of the study is to maximize learning (represented by memory, attention, and preference) from six classroom parameters (height, width, color hue, color saturation, color temperature, and illuminance).

MOLP has already been used to model problems in the field of human behavior, for instance, in psychology, for item calibration/selection in psychometrics [12]. Additionally, very recently, González-Gallardo et al. [13] used a variant of MOLP known as interval multi-objective linear programming (e.g., see Oliveira and Antunes [14]) to analyze the well-being of students in Spain and Finland. Specifically, the main purpose was to study how four indicators (positive feelings, motivation, sense of belonging, and bullying) could be simultaneously improved while taking into account the particularities of both countries. This variation of the MOLP has also been used in order to simultaneously achieve a balanced performance in four measures of academic achievement from their use of the internet [15].

To solve the problem presented here, the help of virtual reality (VR) will be essential because VR allows researchers to overcome the difficulties, including cost, of using real spaces to study any one contextual key (keeping the others unchanged) in a controlled manner. VR can simulate different configurations of classroom characteristics. In recent years, VR has gained particular importance, with 2020 being the turning point at the international level due to the health situation. An increasing body of literature has validated the use of VR [16,17] for perception studies [18] and, specifically, environmental psychology studies [19,20]. Some authors [21–23] have concluded that VR is an efficient tool for measuring attention performance.

To collect the data, the subjects were presented with variations of a VR-based replica of a representative classroom at the Polytechnic University of Valencia. Six classroom parameters were varied: wall height and width, color hue, saturation and temperature, and lighting. Three objective functions were analyzed: memory, attention, and environmental preference. The solutions produced different cognitively efficient design configurations (in terms of memory and attention performance and environmental preference) that allowed the DM to analyze all the information and make a final choice. This is the fundamental contribution of this analytical methodology to the area of human behavioral studies, where statistical approaches have traditionally been applied. MOILP allows researchers to identify combinations of classroom design parameters that can simultaneously optimize cognitive processes taking into account the judgement of an expert.

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The remainder of this work is organized as follows: Section 2 exhaustively reviews the existing literature about the influence of the environment on cognitive processes, including an analysis of how previous studies relate to this work. Section 3 explains the data collection process (procedure, conditions, software used for the VR, etc.). Section 4 presents the mathematical model, that is, the MOILP approach used to optimize the three functions cited above. Section 5 presents the results, which are discussed in Section 6, and finally, some conclusions and possible future research directions are provided in Section 7.

2. Related Cognitive Processing and Environment Works

Over the last decades, increasing attention has been paid to the influence of the environment on human beings at the cognitive–emotional level. Several studies have analyzed the effects of environmental characteristics on mental states and cognitive processes [24–27]. Ulrich's [27] pioneering study found that patients suffered less stress and enjoyed improved recovery in post-surgical scenarios when the windows of their rooms looked out onto natural vistas. Over the years, studies such as these have examined clinical populations and real environments and expanded into other contexts, such as education.

The study of the effects of classroom environments on experimental subjects must address two fundamental questions: first, how to assess improvements in learning, which can be understood as an active mental process of acquisition, retrieval, and use of information [28,29], and second, which classroom characteristics influence this process?

Taking the first question, learning is a complex psychological construct. There is no consensus in the scientific community on its measurement: some studies focus on completing general tasks and others on specific tests of fundamental learning-related processes, such as attention and memory. Attention is a cognitive process that captures information from the environment [30], and memory stores it [31]. The study of attention and memory involves performing specific tasks in specific environments. The nature of the tasks is important because it has been shown that the neuronal load involved in concentration prevents individuals becoming distracted [32,33]. Thus, easier tasks that require less concentration make people pay more attention to their environments. Another line of work, however, has examined subjects' preferences for environments. Perceptions of one's environment are important for two reasons: (1) There is a strong positive correlation between subjects' preferences and improvements in their mood [34]; and (2) a brief, positive mood improvement enhances performance in short- and mediumterm learning tasks [35]. That is, one's environment influences one's mood [36], which, in turn, acts as a cognitive mediator, which results in positive perceptions, which strongly improve student performance [37].

As to the second question, on classroom characteristics, it has been shown that when plants are present in the environment, student performance improves [38–40] and that the arrangement of furniture in classrooms influences teachers' behaviors [41], in-class teaching methodologies [42,43], and how students interact [43–45]. However, the central theme has been the influence of the built space on student learning. Thus, Marchand et al. [46] recently discovered that in a space classified as uncomfortable (temperature of 26.67 °C, lighting of 2500 lx, and ambient sound of 60–65 dBA), university students scored lower in a reading comprehension test than students in a comfortable space (22 °C, 500 lx, 35 dBA); and it has been shown that a green-colored wall simulating vegetation did not improve subjects' performance in a specific test of sustained and selective attention in contrast to the effects of real vegetation found in a school classroom-based longitudinal study [47]. However, most studies have analyzed the visual characteristics of classrooms.

The scientific literature highlights, among others, the visual characteristics of classroom color, lighting, and dimensions. These coincide with three of the seven characteristics of the built environment that have been shown to most influence the progress of primary school students [48]. As for color, it has been shown that in chromatic spaces, fewer errors are made in text correction tasks [49] and that tasks are performed quicker [50]. It has also been shown that cold tones improve the performance of complex tasks [51,52] and enhance the

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cognitive processes, such as attention and memory, of university students [53]. In addition, it has been shown that the contextual key of lighting influences student learning and performance [54]. For example, it has been observed that greater illuminance is associated with greater attention [55,56]. In another line of work, Huiberts [57] observed that very bright, direct light improved the retrieval of numerical information in easy memory tasks, while more muted lighting was better when retrieving similar information in more difficult memory tasks. It has also been observed that higher color-temperature lighting (the light spectrum emitted by a black body heated to a certain temperature) produces faster cognitive processing speed and higher concentration [58] and better attention and memory task performance [56]. As for dimensions, it has been observed that lower ceilings promote cooperation in school classrooms [59] and that smaller classrooms are associated with better performance and higher arousal [60]. In any case, while classroom dimensions have featured in many studies [61–63], they have not been their central focus; thus, there are few relevant conclusive results.

Although many studies have been undertaken in this area, they have all examined the individual characteristics of the classroom and cognitive processes in isolation, ignoring the impact they may have on each other. Classrooms have numerous characteristics, and it is possible that the results obtained from isolated analyses will differ from results obtained from combinations of characteristics. Thus, a good result in terms of lighting may be poorer when it is combined with a certain color or ceiling height, which might make subjects perceive the environment as a whole differently. The existing literature has not examined the effects of different combinations of classroom characteristics. In addition, researchers should simultaneously examine the different cognitive processes underlying learning, as some classroom characteristics might generate positive effects in one cognitive process and negative in others. Thus, for example, a study conducted with university students showed that low illuminance levels (100 lx-200 lx) improved memory but reduced attention [56,64]. An investigation into color showed that the subjects preferred blue and yellow tones, but these were associated with the poorest results in reading comprehension tasks [65]. Thus, classroom designs must optimize the set of these cognitive processes. Table 1 summarizes studies carried out into classrooms, detailing some relevant aspects.

Table 1. Summary of studies of human behavior in classrooms.

Reference	Classroom Design Parameters	Experience Register Behavioral	Experience Methods
Ahrentzen and Evans, 1984	Interior Spaciousness; Degree of Open Perimeter and Amenities	Distraction; Privacy	RC ³
Wheldall and Lam, 1987	Seating Arrangements	Seating Arrangements Classroom Disruption Rate; Task Behavior; Teacher Behavior	
Jago and Tanner, 1999	Lighting; Color	Academic Progress	RC ³
Read, Sugawara, and Brandt, 1999	Ceiling Height; Wall Color	Cooperative Behavior	RC ³
Wannarka Ruhl, 2008	Seating Arrangements	Attention; Instructional Time	RC ³
Doxey, Waliczek, and Zajicek, 2009	Plants	Cognitive Performance; Perception	RC ³
Daly, Burchett, and Torpy, 2010	Plants Classroom Performance		RC ³
Yang, Becerik-Gerber, and Mino, 2013	Temperature; Air quality; Artificial and Natural Lighting; Acoustics; Visibility; Room Layout; Furniture; Hardware and Software.	Satisfaction; Performance	RC ³
Park and Choi, 2014	Seating Arrangements	Motivation; Participation	RC ³
Marchand, Nardi, Reynolds, et al., 2014	Lighting; Sound; Temperature	Student Learning; Mood; Environmental Perception	RC ³

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Table 1. Cont.

Reference	Classroom Design Parameters	Experience Register Behavioral	Experience Methods
Smolders and de Kort, 2014	Lighting (Bright Light)	Alertness; Vitality; Performance and Physiological Arousal (HRV ¹ and Electrodermal Activity)	ER ⁴
Keis, Helbig, Streb, et al., 2014	Lighting (Blue-enriched White Light vs. Standard Lighting)	Speed of Cognitive Processing; Concentration Performance; Visuospatial and Verbal Memory	RC ³
Barrett, Davies, Zhang, et al., 2015	Lighting; Temperature; Air quality; Ownership; Flexibility; Complexity; Color	Academic Progress	RC ³
Huiberts, Smolders, and de Kort, 2015	Lighting (Illuminance Level and Bright Light)	Working Memory	ER ⁴
Xia, Song, Wang, et al., 2016	Color	Cognitive Task Performance	RC ³
Al-Ayash, Kane, Smith, et al., 2016	Wall Color	Reading Task Performance; Emotional Responses; Neurophysiological (HRV ¹)	ER ⁴
Van den Berg, Wesselius, Maas, et al., 2017	Green Walls vs. Plants	Cognitive Performance; Well-being	RC ³
Shernoff, Sannella, Schorr, et al., 2017	Seating Location	Student Engagement; Attention	RC ³
Baum, 2018	Node Classroom vs. Spoke Classroom (Seating Arrangement; Lighting; Audio-visual and Computing equipment)	Classroom Activity; Student Attitude	RC ³
Bernardo, Loupa-Ramos, Matos, et al., 2021	Plants	Sustained and Selective Attention; Working Memory	RC ³
Llinares, Higuera-Trujillo, and Serra, 2021	Wall color	Attention; Memory; Neurophy-siological (HRV ¹ and EEG ²)	VR
Llinares, Castilla, and Higuera-Trujillo, 2021	Lighting (Illuminance; CCT)	Attention; Memory	VR
Llinares, Higuera-Trujillo, Montañana i Aviñó, et al., 2021	Classroom Width	Attention; Memory; Neurophy-siological (HRV ¹ and EEG ²)	VR

 $^{^{\}rm 1}$ Heart rate variability. $^{\rm 2}$ Electroencephalogram. $^{\rm 3}$ Real classroom. $^{\rm 4}$ Experimental room.

The present study analyzes the simultaneous effects of different classroom characteristics on different cognitive processes. That is, an assessment is made of which design configurations of a university classroom (combining the characteristics lighting, color, and size) enhance the set of significant cognitive processes in learning, memory, attention, and preference). To do so, MOILP is used as an analytical tool and VR as an environmental-simulation tool.

3. Materials and Methods

3.1. Participants

The model used data collected from the Polytechnic University of Valencia student population. A total of 112 students participated (50.9% men, 49.1% women, mean age 23.24 years, standard deviation 3.79). To avoid the physical problems associated with VR glasses and to control cultural influences, the subjects were required to: (1) have good vision without glasses (they could wear contact lenses) and (2) be Spanish nationals. The

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procedure, including its non-invasive techniques, was explained to the participants, and they signed the appropriate informed consent form.

3.2. Procedure

The study procedure is shown in Figure 1. In the first phase, a VR replica of a classroom at the Polytechnic University of Valencia was used as the base. Each participant viewed 5 randomly physically modified base classrooms. Each view was modified in the parameters of only one of the classroom characteristics: (a) wall color (hue and saturation); (b) interior lighting (illuminance and color temperature); and (c) dimensions (width and height of the walls). The levels of three psychological metrics and the students' sense of presence were assessed in this phase. Sense of presence is a crucial element of virtual environments. Presence can be understood as the user's illusion of "being there", in a virtual environment [66], where (s) he has lost the sense of being in an environment simulated by a technological medium, and therefore, (s) he responds as if (s) he was in the real world [67]. It has been found that sense of presence is directly related to the validity of the virtual experience [68], understood as the similarity of the simulated experience to one generated by the physical environment represented. For this reason, sense of presence is sometimes used to validate virtual experiences. A detailed description of how the data were obtained is provided in the following two subsections.

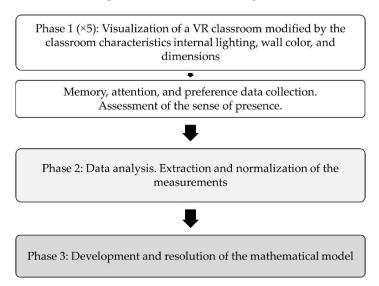


Figure 1. Schema of the study procedure.

3.3. Experimental Design

The same experimental procedure was followed for all conditions, with only one difference, namely the type of classroom each subject viewed in the VR scenario: the base classroom was modified in color, lighting, and dimension. The criteria and processes associated with manipulating these three characteristics are described below.

3.3.1. Conditions

The original characteristics of the real classroom were maintained in each virtual reality scenario, modifications being made on an individual basis only to the parameters of the classroom characteristics. A set of specific values was selected from among the possible values of the different parameters. The criteria for choosing the color, illumination, and dimensions' values were based on the equitable distribution of colors in the Itten chromatic circle [69], on the information provided by the light bulb suppliers, and on the standard measurements used in the construction of removable ceilings.

A total of 29 values for the classroom parameters were obtained: 10 for color hue, 2 for color saturation, 3 for lighting, 4 for color temperature, 4 for height, and 6 for width. These

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were used to assess which combination optimizes memory, attention, and preference. It was not feasible for the experimental procedure to examine the 5760 ($10 \times 2 \times 3 \times 4 \times 4 \times 6$) possible combinations individually. It should be taken into account that each visualization and corresponding data collection took at least 30 min, and each had to be viewed by at least 5 subjects so that the data were statistically validated. Therefore, a comprehensive analysis would take more than 14,400 working hours.

Therefore, the study was simplified by using 56 variables. The variables examined were those resulting from combinations of the two parameters that made up each contextual key. Each variable corresponds to a modification of the base classroom. Thus, each modification involved changing the values of the two parameters that made up the classroom characteristics color, lighting, and dimensions. The changes were applied separately to the two parameters of the classroom characteristics, while the remaining parameters maintained their original values.

3.3.2. Materials Set-Up

The VR classrooms were recreated using Rhinoceros Software v.5.0 and the Corona Renderer Software v.2.0 These provided high-quality modeling and rendering, respectively. ColorMunki TM software was used to translate the 10 colors studied in Munsell notation into RGB notation.

The subjects viewed the classroom simulation, which allowed them to make changes to their visual fields through head movements through a head-mounted device (HTC Vive) connected to the researcher's computer. Unity3D was used to generate the software, which allowed the researcher to show the subjects the different scenarios through the HTC Vive.

The entire experiment was carried out in the same laboratory. The participants undertook the experiment in the same time slots, and the temperature (22.5–23 $^{\circ}$ C) and sound insulation (34 dbA) were kept constant. The researcher's table and the participant's table faced each other in the center of the room.

3.4. Metrics

Data on memory, attention, and preference were collected for each visualization. All were statistically normalized. The data collection is now described.

3.4.1. Memory Task

The memory task was based on the auditory presentation of three out of a total of 16 randomized word lists. The lists were composed of 15 words in the same semantic field. Before the tests, the researcher said: "... then, you will hear a list of words. Try to remember them. Then, you will be asked to repeat them regardless of their order in a time of 30 s. This will be repeated 3 times." Thus, immediately after listening to the words, the subjects were asked to repeat those they could remember. Memory was measured by adding the number of words the subjects repeated from the 45 presented; thus, the more words remembered, the better the result. This is similar to the DRM experimental paradigm [70]. The information was collected through audio recordings, which the researcher played back to calculate the number of correct words recalled by the participants.

3.4.2. Attention Task

The attention task was based on the presentation of 3 lists of 40 sounds of 4 different types; 3 were distracting, and only 1 was objective. The subjects had 750 ms after the presentation of each sound during which to make a mouse click only to the target stimuli. Before the tests the researcher said: "... then, you will hear a series of sounds. You should react as soon as possible to this stimulus (target) by making a single mouse click and avoid clicking when you hear other sounds" (the 3 distractors). Attention was measured by calculating the average of the reaction times to the objective stimuli; thus, shorter times represented better results. This is similar to Seidman's [71] continuous auditory performance test. The information on reaction times was collected through software developed specifically for this research.

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3.4.3. Preference Task

The subjects' subjective perceptions of the environments were assessed through the degree of "I like" that they reported about their experiences of the classroom in a post-experiment written survey. This was rated through a Likert scale, from -4 to 4, addressed by the subjects at the end of their visualizations. The precise question was "Please rate your degree of agreement/disagreement with the following sentence (-4 being a high degree of disagreement, and +4 a high degree of agreement): In general, how much do you like this place?" To avoid any bias, the subjects were first told that there were no correct or incorrect answers. The question is based on that used by Galindo and Corraliza [72] to assess general preference judgements in the Spanish population. The participants gave oral answers to the survey to the interviewers, who incorporated them into a database.

3.4.4. Sense of Presence

The SUS questionnaire [73] was used to quantify sense of presence. This is a sixitem self-report, rated from 1 to 7 on a Likert-type scale. The participants completed the questionnaire at the end of each classroom-simulation experience. The scenarios all achieved reasonably high mean values (mean = 29.38, standard deviation = 8.48). Based on the evidence provided by previous studies into presence [74], it can be concluded that the classroom simulations were satisfactory and that the results obtained are similar to those that might have been obtained by modifying actual classrooms.

3.5. Data Collection Results

The objective of the problem is to maximize learning from classroom design features. Learning was represented by attention (a) and memory (m), cognitive processes directly related to learning, and preference (p), which has an indirect effect. The design features were illumination (x), dimension (y), and color (z), each of which is represented by two design parameters (ij): illuminance and color temperature for x, height and width for y, and hue color and saturation color for z. In the experimental process, these design features (x_{ij} , y_{ij} , and z_{ij}) were viewed by 10 participants, from whom a, m, and p were collected. These data were normalized, subsequently obtaining the mean values by SPSS v. 26 software. These mean values represent the levels of a, m, and p for the study population in each of the design situations. The points below provide a good understanding of the data and the formulae used in this work:

- 1. Let a_{ij}^x , a_{ij}^y , and a_{ij}^z be the means of the levels of attention obtained in the lighting, dimension, and color conditions of the base classroom, respectively, for each combination. These values are shown in column 5 in Tables 2–4, which shows the sets where i and j vary in each case.
- 2. Let m_{ij}^x , m_{ij}^y , and m_{ij}^z be the means of the levels of memory obtained in the lighting, dimensions, and color conditions of the base classroom, respectively, for each combination. These values are shown in column 4 in Tables 2–4, with the same sets for i and j cited above.
- 3. Let p_{ij}^x , p_{ij}^y , and p_{ij}^z be the means of the levels of preference obtained for the lighting, dimensions, and color conditions of the base classroom, respectively, for each combination. These values are shown in column 6 in Tables 2–4, with the same sets for i and j cited above.
- 4. Let \overline{a}^x , \overline{a}^y , \overline{a}^z , \overline{m}^x , \overline{m}^y , \overline{m}^z , \overline{p}^x , \overline{p}^y , and \overline{p}^z be the means of the values a^x_{ij} , a^y_{ij} , a^y_{ij} , m^x_{ij} , m^y_{ij} , m^z_{ij} , p^x_{ij} , and p^z_{ij} , respectively, with their respective variations being i and j. These values are shown at the end of the respective columns in Tables 2–4;

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5. Let x_{ij} , y_{ij} , and z_{ij} be the 0–1 variables whose values of 1 indicate that the classroom has lighting with illuminance of type i, color temperature of type j, dimensions with height type i, and width type j, and walls colored with hue type i, and with saturation type j, respectively. Conversely, a 0 value indicates that the classroom does not have type i lighting and j-type color temperature, dimensions of height type i and width type j, and walls colored with tone type i and saturation of type j, respectively. These variables are shown in column 3 of Tables 1–3, respectively.

Tables 2–4 provide the data obtained on illumination, dimensions, and color, respectively.

Table 2. Grouping of variables for each parameter of the classroom lighting characteristic.

Illuminance	Color Temperature	Variable	m_{ij}^x	a_{ij}^x	p_{ij}^x
	10,500 K	<i>x</i> ₁₁	-0.3573	0.4647	0.7778
F00.1	6500 K	x_{12}	0.1104	-0.2373	0.6154
500 lx	4000 K	<i>x</i> ₁₃	-0.0857	0.208	0.6429
	3000 K	x_{14}	-0.1174	0.0887	-0.3571
	10,500 K	<i>x</i> ₂₁	-0.5019	-0.1531	0.3333
200.1	6500 K	x_{22}	0.4349	-0.7734	0.0714
300 lx	4000 K	x_{23}	-0.0525	0.0225	0.011
	3000 K	x_{24}	0.2053	-0.2405	0.4615
	10,500 K	<i>x</i> ₃₁	-0.1168	0.0298	0.5714
100 lx	6500 K	x_{32}	0.5459	-0.2542	1.3846
	4000 K	x ₃₃	0.4598	0.1283	1
	3000 K	x ₃₄	-0.1188	0.2714	0.7059
			$\overline{m}^x = 0.0338$	$\overline{a}^x = -0.0371$	$\overline{p}^x = 0.5182$

Table 3. Grouping of variables for each parameter of the classroom characteristic dimensions.

Height	Width	Variable	m_{ij}^y	a_{ij}^y	p_{ij}^y
	8.4 m	y_{11}	0.2123	-0.4279	1
	6.2 m	y_{12}	0.964	0.0556	0.1333
3.2 m	6 m	y_{13}	0.3807	-0.1804	1.1429
3.2 m	4.8 m	y_{14}	-0.3999	0.2504	-0.625
	3.6 m	y_{15}	-0.1748	0.4341	-2
	2.4 m	y_{16}	-0.6995	0.6277	0
	8.4 m	y_{21}	-0.0828	0.1032	-0.0814
	6.2 m	y ₂₂	0.1614	-0.0116	0.1333
3.8 m	6 m	<i>y</i> ₂₃	-0.3945	0.5679	0.3333
3.8 m	4.8 m	y_{24}	0.0007	0.5091	-0.375
	3.6 m	y_{24}	-0.4977	-0.5824	0
	2.4 m	y_{26}	-0.0565	-0.1736	-2.1667
	8.4 m	<i>y</i> ₃₁	0.1639	-0.6203	1.1429
	6.2 m	<i>y</i> ₃₂	0.0575	-0.27	-0.1
1.1	6 m	<i>y</i> 33	-0.5299	0.0587	-1
4.4 m	4.8 m	y_{34}	0.0537	0.1453	-0.1429
	3.6 m	<i>y</i> ₃₅	-0.1942	-0.2556	-1.3333
	2.4 m	<i>y</i> ₃₆	-0.5925	0.916	-0.8333
	8.4 m	<i>y</i> ₄₁	-032256	-0.432	0.5455
	6.2 m	<i>y</i> ₄₂	-0.1003	-0.2714	0.3077
26.	6 m	y_{43}	-0.1668	2.2663	-1.2857
2.6 m	4.8 m	y_{44}	-0.2615	0.117	1
	3.6 m	y_{45}	0.1623	-0.0309	-2.625
	2.4 m	<i>y</i> 46	0.0421	0.7354	-1.7143
			$\overline{m}^y = -0.1269$	$\overline{a}^y = 0.1471$	$\overline{p}^y = -0.3417$

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Table 4. Grouping of variables for each	parameter of the classroom	characteristic color.
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Hue	Saturation	Variable	m_{ij}^z	a_{ij}^z	p_{ij}^z
ED	High	z ₁₁	0.9211	0.0702	-0.3333
5B	Low	z_{12}	-0.0039	0.2269	-0.1667
5G	High	z_{21}	-0.4155	0.2138	-0.2
	Low	z ₂₂	-0.2939	0.4916	-2.1429
5GY	High	z_{31}	-0.0421	-0.0328	-0.8333
JG1	Low	z ₃₂	-0.0758	0.3397	1.4286
5Y	High	z_{41}	-0.6764	-0.3718	1.1429
	Low	z_{42}	-0.1845	-0.6578	1.8571
EV/D	High	z_{51}	0.0605	-0.1496	0.5
5YR	Low	z_{52}	0.0816	-0.3638	-0.3333
5R	High	z ₆₁	-0.1929	0.3043	-2.125
	Low	z_{62}	-0.309	0.2662	1.5
5RP	High	z_{71}	-0.2257	0.6897	-0.625
JKI -	Low	z ₇₂	-0.544	-0.3037	-2.1429
5P	High	z_{81}	0.3314	0.0233	-1.1667
	Low	z_{82}	0.9799	-0.127	-0.8333
EDD	High	z_{91}	-0.1766	-0.3066	0.2222
5PB	Low	<i>z</i> ₉₂	0.249	-0.073	1.5714
5GB	High	z_{101}	-0.1321	-0.0261	0.6667
3GD	Low	z_{102}	0.0734	0.0751	-0.1429
			$\overline{m}^z = -0.0288$	$\overline{a}^z = 0.0144$	$\overline{p}^z = -0.1078$

4. Mathematical Model

The problem of identifying the combinations of the six classroom parameters (height, width, color hue, color saturation, color temperature, and lighting) that provide the best results for memory and attention performance and preference was addressed by modeling a MOILP problem designed to optimize these functions. It should be noted that the aim is to maximize the memory and preference values and minimize the attention values. Therefore, equality $Maximize\ f(x) = -Minimize\ (-f(x))$ is used for the attention function. With this equation, all the functions can be maximized.

Taking into account the information given in Section 3.3.1, the ideal MOILP problem should consider 5760 binary variables, each one corresponding to a different combination of color hue, color saturation, lighting, color temperature, height, and width. Its aim should be maximizing the three-dimensional vector corresponding to the average (normalized) results of memory, attention, and preference obtained for each one of the 5760 combinations. However, to pose and solve this problem is practically impossible due to the data collection time (more than 14,400 h) and to the number of variables. Instead, a more realistic situation is to consider the 56 variables introduced in Section 3.3.1 and detailed in Section 3.5 so that the three-dimensional vector to maximize has as components the sum of the means of the results of memory, the sum of the means of the results of attention, and the sum of the means of the results of preference (always normalized values) for each one of the 12 combinations of lighting and color temperature, 24 combinations of height and width, and 20 combinations of color hue and color saturation (a total of 56 combinations).

Therefore, using the notation given in Section 3.5, the following MOILP problem was formulated:

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Maximize
$$\left(\sum_{i=1}^{3} \sum_{j=1}^{4} m_{i,j}^{x} \cdot x_{i,j} + \sum_{i=1}^{4} \sum_{j=1}^{6} m_{i,j}^{y} \cdot y_{i,j} + \sum_{i=1}^{10} \sum_{j=1}^{2} m_{i,j}^{z} \cdot z_{i,j}, - \sum_{i=1}^{3} \sum_{j=1}^{4} a_{i,j}^{x} \cdot x_{i,j} - \sum_{i=1}^{4} \sum_{j=1}^{6} a_{i,j}^{y} \cdot y_{i,j} - \sum_{i=1}^{10} \sum_{j=1}^{2} a_{i,j}^{z} \cdot z_{i,j}, \right)$$

$$\sum_{i=1}^{3} \sum_{j=1}^{4} p_{i,j}^{x} \cdot x_{i,j} + \sum_{i=1}^{4} \sum_{j=1}^{6} p_{i,j}^{y} y_{i,j} + \sum_{i=1}^{10} \sum_{j=1}^{2} p_{i,j}^{z} \cdot z_{i,j} \right)$$

$$(1)$$

s.t:

$$\sum_{i=1}^{3} \sum_{j=1}^{4} x_{i,j} = 1 \tag{2}$$

$$\sum_{i=1}^{4} \sum_{j=1}^{6} y_{i,j} = 1 \tag{3}$$

$$\sum_{i=1}^{10} \sum_{j=1}^{2} z_{i,j} = 1 \tag{4}$$

$$\sum_{i=1}^{3} \sum_{j=1}^{4} m_{i,j}^{x} \cdot x_{i,j} + \sum_{i=1}^{4} \sum_{j=1}^{6} m_{i,j}^{y} \cdot y_{i,j} + \sum_{i=1}^{10} \sum_{j=1}^{2} m_{i,j}^{z} \cdot z_{i,j} \ge \overline{m}^{x} + \overline{m}^{y} + \overline{m}^{z}$$
 (5)

$$-\sum_{i=1}^{3}\sum_{j=1}^{4}a_{i,j}^{x}\cdot x_{i,j} - \sum_{i=1}^{4}\sum_{j=1}^{6}a_{i,j}^{y}\cdot y_{i,j} - \sum_{i=1}^{10}\sum_{j=1}^{2}a_{i,j}^{z}\cdot z_{i,j} \ge -\overline{a}^{x} - \overline{a}^{y} - \overline{a}^{z}$$
 (6)

$$\sum_{i=1}^{3} \sum_{j=1}^{4} p_{i,j}^{x} \cdot x_{i,j} + \sum_{i=1}^{4} \sum_{j=1}^{6} p_{i,j}^{y} \cdot y_{i,j} + \sum_{i=1}^{10} \sum_{j=1}^{2} p_{i,j}^{z} \cdot z_{i,j} \ge \overline{p}^{x} + p^{y} + \overline{p}^{z}$$
 (7)

$$x_{i,j}, y_{i,j}, z_{i,j} \in \{0,1\} \ \forall \ i,j$$
 (8)

where:

Equation (1) represents the multi-objective function, that is, the vector with components students' memory, attention, and preference.

Equations (2)–(4) guarantee that each classroom is composed of a single parameter of illuminance and temperature in terms of lighting, height, and width (in terms of size) and a single parameter of hue and saturation (in terms of wall color), respectively.

Equations (5)–(7) ensure that the total memory value is higher than the sum of the memory means, the total attention value is higher than the sum of the attention means, and the total preference value is higher than the sum of the preference means, respectively. Note that these three inequations represent the logical lower bounds for the functions, and they can be changed for other more (or less) demanding inequations [11]. The reason why these three restrictions were included in the formulation will be discussed later.

Equation (8) defines the problem variables as binaries.

Other linear restrictions could be added to the formulation if considered opportune. Moreover, it is obvious that this formulation can be extended to a more general formulation, with general bounds for i and j in each case, to more objective functions and to more variable types.

As stated in Section 1, there are several ways to obtain the set of efficient solutions to a MOILP problem. In this case, the following scalarization of the MOILP problem was used: Given $\lambda_1, \lambda_2, \lambda_3 \in \mathbb{R}^+$ with $\lambda_1 + \lambda_2 + \lambda_3 = 100$ and $b_a, b_m, b_p \in \mathbb{R}$ with $b_a \geq -\overline{a}^x - \overline{a}^y - \overline{a}^z$, $b_m \geq \overline{m}^x + \overline{m}^y + \overline{m}^z$ and $b_p \geq \overline{p}^x + p^y + \overline{p}^z$, the optimal solutions corresponding to each ILP problem formulated as follows are efficient solutions to the MOILP problem [11], with objective function given by Equation (9) and restrictions given by Equations (2)–(4), (8) and (10)–(12).

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Maximize
$$\lambda_{1}\left(\sum_{i=1}^{3}\sum_{j=1}^{4}m_{i,j}^{x}\cdot x_{i,j} + \sum_{i=1}^{4}\sum_{j=1}^{6}m_{i,j}^{y}\cdot y_{i,j} + \sum_{i=1}^{10}\sum_{j=1}^{2}m_{i,j}^{z}\cdot z_{i,j}\right)$$

$$+ \lambda_{2}\left(-\sum_{i=1}^{3}\sum_{j=1}^{4}a_{i,j}^{x}\cdot x_{i,j} - \sum_{i=1}^{4}\sum_{j=1}^{6}a_{i,j}^{y}\cdot y_{i,j} - \sum_{i=1}^{10}\sum_{j=1}^{2}a_{i,j}^{z}\cdot z_{i,j}\right)$$

$$+ \lambda_{3}\left(\sum_{i=1}^{3}\sum_{j=1}^{4}p_{i,j}^{x}\cdot x_{i,j} + \sum_{i=1}^{4}\sum_{j=1}^{6}p_{i,j}^{y}y_{i,j} + \sum_{i=1}^{10}\sum_{j=1}^{2}p_{i,j}^{z}\cdot z_{i,j}\right)$$
(9)

$$\sum_{i=1}^{3} \sum_{i=1}^{4} m_{i,j}^{x} \cdot x_{i,j} + \sum_{i=1}^{4} \sum_{i=1}^{6} m_{i,j}^{y} \cdot y_{i,j} + \sum_{i=1}^{10} \sum_{j=1}^{2} m_{i,j}^{z} \cdot z_{i,j} \ge b_{m}$$
 (10)

$$-\sum_{i=1}^{3}\sum_{j=1}^{4}a_{i,j}^{x}\cdot x_{i,j} - \sum_{i=1}^{4}\sum_{j=1}^{6}a_{i,j}^{y}\cdot y_{i,j} - \sum_{i=1}^{10}\sum_{j=1}^{2}a_{i,j}^{z}\cdot z_{i,j} \ge b_{a}$$

$$\tag{11}$$

$$\sum_{i=1}^{3} \sum_{j=1}^{4} p_{i,j}^{x} \cdot x_{i,j} + \sum_{i=1}^{4} \sum_{j=1}^{6} p_{i,j}^{y} \cdot y_{i,j} + \sum_{i=1}^{10} \sum_{j=1}^{2} p_{i,j}^{z} \cdot z_{i,j} \ge b_{l}$$
 (12)

Note that Equations (5)–(7) are particular cases of Equations (10)–(12), respectively. This is the fact by which they have been considered in the formulation of the problem. Therefore, from a theoretical point of view, solving a MOILP problem involves solving infinite ILP problems, as shown above (one for each combination of λ_1 , λ_2 , λ_3 , b_a , b_m and b_p). However, it is likely that the vast majority of these ILP problems will have the same optimal solutions, and the total number of different efficient solutions will not be very high because the variables are integers and particularly in this case, where they are binary.

An interactive procedure based on the scalarization described above is used here to obtain good (according to the DM), efficient solutions to the MOILP problem. As usual with this method, a subset of efficient solutions is generated, and from this subset, the DM draws conclusions and proposes, for instance, new bounds for the objective functions to generate a new subset of efficient solutions, which are in turn analyzed by the DM and so on, until the DM decides which are the most efficient solutions to the MOILP.

The proposed heuristic is shown below. This procedure does not guarantee identification of the complete set of efficient solutions, but based on the problem's characteristics, it is expected that it will obtain a representative set of solutions.

Heuristic:

Step 1. For each combination of even numbers $\lambda_1, \lambda_2, \lambda_3 \in \mathbb{N}^+$ with $\lambda_1 + \lambda_2 + \lambda_3 = 100$ (1176 ILP combinations), solve (with *Mathematica* v12.1) the ILP problem with Equation (9) as the objective function, using the restrictions of Equations (2)–(8). Solve the same problem but with $\lambda_1 = \lambda_2 = \lambda_3 = 100/3$ (the same weight for all three objective functions). The DM assesses the set of optimal solutions obtained (all of them are efficient solutions to the MOILP problem) and makes decisions, such as removing from the list of solutions those not considered adequate, and establishes (increases or decreases) the percentile to be used in Step 2.

Step 2. Repeat Step 1, changing the right-hand side of Equations (5)–(7) by the percentile i provided by the DM of the list of 1177 non-dominated objective vectors corresponding to each objective function (in the first execution of Step 1). The new solutions obtained are saved (although sometimes no solution is obtained). This step is repeated until no further solutions emerge, and all the percentiles j with j < i have been considered along the heuristic or until the DM decides that even if more solutions may exist, it is not important to identify them.

Step 3. The DM draws conclusions about those efficient solutions obtained in the process that remain in the list.

The criteria to be used by the DM to remove or maintain efficient solutions in the list in Step 2 and to decide which are the best solutions in Step 3 depend on the situation. Thus, in general, the classroom will be designed to take account of all three cognitive processes

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(memory, attention, and preference), but it is possible that, in circumstances in which a high cognitive load is required, such as taking an exam, the DM will choose a criterion prioritizing memory and attention over preference.

To give a general idea of how the heuristic works, in Step 2, the lower bounds for the values of the objective functions vary; these variations are the percentiles of the 1177 initial solutions. The higher the percentile, the greater the requirement for the objective functions. If, for instance, the DM introduces a high percentile, and none of the 1177 ILP problems has a feasible solution, during the next Step 2 run, the DM might lower the percentile to try to obtain new solutions or even to terminate the process. The number of iterations will be finite, as it is obvious that from a certain percentile (unknown a priori), no new solutions will be found.

Note that, in each iteration, the proposed heuristic must optimally solve 1177 ILP problems, each of which theoretically has exponential complexity. Therefore, from a computational viewpoint, this is a complex heuristic; but the *Mathematica* V.12 tool has been shown to be very effective in this regard. This fact, together with the simplicity of the alternation and connection between the computation phases and the intervention phases of the DM, has prompted the authors of the present study to opt for this heuristic although, obviously, other procedures could have been applied to obtain a reasonable set of "good", efficient solutions to the MOILP problem here formulated.

5. Results

The results obtained by the heuristic are shown in Table 5. On the 1st run of Step 1, only 10 efficient solutions were obtained from the 1177 ILP problems. This caused the DM to adopt a conservative stance and advance from percentile to percentile. The DM decided not to discard any of the 10 solutions and to apply the 1st percentile in the 1st run of Step 2. On the 2nd run of Step 1, seven efficient solutions to the MOILP problem were obtained from the new 1177 ILP problems, only two of which were new. The DM decided not to discard any of the 12 solutions and applied the 2nd percentile in the 2nd run of Step 2. Only two efficient solutions were obtained during the 3rd run of Step 1 from the new 1177 ILP problems, only one of which differed from those previously obtained. The DM decided not to discard any of the 13 solutions and applied the 3rd percentile in the 3rd run of Step 2. None of the 1177 ILP problems assessed in the 4th run of Step 1 had a feasible solution; therefore, the heuristic procedure was terminated after the DM provided conclusions on the 13 efficient solutions obtained (provided below).

The *Mathematica* software was run on a PC Intel[®]CoreTM I5-7500 with 3.40 GHz and 16GB RAM. The average CPU time to obtain the optimal solution on all ILP problems was 0.0035 s, with a maximum value of 0.0156 s and a minimum value of 0 s, which, according to *Mathematica*'s assumptions, means that the calculation took no measurable CPU time.

Figure 2 represents the position in three-dimensional space of the points corresponding to the 13 non-dominated objective vectors obtained by the heuristic. Note that the *Mathematica* software automatically scales the points. Figure 3 shows the points of the vectors of the three objective functions on the same scale. It is worth remembering that the values entered in Equation (1) are normalized.

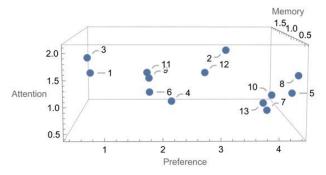


Figure 2. Three-dimensional graph of the efficient solution set.

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Table 5.	Set of	efficient	solutions	obtained	with th	e heuristic.

Solutions	n ¹	Memory	Attention	Preference	R-h Side ²	Step 1
1: $x_{22} = 1, y_{13} = 1, z_{82} = 1$	9	1.7955	1.0808	0.381		
2: $x_{22} = 1$, $y_{31} = 1$, $z_{42} = 1$	200	0.4143	2.0515	3.0714		
$3: x_{22} = 1, y_{31} = 1, z_{82} = 1$	55	1.5787	1.5207	0.381		
4: $x_{32} = 1$, $y_{13} = 1$, $z_{11} = 1$	50	1.8477	0.3644	2.1942	$b_m = -0.1218;$	
5: $x_{32} = 1$, $y_{13} = 1$, $z_{42} = 1$	108	0.7421	1.0924	4.3846	$b_m = -0.1218,$ $b_a = -0.1244;$	1.1
$6: x_{32} = 1, y_{13} = 1, z_{82} = 1$	86	1.9065	0.5616	1.6942	$b_a = -0.1244,$ $b_p = 0.0686$	1st run
7: $x_{32} = 1$, $y_{13} = 1$, $z_{92} = 1$	121	1.1756	0.5076	4.0989	$\nu_p = 0.0000$	
8: $x_{32} = 1$, $y_{31} = 1$, $z_{42} = 1$	527	0.5253	1.5323	4.3846		
9: $x_{32} = 1$, $y_{31} = 1$, $z_{82} = 1$	18	1.6897	1.0015	1.6942		
10: $x_{32} = 1$, $y_{31} = 1$, $z_{92} = 1$	3	0.9588	0.9475	4.0989		
$11: x_{22} = 1, y_{31} = 1, z_{51} = 1$	16	0.6593	1.5433	1.7143		
12: $x_{22} = 1$, $y_{31} = 1$, $z_{92} = 1$	131	0.8478	1.4667	2.7857		
5: $x_{32} = 1$, $y_{13} = 1$, $z_{42} = 1$	672	0.7421	1.0924	4.3846	$b_m = 0.4143;$	
6: $x_{32} = 1$, $y_{13} = 1$, $z_{82} = 1$	140	1.9065	0.5616	1.6942	$b_a = 0.5076;$	2nd run
7: $x_{32} = 1$, $y_{13} = 1$, $z_{92} = 1$	127	1.1756	0.5076	4.0989	$b_p = 1.6942$	
9: $x_{32} = 1$, $y_{31} = 1$, $z_{82} = 1$	83	1.6897	1.0015	1.6942	•	
10: $x_{32} = 1$, $y_{31} = 1$, $z_{92} = 1$	8	0.9588	0.9475	4.0989		
$10: x_{32} = 1, y_{31} = 1, z_{92} = 1$	1124	0.9588	0.9475	4.0989	$b_m = 0.5253;$ $b_a = 0.5616;$	3rd run
13: $x_{32} = 1$, $y_{11} = 1$, $z_{92} = 1$	53	1.0072	0.7551	3.956	$b_p = 3.0714$	2227

 $^{^{1}}$ *n* represents the frequency with which the solution was repeated among the 1177 possible. 2 r-h side column shows the right-hand side values of Equations (5)–(7).

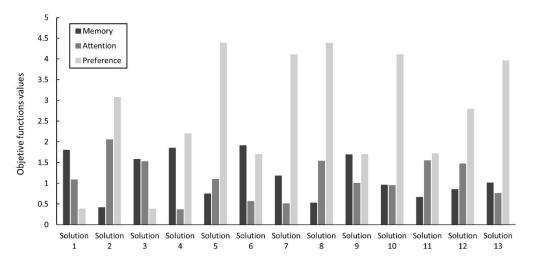


Figure 3. Levels of psychological metrics of each efficient solution.

In Step 3, the DM identified the four best solutions based on the different selection criteria. The first criterion is to ensure that the three objective functions are broadly in balance, that is, that one is not more salient than the others. Based on this decision, the DM chose solutions 9 and 10. It is worth mentioning that solution 10 attached equal importance to the three metrics ($\lambda_1 = \lambda_2 = \lambda_3 = 100/3$) and was the most frequently cited solution to the 1770 problems. Another possible criterion would be to prioritize memory and attention over preference, particularly relevant in the design of classrooms where important cognitive effort is required, such as for exams. In this case, the DM would select solutions 1 and 3. Table 6 lists the design configurations of these four solutions.

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			Decision Cr	iterion	
Design Parameters		Better (Memory, At	Better (Memory, Attention, Preference) Bett		
G		10	9	1	3
Tipletin -	Color Temperature	6500 K	6500 K	6500 K	6500 K
Lighting	Illuminance	100 lx	100 lx	300 lx	300 lx
0.1	Hue	5PB	5P	5P	5P
Color	Saturation	low	low	low	low
Dimension	Height	4.4 m	4.4 m	3.2 m	4.4 m
	Width	8.4 m	8.4 m	6 m	8.4 m

Table 6. Design configurations for the best solutions.

As shown in Table 6, the best solutions have the same illumination color temperature (6500 K) and the same wall color saturation (low). The difference between the solutions, according to the decision criteria, is in illuminance. A higher illuminance level (300 lx) enhances attention and memory processes although it is regarded as less pleasant, decreasing preference level. The solutions with the best results in attention, memory, and preference levels (9 and 10) have the same characteristics in terms of lighting, color saturation, and dimension but feature different hues. On the other hand, the solutions that enhance attention and memory feature the same configurations except in dimensions.

6. Discussion

The present study aims to identify the combination of classroom design parameters that optimize students' internal psychological processes by applying MOLP as an analysis method and using VR as an environmental-simulation tool. The study makes three fundamental contributions: one methodological, one at the results level, and one at the application level.

As for the methodological contribution, a mathematical analysis was undertaken to complement the traditional statistical approach of the behavioral study. In this sense, this methodology is novel and ideal for: (1) identifying the design configurations that take into account the combination of parameters that make up spaces, as in real scenarios; (2) optimizing several objective psychological metrics, which is of special interest for multifunctional spaces; and (3) considering the judgement of an expert, which is important in human behavior studies.

The results of the present study provided a small set of efficient solutions to be evaluated by the DM. As shown in Table 5, there are two possible selection criteria. On the one hand, taking into account the three cognitive processes, the DM would select solutions 10 and 9, which combine dimensions of 4.4 m height and 8.4 m width, interior artificial lighting of 6500 K and 100 lx, and low-saturation blue or purple wall color. These solutions are suitable, as they maintain a high level of preference and the best combination of attention and memory levels. Moreover, solution 10 gives all three psychological metrics equal importance ($\lambda_1 = \lambda_2 = \lambda_3 = 100/3$). Another possible criterion would be to prioritize memory and attention over preference given their importance in learning [28,29]. Both are important for class sessions with activities that require different levels of cognitive load, such as taking exams [75], undertaking projects [76], and teaching through alternative educational methodologies, such as the flipped classroom [77]. In this case, the DM would choose solutions 1 and 3, which combine interior artificial lighting of 6500 K and 300 lx, low-saturation purple wall color, and dimensions of 3.2 m height and 6 m width or 4.4 m height and 8.4 m width. These solutions, while subject to the constraints of the model, are efficient for this set of cognitive processes; while they present the lowest preference values, they achieved the highest values in the combination of attention and memory. The preference-performance relationship has been examined in many studies, but no conclusive results have been achieved. Some authors have negatively correlated the two [65], and others have done so positively [78].

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In any case, this research provides concrete design results to consider. The benefit of using a color temperature of 6500 K was clear for attention, memory, and preference. In addition, some authors have observed that blue-enriched white light has a positive effect on performance [58,79]. The combination of high lighting color temperature (6500 K) and low illuminance (100 lx and 300 lx) was common to all 13 solutions. Several authors have shown that lower illuminance improves cognitive performance [80,81]. This lighting effect is repeated with color. Virtually all the solutions featured low-saturation colors. Similarly, Kwallek et al. [49] observed that fewer errors were made in performance tasks in environments with colors of saturations similar to or lower than those used in the present study. The solutions included both cold and warm hues. On this issue, the literature is conflicting. Some authors, for example, Mahnke [82], have argued that better academic performance is achieved with blue colors in high/secondary school classrooms, while Barret [83] observed that warm-hue colors are more appropriate for senior grades and that cold-hue colors are more appropriate for junior grades. As to the dimensions contextual key, in no case did lower ceilings (2.4 m) or narrow classrooms (3.6 m) improve student performance. Specifically, heights between 3.2 m and 4.4 m and widths between 6 m and 8.4 m provided efficient solutions. This outcome may be consistent with the results obtained by Vartanian et al. [84], in which high ceilings were evaluated as more beautiful than low ceilings because they expanded the viewers' fields of view.

Regarding the application contribution, this article connects two different fields of study: architecture and psychology. In general, from the architectural perspective, studies have analyzed human responses to built spaces taking preference as the main decision-making criterion [85,86] and used psychological metrics other than task performance, that is, through self-reports [62,87]; this may be a limited approach. The present study proposes using environmental preferences (typical in architectural studies) and the results of tests analyzing cognitive processes (typical in psychology studies) to bridge the gap between the two disciplines.

7. Conclusions and Future Research

This study addresses the complexity of the analysis of the effects of classroom environments on subjects. This complexity drives the need to analyze the various environmental characteristics of classrooms and the cognitive processes involved in learning. This process requires the application of techniques that can simultaneously analyze a large number of variables. In the present study, the MOLP analysis technique was applied to optimize this set of variables. The advantages of this method for this work are: (1) it takes into account the interdependencies between classroom characteristics; (2) it maximizes the levels of the cognitive processes attention, memory, and preference; and (3) it provides several efficient solutions to allow the DM to select the most appropriate depending on the situation. In this case, the best solutions share interior artificial lighting of 6500 K and low-saturation wall colors. If the DM wants to achieve high levels in the three psychological metrics (memory, attention, and preference), the classroom should also be 4.4 m high and 8.4 m wide and have 100 lx of lighting and blue or purple wall color. If, on the other hand, the requirement is to enhance only the cognitive functions of attention and memory, the DM will choose a classroom combining 300 lx and low-saturation purple wall color, with dimensions of 3.2 m height and 6 m width or 4.4 m height and 8.4 m width. These results may be of interest to researchers and professionals involved in the design of educational centers.

Finally, as for future research lines, three aspects should be considered. First, the present study used auditory tasks to examine the processing of classroom characteristics. In future works, it would be interesting to include other types of tasks, as information processing through other sensory pathways requires different neural bases [88–90]. Second, regarding the stimuli presented, it would be interesting to address the influence of multisensory contextual cues [91]. Synergies between interior design parameters may involve senses other than sight, such as auditory and tactile temperature receptors [46]. Third, regarding the sample, it would be interesting to analyze whether differences exist between

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men and women [92,93]. In all these lines, MOILP analyses with appropriate interactive methods can help researchers to obtain good results.

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