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Annual Operating Costs Minimization in Electrical Distribution Networks via the Optimal Selection and Location of Fixed-Step Capacitor Banks Using a Hybrid Mathematical Formulation

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Abstract: The minimization of annual operating costs in radial distribution networks with the optimal selection and siting of fixed-step capacitor banks is addressed in this research by means of a two-stage optimization approach. The first stage proposes an approximated mixed-integer quadratic model to select the nodes where the capacitor banks must be installed. In the second stage, a recursive power flow method is employed to make an exhaustive evaluation of the solution space. The main contribution of this research is the use of the expected load curve to estimate the equivalent annual grid operating costs. Numerical simulations in the IEEE 33- and IEEE 69-bus systems demonstrate the effectiveness of the proposed methodology in comparison with the solution of the exact optimization model in the General Algebraic Modeling System software. Reductions of 33.04% and 34.29% with respect to the benchmark case are obtained with the proposed two-stage approach, with minimum investments in capacitor banks. All numerical implementations are performed in the MATLAB software using the convex tool known as CVX and the Gurobi solver. The main advantage of the proposed hybrid optimization method lies in the possibility of dealing with radial and meshed distribution system topologies without any modification on the MIQC model and the recursive power flow approach.

Keywords: fixed-step capacitor banks; daily load variations; annual operating costs minimization; two-stage optimization approach; successive approximation power flow method

MSC: 94C15; 90C27; 90C26

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1. Introduction

Electrical distribution networks are constantly growing due to the requirements of commercial, industrial, and residential users in urban and rural areas [1]. Most of the investments in electrical systems are condensed into medium- and low-voltage level applications to satisfy the growing electric demand [2]. For this reason, utilities are intended to improve the quality service in their grids in order to enable the interconnection of new users with minimum investment and operating costs [3]. One of the key aspects in the construction of electrical distribution grids is their topology. These networks are typically constructed with a radial configuration to minimize the investment in electrical infrastructure (conductors, isolators, protective devices, and so on) [4], but these investment reductions are counteracted by increments in the operating costs, since radial configurations have higher power losses in comparison with meshed grid configurations [5].

To deal with the costs of energy losses distribution networks, electric distribution companies typically employ shunt power compensation, i.e., with active and reactive power sources [6,7]. In the case of active power compensation, dispersed generators and/or battery energy storage systems are typically considered [8,9]. Nevertheless, the investment costs of these devices are not compensated by a reduction in power losses, which implies that said devices are installed in the grid to minimize the total energy purchasing costs in the substation bus or to minimize the total greenhouse gases emitted into the atmosphere [10,11]. However, in some cases, with these objective functions, the energy losses can increase with respect to the benchmark case [12]. In the case of reactive power compensation, the most commonly used devices are fixed-step capacitor banks and distribution static compensators (i.e., D-STATCOMs) [13], both of which can be installed in order to reduce energy losses in distribution networks. However, capacitor banks are simple, economic, and reliable devices that require little maintenance and can continue to work for 20–25 years [14], whereas D-STATCOMs imply continuous maintenance costs and are based on power electronics, which implies that the rate of failure is higher in comparison with capacitor banks [15].

In the scientific literature, multiple approaches to locate and size fixed-step capacitor banks in distribution networks have been proposed. Some of these works are discussed below. Ref. [16] proposed the application of the tabu search algorithm to locate and size fixed-step capacitor banks including the exploration and exploitation aspects of genetic algorithms as well as simulated annealing methods. Ref. [14] presented the application of the flower pollination algorithm to locate and size fixed-step capacitor banks in distribution grids with radial structures. Numerical results in IEEE 33-, 34-, 69-, and 85-bus systems demonstrated the efficiency of this proposal when compared with different literature reports based on genetic algorithms and fuzzy logic. Ref. [17] proposed the implementation of the discrete version of the vortex search algorithm to locate and size fixed-step capacitor banks in radial distribution networks. Numerical results in the IEEE 33- and 69-bus grids showed the efficiency of said algorithm when compared with the flower pollination algorithm reported in [14]. Ref. [18] presented the application of the Chu and Beasley genetic algorithm (CBGA) with an integer codification to locate and size fixed-step capacitor banks in distribution networks considering radial and meshed configurations. Numerical results in the IEEE 33- and 69-bus systems demonstrated the efficiency of this optimization approach when compared with the exact solution reached in the General Algebraic Modeling System (GAMS) software. Ref. [19] presented the reformulation of the exact mixed-integer nonlinear programming (MINLP) model in order to locate and size fixed-step capacitor banks into a mixed-integer, second-order cone programming approach. Numerical results in the IEEE 33- and 69-bus grids demonstrated the effectiveness of this solution methodology by improving results obtained in the literature with the CBGA, the GAMS software, and the flower pollination approaches. Other optimization algorithms that can be found in the current literature to locate and size fixed-step capacitor banks are: artificial bee colony optimization [20], particle swarm optimization [21], gravitational search algorithms [22], cuckoo search algorithm [23], and modifications of genetic and tabu search algorithms [24,25], among others.

The main characteristic of the aforementioned literature reports (except for the GAMS and conic approximations) is that metaheuristic approaches work with a master–slave optimization structure, where the master stage is entrusted with selecting the optimal location and sizes of the capacitor banks and the slave stage evaluates the power losses of each configuration provided by the master stage [17]. In some cases, the master stage only provides the set of nodes where the capacitor banks will be installed, and the slave stage solves the optimal power flow problem to find their optimal sizes [26,27].

Considering the advantages of fixed-step capacitor banks and their widespread use to reduce power in distribution networks, as well as the slave optimization methods reported in the literature, this research focuses on proposing a new two-stage optimization approach to locate and size these devices in distribution networks. The main contribution of this research is the formulation of a mixed-integer quadratic convex (MIQC) model to identify the nodes where the capacitor banks will be located. The main advantage of the MIQC formulation is that it ensures the optimal global optimal solution of the relaxed model. This implies that no statistical evaluations confirm the location of the capacitors, which is necessary for metaheuristic-based optimizers. Once the location of the fixed-step capacitor banks is defined by the MIQC model, a recursive power flow solution is implemented in order to evaluate all possible sizes (i.e., exhaustive evaluation of the solution space), which allows for determining the best possible sizes for these capacitors. It is worth mentioning that the exhaustive exploration of the solution space is possible since it has a few thousand options, which are easily assessed with any personal or desktop computer.

Note that the selection of the MIQC formulation to determine the set of nodes where the fixed-step capacitor banks will be located is motivated by the convexity of the solution space for each binary variable combination, which implies that, through the combination of a Branch and Bound method with the interior point approach, it is possible to ensure that the global optimum is found [28]. Even if some metaheuristic models perform efficiently in electrical engineering problems such as the reconfiguration of distribution grids including dispersed generation [29,30], more studies are required in this research field to make metaheuristics competitive against MIQC models.

The remainder of this article is organized as follows: Section 2 presents the proposed MIQC model that defines the optimal location of the fixed-step capacitor banks. This model corresponds to a quadratic objective function with linear integer constraints. Section 3 presents the recursive power flow solution method based on the successive approximation approach to define the optimal sizes of the fixed-step capacitor banks. Section 5 shows the main characteristics of the IEEE 33- and 69-bus systems. Section 6 presents the numerical results, their analysis, and discussions, as well as the comparison between the proposed two-stage solution methodology and the literature reports. Finally, Section 7 lists the main concluding remarks derived from this research, as well as some proposals for future work.

2. Nodal Selection Strategy

To select the nodes where the fixed-step capacitor banks will be installed, a mixed-integer quadratic convex (MIQC) formulation is proposed by using the equivalent linear power flow formulation for electrical distribution networks obtained by simplifying the MINLP model for distribution system reconfiguration reported in [31]. The simplifications made to the MINLP model in order to obtain an MIQC are the following:

i. All the voltage magnitudes are assumed to be known, i.e., these can be assigned as plane voltages $(1\angle 0^\circ)$or set as the power flow solution without capacitor banks (i.e., the benchmark case).
ii. The magnitude of the currents through the distribution lines is mainly governed by the active and reactive power consumption, which implies that the effect of the second Kirchhoff law at each line is negligible in comparison with the first Kirchhoff law at each node.

By considering the aforementioned assumptions on the exact MINLP model proposed by [31], the MIQC model to define the set of nodes where the fixed-step capacitor banks will be installed is obtained (see Figure 1). This model is defined in Equations (1)–(5):

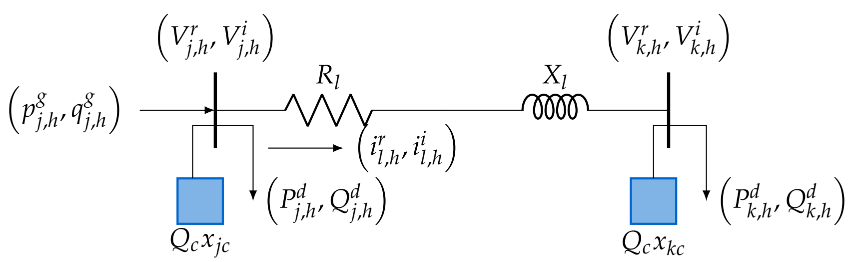

Figure 1. Representation of a distribution branch with the main variables of the MIQC model.

Obj. fun.:

$$\min z_{\text{approx}} = C_{\text{kWh}} T \sum_{l\in\mathcal{L}} \sum_{h\in\mathcal{H}} R_l \left(\left(i^r_{l,h} \right)^2 + \left(i^i_{l,h} \right)^2 \right) \Delta h + \sum_{j\in\mathcal{N}} \sum_{c\in\mathcal{C}} C^{\text{cap}}_c Q_c x_{jc} \quad (1)$$

Subj. to.:

$$p^g_{j,h} - P^d_{j,h} = \sum_{l\in\mathcal{L}} \mathcal{A}_{jl} \left(V^r_{j,h} i^r_{l,h} + V^i_{j,k} i^i_{l,h} \right), \ \{\forall j \in \mathcal{N}, \ \forall h \in \mathcal{H}\}, \quad (2)$$

$$q^g_{j,h} + \sum_{c\in\mathcal{C}} Q_c x_{jc} - Q^d_{j,h} = -\sum_{l\in\mathcal{L}} \mathcal{A}_{jl} \left(V^r_{j,h} i^i_{l,h} - V^i_{j,h} i^r_{l,h} \right), \ \{\forall j \in \mathcal{N}, \ \forall h \in \mathcal{H}\}, \quad (3)$$

$$\sum_{c\in\mathcal{C}} x_{jc} \leq 1, \ \{\forall j \in \mathcal{N}\}, \quad (4)$$

$$\sum_{j\in\mathcal{N}} \sum_{c\in\mathcal{C}} x_{jc} \leq N^{\text{cap}}_{\text{ava}}, \ \{\forall j \in \mathcal{N}\}. \quad (5)$$

Note that all the mathematical symbols, parameters, and variables are contained in the nomenclature list presented at the end of this document.

Note that the proposed MIQC formulation defined in Equations (1)–(5) has the following interpretation: Equation (1) defines the approximated annual grid operative costs of the network, which correspond to the sum of the expected costs concerning the energy losses and the installation of the fixed-step capacitor banks. Equation (2) defines the active power balance equilibrium at node j for each period of time h, which is linear when the voltages $V^r_{j,h}$ and $V^i_{j,h}$ are assumed to be known. Equation (3) defines the reactive power equilibrium at each node for each period of time. Inequality constraint (4) defines the possibility of installing as much as one fixed-step capacitor bank type at node j. Finally, inequality constraint (5) limits the maximum number of fixed-step capacitor banks that can be installed in the distribution grid.

In order to characterize the optimization model defined in Equations (1)–(5), the classification and type of variables are presented in Table 1, including the number and type of constraints. It is important to mention that, in this classification, n is the number of nodes, h represents the periods of time, c corresponds to the number of capacitor available, and l is the number of lines.

Table 1. Characterization of the MIQC model (1)–(5).

Variables	**Type**	**Number**
Capacitor locations and sizes	Binary	nc
Currents (real and imaginary parts)	Real	$2lh$
Active and reactive power generation	Real	$2nh$
Objective function	Real	1
Total number of variables	Real + binary	$2(l+n)h + nc + 1$
Constraints	**Type**	**Number**
Active power balance	Equality	np
Reactive power balance	Equality	np
Capacitors per node	Inequality	n
Number of capacitors available	Inequality	1
Objective function	Equality	1
Total number of constraints	Equalities + inequalities	$(2p+1)n + 1$

Remark 1. *The solution of the MIQC model defined in Equations (1)–(5) provides the nodes where the capacitor banks will be installed along with their sizes, since it completely defines the final values for the variables x_{jc}. However, only the nodes where these capacitors will be installed are taken from this solution, since their sizes correspond to an approximate solution of the exact MINLP model due to the approximation introduced by the voltage magnitude simplification.*

To refine the solution provided by the MIQC model regarding the sizes of the fixed-step capacitor banks, the next section presents the methodology for finding the fixed-step capacitor banks, which is based on the recursive power flow solution for each possible fixed-step capacitor size combination.

It is important to mention that the dimension of the solution space defined by the binary variables in the MIQC model in Equations (1)–(5) takes a combination form, where it depends on the number of capacitors available and the number of nodes in the distribution network. Note that, for the IEEE 33- and 69-bus grids, these dimensions are 4960 and 50,116 when three capacitors are considered for installation [32].

3. Assigning the Optimal Sizes

Once the MIQC model presented in Section 2 has been solved, the values of the variables x_{jc} are known. However, in the second stage of the proposed optimization methodology, the optimal sizes of the fixed-step capacitor banks are refined by fixing these capacitors in the j nodes.

Given that the optimization problem can take a maximum of $N^{\text{cap}}_{\text{ava}}$ to be installed in the distribution network (one fixed-step capacitor bank per node), in this stage, an integer combination is proposed to represent each fixed-step capacitor size. Note that, if the small fixed-step capacitor bank is assigned to node i with 1 and the largest fixed-step capacitor bank for the k node is set as c, then the proposed codification for the selected nodes has the following structure:

$$x_{\text{sol}} = [i,\ j,\ k,\ |\ 1,\ 7,\ c], \tag{6}$$

where x_{sol} represents the vector associated with the selected nodes j, k, and m, along with their corresponding sizes (possible solution). It is important to mention that, for each solution in Equation (6), the total costs of the fixed-step capacitors are easily determined with the second component of the objective function (1).

Remark 2. *Due to the fact that the maximum size of the capacitor bank is assigned as c and the maximum number of devices installed is N^{cap}_{ava}, the size of the solution space for the optimal sizing problem of the fixed-step capacitor banks is $c^{N^{cap}_{ava}}$. This number is $14^3 = 2774$ for the studied set of alternatives regarding the fixed-step capacitor banks.*

Considering a solution space with $c = 14$ and $N^{\text{cap}}_{\text{ava}} = 3$, the proposed methodology to determine the optimal sizes of these capacitor banks is exhaustive, which implies that the 2774 options are evaluated in a conventional power flow formula. The proposed power flow methodology is the successive approximation power flow method reported in [33].

The general recursive power flow formula for the successive approximation method is defined in Equation (7):

$$\mathbb{V}^{m+1}_{d,h} = \mathbb{Y}^{-1}_{dd}\left[\text{diag}^{-1}\left(\mathbb{V}^{m}_{d,h}\right)\left(\mathbb{S}^{\star}_{cap,h} - \mathbb{S}^{\star}_{d,h}\right) - \mathbb{Y}_{ds}\mathbb{V}_{s,h}\right], \tag{7}$$

where m is the iterative counter, $\mathbb{V}_d$ is the vector that contains all the voltage variables in the complex domain for all the demand nodes in each period of time h, $\mathbb{S}_{cap,h}$ is the complex vector that contains all the power outputs in the fixed-step capacitor banks in each period of time h (note that this vector is provided for each of the 2744 size combinations), $\mathbb{S}_{d,h}$ is the complex demand vector with the active and reactive power consumption in the demand nodes for each period of time, $\mathbb{V}_{s,h}$ is the complex voltage output at the substation bus, $\mathbb{Y}_{dd}$

is a complex square matrix that contains all the admittances among the demand nodes, and $\mathbb{Y}_{ds}$ is a rectangular complex matrix that contains the admittances between the demand and the substation buses. Note that $\mathrm{diag}(z)$ and $z^{\star}$ are a matrix with all the elements of the z at its diagonal and the conjugate operator of the complex vector z, respectively.

It is important to emphasize that $\mathbb{S}^{\star}_{cap,h}$ is 0 for the set of nodes different from j, k, and m. Regarding the example codification presented in Equation (7), these nodes are listed below:

$$\begin{bmatrix} \mathbb{S}_{i,h} \\ \mathbb{S}_{j,h} \\ \mathbb{S}_{k,h} \end{bmatrix} = \begin{bmatrix} jQ_1 \\ jQ_7 \\ jQ_c \end{bmatrix}$$

The main characteristic of the recursive power flow Formula (7) is that its convergence to the power flow solution can be ensured by applying the Banach fixed-point theorem [34]. To determine if the power flow Formula (7) has converged, the difference between the voltage magnitudes between two consecutive iterations is used, i.e.,

$$\max_{h}\left\{ ||\mathbb{V}^{m+1}_{d,h}| - |\mathbb{V}^{m}_{d,h}|| \right\} \leq \varepsilon, \tag{8}$$

where ε is the tolerance value, which is assigned as 1×10^{-10}, as recommended in [33].

Note that, once the power flow problem is solved with Equation (7), the amount of power losses is calculated for each period of time, as presented in Equation (9):

$$P_{\mathrm{loss},h} = \mathrm{real}\left\{ \mathbb{V}_h^{\top} \left(\mathbb{Y}_{\mathrm{bus}} \mathbb{V}_h\right)^{\star} \right\}, \tag{9}$$

where $\mathbb{V}_h$ is the vector that contains the substation and demand voltages, ordered as $[\mathbb{V}_{s,h}\ \mathbb{V}_{d,h}]^{\top}$; and $\mathbb{Y}_{\mathrm{bus}}$ is the nodal admittance matrix of the distribution grid. With the power losses at each period of time, the exact operational costs of the distribution systems with fixed-step capacitor banks (i.e., $\min z_{\mathrm{costs}}$) can obtained as defined in Equation (10):

$$\min z_{\mathrm{costs}} = C_{\mathrm{kWh}} T \sum_{ij \in \mathcal{L}} \sum_{h \in \mathcal{H}} P_{\mathrm{loss},h} \Delta h + \sum_{j \in \mathcal{N}} \sum_{c \in \mathcal{C}} C_c^{\mathrm{cap}} Q_c x_{jc} \tag{10}$$

4. Summary of the Solution Methodology

The proposed solution methodology for locating and selecting fixed-step capacitor banks is summarized in Algorithm 1. Here, the problem of location is solved with a mixed-integer convex formulation, and the selection (i.e., sizing) is determined with the recursive evaluations of the power flow problem for each one of the $N^{\mathrm{cap}}_{\mathrm{ava}}$ possible combinations.

Algorithm 1: Proposed two-stage solution methodology to select and locate fixed-step capacitor banks in distribution networks.

```
Data: Define the distribution network under study
Obtain the per-unit equivalent of the distribution grid;
Define the number of capacitor banks available for installation, i.e., N^cap_ava;
Solve the MIQC model (1)–(5);
Select the positions where the capacitor banks will be located from the variable x_jc;
for All the capacitor size combinations do
    Define the values for the vector S*_cap,h;
    Solve the recursive power flow Formula (7);
    Calculate power losses at each period of time with Equation (9);
    Determine the objective function with Equation (10);
end
Order all the solutions in ascending form with the values of the objective function;
Result: Report the optimal solution
```

5. Test Feeder Information

To validate the proposed two-stage optimization methodology, two classical test feeders known as the IEEE 33- and 69-bus systems are employed. The parametric information for these grids is presented below.

5.1. IEEE 33-Bus Grid

The IEEE 33-bus grid is an electrical network operated with 12.66 kV at the substation bus located at node 1. This system has a radial structure, i.e., 33 buses and 32 lines. The electrical configuration of this test feeder is presented in Figure 2a, and its electrical parameters are listed in Table 2.

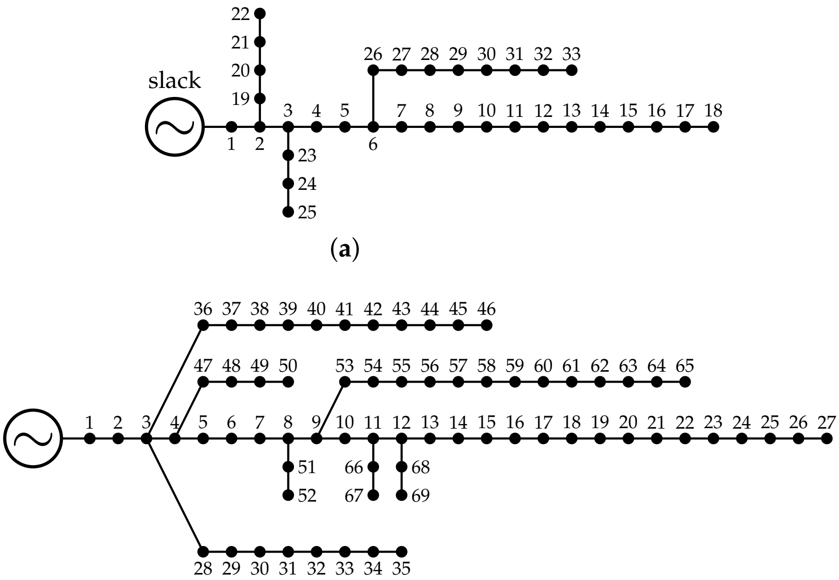

Figure 2. Grid configuration of the test feeders: (**a**) IEEE 33- and (**b**) IEEE 69-node system.

Table 2. Parametric information for the IEEE 33-bus system.

Node i	Node j	R_{ij} (Ω)	X_{ij} (Ω)	P_j (kW)	Q_j (kvar)	Node i	Node j	R_{ij} (Ω)	X_{ij} (Ω)	P_j (kW)	Q_j (kvar)
1	2	0.0922	0.0477	100	60	17	18	0.7320	0.5740	90	40
2	3	0.4930	0.2511	90	40	2	19	0.1640	0.1565	90	40
3	4	0.3660	0.1864	120	80	19	20	1.5042	1.3554	90	40
4	5	0.3811	0.1941	60	30	20	21	0.4095	0.4784	90	40
5	6	0.8190	0.7070	60	20	21	22	0.7089	0.9373	90	40
6	7	0.1872	0.6188	200	100	3	23	0.4512	0.3083	90	50
7	8	1.7114	1.2351	200	100	23	24	0.8980	0.7091	420	200
8	9	1.0300	0.7400	60	20	24	25	0.8960	0.7011	420	200
9	10	1.0400	0.7400	60	20	6	26	0.2030	0.1034	60	25
10	11	0.1966	0.0650	45	30	26	27	0.2842	0.1447	60	25
11	12	0.3744	0.1238	60	35	27	28	1.0590	0.9337	60	20
12	13	1.4680	1.1550	60	35	28	29	0.8042	0.7006	120	70
13	14	0.5416	0.7129	120	80	29	30	0.5075	0.2585	200	600
14	15	0.5910	0.5260	60	10	30	31	0.9744	0.9630	150	70
15	16	0.7463	0.5450	60	20	31	32	0.3105	0.3619	210	100
16	17	1.2860	1.7210	60	20	32	33	0.3410	0.5302	60	40

5.2. IEEE 69-Bus Grid

The IEEE 69-bus grid is an electrical network operated with 12.66 kV at the substation bus located at node 1. This system has a radial structure, i.e., 69 buses and 68 lines.

The electrical configuration of this test feeder is presented in Figure 2b, and its electrical parameters are listed in Table 3.

Table 3. Parametric information for the IEEE 69-bus system.

Node i	Node j	R_{ij} (Ω)	X_{ij} (Ω)	P_j (kW)	Q_j (kvar)	Node i	Node j	R_{ij} (Ω)	X_{ij} (Ω)	P_j (kW)	Q_j (kvar)
1	2	0.0005	0.0012	0	0	3	36	0.0044	0.0108	26	18.55
2	3	0.0005	0.0012	0	0	36	37	0.0640	0.1565	26	18.55
3	4	0.0015	0.0036	0	0	37	38	0.1053	0.1230	0	0
4	5	0.0251	0.0294	0	0	38	39	0.0304	0.0355	24	17
5	6	0.3660	0.1864	2.6	2.2	39	40	0.0018	0.0021	24	17
6	7	0.3810	0.1941	40.4	30	40	41	0.7283	0.8509	1.2	1
7	8	0.0922	0.0470	75	54	41	42	0.3100	0.3623	0	0
8	9	0.0493	0.0251	30	22	42	43	0.0410	0.0475	6	4.3
9	10	0.8190	0.2707	28	19	43	44	0.0092	0.0116	0	0
10	11	0.1872	0.0619	145	104	44	45	0.1089	0.1373	39.22	26.3
11	12	0.7114	0.2351	145	104	45	46	0.0009	0.0012	39.22	26.3
12	13	1.0300	0.3400	8	5	4	47	0.0034	0.0084	0	0
13	14	1.0440	0.3450	8	5.5	47	48	0.0851	0.2083	79	56.4
14	15	1.0580	0.3496	0	0	48	49	0.2898	0.7091	384.7	274.5
15	16	0.1966	0.0650	45.5	30	49	50	0.0822	0.2011	384.7	274.5
16	17	0.3744	0.1238	60	35	8	51	0.0928	0.0473	40.5	28.3
17	18	0.0047	0.0016	60	35	51	52	0.3319	0.1114	3.6	2.7
18	19	0.3276	0.1083	0	0	9	53	0.1740	0.0886	4.35	3.5
19	20	0.2106	0.0690	1	0.6	53	54	0.2030	0.1034	26.4	19
20	21	0.3416	0.1129	114	81	54	55	0.2842	0.1447	24	17.2
21	22	0.0140	0.0046	5	3.5	55	56	0.2813	0.1433	0	0
22	23	0.1591	0.0526	0	0	56	57	1.5900	0.5337	0	0
23	24	0.3460	0.1145	28	20	57	58	0.7837	0.2630	0	0
24	25	0.7488	0.2475	0	0	58	59	0.3042	0.1006	100	72
25	26	0.3089	0.1021	14	10	59	60	0.3861	0.1172	0	0
26	27	0.1732	0.0572	14	10	60	61	0.5075	0.2585	1244	888
3	28	0.0044	0.0108	26	18.6	61	62	0.0974	0.0496	32	23
28	29	0.0640	0.1565	26	18.6	62	63	0.1450	0.0738	0	0
29	30	0.3978	0.1315	0	0	63	64	0.7105	0.3619	227	162
30	31	0.0702	0.0232	0	0	64	65	1.0410	0.5302	59	42
31	32	0.3510	0.1160	0	0	11	66	0.2012	0.0611	18	13
32	33	0.8390	0.2816	14	10	66	67	0.0047	0.0014	18	13
33	34	1.7080	0.5646	19.5	14	12	68	0.7394	0.2444	28	20
34	35	1.4740	0.4873	6	4	68	69	0.0047	0.0016	28	20

5.3. Parameters for the Economic Assessment

To determine the annual grid operating costs in the network when the fixed-step capacitor banks are installed, the information regarding sizes and costs is presented in Table 4. Note that this information was adapted from [14].

Table 4. Costs of the capacitors per capacity.

Option	Q_c (kvar)	Cost ($/kvar-Year)	Option	Q_c (kvar)	Cost ($/kvar-Year)
1	150	0.500	8	1200	0.170
2	300	0.350	9	1350	0.207
3	450	0.253	10	1500	0.201
4	600	0.220	11	1650	0.193
5	750	0.276	12	1800	0.870
6	900	0.183	13	1950	0.211
7	1050	0.228	14	2100	0.176

6. Computational Implementation

The proposed two-stage optimization approach was implemented in the MATLAB software, version 2019*b*. The mixed-integer linear programming model was implemented using the CVX and the Gurobi solver. The recursive power flow solution was implemented with our own scripts, which used the successive approximation power flow formulation. All the simulations were run on a desktop computer with an Intel(R) Core(TM) i7-7700 2.8-GHz processor and 16.0 GB of RAM on a 64-bit version of Microsoft Windows 10 Home.

6.1. IEEE 33-Bus Grid

For this test feeder, considering that, throughout the year, the system operates under peak load conditions, as analyzed by [14], the first stage of our proposed optimization method identifies nodes 13, 24, and 30 as the optimal location for the capacitor banks.

Now, by fixing the sizes provided by the MIQC model and applying the refinement stage of our proposed optimization model, the optimal sizes for these locations are 450, 450, and 1050 kvar. Note that this solution has an expected annual operative cost of US$/year 23,747.317, with an investment of US$/year 467.10. These values imply a reduction of 33.04% with respect to the annual cost of the benchmark case (i.e., US$/year 35,445.909 without capacitors). Table 5 presents a comparison with the solution of the exact MINLP model in the GAMS software and the best three solutions reported for our two-stage proposed optimization approach.

Table 5. Optimal location, sizes, and annual expected costs for the IEEE 33-bus system under peak load conditions.

Method	Size (Node) (Mvar)	Losses (kW)	C. Caps. US$	C. Total US$
GAMS	{0.30 (14), 0.45 (24), 1.05 (30)}	139.292	458.25	23,859.313
MIQC (sol. 1)	{0.45 (13), 0.45 (24), 1.05 (30)}	138.473	467.10	23,747.317
MIQC (sol. 2)	{0.45 (13), 0.60 (24), 0.90 (30)}	138.917	410.55	23,748.531
MIQC (sol. 3)	{0.45 (13), 0.45 (24), 0.90 (30)}	139.075	392.40	23,757.083

The most important result in Table 5 is that, with the proposed two-stage optimization method, it is possible to generate a list with the alternative solutions. Note that, in this list, there are three solutions with better final objective function value than the solution obtained with the GAMS software.

6.2. IEEE 69-Bus Grid

For this test feeder, considering that, throughout the year, the system operates under peak load conditions, as analyzed by [14], the first stage of our proposed optimization method identifies nodes 11, 21, and 61 as the optimal location for the capacitor banks. In addition, by fixing the sizes provided by the MIQC model and applying the refinement stage of our proposed optimization model, the optimal sizes for these locations are 450, 150, and 1200 kvar. Note that this solution has an expected annual operative cost of US$/year 24,845.246, with an investment of US$/year 392.85. These values imply a reduction of 34.29% with respect to the annual cost of the benchmark case (i.e., US$/year 37,812.056 without capacitors). Table 5 presents a comparison with the solution of the exact MINLP model in the GAMS software and the best three solutions reported for our two-stage proposed optimization approach.

Note that the main result in Table 6 is that the proposed two-stage optimization method allows for identifying at least four solutions with better objective function values than the solution reached with the GAMS software.

Table 6. Optimal location, sizes, and annual expected costs for the IEEE 69-bus system under peak load conditions.

Method	Size (Node) (Mvar)	Losses (kW)	C. Caps. US$	C. Total US$
GAMS	{0.45 (11), 0.15 (27), 1.20 (61)}	145.738	392.85	24,876.910
MIQC (sol. 1)	{0.45 (11), 0.15 (21), 1.20 (61)}	145.550	392.85	24,845.246
MIQC (sol. 2)	{0.30 (11), 0.30 (21), 1.20 (61)}	145.492	414.00	24,856.573
MIQC (sol. 3)	{0.60 (11), 0.15 (21), 1.20 (61)}	145.614	411.00	24,874.173
MIQC (sol. 4)	{0.45 (11), 0.30 (21), 1.20 (61)}	145.556	422.85	24,876.229

Table 6 confirms that performing an exhaustive exploration once the nodes where the fixed-step capacitor banks will be installed have been identified enables the identification of additional solutions that cannot be found by means of commercial approaches. This situation is particularly important for utilities since additional investment alternatives can be identified prior to making the final decision regarding installation in the grids.

6.3. Numerical Results Considering Daily Load Variations

To verify the effectiveness and robustness of the proposed optimization approach, in this simulation scenario, the daily variations of the active and reactive power curve in 30-min periods were considered [35]. The active and reactive power variations are listed in Table 7.

Table 7. Behavior of the daily active and reactive power consumption.

Time (h)	Active (pu)	Reactive (pu)	Time (h)	Active (pu)	Reactive (pu)
1	0.34	0.2954	25	0.94	0.6764
2	0.28	0.2238	26	0.94	0.7228
3	0.22	0.1964	27	0.90	0.7754
4	0.22	0.1666	28	0.84	0.6868
5	0.22	0.1478	29	0.86	0.7542
6	0.20	0.1654	30	0.90	0.8538
7	0.18	0.1662	31	0.90	0.8448
8	0.18	0.1274	32	0.90	0.7294
9	0.18	0.1404	33	0.90	0.8452
10	0.20	0.1750	34	0.90	0.6162
11	0.22	0.1456	35	0.90	0.5988
12	0.26	0.2428	36	0.90	0.6672
13	0.28	0.2462	37	0.86	0.7086
14	0.34	0.2780	38	0.84	0.6798
15	0.40	0.2820	39	0.92	0.8468
16	0.50	0.3996	40	1	0.8122
17	0.62	0.4994	41	0.98	0.7640
18	0.68	0.6448	42	0.94	0.7640
19	0.72	0.6526	43	0.90	0.7774
20	0.78	0.7322	44	0.84	0.5502
21	0.84	0.7170	45	0.76	0.6766
22	0.86	0.6632	46	0.68	0.4710
23	0.90	0.8374	47	0.58	0.4602
24	0.92	0.7304	48	0.50	0.3636

Considering the daily information in Table 7 for the active and reactive power demands, in the case of the IEEE 33-bus grid, the best solution provided by the proposed two-stage optimization method corresponds to locating the fixed-step capacitor banks in nodes 2, 7, and 30, with sizes of 150, 450, and 450 kvar, respectively. These capacitor banks have an investment cost of US$/year 302.70, with total annual operative costs of US$/year 12,763.112. This value corresponds to a reduction of 17.95% with respect to the benchmark case (i.e., US$/year 15,555.063 without installing fixed-step capacitor banks).

For the IEEE 69-bus system, the proposed methodology identifies the nodes 11, 24, and 61 and fixed-step capacitor banks with sizes of 150, 150, and 600 kvar, respectively. These capacitor banks have an investment cost of US$/year 282, with total annual operative costs of US$/year 13,141.378. This implies a reduction of 20.44% with respect to the benchmark case (i.e., US$/year 16,517.383 without installing fixed-step capacitor banks).

6.4. Applicability in Meshed Distribution Networks

To demonstrate the applicability of the proposed hybrid optimization model to deal with the problem regarding the optimal placement and sizing of fixed-step capacitor banks in electrical distribution grids with meshed configurations, in this simulation scenario, the meshed configuration of the IEEE 33-bus grid was considered with all the tie-lines in the reconfiguration problem studied in [36] permanently closed. For the sake of completeness, the lines added to the IEEE 33-bus system are listed in Table 8.

Table 8. Distribution lines added to the IEEE 33-bus grid.

Node i	Node j	R_{ij} (Ω)	X_{ij} (Ω)	Node i	Node j	R_{ij} (Ω)	X_{ij} (Ω)
8	21	2.0	2.0	12	22	2.0	2.0
9	15	2.0	2.0	18	33	0.5	0.5
25	29	0.5	0.5	—	—	—	—

For the IEEE 33-bus system, where the distribution lines in Table 8 are added to the distribution system topology presented in Figure 2a and the daily active and reactive power behavior in Table 7 is considered, the solution of the MIQC model (1)–(5) defines the location of the fixed-step capacitor banks at nodes 2, 8, and 30, respectively. In addition, by fixing these sizes in the recursive power flow solution methodology, the optimal sizes assigned for these nodes are 150, 300, and 600 kvar, respectively. When these fixed-step capacitor banks are installed, the annual grid operative costs take a value of US$/year 7927.316, i.e., a reduction of 14.88% with respect to the benchmark case.

Note that the most important result in the meshed scenario is as expected: the annual grid operating costs is low in comparison with the radial operative case, since the benchmark case in the radial scenario was US$/year 15,555.063, while, in the full meshed operation scenario, this value decreased to US$/year 9313.495. This is due to the presence of meshes in the distribution network allowing for a better power flow distribution and improving voltage regulation in the network, which are directly related with the reductions in the total energy losses of the network compared to the radial topology.

7. Conclusions and Future Work

The problem regarding the optimal placement and sizing of fixed-step capacitor banks in electrical distribution networks with radial structure was addressed in this research via the application of a two-stage optimization methodology. The first stage of the proposed optimization approach dealt with selecting the nodes where the fixed-step capacitor banks would be installed through the implementation of an MIQC model. The second stage corresponded to an exhaustive assessment of the solution space by using the successive approximation power flow method for each possible size combination of the fixed-step capacitor banks on the nodes, which was provided by the MIQC model.

The numerical results in the IEEE 33- and 69-bus grids showed that the proposed two-stage optimization approach finds better objective function values than the GAMS software with its exact MINLP solvers. For the IEEE 33-bus grid, the difference between the proposed method and the GAMS software was US$/year 111.996. In addition, for this test feeder, the proposed MIQC model found two additional solutions with better objective function values when compared to the GAMS optimal solution. In the case of the IEEE 69-bus system, the proposed two-stage approach found an additional gain of US$/year 31.664. In addition, the proposed MIQC model found three alternative solutions with better objective function values in comparison with the GAMS optimal solution.

As for the reductions with respect to the benchmark case, in the scenario involving a year-long operation under peak load conditions, for the IEEE 33-bus, the annual expected improvement was about 33.04%, and, for the IEEE 69-bus system, it was 34.29%. However, when the daily active and reactive power consumption was considered, the expected improvement was about 17.95% for the IEEE 33-bus grid, and, for the IEEE 69-bus grid, it was about 20.44%. Note that these behaviors were expected in the context of this simulation, given that the daily behavior of the active and reactive power curves corresponded to a realistic operative scenario, whereas the peak load operation represented a theoretical operative scenario that only served to validate new solution methodologies with respect to literature reports.

The main limitation of the proposed optimization method is related to the possibility of evaluating the entirety of the solution space regarding the possible sizes of the fixed-step capacitor banks, since, for large test feeders where more than three capacitor banks are available, the proposed recursive solution methodology entails long processing times. Note that, for a system with an availability of 7 capacitor banks and 14 possible sizes, the dimension of the solution space is 105,413,504, i.e., more than 105 million possible solutions. Therefore, for large solution spaces, it is recommended to replace the recursive power flow evaluation method with a specialized metaheuristic optimization technique that can efficiently deal with discrete variables with reduced computational effort.

As future work, it will be possible to conduct the following studies: (i) to propose a mixed-integer conic model that allows for integrating fixed-step capacitor banks with the optimal grid reconfiguration of the network while considering daily active and reactive power variations; (ii) to extend the proposed MIQC model in order to locate and size renewable energy resources and batteries in distribution networks; and (iii) to consider more realistic models to represent capacitor banks, including energy losses and reactive power injection variability as a function of the voltage at the nodes where the banks are connected.

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Nomenclature

$\mathbb{S}_{cap,h}$	Complex vector that contains all the power outputs in the fixed-step capacitor banks for each period of time h (var).
$\mathbb{S}_{d,h}$	Complex vector with the active and reactive power consumption in the demand nodes for each period of time (VA).
$\mathbb{V}_{d,h}$	Complex vector that contains all the voltages in the demanded nodes for each period of time (V).
$\mathbb{V}_{s,h}$	Complex variable associated with the voltage output at the slack source (V).
$\mathbb{Y}_{\text{bus}}$	Nodal admittance matrix (S).
$\mathbb{Y}_{dd}$	Component of the nodal admittance matrix that associates demand nodes with each other (S).

$\mathcal{A}_{jl}$	Component of the node-to-branch incidence matrix that associates node j with line l.
$\mathcal{C}$	Set that contains all fixed-step capacitor bank types available for installation in the distribution grid.
$\mathcal{H}$	Set that contains all hours of the operation period (typically 24 h).
$\mathcal{L}$	Set that contains all distribution lines of the network.
$\mathcal{N}$	Set that contains all the nodes of the network.
ε	Parameter associated with the maximum convergence error admissible for the power flow solution (V).
C_{kWh}	Expected costs of the energy losses (US$/kWh-year).
C_c^{cap}	Installation cost of the fixed-step capacitor bank type c (US$/kvar).
h	Subscript associated with the set $\mathcal{H}$.
$i_{l,h}^i$	Imaginary component of the current flowing through line l in the period of time h (A).
$i_{l,h}^r$	Real component of the current flowing through line l in the period of time h (A).
j	Subscript associated with the set $\mathcal{N}$.
l	Subscript associated with the set $\mathcal{L}$.
m	Superscript associated with the number of iterations.
$N_{\text{ava}}^{\text{cap}}$	Number of fixed-step capacitor banks available for installation.
$P_{\text{loss},h}$	Active power losses in the distribution network for each period of time (W).
$P_{j,h}^d$	Active power generation consumed at node j in period of time h (W).
$p_{j,h}^g$	Active power generation injected at node j in period of time h (W).
Q_c	Reactive power capacity of a type c fixed-step capacitor bank (kvar).
$Q_{j,h}^d$	Reactive power generation consumed at node j in period of time h (var).
$q_{j,h}^g$	Reactive power generation injected at node j in period of time h (var).
R_l	Resistive parameter of the distribution line l (Ω).
T	Length of the planning period (days).
$V_{j,h}^i$	Imaginary component of the voltage magnitude at node j in the period of time h (V).
$V_{j,h}^r$	Real component of the voltage magnitude at node j in the period of time h (V).
x_{sol}	Solution vector that contains the nodes where the fixed-step capacitor banks will be located along with their possible sizes.
x_{jc}	Binary variable associated with the installation ($x_{jc} = 1$) or not ($x_{jc} = 0$) of a fixed-step capacitor bank type c at node j.
z_{approx}	Approximate objective function value associated with the expected annual grid operating costs (US$).
z_{costs}	Expected annual operating costs of the network (US$).

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