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Editorial

Mathematics in Finite Element Modeling of Computational Friction Contact Mechanics 2021–2022

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Abstract: In engineering practice, structures with identical components or parts are useful from several points of view: less information is needed to describe the system; designs can be conceptualized quicker and easier; components are made faster than during traditional complex assembly; and finally, the time needed to achieve the structure and the cost involved in manufacturing decrease. Additionally, the subsequent maintenance of this system then becomes easier and cheaper. The aim of this Special Issue is to provide an opportunity for international researchers to share and review recent advances in the finite element modeling of computational friction contact mechanics. Numerical modeling in mathematics, mechanical engineering, computer science, computers, etc. presents many challenges. The finite element method applied in solid mechanics was designed by engineers to simulate numerical models in order to reduce the design costs of prototypes, tests and measurements. This method was initially validated only by measurements but gave encouraging results. After the discovery of Sobolev spaces, the abovementioned results were obtained, and today, numerous researchers are working on improving this method. Some of applications of this method in solid mechanics include mechanical engineering, machine and device design, civil engineering, aerospace and automotive engineering, robotics, etc. Frictional contact is a complex phenomenon that has led to research in mechanical engineering, computational contact mechanics, composite material design, rigid body dynamics, robotics, etc. A good simulation requires that the dynamics of contact with friction be included in the formulation of the dynamic system so that an approximation of the complex phenomena can be made. To solve these linear or nonlinear dynamic systems, which often have non-differentiable terms, or discontinuities, software that considers these high-performance numerical methods and computers with high computing power are needed. This Special Issue is dedicated to this kind of mechanical structure and to describing the properties and methods of analysis of these structures. Discrete or continuous structures in static and dynamic cases are also considered. Additionally, theoretical models, mathematical methods and numerical analysis of these systems, such as the finite element method and experimental methods, are used in these studies. Machine building, automotive, aerospace and civil engineering are the main areas in which such applications appear, but they can also be found in most other engineering fields. With this Special Issue, we want to disseminate knowledge among researchers, designers, manufacturers and users in this exciting field.

Keywords: finite element analysis; weak solutions; elastic material; composites; contact problems; variational inequalities; friction laws; statics; kinetic or sliding friction; optimal control

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1. Introduction

The application of mathematics in the industry has led to interesting problems in the study of the mechanical systems and has revealed interesting properties and various methods in the field of engineering. There are many areas where mathematics is applied in the design and calculus of mechanical systems such as in automotive engineering, airspace engineering, construction, manufacturing, etc. [1–12]. All these reasons have resulted in continuous research, which has led to further developments in the field. Some such research is presented in this Special Issue, in which many researchers present their latest findings. We hope that other researchers will find the presented information interesting and useful for their future work and the results useful to engineers in practical applications.

2. Statistics of this Special Issue

The submission statistics of this Special Issue were as follows: total submissions, 22; published, 10 (45%); and rejected, 12 (55%) (see [13–22]). The geographical distribution, by country of authors whose papers were published is shown in Table 1, and it can be seen that the 25 authors are from 6 different countries. Note that it was usual for papers to be written by more than one author and for authors to collaborate with those from different affiliations or to have more than one affiliation.

Table 1. Geographic distribution of authors by country.

Country	Number of Authors
Romania	12
Saudi Arabia	3
China	6
UK	1
Egypt	2
Slovakia	1
Total	25

3. Brief Overview of the Contributions to this Special Issue

This section classifies the manuscripts according to the topics proposed in this Special Issue. An analysis of the topics identified and summarized the research undertaken. It was observed that there are three topics that dominated: mathematics applied in mechanical engineering, in the field of materials (continuous mechanics) and in civil engineering.

In [13], a generalized Lord and Shulman-type thermoelastic model created based on Eringen's non-local continuum theory was presented. The viscoelastic properties of the studied isotropic material were determined using the Kelvin–Voigt linear model. Analytical solutions for the fields of displacement, temperature and thermal stresses were then obtained, which were then represented graphically for local and non-local viscothermoelastic materials in order to evaluate the quality of wave propagation in different studied media. In their work, comparisons were made between the situations with and without thermal relaxation time. The practical application of the obtained results can be promising in the analysis of nanostructures, taking into account the effects of a small size.

Based on a modern dimensional analysis (MDA), in [14], a model is developed and experimentally validated to study heat transfer in rectangular hole beams. After presenting the basic elements of model law (ML), the authors conceived a set of prototypes and models at different scales. In this paper, the following were presented: the original test stand of these structural elements; a block diagram of the original electronic heating and control system; basic considerations regarding the particularity of this heating system from the point of view of heat transfer; and measurement data, obtained for both nonthermally protected elements and for those protected with layers of intumescent paints. In the last part of the paper, the values obtained by rigorous direct measurements using ML on the prototypes and models were compared. Almost identical values were obtained.

Thermophotovoltaic interactions using a new mathematical model of thermoelasticity established on a modified type III Green–Naghdi model (GN-III) were presented in [15]. The basic equations, in which heat transfer is in the form of the Moore–Gibson–Thompson (MGT) equation, were derived by adding a single delay factor to the GN-III model. The impact of temperature and electrical elastic displacement of semiconductors throughout the excited thermoelectric mechanism were studied theoretically using this model. The influence of rotation parameters on various photothermal characteristics of a silicon solid was presented and explored using the Laplace technique.

An interesting study concerning the thermo-diffusion interaction in an unbounded material with spherical cavities in the context dual phase lag model was performed in [16]. The applied model was a finite element model. The bounding surface of the inner hole was loaded thermally by external heat flux and was traction-free. The delay times in the microstructural interactions caused thermal physics to need to take into account the hyperbolic effects within the medium, and the phase lags of chemical potential and diffusing mass flux vector were interpreted. A comparison of the presence and absence of mass diffusions during coupling was made using the Lord–Shulman and dual phase lag theories in [13].

In [17], the authors proved that the presence of voids and internal state variables in an elastic body with a dipolar structure has no effect on the uniqueness regarding the solution of the initial-boundary value problem within this context. Therefore, some auxiliary estimates underlying the result of uniqueness were obtained. Finally, by means of these estimates and by using Gronwall's inequality, the main result is proven.

Based on various models, in [18], a new model for porothermoelastic waves under a fractional time derivative and two time delays was utilized to study temperature increments, stress and the displacement components of the solid and fluid phases in porothermoelastic media. The governing equations were presented under Lord–Shulman theory with thermal relaxation time and the finite element method was used to solve these equations. The effects of fractional parameter and porosity in porothermoelastic media were also studied. These results will allow future studies to gain detailed insights into non-simple porothermoelasticity with various phases.

The paper in [19] can provide particularly useful results in biology. Numerical estimations of a nonlinear hyperbolic bioheat equation under various boundary conditions for medicinal treatments of tumor cells were constructed. The heating source components in a nonlinear hyperbolic bioheat transfer model, such as the rate of blood perfusions and the metabolic heating generations, were considered experimentally temperature-dependent functions. A parametric analysis was then performed to identify an appropriate procedure for selecting significant design variables in order to yield further accuracy to thus achieve efficient thermal power in hyperthermia treatments.

In [20], optimal control for the solutions to the problem with dry friction quasistatic contact is studied. The optimal control problem consists, in our case, of minimizing a sequence of optimal control problems, where the control variable is given by a Neumann-type boundary condition. The state system is represented by the limit of a sequence, whose terms are obtained from the discretization, in time, with the finite difference, and space, with the finite element method, of a regularized quasistatic contact problem. The purpose of this optimal control problem is to ensure that the traction force (the control variable) acting on one side of the boundary (the Neumann boundary condition) of the elastic body produces a displacement field (the state system solution) close enough to the imposed displacement field and that the traction force from the boundary remains small enough.

An interesting problem with future practical application is studied in [21]. The vibration behavior of a concrete structure currently used in civil engineering is presented. The truss structure considered has symmetries that can be used to facilitate both the design and construction of a building. Moreover, the symmetries encountered can be used to simplify the calculation of vibrations in the system. In this way, the dynamic analysis of the structure, and the design and the costs related to this stage can be simplified.

Finally, the last article presents another application with possible practical developments. Rotary piezoelectric motors based on the converse piezoelectric effect [22] are very competitive in the fields of precision driving and positioning. Miniaturization and larger output capability are crucial design objectives, and efforts have been persistent made to study structural modification, new materials application and optimization of control systems, but their effectiveness has been limited. The resonance rotor excited by a stator is investigated, and the meshing drive mechanism behind double traveling waves is proposed. Based on a theoretical analysis of bending vibration, the finite element method (FEM) is used to compare the modal shape and modal response in the peripheric, axial, and radial directions for the stator and three rotors. Verified by the prototype experimental results, the speed of the proposed motor is the sum of the velocity of the stator's contact particle and the resonance rotor's contact particle, while the torque is less than twice the motor using the reference rotor.

A number of papers on very interesting topics within the theme covered by this Special Issue and some possible practical applications were published. Rigorous mathematical methods were used in the studies, and the obtained results are consistent and, in some cases, experimentally verified.

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