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Yield Curve Models with Regime Changes: An Analysis for the Brazilian Interest Rate Market

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Abstract: This study examines the effectiveness of various specifications of the dynamic Nelson–Siegel term structure model in analyzing the term structure of Brazilian interbank deposits. A key contribution of our research is the incorporation of regime changes and other time-varying parameters in the model, both when relying solely on observed yields and when incorporating macroeconomic variables. By allowing parameters in the latent factors to adapt to changes in persistence patterns and the overall shape of the yield curve, these mechanisms enhance the model's flexibility. To evaluate the performance of the models, we conducted assessments based on their in-sample fit and out-of-sample forecast accuracy. Our estimation approach involved Bayesian procedures utilizing Markov Chain Monte Carlo techniques. The results highlight that models incorporating macro factors and greater flexibility demonstrated superior in-sample fit compared to other models. However, when it came to out-of-sample forecasts, the performance of the models was influenced by the forecast horizon and maturity. Models incorporating regime switching exhibited better performance overall. Notably, for long maturities with a one-month ahead forecast horizon, the model incorporating regime changes in both the latent and macro factors emerged as the top performer. On the other hand, for a twelve-month horizon, the model incorporating regime switching solely in the macro factors demonstrated superior performance across most maturities. These findings have significant implications for the development of trading and hedging strategies in interest rate derivative instruments, particularly in emerging markets that are more prone to regime changes and structural breaks.

Keywords: yield curve; forecasting; regime switching; MCMC

MSC: 62F15; 91B84; 60J05

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1. Introduction

Understanding the yield curve's evolution is crucial for various financial activities, such as risk management, pricing financial assets, portfolio allocation, and debt structuring. Additionally, comprehension of the yield curve in conjunction with economic activity is essential for driving a country's monetary and fiscal policy.

The mathematical contribution of this work is the combination of several mathematical methods in the analysis of a very relevant financial market problem, the modeling and forecasting of yield curves using a generalized Nelson–Siegel [1] representation. The modeling framework used combines elements of function approximation theory [2,3], interpolation theory ([4], dimensionality reduction [5,6]), prediction of stochastic processes [7], non-linear time series models [8], Markov chains and hidden Markov models [9,10], structural breaks [11,12] and the use of Bayesian methods through Markov Chain Monte Carlo [13].

The term structure of interest rates is defined as the dynamic evolution of a function that relates the bond yields to each possible continuous valued time to maturity, and thus our object of analysis is the realization of an infinite-dimensional process, as discussed for example in [14] or [15]. The dynamics of the yield curve over time can be represented by an infinite number of stochastic processes in the form of a stochastic differential equation for each maturity using the Heath–Jarrow–Morton framework [16], or through the so-called Musiela parameterization (e.g., [15]), which represents the evolution of the forward yield curve through a stochastic partial differential equation. Our work tries to represent the evolution of the yield curve through a representation structure based on polynomial approximations that simultaneously performs a dimension reduction procedure and allows performing both the interpolation of the curve for unobserved maturities and the forecasting (extrapolation) of the yield curve for future periods.

Using elements from the dimension reduction literature, our curve representation framework uses an approximation of the function that relates yields to maturity through a representation using orthogonal Laguerre polynomials with time-varying loadings based on the Nelson–Siegel [1] representation. In obtaining empirical estimates for the coefficients of this polynomial, we are estimating a finite dimensional approximation that is computationally simple, and this approximation is quite accurate for observed yield curves, as discussed for example in [17]. It is also important to note that the dynamic Nelson–Siegel function used in the work, which is a reparametrization of Laguerre polynomials, allows approximating the first three principal components of the spectral decomposition of the covariance function of the term structure of interest rates, recovering the so-called level, slope, and curvature components of the yield curve, in the sense of [18]. Central banks such as those in Belgium, Finland, France, Italy, Germany, Norway, Spain, Sweden, Switzerland, frequently employ this approach (e.g., [19]).

Ref. [20] proposed a dynamic model for the [1] yield curve, treating each parameter of the cross-section fit of the Nelson–Siegel model as a latent factor. By modeling and forecasting these latent factors, it is possible to obtain forecasts for the entire yield curve. This representation is useful since these estimated components can be used in procedures for hedging and immunizing movements in interest rates, which are essential for the management of portfolios of fixed income assets. It is also important to note that by estimating the dynamic coefficients of the polynomial approximation of the Nelson–Siegel function, we can obtain estimates for yields for any maturity, including unobserved maturities, and thus we are performing a yield curve interpolation/smoothing procedure. Nelson–Siegel polynomials can be interpreted as a second-order Taylor expansion, and thus also impose smoothness on the estimated curve, which is a necessary condition for imposing mathematical constraints of no arbitrage between yields for the different maturities of the curve, as discussed in [17]. Another relevant mathematical connection is the property that the Laguerre polynomial structure of the Nelson–Siegel approximation can be justified as an approximation for a multifactor parameterization of the yield curve [21], using the exponential-affine model structure proposed in [22]. The Nelson–Siegel parameterization appears as the solution of a system of Riccati difference equations that generate the dynamics of an arbitrage-free yield curve, assuming that the structure for the risk premium is given by an exponential-affine formulation, which connects the model used with the literature of dynamic processes.

Our particular contribution is to verify how the modification of the assumed dynamic structure for the latent components (level, slope and curvature in the Nelson–Siegel dynamic representation) affects the in-sample fit and the model out-of-sample forecasts. In particular, we use regime-switching structures to represent the latent components of the process, replacing the structure of an autoregressive process with fixed parameters in time with several representations of processes with varying parameters in time. A regime switching model is a statistical framework used to analyze economic or financial time series data, wherein the underlying data-generating process switches between different regimes or states. In this model, the observed data are assumed to be generated by multiple regimes, each characterized by a distinct set of parameters or relationships. The term “regime” refers to a specific state of the system, which could represent different economic conditions, market regimes, policy regimes, or any other relevant factors affecting the data. The regime switching model allows for the possibility that the behavior of the data changes over time, switching between these different regimes. The key idea behind regime switching models is to capture the nonlinear and time-varying nature of economic or financial data, which cannot be adequately explained by traditional linear models. By allowing for regime shifts, these models can account for structural changes, volatility clustering, asymmetry, and other nonlinear patterns often observed in real-world data.

This modification is relevant since it allows incorporating into the mathematical structure of the yield curve representation the effect of changes in the economic environment that can affect both the persistence structure of the yield curve and, in fact, the shape of the yield curve itself, since we also allow the decay/shape parameter of the Nelson–Siegel function to be time-varying. Thus, the representations with time-varying parameters studied in this work can capture the effect of economic changes, structural breaks and other events that affect the persistence patterns and shape of the yield curve over time. These modifications are especially important for yield curves in emerging markets, which are more sensitive to changes in macroeconomic/monetary policy management both in the country itself and in central countries. The use of regime change frameworks is especially useful in these markets, as seen for example in [23], but it is also relevant in developed markets in the presence of systemic shocks, as discussed for example in [24].

Several approaches have been developed over the years to estimate the yield curve, including no-arbitrage models, equilibrium models, and statistical models. No-arbitrage models are significant as they align with asset pricing theory's constraints. Nevertheless, this approach may be unsuitable for out-of-sample forecasting, according to [25], motivating the use of statistical models to forecast the term structure of interest rates. Given the significance of the Nelson–Siegel model and its extensions in the empirical analysis of the yield curve, we aim to estimate various versions of this model for the Brazilian market, incorporating different methods of time-varying parameters, regime switches and macroeconomic variables. While previous studies such as [26,27] have incorporated macroeconomic variables using Brazilian data, none have used discrete and/or continuous regime switches in the Nelson–Siegel dynamic model. Therefore, we aim to incorporate these parameter changes and examine the model's adjustment and yield curve forecast. The Bayesian estimation method is based on Markov Chain Monte Carlo (MCMC) with a hybrid Gibbs/Metropolis Hastings sampling, following [28].

Nelson–Siegel's dynamic model extensions, which embody macroeconomic variables [29], discrete regime changes [30,31], and continuous regime changes and time-varying parameters [32–34], show an improvement in the adjustment and/or prediction of US term structure and have enriched the research in this area. In addition, Nelson–Siegel's dynamic model with macroeconomic variables and regime changes provides great flexibility for understanding the interactions between the yield curve and macroeconomics [17].

Although out-of-sample forecasts utilizing extensions of the Nelson–Siegel dynamic model have been widely studied for US data, which is generally more stable than Brazilian data, different approaches incorporating Markovian regime switches into the yield curve have been motivated by evidence of regime switches in both US yield curve data [30–33] and Brazilian macroeconomic data [35–37]. This suggests that our proposed models have the potential to be useful in both markets. Furthermore, the significant changes in the shape of the yield curve over time indicate that the latent factors have substantial variability over time, making our proposed models more compelling.

The empirical results demonstrate that models that incorporate regime switches and macroeconomic variables provide a better in-sample fit for the inference of the Brazilian Interbank Deposits curve, highlighting the relevance of the proposed approaches. With regards to out-of-sample forecasting, the models with better predictive power vary depending on the forecast horizon and the maturity analyzed. For the 1-month forecast horizon, the incorporation of macroeconomic factors improves the forecasts, while for higher maturities, the incorporation of regime switches also enhances predictive accuracy. Specifically, for the 12-month horizon, the regime switch model based on [28] for macroeconomic factors shows superior performance.

The structure of this paper is as follows: in Section 2, we present the methodology used for the modeling, describing the modifications of [20] incorporating the possibility of regime switches and macroeconomic variables into the model. In Section 3 is shown the descriptive analyses of the database. In Section 4, the results of in-sample analyses and out-of-sample forecasts are presented. Finally, in Section 5, we discuss the main results and their conclusions.

2. The Dynamic Nelson–Siegel Model and Its Extensions

2.1. The Nelson–Siegel Static Formulation and Its Dynamic Version

The Dynamic Nelson-Siegel Model, introduced by [20], employs variations in the exponential components of the Nelson–Siegel [1] model to capture the entire yield curve on a period-to-period basis using three time-varying parameters. The static formulation of the Nelson–Siegel model for the spot rate y curve on a given day is:

$$y(\tau) = \beta_1 + \beta_2\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_3\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right)$$

The model interprets the three yield curve coefficients of the Nelson–Siegel model as latent factors representing the level, slope, and curvature components of the yield curve, using a structure of a constant plus Laguerre polynomials. With the reinterpretation given by [20], this curve took the following form:

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}\right) + \epsilon_t(\tau) \tag{1}$$

where λ is responsible for the exponential decay rate, and β_{1t}, β_{2t}, β_{3t} are the three dynamic latent factors. These βs represent, respectively, long-, short- and medium-term components. The evolution of the latent factors of this model is given by a first-order autoregressive vector:

$$\begin{bmatrix}\beta_{1t}\\ \beta_{2t}\\ \beta_{3t}\end{bmatrix} = \begin{bmatrix}\mu_1\\ \mu_2\\ \mu_3\end{bmatrix} + \Phi\begin{bmatrix}\beta_{1t-1}\\ \beta_{2t-1}\\ \beta_{3t-1}\end{bmatrix} + \begin{bmatrix}\eta_{1t}\\ \eta_{2t}\\ \eta_{3t}\end{bmatrix} \tag{2}$$

where Φ denotes the matrix (3×3) of autoregressive vector parameters and $\eta \sim \mathrm{N}(0, \Omega)$.

The model's effectiveness in predicting out-of-sample yield curves is investigated through autoregressive models for the factors, enabling the projection of the entire yield curve by predicting the factors. The good out-of-sample forecasting properties of this model are analyzed in [20], showing its superiority over several other yield curve prediction methods.

Several methods are available for the estimation of the Nelson–Siegel dynamic model, ranging from the ordinary least squares estimation based on a two-step procedure [20], to the maximum likelihood estimation using space-state representation given by Equations (2) and (3) in conjunction with the Kalman filter [29], to Bayesian analyzes using Markov Chain Monte Carlo (MCMC) (e.g., [38,39]) and Laplace approximation [40] methods. The state space representation of the model facilitates its generalization to incorporate a layered hierarchical structure, which is useful for modeling joint yield curves. These layers naturally arise in multinational analyzes, where a country's yield curves may depend on its own factors, and these factors may depend on global factors. Papers such as [41,42] do this modeling.

To incorporate no-arbitrage restrictions into the model, it is necessary to insert a yield curve adjustment term. Ref. [43] shows how Nelson–Siegel's dynamic model can be adjusted to fit the Duffie–Kan [22] class of no-arbitrage models, which by definition do not allow the possibility of gain without risk by corresponding price differences of the same asset with different maturities. Incorporation of the adjustment term significantly increases the complexity of the model. In addition, there is empirical evidence that the

Nelson–Siegel dynamic model has almost no arbitrage without the need for the adjustment term (e.g., [43]), and the no-arbitrage restrictions do not represent significant gains in forecasting performance, as discussed by [25].

The Nelson–Siegel dynamic model has been primarily explored from a finance perspective, but it also offers opportunities for economic analysis in macro finance. One approach is to use the yield curve to explain or predict macroeconomic variables or vice versa. However, incorporating macroeconomic variables into the model can lead to significant gains as it captures the dynamic interactions between yield curve factors and macroeconomic fundamentals.

One example of incorporating macroeconomic variables into the Nelson–Siegel dynamic model is the work of [29]. This paper estimates the yield curve by including observable macroeconomic variables such as real activity, inflation, and monetary policy instruments to capture macroeconomic dynamics. Their findings suggest a strong two-way causality between yield curve factors and macroeconomic variables.

Studies have also analyzed the relationship between the level, slope, and curvature factors of the yield curve and macroeconomic fundamentals. Economic theory suggests that the nominal yield curve level should be related to the expected inflation level, and therefore, inflation is considered in the work of [29]. Similarly, the slope of the yield curve is related to real economic activity, and manufacturing utilization is used as a macroeconomic foundation in the same paper. However, the role of curvature is argued to be limited, so it is not included in the macroeconomic basis. Finally, macroeconomic policy can affect all factors in the yield curve, and therefore, ref. [29] includes a variable for the federal funds rate. Overall, incorporating macroeconomic variables into the Nelson–Siegel dynamic model can provide a more comprehensive understanding of the yield curve's behavior and its relationship with macroeconomic fundamentals.

2.2. The Dynamic Nelson–Siegel Model with Time-Varying Parameters and Regime Switching

Several articles, discussed in what follows, found empirical evidence that the dynamic Nelson–Siegel model also significantly improved with the incorporation of regime switching. In these cases, the idea was to change the basic linear dynamics of latent factors, previously represented by β_{1t}, β_{2t}, and $\beta_3 t$ by a vector of latent factors f_t given by:

$$f_t = c + A f_{t-1} + \eta_t \tag{3}$$

by the dynamics of regime switching along the lines:

$$\begin{gathered} f_t = c_{s_t} + A_{s_t} f_{t-1} + \eta_t, \\ \eta_t \sim N(0, \sigma^2_{s_t}) \end{gathered} \tag{4}$$

where s_t = 1, 2 for two-state models. Standard linear dynamics is a special case when $c_1 = c_2$, $A_1 = A_2$ and $\sigma^2_1 = \sigma^2_2$.

In addition, s_t can be a first-order Markov process, where regimes are determined by an unobserved state or regime variable that follows a discrete state Markov process, which is one of the cases that is explored in our estimations. Discrete state Markov processes (Markov chains) are very popular for state-dependent behaviors. They are applied to modeling and forecasting business cycles, term interest rate structure, volatility in economic and financial variables, exchange rate dynamics, and inflation rate dynamics (e.g., [17]). Thus, one can go further, with the possibility of variation over time through a probability transition matrix, with s_t guided by (two regimes case):

$$P_t = \begin{pmatrix} p_{11,t} & 1 - p_{11,t} \\ 1 - p_{22,t} & p_{22,t} \end{pmatrix} \tag{5}$$

where $p_{11,t}$ and $p_{22,t}$ are the probabilities of regime switch in each period.

One paper that incorporates regime switches into the Nelson–Siegel dynamic model is [31], which uses a model with a latent variable with Markov switching that allows jumps in the stochastic process followed by interest rates. They derive discretionary time no-arbitrage constraints for this model and evaluate if yield curve breaks from variations in the λ decay parameter can be captured using a regime switch model to avoid potential overfitting. They assess which extensions allow variations in the shape of the yield curve over time while incorporating non-arbitrage constraints into Markov's switch structure. They conclude that the single-regime model, which treats λ as a continuous time-varying factor, performs very well in fit and forecasting only over a very short-term horizon. Models with switches in the decay parameter have good forecasting performance in the medium- and long-term horizons. Thus, the performance of the model depends on the forecast horizon, where several extensions have better performance than the dynamic Nelson–Siegel single regime model.

Ref. [32] also incorporates regime switches into the term structure of Nelson–Siegel's dynamic model, and they include macroeconomic factors that have already been incorporated in texts such as [29]. They use an MCMC procedure and find that macroeconomic factors help to explain yield curve movements and improve the forecast performance of the yield curve. Additionally, as the forward interest rate structure contains important information about economic activity, the yield curve with macroeconomic factors also helps to explain macroeconomic problems and produce higher GDP growth forecasts. The MCMC method allows for the efficient estimation of the model and the extraction of latent factors from the yield curve and unobserved regimes simultaneously. They conclude that regime switches are important to understand the interaction between the yield curve and economic activity.

Ref. [30] proposed a model that incorporates regime switch in a more appropriate way for the US forward interest rate, which has been at its effective lower limit (zero lower bound) since December 2008. Unlike the previous papers by [31,32], ref. [30]'s model estimates a normal state and a zero-bound state. To generate variation even at short-term rates, a positive probability of the zero-threshold output is given at any time, which is modeled as a time-varying process and a point Poisson process whose first jump indicates the exit from the zero lower bound. For the normal state, the yield curve is modeled using the non-arbitrage version of the Nelson–Siegel model developed by [43]. When the economy leaves the zero-bound state, it is assumed to return to the normal state, representing a switch in the dynamic structure of the yield curve rather than a discrete switch.

Refs. [33,34] also include regime switches in Nelson–Siegel's dynamic model but focus only on the in-sample analysis of the adjustment. Ref. [33] uses US data and finds that the inclusion of regime switches in the decay factor improves the in-sample forecast compared to both the standard Nelson–Siegel dynamic model and the model with the inclusion of regime switch in volatility. On the other hand, ref. [34] uses Japanese corporate bond data and incorporates a probability transition matrix guided by macroeconomic indicators, which improves the yield curve adjustment.

It is important to highlight that Brazil has characteristics distinct from the US financial market, with lower securities liquidity and a greater number of interventions, which may lead to more frequent regime switches and higher price volatility due to macroeconomic uncertainties. Moreover, for the Brazilian market, several papers (e.g., [35–37]) have found that regime switches have a better fit for estimating monetary policy parameters, Taylor rules and main parameters of the Brazilian economy used in DSGE models.

However, even in the Brazilian case, the literature finds empirical evidence that the dynamic Nelson–Siegel model and its extensions have greater predictive power compared to other term structure and random walk models (e.g., [26,39,44,45]). With the introduction of macroeconomic factors to the dynamic Nelson–Siegel model, ref. [27] found a significant improvement for horizons greater than 3 months, with no significant losses for 1-month horizons when compared to the dynamic model without macroeconomic factors.

This paper proposes to estimate the dynamic model while taking into account these regime switches to verify if the incorporation of regime switches in macroeconomic variables improves predictive accuracy and adjustment. In the Brazilian case, Nelson–Siegel term interest rate models with macro factors and the incorporation of regime switches have not yet been estimated.

2.3. Proposed Extensions

In this section, we present various extensions of the Nelson–Siegel dynamic model used for yield curve parameterization. The estimation of the yield curve is based on Equations (1) and (2), which provide its basic structure. To account for regime switches, we employ a dynamic Nelson–Siegel model that utilizes a Markov process to model the probabilities in a transition matrix. Our proposed models incorporate regime switches in the mean and persistence, as well as in conjunction with regime switches in macroeconomic variables.

The macroeconomic extension of the dynamic Nelson–Siegel model has three observable macro factors: manufacturing utilization capacity (CU), the federal funds rate (FFR), and inflation (INFL), which, according to [29], represent a real factor for capturing real economic activity, a monetary policy factor for capturing the effect of policy, and finally a factor for describing the price level, respectively. State variables are represented in a vector (7×1), $X'_t = (\beta_{1t}\ \beta_{2t}\ \beta_{3t}\ \lambda_t\ CU_t\ FFR_t\ INFL_t)$ and $\mathbf{y}'_t = (y_{t(\tau_1)}\ \cdots\ y_{t(\tau_n)})$, so the model can be succinctly written as:

$$\mathbf{y}_t = \Lambda X_t + \epsilon_t \tag{6}$$

where Λ is an array of coefficients $(\mathrm{N} \times 7)$.

To enable model state switching, based on [28], suppose s_t is a discrete random variable with domain 1, 2, ..., K, and assume a first-order Markov process with K-states. The transition matrix can be written as:

$$\begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1K} \\ p_{21} & p_{22} & \cdots & p_{2K} \\ . & . & & . \\ . & . & \cdots & . \\ p_{K1} & p_{K2} & \cdots & p_{KK} \end{pmatrix} \tag{7}$$

where $p_{ij} = Pr(s_t = j | s_{t-1} = i)$ with $\sum_{j=1}^{K} p_{ij} = 1$ for $i = 1, \ldots, \mathrm{K}$ and the state variable s_t defines a particular regime for parameter values. For example, $s_t = 1$ corresponds to the first scheme, and $s_t = K$ corresponds to the last scheme. The state equation with regime switching is given by:

$$X_{t+1} = \alpha_{s_{t+1}} + \phi X_t + \eta_t \tag{8}$$

where

$$\alpha_{s_t} = \gamma_1 + \sum_{j=2}^{K} \gamma_j I_{jt} \tag{9}$$

and I_{jt} is a variable equal to 1 when s_t is larger or equal to j.

This model can capture the changes in behavior due to economic forces as well as discrete abrupt changes due to atypical events, as it allows different α in different regimes, which can capture occasional discrete switches (jumps).

Additionally, the following assumptions are made: (i) the coefficient ϕ must be within the unit circle; (ii) each regime must correspond to at least one point in time; and (iii) every $\gamma_j (j = 2, \ldots, \mathrm{K})$ is negative. Hypothesis (i) guarantees stationarity given the state s_t, and assumption (ii) ensures that γ_j's will not be unidentified. Finally, hypothesis (iii) implies that the higher the value of s_t, the lower the level. Thus, the first regime defines the state with the highest level, and the last regime defines the state with the lowest level.

2.4. Bayesian Estimation

Bayesian inference offers several advantages when applied to non-linear and latent models. This methodology addresses problems in traditional estimation mechanisms, such as nonlinearity, identification, and dimensionality. It allows for the incorporation of prior beliefs about the parameters and latent variables, and it provides a mechanism to update these beliefs based on observed data using Bayes' theorem. This leads to more robust and accurate estimates of the underlying quantities of interest. In Bayesian inference, the focus is not only on estimating the point estimates of parameters, but also on obtaining the full posterior distribution. This distribution provides a complete characterization of the uncertainty associated with the parameter estimates. It allows for the computation of various summaries such as credible intervals, which provide a range of plausible values for the parameters. These summaries are valuable for making informed decisions and understanding the variability in the model's predictions.

Bayesian inference allows for the incorporation of prior knowledge or information about the parameters and latent variables into the analysis. This is particularly useful when data are scarce or limited, as it helps in leveraging existing knowledge to improve the estimates. Prior information can be based on expert opinions, previous studies, or even results from related models. By combining prior information with observed data, Bayesian inference provides a coherent and principled framework for inference. Bayesian inference naturally lends itself to hierarchical modeling, which is commonly used in non-linear and latent models. Hierarchical models allow for capturing dependencies and heterogeneity across different levels of the data.

Bayesian inference methods have been successfully used in many problems with complex structures in several application areas. For example, in environmental applications, we can cite [46,47]. Applications of Bayesian methods in prediction and forecasting procedures can be seen in [48–52]. Bayesian methods are also important in estimating term structure models of interest rates, as can be seen in [38,39,42] by means of Markov Chain Monte Carlo using Gibbs and Metropolis–Hastings sampling, ref. [40] using analytical Laplace approximations, and [53] using Hamiltonian Monte Carlo methods.

To account for the non-linear structure and the presence of latent factors, we adopt a combination of Gibbs and Metropolis–Hastings to generate samples from the posterior joint distribution of unknown parameters and latent variables. This combines Markov Chain Monte Carlo (e.g., [13]) and data augmentation for parameter estimation, while the Gibbs sampling algorithm is used for latent factors. The use of Bayesian methods with MCMC algorithms enables estimation using all available information from the yield curve, without the imposition of ad hoc constraints. With MCMC estimation, both linear and nonlinear models are treated similarly. A significant advantage of the Bayesian methodology is its ability to treat latent factors as additional parameters for estimation.

In Bayesian inference, the objective is to find the so-called posterior distribution of parameters of interest conditioned to the observed sample, denoted by $p(\Theta|y)$. This posterior distribution is the result of updating the assumed prior distribution for the parameters with the information in the sample, represented by the likelihood function. To find the distribution of the conditioned parameters to the sample, the following Bayes theorem is used:

$$p(\Theta|y) = p(\Theta, y)/p(y) = p(y|\Theta)p(\Theta)/p(y) \tag{10}$$

where $p(y|\Theta)$ is the model likelihood, $p(\Theta)$ denotes the assumed prior distribution for the parameter, and $p(y)$ is the marginal distribution of the sample, which needs to be known until a constant of integration. This can be represented by:

$$p(\Theta|y) = p(\Theta, y)/p(\Theta) = p(y|\Theta)p(\Theta)/c \tag{11}$$

Thus, the posterior distribution is proportional to the likelihood product by the prior distribution:

$$p(\Theta|y) \propto p(y|\Theta)p(\Theta) \tag{12}$$

The primary objective of Bayesian estimation is to obtain the posterior distribution, which incorporates prior information updated by the available information in the sample, as given by the likelihood function. A Monte Carlo methodology commonly used in Bayesian estimation is Markov Chain Monte Carlo (MCMC) algorithms. MCMC methods simulate a Markov chain with a stationary distribution that converges to the distribution $p(\Theta|y)$. A crucial result is that the estimation of $p(\Theta|y)$ can be factored using a method of sampling the conditional parameter distributions, which have smaller dimensions and can be simulated more easily. This method avoids numerical maximization methodology, which can be problematic for nonlinear functions. The empirical validity of this methodology is verified using methods to determine the convergence of Markov chains to their stationary distribution. Another advantage of Bayesian inference methods is that the use of prior information can help address some problems, such as the estimation of poorly specified models. For space reasons, we do not present the full details of the estimation procedures, but details can be found in [39], for example. The prior structure for the analyzed models is described in Section 4.

Due to space constraints and the overall relevance to other aspects of the text, we have not included the convergence diagnostics of each estimated model in this presentation of results. Additionally, considering that each model involves a large number of parameters, it would be impractical to present the typical convergence results for each parameter. Furthermore, as we perform out-of-sample forecasting procedures, each model is re-estimated in different samples, making it challenging to provide diagnoses for each model–sample combination.

Nevertheless, we adopted a comprehensive strategy to ensure convergence of the analyzed models. We conducted various standard convergence diagnostics, including visual methods such as trace and autocorrelation plots, as well as formal tests such as the Gelman–Rubin diagnostic and the Geweke diagnostic. Based on the results of these analyses, we established a minimum chain size (i.e., number of iterations) that would guarantee convergence across all estimated models.

To this end, we set a chain size of 100,000 iterations, discarding the initial 10,000 samples as a burn-in procedure. This approach helps eliminate any potential bias introduced by the initial values of the chains and allows the subsequent samples to better represent the target posterior distribution. While specific convergence diagnostics for individual models are not reported here, we have taken appropriate measures to ensure the convergence and reliability of our results.

3. Descriptive Analysis

The analyzed database contains monthly closing data (last day of each month) of Interbank Deposit's (ID's) future contracts with different maturities (one, three, six, nine, twelve, fifteen, eighteen, twenty, twenty-four, twenty-seven, thirty, thirty-three, thirty-six, thirty-nine, forty-eight, sixty, and seventy-two months), available in B3, former BM&FBovespa (http://www.bmfbovespa.com.br/pt_br/servicos/market-data/consultas/mercado-de-derivativos/precos-referenciais/taxas-referenciais-bm-fbovespa/ (accessed on 1 October 2018), from February 2003 to August 2017. The yields for fixed maturities are constructed using cubic splines interpolation for the observed yields.

The ID futures contract is a financial instrument that represents a hypothetical security with a face value of BRL 100,000.00 that is due on a specific date in the future. These contracts are traded at a unit price (PU) that reflects the present value of a security, which would be redeemed at maturity for its nominal value based on the prevailing interest rate with a period of n business days until maturity. Unlike other countries where the main reference for the term structure is given by government bonds, in Brazil, the DI futures contract is the main interest reference due to the great liquidity of this contract, the ease of trading on an exchange, and the fact that by the use of margin structures and collateral, the contract is a benchmark for an asset free of default risk. It is often used as an interest rate benchmark in pricing other financial instruments, and the market benchmark in the fixed income market.

Macroeconomic variables were incorporated with monthly data following the [29] specification, which include manufacturing utilization capacity, the proxy for federal funds rate as the Selic rate, and inflation rate measured by the IPCA (*Indice de Preços ao Consumidor Agregado*). The data are available at National Confederation of Industry, Central Bank of Brazil and *nstituto Brasileiro de Geografia e Estatística* (IBGE), respectively, for February 2003 to August 2017.

Figure 1 shows the observed ID data at different maturities in time. The temporal variations, mainly of slope and curvature, are visible in this graph. Table 1 shows the descriptive statistics of the yield curve and the empirical measures of slope and curvature. Both slope and curvature have a lower standard deviation when compared to individual forward interest rates, and they are also less persistent. The pattern presented by Figure 1 makes its analysis interesting for the proposed models. It can be observed that the shape of the yield curves changes over time, indicating that latent factors should have great variability. It is also evident that the curve decay pattern changes considerably over time, justifying the use of time-varying parameters as opposed to the fixed decay parameter assumed in the usual estimation of the [20] model. Figure 2 shows the median yields for each maturity. Even long-term interest rates show oscillatory movements, which is more common in developing countries since in developed countries, long-term maturities tend to be more stable due to their greater economic stability.

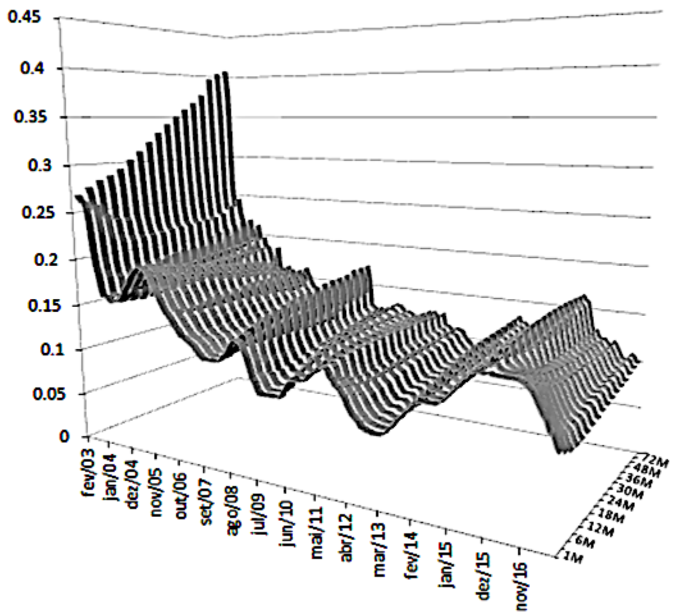

Figure 1. Observed Interbank Deposit Rates for the 17 Different Maturities.

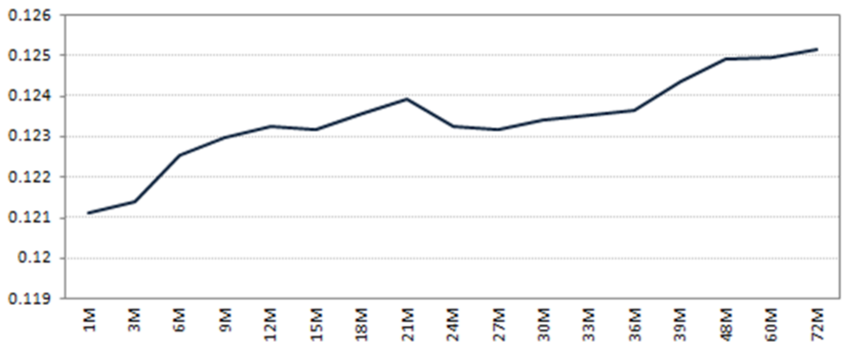

Figure 2. Median Yields by Maturity.

Table 1. Descriptive Data Analysis.

Maturity (Month)	Mean	Std Dev.	Maximum	Minimum	$\rho(1)$	$\rho(12)$	$\rho(24)$
1M	0.1293	0.0404	0.2682	0.0697	0.9561	0.4454	0.3688
3M	0.1290	0.0397	0.2758	0.0704	0.9487	0.4421	0.3446
6M	0.1290	0.0388	0.2825	0.0707	0.9389	0.4386	0.3193
9M	0.1292	0.0380	0.2866	0.0708	0.9313	0.4293	0.2993
12M	0.1299	0.0375	0.2933	0.0715	0.9232	0.4179	0.2854
15M	0.1306	0.0371	0.3026	0.0722	0.9130	0.4061	0.2775
18M	0.1314	0.0368	0.3118	0.0740	0.9024	0.3965	0.2722
21M	0.1320	0.0366	0.3217	0.0759	0.8915	0.3893	0.2683
24M	0.1327	0.0366	0.3324	0.0776	0.8799	0.3816	0.2646
27M	0.1333	0.0367	0.3418	0.0791	0.8700	0.3764	0.2605
30M	0.1337	0.0369	0.3509	0.0800	0.8611	0.3735	0.2563
33M	0.1342	0.0371	0.3593	0.0810	0.8533	0.3724	0.2515
36M	0.1346	0.0374	0.3673	0.0820	0.8458	0.3700	0.2459
39M	0.1350	0.0378	0.3768	0.0830	0.8379	0.3665	0.2391
48M	0.1360	0.0392	0.3965	0.0845	0.8286	0.3548	0.2230
60M	0.1370	0.0409	0.4034	0.0868	0.8352	0.3467	0.2075
72M	0.1378	0.0420	0.4072	0.0881	0.8371	0.3406	0.2072
Slope	−0.0085	0.0235	0.0430	−0.1391	0.7756	0.0140	−0.0696
Curvature	−0.0013	0.0152	0.0314	−0.0823	0.8054	0.0211	0.0698

Note: Slope is defined as the difference between the 1-month and 72-month maturity interest rate, and the curvature is defined as twice the 24-month maturity interest rate minus the sum of the maturity rates at 3 months and 72 months. Autocorrelations for 1, 12 and 24 months are shown.

4. Estimation Results—In-Sample Analysis

Several versions of the Nelson–Siegel dynamic model (MDNS) are estimated for the Brazilian yield curve with the incorporation of macroeconomic variables and/or regime switches in the stochastic process. We estimate 13 different specifications, which are detailed below:

1. Dynamic Nelson–Siegel Model (MDNS);
2. Dynamic Nelson–Siegel Model with regime switching in the mean (MDNS-M);
3. Dynamic Nelson–Siegel Model with regime switching in the persistence (MDNS-P);
4. Dynamic Nelson–Siegel Model with regime switching in the loading factor (MDNS-λ);
5. Dynamic Nelson–Siegel Model with exogenous macroeconomics variables (MDNS-Macro);
6. Dynamic Nelson–Siegel Model with regime switching in the mean and with exogenous macroeconomics variables (MDNS-MMacro);
7. Dynamic Nelson–Siegel Model with regime switching in the persistence and with exogenous macroeconomics variables (MDNS-PMacro);
8. Dynamic Nelson–Siegel Model with endogenous macroeconomic variables and regime switching in the mean and in the macroeconomic variables (MDNS-MMacroEnd);
9. Dynamic Nelson–Siegel Model with endogenous macroeconomics variables and regime switching in the persistence and in the macroeconomic variables (MDNS-PMacroEnd);
10. Dynamic Nelson–Siegel Model with regime switching based on [28] in the mean (MDNS-S);
11. Dynamic Nelson–Siegel Model with regime switching based on [28] in the loading factor (MDNS-Sλ);
12. Dynamic Nelson–Siegel Model with regime switching based on [28] in the mean and with macroeconomic variables (MDNS-SmediaMacro);
13. Dynamic Nelson–Siegel Model with regime switching based on [28] in the macroeconomic variables of the slope factor (MDNS-Smacro).

These specifications allow us to analyze how the different characteristics of the models affect the fit and the results obtained.

As discussed in Section 2, the method used for the estimation in all approaches is MCMC using a combination of Gibbs and Metropolis–Hastings samplers, in which 100,000 iterations were performed to arrive at the posterior distribution of parameters and latent factors, with a burn-in (number of samples discarded) of 10,000, and 90,000 iterations for the construction of the posterior distributions. The prior distribution for the mean and autoregressive parameters is given by Gaussian distributions, and for the variance of the latent variables, we assume Gamma distributions. The prior distribution for the latent factors β_ts and λ_t are Gaussian and Log–Gaussian, respectively, where the Log–Gaussian is used to impose positivity for λ_t coefficients. For the Markov transition probabilities in the model with Markov switching, we assume beta distributions, and for the transition parameters in the models using [28], we assume uniform distributions. The specific hyperparameter's values can be obtained from the authors. Due to the high number of specifications analyzed, and the focus on fit results, we do not present the estimated parameters in each model, but these results are also available from the authors.

The posterior mean of the estimated latent factors β_{1t}, β_{2t} and β_{3t} in each model are shown in Figure 3. The latent level factors, β_{1t}, are very similar in all models estimated here. Slope and curvature parameters present divergences between the models, although the first three models (MDNS, MDNS-M, MDNS-P) present very similar factors. Models in which these parameters present a greater displacement are MDNS-λ and MDNS-Sλ, that is, the models with regime changes in the parameters and in the decay factor.

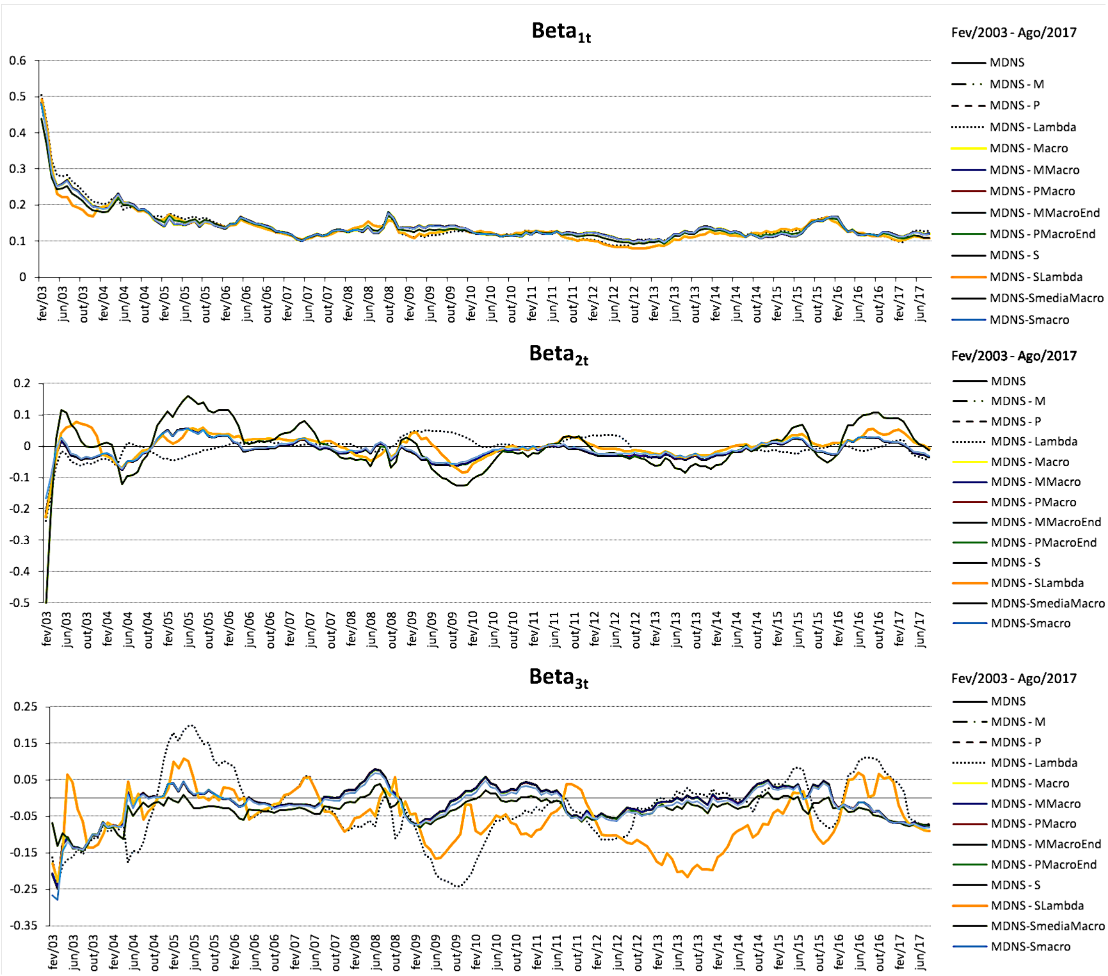

Figure 3. Posterior Distribution of Latent Variables.

We compare the in-sample fit of all estimated models using model error measures—ME (*Mean Error*), RMSE (Root Mean Squared Error) and MAE (Mean Absolute Error)—for each maturity in the sample. Table 2 shows the in-sample error measures. At the 1-month maturity, the model with the best performance in the three criteria is the Nelson–Siegel dynamic model with regime switch in the decay factor (MDNS-λ). For the 6-month and 48-month maturities, it can be observed that the incorporation of macroeconomic variables is relevant for a better adjustment since the highlighted best models incorporate regime change in persistence (MDNS-PMacro and MDNS-PMacroEnd models). In addition, more flexible models, MDNS-PMacroEnd and MDNS-SMacro, have a better fit in most maturities greater than 15 months.

Out-of-Sample Forecast Analysis

In this section we analyze the performance of the out-of-sample forecast for the thirteen models previously specified plus the random walk (RW) model, a usual benchmark for interest rate forecasts. The forecasts in this study cover one- and twelve-month horizons for each of the analyzed maturities. The one and twelve-month forecast horizons represent short and long forecast horizons, and are the most used forecast horizons used in asset pricing and other financial applications of term-structure models.

Previous research, such as [29,32], suggests that incorporating macroeconomic variables can enhance the predictive power of the interest curve. We aim to investigate this proposition using Brazilian ID data. Furthermore, regime changes have been shown to have a significant impact on data behavior, particularly for interest curves with frequent format changes, as evidenced by studies such as [30–32]. Thus, we also want to check if regime switches will be a crucial factor in out-of-sample forecasting for interest curves with greater shape variability. To construct the out-of-sample forecasts, we re-estimate the models using a recursive prediction approach from September 2015 to August 2017. This approach involves extending the sample by one month at a time, and we have an out-of-sample interval of 24 months.

Based on the forecasts, we also compute a squared loss function, given by the squared difference between the observed and the forecasted yield. With this loss function, we use the Model Confidence Set (MCS) procedure, proposed by [54], which determines the best models among the analyzed set according to a confidence level, to compare the predictive accuracy of all models. MCS is built on the basis of a set of models, $\mathcal{M}^0$, and a criterion for evaluating such models empirically, using a loss function. The procedure is based on an equivalence test, $\delta_{\mathcal{M}}$, and an elimination rule, $e_{\mathcal{M}}$. The equivalence test is performed on the set $\mathcal{M} = \mathcal{M}^0$. If $\delta_{\mathcal{M}}$ is rejected, there is evidence that the models in $\mathcal{M}$ are not equally good, and $e_{\mathcal{M}}$ is used to eliminate the models with poor performance. This procedure is repeated until $\delta_{\mathcal{M}}$ is no longer rejected, and the "chosen" models are classified according to their predictive accuracy. The MCS controls for the small sample properties using a bootstrap procedure, and also for the False Discovery Rate generated by the multiple comparisons in the sequential elimination procedure.

Table 3 displays forecast error measures for the 1-Month forecast horizon. Among the considered models, the MDNS-Macro model performs best for the shortest maturities, while the models with greater flexibility, MDNS-MMacroEnd and MDNS-PMacroEnd, stand out for the longest maturities based on error choice criteria. The results of the Model Confidence Set, presented in Table A1 of the Appendix A, confirm the superior performance of the MDNS-Macro model for lower maturities and the MDNS-MMacroEnd model for higher maturities. Therefore, the MDNS-Macro is the recommended choice for shorter maturities, and the MDNS-MMacroEnd is the recommended choice for higher maturities.

Regarding the 12-month forecast, MDNS-Macro shows lower forecasting errors (Table 4) for the first 14 maturities, according to all the chosen criteria. However, for higher maturities, MDNS-S shows less forecast error due to its greater flexibility, despite incorporating fewer external data. In the Model Confidence Set for the 12-month horizon presented in Table A2 in the Appendix A, the MDNS, MDNS-PMacro, and MDNS-PMacroEnd are the standout models for maturities up to 6 months. However, the most prominent model for most maturities is the MDNS-Smacro, which includes both macroeconomic variables and a regime switch based on the work of [28].

Table 2. Error Measures.

Mean Error													
	MDNS	**MDNS-M**	**MDNS-P**	**MDNS-λ**	**MDNS-Macro**	**MDNS-MMacro**	**MDNS-PMacro**	**MDNS-MMacro End**	**MDNS-PMacro End**	**MDNS-S**	**MDNS-SLambda**	**MDNS-Smedia Macro**	**MDNS-Smacro**
1M	$3.67 \cdot 10^{-4}$	$3.64 \cdot 10^{-4}$	$3.68 \cdot 10^{-4}$	$3.04 \cdot 10^{-4}$	$3.57 \cdot 10^{-4}$	$3.56 \cdot 10^{-4}$	$3.52 \cdot 10^{-4}$	$3.44 \cdot 10^{-4}$	$3.53 \cdot 10^{-4}$	$3.68 \cdot 10^{-4}$	$1.133 \cdot 10^{-3}$	$8.40 \cdot 10^{-4}$	$3.14 \cdot 10^{-4}$
3M	$3.0 \cdot 10^{-5}$	$2.9 \cdot 10^{-5}$	$3.0 \cdot 10^{-5}$	$8.98 \cdot 10^{-4}$	$2.8 \cdot 10^{-5}$	$2.9 \cdot 10^{-5}$	$2.7 \cdot 10^{-5}$	$2.1 \cdot 10^{-5}$	$2.8 \cdot 10^{-5}$	$3.1 \cdot 10^{-5}$	$-1.043 \cdot 10^{-3}$	$-7.13 \cdot 10^{-4}$	$1.68 \cdot 10^{-4}$
6M	$-2.45 \cdot 10^{-4}$	$-2.43 \cdot 10^{-4}$	$-2.44 \cdot 10^{-4}$	$-5.29 \cdot 10^{-4}$	$-2.39 \cdot 10^{-4}$	$-2.38 \cdot 10^{-4}$	$-2.37 \cdot 10^{-4}$	$-2.41 \cdot 10^{-4}$	$-2.37 \cdot 10^{-4}$	$-2.44 \cdot 10^{-4}$	$-2.045 \cdot 10^{-3}$	$-1.227 \cdot 10^{-3}$	$-2.46 \cdot 10^{-4}$
9M	$-3.98 \cdot 10^{-4}$	$-3.95 \cdot 10^{-4}$	$-3.97 \cdot 10^{-4}$	$-1.109 \cdot 10^{-3}$	$-3.90 \cdot 10^{-4}$	$-3.89 \cdot 10^{-4}$	$-3.86 \cdot 10^{-4}$	$-3.89 \cdot 10^{-4}$	$-3.87 \cdot 10^{-4}$	$-3.97 \cdot 10^{-4}$	$-2.251 \cdot 10^{-3}$	$-1.359 \cdot 10^{-3}$	$-4.43 \cdot 10^{-4}$
12M	$-3.00 \cdot 10^{-4}$	$-2.98 \cdot 10^{-4}$	$-3.00 \cdot 10^{-4}$	$-1.070 \cdot 10^{-3}$	$-2.93 \cdot 10^{-4}$	$-2.92 \cdot 10^{-4}$	$-2.90 \cdot 10^{-4}$	$-2.91 \cdot 10^{-4}$	$-2.90 \cdot 10^{-4}$	$-2.99 \cdot 10^{-4}$	$-1.983 \cdot 10^{-3}$	$-1.157 \cdot 10^{-3}$	$-3.59 \cdot 10^{-4}$
15M	$-1.19 \cdot 10^{-4}$	$-1.16 \cdot 10^{-4}$	$-1.18 \cdot 10^{-4}$	$-7.99 \cdot 10^{-4}$	$-1.13 \cdot 10^{-4}$	$-1.13 \cdot 10^{-4}$	$-1.11 \cdot 10^{-4}$	$-1.11 \cdot 10^{-4}$	$-1.11 \cdot 10^{-4}$	$-1.18 \cdot 10^{-4}$	$-1.535 \cdot 10^{-3}$	$-8.33 \cdot 10^{-4}$	$-1.80 \cdot 10^{-4}$
18M	$3.7 \cdot 10^{-5}$	$3.8 \cdot 10^{-5}$	$3.7 \cdot 10^{-5}$	$-4.95 \cdot 10^{-4}$	$4.0 \cdot 10^{-5}$	$4.0 \cdot 10^{-5}$	$4.1 \cdot 10^{-5}$	$4.2 \cdot 10^{-5}$	$4.1 \cdot 10^{-5}$	$3.7 \cdot 10^{-5}$	$-1.070 \cdot 10^{-3}$	$-5.16 \cdot 10^{-4}$	$-2.0 \cdot 10^{-5}$
21M	$1.27 \cdot 10^{-4}$	$1.28 \cdot 10^{-4}$	$1.27 \cdot 10^{-4}$	$-2.39 \cdot 10^{-4}$	$1.28 \cdot 10^{-4}$	$1.28 \cdot 10^{-4}$	$1.28 \cdot 10^{-4}$	$1.29 \cdot 10^{-4}$	$1.28 \cdot 10^{-4}$	$1.27 \cdot 10^{-4}$	$-6.52 \cdot 10^{-4}$	$-2.57 \cdot 10^{-4}$	$7.9 \cdot 10^{-5}$
24M	$2.18 \cdot 10^{-4}$	$2.19 \cdot 10^{-4}$	$2.18 \cdot 10^{-4}$	$1.7 \cdot 10^{-5}$	$2.17 \cdot 10^{-4}$	$2.17 \cdot 10^{-4}$	$2.16 \cdot 10^{-4}$	$2.18 \cdot 10^{-4}$	$2.17 \cdot 10^{-4}$	$2.19 \cdot 10^{-4}$	$-2.29 \cdot 10^{-4}$	$4 \cdot 10^{-6}$	$1.80 \cdot 10^{-4}$
27M	$2.36 \cdot 10^{-4}$	$2.35 \cdot 10^{-4}$	$2.35 \cdot 10^{-4}$	$1.87 \cdot 10^{-4}$	$2.33 \cdot 10^{-4}$	$2.32 \cdot 10^{-4}$	$2.31 \cdot 10^{-4}$	$2.32 \cdot 10^{-4}$	$2.31 \cdot 10^{-4}$	$2.36 \cdot 10^{-4}$	$1.14 \cdot 10^{-4}$	$1.87 \cdot 10^{-4}$	$2.09 \cdot 10^{-4}$
30M	$1.84 \cdot 10^{-4}$	$1.83 \cdot 10^{-4}$	$1.84 \cdot 10^{-4}$	$2.72 \cdot 10^{-4}$	$1.80 \cdot 10^{-4}$	$1.79 \cdot 10^{-4}$	$1.78 \cdot 10^{-4}$	$1.79 \cdot 10^{-4}$	$1.78 \cdot 10^{-4}$	$1.84 \cdot 10^{-4}$	$3.76 \cdot 10^{-4}$	$2.95 \cdot 10^{-4}$	$1.69 \cdot 10^{-4}$
33M	$1.23 \cdot 10^{-4}$	$1.21 \cdot 10^{-4}$	$1.22 \cdot 10^{-4}$	$3.29 \cdot 10^{-4}$	$1.18 \cdot 10^{-4}$	$1.17 \cdot 10^{-4}$	$1.15 \cdot 10^{-4}$	$1.17 \cdot 10^{-4}$	$1.16 \cdot 10^{-4}$	$1.22 \cdot 10^{-4}$	$6.12 \cdot 10^{-4}$	$3.86 \cdot 10^{-4}$	$1.19 \cdot 10^{-4}$
36M	$8.8 \cdot 10^{-5}$	$8.7 \cdot 10^{-5}$	$8.8 \cdot 10^{-5}$	$3.95 \cdot 10^{-4}$	$8.3 \cdot 10^{-5}$	$8.2 \cdot 10^{-5}$	$8.1 \cdot 10^{-5}$	$8.1 \cdot 10^{-5}$	$8.1 \cdot 10^{-5}$	$8.7 \cdot 10^{-5}$	$8.58 \cdot 10^{-4}$	$4.95 \cdot 10^{-4}$	$9.7 \cdot 10^{-5}$
39M	$1.04 \cdot 10^{-4}$	$1.03 \cdot 10^{-4}$	$1.04 \cdot 10^{-4}$	$4.95 \cdot 10^{-4}$	$9.9 \cdot 10^{-5}$	$9.8 \cdot 10^{-5}$	$9.7 \cdot 10^{-5}$	$9.7 \cdot 10^{-5}$	$9.7 \cdot 10^{-5}$	$1.03 \cdot 10^{-4}$	$1.137 \cdot 10^{-3}$	$6.46 \cdot 10^{-4}$	$1.24 \cdot 10^{-4}$
48M	$-4.6 \cdot 10^{-5}$	$-4.8 \cdot 10^{-5}$	$-4.6 \cdot 10^{-5}$	$5.07 \cdot 10^{-4}$	$-4.9 \cdot 10^{-5}$	$-4.9 \cdot 10^{-5}$	$-5.0 \cdot 10^{-5}$	$-5.1 \cdot 10^{-5}$	$-5.0 \cdot 10^{-5}$	$-4.7 \cdot 10^{-5}$	$1.666 \cdot 10^{-3}$	$8.46 \cdot 10^{-4}$	$5 \cdot 10^{-6}$
60M	$-1.95 \cdot 10^{-4}$	$-1.96 \cdot 10^{-4}$	$-1.95 \cdot 10^{-4}$	$4.33 \cdot 10^{-4}$	$-1.94 \cdot 10^{-4}$	$-1.93 \cdot 10^{-4}$	$-1.92 \cdot 10^{-4}$	$-1.95 \cdot 10^{-4}$	$-1.93 \cdot 10^{-4}$	$-1.96 \cdot 10^{-4}$	$2.205 \cdot 10^{-3}$	$1.057 \cdot 10^{-3}$	$-1.10 \cdot 10^{-4}$
72M	$-2.11 \cdot 10^{-4}$	$-2.10 \cdot 10^{-4}$	$-2.10 \cdot 10^{-4}$	$4.07 \cdot 10^{-4}$	$-2.04 \cdot 10^{-4}$	$-2.03 \cdot 10^{-4}$	$-2.00 \cdot 10^{-4}$	$-2.05 \cdot 10^{-4}$	$-2.02 \cdot 10^{-4}$	$-2.12 \cdot 10^{-4}$	$2.692 \cdot 10^{-3}$	$1.308 \cdot 10^{-3}$	$-1.00 \cdot 10^{-4}$
Root Mean Squared Error													
	MDNS	**MDNS-M**	**MDNS-P**	**MDNS-λ**	**MDNS-Macro**	**MDNS-MMacro**	**MDNS-PMacro**	**MDNS-MMacro End**	**MDNS-PMacro End**	**MDNS-S**	**MDNS-SLambda**	**MDNS-Smedia Macro**	**MDNS-Smacro**
1M	$1.994 \cdot 10^{-3}$	$1.993 \cdot 10^{-3}$	$1.993 \cdot 10^{-3}$	$1.532 \cdot 10^{-3}$	$1.987 \cdot 10^{-3}$	$1.978 \cdot 10^{-3}$	$1.961 \cdot 10^{-3}$	$1.943 \cdot 10^{-3}$	$1.960 \cdot 10^{-3}$	$1.994 \cdot 10^{-3}$	$2.699 \cdot 10^{-3}$	$2.539 \cdot 10^{-3}$	$1.973 \cdot 10^{-3}$
3M	$6.39 \cdot 10^{-4}$	$6.42 \cdot 10^{-4}$	$6.46 \cdot 10^{-4}$	$4.084 \cdot 10^{-3}$	$6.40 \cdot 10^{-4}$	$6.44 \cdot 10^{-4}$	$6.47 \cdot 10^{-4}$	$6.50 \cdot 10^{-4}$	$6.56 \cdot 10^{-4}$	$6.40 \cdot 10^{-4}$	0.004070	$2.724 \cdot 10^{-3}$	$7.44 \cdot 10^{-4}$
6M	$1.567 \cdot 10^{-3}$	$1.566 \cdot 10^{-3}$	$1.572 \cdot 10^{-3}$	$2.208 \cdot 10^{-3}$	$1.564 \cdot 10^{-3}$	$1.570 \cdot 10^{-3}$	$1.566 \cdot 10^{-3}$	$1.585 \cdot 10^{-3}$	$1.574 \cdot 10^{-3}$	$1.572 \cdot 10^{-3}$	$4.708 \cdot 10^{-3}$	$3.436 \cdot 10^{-3}$	$1.605 \cdot 10^{-3}$
9M	$1.670 \cdot 10^{-3}$	$1.665 \cdot 10^{-3}$	$1.670 \cdot 10^{-3}$	$3.853 \cdot 10^{-3}$	$1.661 \cdot 10^{-3}$	$1.663 \cdot 10^{-3}$	$1.653 \cdot 10^{-3}$	$1.670 \cdot 10^{-3}$	$1.659 \cdot 10^{-3}$	$1.672 \cdot 10^{-3}$	$4.736 \cdot 10^{-3}$	$3.313 \cdot 10^{-3}$	$1.711 \cdot 10^{-3}$
12M	$1.399 \cdot 10^{-3}$	$1.393 \cdot 10^{-3}$	$1.396 \cdot 10^{-3}$	$4.012 \cdot 10^{-3}$	$1.389 \cdot 10^{-3}$	$1.387 \cdot 10^{-3}$	$1.376 \cdot 10^{-3}$	$1.390 \cdot 10^{-3}$	$1.380 \cdot 10^{-3}$	$1.399 \cdot 10^{-3}$	$4.307 \cdot 10^{-3}$	$2.800 \cdot 10^{-3}$	$1.437 \cdot 10^{-3}$
15M	$1.060 \cdot 10^{-3}$	$1.055 \cdot 10^{-3}$	$1.057 \cdot 10^{-3}$	$3.542 \cdot 10^{-3}$	$1.051 \cdot 10^{-3}$	$1.049 \cdot 10^{-3}$	$1.038 \cdot 10^{-3}$	$1.049 \cdot 10^{-3}$	$1.040 \cdot 10^{-3}$	$1.060 \cdot 10^{-3}$	$3.609 \cdot 10^{-3}$	$2.138 \cdot 10^{-3}$	$1.075 \cdot 10^{-3}$
18M	$8.08 \cdot 10^{-4}$	$8.05 \cdot 10^{-4}$	$8.05 \cdot 10^{-4}$	$2.846 \cdot 10^{-3}$	$8.03 \cdot 10^{-4}$	$8.02 \cdot 10^{-4}$	$7.95 \cdot 10^{-4}$	$8.01 \cdot 10^{-4}$	$7.93 \cdot 10^{-4}$	$8.09 \cdot 10^{-4}$	$2.851 \cdot 10^{-3}$	$1.580 \cdot 10^{-3}$	$8.03 \cdot 10^{-4}$
21M	$6.92 \cdot 10^{-4}$	$6.92 \cdot 10^{-4}$	$6.91 \cdot 10^{-4}$	$2.137 \cdot 10^{-3}$	$6.92 \cdot 10^{-4}$	$6.91 \cdot 10^{-4}$	$6.92 \cdot 10^{-4}$	$6.92 \cdot 10^{-4}$	$6.88 \cdot 10^{-4}$	$6.92 \cdot 10^{-4}$	$2.158 \cdot 10^{-3}$	$1.182 \cdot 10^{-3}$	$6.79 \cdot 10^{-4}$
24M	$7.70 \cdot 10^{-4}$	$7.71 \cdot 10^{-4}$	$7.70 \cdot 10^{-4}$	$1.498 \cdot 10^{-3}$	$7.72 \cdot 10^{-4}$	$7.73 \cdot 10^{-4}$	$7.78 \cdot 10^{-4}$	$7.77 \cdot 10^{-4}$	$7.75 \cdot 10^{-4}$	$7.70 \cdot 10^{-4}$	$1.661 \cdot 10^{-3}$	$1.042 \cdot 10^{-3}$	$7.61 \cdot 10^{-4}$
27M	$9.35 \cdot 10^{-4}$	$9.36 \cdot 10^{-4}$	$9.35 \cdot 10^{-4}$	$1.128 \cdot 10^{-3}$	$9.37 \cdot 10^{-4}$	$9.39 \cdot 10^{-4}$	$9.43 \cdot 10^{-4}$	$9.43 \cdot 10^{-4}$	$9.41 \cdot 10^{-4}$	$9.35 \cdot 10^{-4}$	$1.539 \cdot 10^{-3}$	$1.193 \cdot 10^{-3}$	$9.40 \cdot 10^{-4}$
30M	$1.094 \cdot 10^{-3}$	$1.094 \cdot 10^{-3}$	$1.094 \cdot 10^{-3}$	$1.114 \cdot 10^{-3}$	$1.095 \cdot 10^{-3}$	$1.097 \cdot 10^{-3}$	$1.099 \cdot 10^{-3}$	$1.100 \cdot 10^{-3}$	$1.098 \cdot 10^{-3}$	$1.094 \cdot 10^{-3}$	$1.669 \cdot 10^{-3}$	$1.417 \cdot 10^{-3}$	$1.106 \cdot 10^{-3}$
33M	$1.264 \cdot 10^{-3}$	$1.262 \cdot 10^{-3}$	$1.262 \cdot 10^{-3}$	$1.390 \cdot 10^{-3}$	$1.263 \cdot 10^{-3}$	$1.264 \cdot 10^{-3}$	$1.264 \cdot 10^{-3}$	$1.267 \cdot 10^{-3}$	$1.264 \cdot 10^{-3}$	$1.263 \cdot 10^{-3}$	$1.962 \cdot 10^{-3}$	$1.647 \cdot 10^{-3}$	$1.276 \cdot 10^{-3}$
36M	$1.434 \cdot 10^{-3}$	$1.431 \cdot 10^{-3}$	$1.431 \cdot 10^{-3}$	$1.795 \cdot 10^{-3}$	$1.432 \cdot 10^{-3}$	$1.432 \cdot 10^{-3}$	$1.430 \cdot 10^{-3}$	$1.434 \cdot 10^{-3}$	$1.429 \cdot 10^{-3}$	$1.433 \cdot 10^{-3}$	$2.332 \cdot 10^{-3}$	$1.901 \cdot 10^{-3}$	$1.441 \cdot 10^{-3}$
39M	$1.619 \cdot 10^{-3}$	$1.615 \cdot 10^{-3}$	$1.613 \cdot 10^{-3}$	$2.272 \cdot 10^{-3}$	$1.615 \cdot 10^{-3}$	$1.613 \cdot 10^{-3}$	$1.609 \cdot 10^{-3}$	$1.613 \cdot 10^{-3}$	$1.608 \cdot 10^{-3}$	$1.617 \cdot 10^{-3}$	$2.790 \cdot 10^{-3}$	$2.216 \cdot 10^{-3}$	$1.623 \cdot 10^{-3}$
48M	$1.220 \cdot 10^{-3}$	$1.211 \cdot 10^{-3}$	$1.209 \cdot 10^{-3}$	$3.038 \cdot 10^{-3}$	$1.209 \cdot 10^{-3}$	$1.204 \cdot 10^{-3}$	$1.198 \cdot 10^{-3}$	$1.202 \cdot 10^{-3}$	$1.195 \cdot 10^{-3}$	$1.214 \cdot 10^{-3}$	$3.713 \cdot 10^{-3}$	$2.611 \cdot 10^{-3}$	$1.228 \cdot 10^{-3}$
60M	$1.559 \cdot 10^{-3}$	$1.575 \cdot 10^{-3}$	$1.576 \cdot 10^{-3}$	$4.157 \cdot 10^{-3}$	$1.580 \cdot 10^{-3}$	$1.589 \cdot 10^{-3}$	$1.611 \cdot 10^{-3}$	$1.592 \cdot 10^{-3}$	$1.600 \cdot 10^{-3}$	$1.569 \cdot 10^{-3}$	0.005351	$3.677 \cdot 10^{-3}$	$1.626 \cdot 10^{-3}$
72M	$2.833 \cdot 10^{-3}$	$2.852 \cdot 10^{-3}$	$2.855 \cdot 10^{-3}$	$5.033 \cdot 10^{-3}$	$2.861 \cdot 10^{-3}$	$2.873 \cdot 10^{-3}$	$2.891 \cdot 10^{-3}$	$2.880 \cdot 10^{-3}$	$2.881 \cdot 10^{-3}$	$2.847 \cdot 10^{-3}$	0.006927	$4.676 \cdot 10^{-3}$	$2.864 \cdot 10^{-3}$

Table 2. *Cont.*

	Mean Absolute Error												
	MDNS	**MDNS-M**	**MDNS-P**	**MDNS-λ**	**MDNS-Macro**	**MDNS-MMacro**	**MDNS-PMacro**	**MDNS-MMacro End**	**MDNS-PMacro End**	**MDNS-S**	**MDNS-SLambda**	**MDNS-Smedia Macro**	**MDNS-Smacro**
1M	$1.531 \cdot 10^{-3}$	$1.527 \cdot 10^{-3}$	$1.524 \cdot 10^{-3}$	$9.30 \cdot 10^{-4}$	$1.515 \cdot 10^{-3}$	$1.508 \cdot 10^{-3}$	$1.501 \cdot 10^{-3}$	$1.483 \cdot 10^{-3}$	$1.498 \cdot 10^{-3}$	$1.527 \cdot 10^{-3}$	$2.123 \cdot 10^{-3}$	$1.994 \cdot 10^{-3}$	$1.497 \cdot 10^{-3}$
3M	$4.53 \cdot 10^{-4}$	$4.55 \cdot 10^{-4}$	$4.58 \cdot 10^{-4}$	$2.865 \cdot 10^{-3}$	$4.55 \cdot 10^{-4}$	$4.58 \cdot 10^{-4}$	$4.63 \cdot 10^{-4}$	$4.59 \cdot 10^{-4}$	$4.63 \cdot 10^{-4}$	$4.57 \cdot 10^{-4}$	$3.118 \cdot 10^{-3}$	$2.175 \cdot 10^{-3}$	$5.29 \cdot 10^{-4}$
6M	$1.237 \cdot 10^{-3}$	$1.236 \cdot 10^{-3}$	$1.244 \cdot 10^{-3}$	$1.719 \cdot 10^{-3}$	$1.232 \cdot 10^{-3}$	$1.238 \cdot 10^{-3}$	$1.236 \cdot 10^{-3}$	$1.248 \cdot 10^{-3}$	$1.242 \cdot 10^{-3}$	$1.244 \cdot 10^{-3}$	$3.778 \cdot 10^{-3}$	$2.780 \cdot 10^{-3}$	$1.270 \cdot 10^{-3}$
9M	$1.305 \cdot 10^{-3}$	$1.300 \cdot 10^{-3}$	$1.307 \cdot 10^{-3}$	$2.970 \cdot 10^{-3}$	$1.296 \cdot 10^{-3}$	$1.296 \cdot 10^{-3}$	$1.292 \cdot 10^{-3}$	$1.301 \cdot 10^{-3}$	$1.295 \cdot 10^{-3}$	$1.306 \cdot 10^{-3}$	$3.773 \cdot 10^{-3}$	$2.639 \cdot 10^{-3}$	$1.350 \cdot 10^{-3}$
12M	$1.039 \cdot 10^{-3}$	$1.036 \cdot 10^{-3}$	$1.043 \cdot 10^{-3}$	$3.056 \cdot 10^{-3}$	$1.031 \cdot 10^{-3}$	$1.033 \cdot 10^{-3}$	$1.024 \cdot 10^{-3}$	$1.033 \cdot 10^{-3}$	$1.022 \cdot 10^{-3}$	$1.043 \cdot 10^{-3}$	$3.338 \cdot 10^{-3}$	$2.169 \cdot 10^{-3}$	$1.078 \cdot 10^{-3}$
15M	$7.02 \cdot 10^{-4}$	$7.00 \cdot 10^{-4}$	$7.06 \cdot 10^{-4}$	$2.675 \cdot 10^{-3}$	$6.95 \cdot 10^{-4}$	$6.97 \cdot 10^{-4}$	$6.89 \cdot 10^{-4}$	$6.97 \cdot 10^{-4}$	$6.88 \cdot 10^{-4}$	$7.07 \cdot 10^{-4}$	$2.772 \cdot 10^{-3}$	$1.606 \cdot 10^{-3}$	$7.46 \cdot 10^{-4}$
18M	$5.33 \cdot 10^{-4}$	$5.31 \cdot 10^{-4}$	$5.32 \cdot 10^{-4}$	$2.129 \cdot 10^{-3}$	$5.31 \cdot 10^{-4}$	$5.31 \cdot 10^{-4}$	$5.32 \cdot 10^{-4}$	$5.32 \cdot 10^{-4}$	$5.26 \cdot 10^{-4}$	$5.33 \cdot 10^{-4}$	$2.207 \cdot 10^{-3}$	$1.127 \cdot 10^{-3}$	$5.49 \cdot 10^{-4}$
21M	$5.31 \cdot 10^{-4}$	$5.33 \cdot 10^{-4}$	$5.31 \cdot 10^{-4}$	$1.608 \cdot 10^{-3}$	$5.33 \cdot 10^{-4}$	$5.34 \cdot 10^{-4}$	$5.35 \cdot 10^{-4}$	$5.36 \cdot 10^{-4}$	$5.32 \cdot 10^{-4}$	$5.32 \cdot 10^{-4}$	$1.652 \cdot 10^{-3}$	$8.34 \cdot 10^{-4}$	$5.20 \cdot 10^{-4}$
24M	$6.40 \cdot 10^{-4}$	$6.41 \cdot 10^{-4}$	$6.40 \cdot 10^{-4}$	$1.189 \cdot 10^{-3}$	$6.42 \cdot 10^{-4}$	$6.43 \cdot 10^{-4}$	$6.46 \cdot 10^{-4}$	$6.44 \cdot 10^{-4}$	$6.41 \cdot 10^{-4}$	$6.41 \cdot 10^{-4}$	$1.257 \cdot 10^{-3}$	$7.63 \cdot 10^{-4}$	$6.27 \cdot 10^{-4}$
27M	$7.20 \cdot 10^{-4}$	$7.20 \cdot 10^{-4}$	$7.20 \cdot 10^{-4}$	$9.07 \cdot 10^{-4}$	$7.19 \cdot 10^{-4}$	$7.21 \cdot 10^{-4}$	$7.20 \cdot 10^{-4}$	$7.23 \cdot 10^{-4}$	$7.20 \cdot 10^{-4}$	$7.20 \cdot 10^{-4}$	$1.160 \cdot 10^{-3}$	$8.93 \cdot 10^{-4}$	$7.13 \cdot 10^{-4}$
30M	$7.48 \cdot 10^{-4}$	$7.48 \cdot 10^{-4}$	$7.49 \cdot 10^{-4}$	$8.26 \cdot 10^{-4}$	$7.45 \cdot 10^{-4}$	$7.47 \cdot 10^{-4}$	$7.45 \cdot 10^{-4}$	$7.47 \cdot 10^{-4}$	$7.46 \cdot 10^{-4}$	$7.49 \cdot 10^{-4}$	$1.296 \cdot 10^{-3}$	$1.068 \cdot 10^{-3}$	$7.49 \cdot 10^{-4}$
33M	$7.43 \cdot 10^{-4}$	$7.42 \cdot 10^{-4}$	$7.45 \cdot 10^{-4}$	$9.74 \cdot 10^{-4}$	$7.39 \cdot 10^{-4}$	$7.40 \cdot 10^{-4}$	$7.37 \cdot 10^{-4}$	$7.40 \cdot 10^{-4}$	$7.37 \cdot 10^{-4}$	$7.46 \cdot 10^{-4}$	$1.480 \cdot 10^{-3}$	$1.202 \cdot 10^{-3}$	$7.53 \cdot 10^{-4}$
36M	$7.56 \cdot 10^{-4}$	$7.56 \cdot 10^{-4}$	$7.61 \cdot 10^{-4}$	$1.243 \cdot 10^{-3}$	$7.53 \cdot 10^{-4}$	$7.53 \cdot 10^{-4}$	$7.50 \cdot 10^{-4}$	$7.53 \cdot 10^{-4}$	$7.49 \cdot 10^{-4}$	$7.61 \cdot 10^{-4}$	$1.723 \cdot 10^{-3}$	$1.350 \cdot 10^{-3}$	$7.77 \cdot 10^{-4}$
39M	$7.81 \cdot 10^{-4}$	$7.79 \cdot 10^{-4}$	$7.85 \cdot 10^{-4}$	$1.562 \cdot 10^{-3}$	$7.77 \cdot 10^{-4}$	$7.77 \cdot 10^{-4}$	$7.73 \cdot 10^{-4}$	$7.76 \cdot 10^{-4}$	$7.71 \cdot 10^{-4}$	$7.85 \cdot 10^{-4}$	$2.040 \cdot 10^{-3}$	$1.508 \cdot 10^{-3}$	$8.05 \cdot 10^{-4}$
48M	$5.72 \cdot 10^{-4}$	$5.72 \cdot 10^{-4}$	$5.78 \cdot 10^{-4}$	$2.200 \cdot 10^{-3}$	$5.72 \cdot 10^{-4}$	$5.75 \cdot 10^{-4}$	$5.73 \cdot 10^{-4}$	$5.75 \cdot 10^{-4}$	$5.64 \cdot 10^{-4}$	$5.76 \cdot 10^{-4}$	$2.855 \cdot 10^{-3}$	$1.679 \cdot 10^{-3}$	$5.95 \cdot 10^{-4}$
60M	$7.99 \cdot 10^{-4}$	$8.08 \cdot 10^{-4}$	$8.04 \cdot 10^{-4}$	$2.934 \cdot 10^{-3}$	$8.11 \cdot 10^{-4}$	$8.16 \cdot 10^{-4}$	$8.16 \cdot 10^{-4}$	$8.18 \cdot 10^{-4}$	$8.11 \cdot 10^{-4}$	$8.03 \cdot 10^{-4}$	$3.919 \cdot 10^{-3}$	$2.106 \cdot 10^{-3}$	$7.39 \cdot 10^{-4}$
72M	$1.524 \cdot 10^{-3}$	$1.529 \cdot 10^{-3}$	$1.525 \cdot 10^{-3}$	$3.615 \cdot 10^{-3}$	$1.526 \cdot 10^{-3}$	$1.530 \cdot 10^{-3}$	$1.527 \cdot 10^{-3}$	$1.534 \cdot 10^{-3}$	$1.528 \cdot 10^{-3}$	$1.524 \cdot 10^{-3}$	$4.866 \cdot 10^{-3}$	$2.419 \cdot 10^{-3}$	$1.448 \cdot 10^{-3}$

Highlighted values are the minimum mean error, root of the mean square error or mean absolute error, according to the indication of the table, for each maturity.

Table 3. Out-of-Sample Error Measures—1-Month Forecast Horizon *.

	Mean Error												
	MDNS	**MDNS-M**	**MDNS-P**	**MDNS-λ**	**MDNS-Macro**	**MDNS-MMacro**	**MDNS-PMacro**	**MDNS-MMacro End**	**MDNS-PMacro End**	**MDNS-S**	**MDNS-Smedia Macro**	**MDNS-Smacro**	**RW**
1M	$-3.939 \cdot 10^{-3}$	$-2.799 \cdot 10^{-3}$	$-2.448 \cdot 10^{-3}$	$-2.820 \cdot 10^{-3}$	$2.611 \cdot 10^{-3}$	$1.749 \cdot 10^{-3}$	$-1.915 \cdot 10^{-3}$	$2.165 \cdot 10^{-3}$	$-2.06 \cdot 10^{-4}$	$-3.751 \cdot 10^{-3}$	$2.248 \cdot 10^{-3}$	$-4.37 \cdot 10^{-4}$	$1.6458 \cdot 10^{-2}$
3M	$-6.278 \cdot 10^{-3}$	$-5.204 \cdot 10^{-3}$	$-4.893 \cdot 10^{-3}$	$2.749 \cdot 10^{-3}$	$-2.8 \cdot 10^{-5}$	$-8.02 \cdot 10^{-4}$	$-4.158 \cdot 10^{-3}$	$-1.366 \cdot 10^{-3}$	$-2.593 \cdot 10^{-3}$	$-6.113 \cdot 10^{-3}$	$-4.07 \cdot 10^{-4}$	$-8.39 \cdot 10^{-4}$	$2.2750 \cdot 10^{-2}$
6M	$-7.760 \cdot 10^{-3}$	$-6.778 \cdot 10^{-3}$	$-6.514 \cdot 10^{-3}$	$-5.651 \cdot 10^{-3}$	$-1.947 \cdot 10^{-3}$	$-2.616 \cdot 10^{-3}$	$-5.569 \cdot 10^{-3}$	$-3.678 \cdot 10^{-3}$	$-4.194 \cdot 10^{-3}$	$-7.622 \cdot 10^{-3}$	$-2.328 \cdot 10^{-3}$	$-7.621 \cdot 10^{-3}$	$2.8095 \cdot 10^{-2}$
9M	$-7.349 \cdot 10^{-3}$	$-6.450 \cdot 10^{-3}$	$-6.222 \cdot 10^{-3}$	$-9.367 \cdot 10^{-3}$	$-1.946 \cdot 10^{-3}$	$-2.535 \cdot 10^{-3}$	$-5.145 \cdot 10^{-3}$	$-3.820 \cdot 10^{-3}$	$-3.933 \cdot 10^{-3}$	$-7.229 \cdot 10^{-3}$	$-2.313 \cdot 10^{-3}$	$-9.697 \cdot 10^{-3}$	$3.0831 \cdot 10^{-2}$
12M	$-6.708 \cdot 10^{-3}$	$-5.885 \cdot 10^{-3}$	$-5.684 \cdot 10^{-3}$	$-1.0598 \cdot 10^{-2}$	$-1.684 \cdot 10^{-3}$	$-2.212 \cdot 10^{-3}$	$-4.528 \cdot 10^{-3}$	$-3.535 \cdot 10^{-3}$	$-3.455 \cdot 10^{-3}$	$-6.602 \cdot 10^{-3}$	$-2.028 \cdot 10^{-3}$	$-1.0036 \cdot 10^{-2}$	$3.4132 \cdot 10^{-2}$
15M	$-5.828 \cdot 10^{-3}$	$-5.072 \cdot 10^{-3}$	$-4.893 \cdot 10^{-3}$	$-1.0329 \cdot 10^{-2}$	$-1.151 \cdot 10^{-3}$	$-1.629 \cdot 10^{-3}$	$-3.693 \cdot 10^{-3}$	$-2.862 \cdot 10^{-3}$	$-2.740 \cdot 10^{-3}$	$-5.732 \cdot 10^{-3}$	$-1.467 \cdot 10^{-3}$	$-9.230 \cdot 10^{-3}$	$3.6183 \cdot 10^{-2}$
18M	$-4.868 \cdot 10^{-3}$	$-4.172 \cdot 10^{-3}$	$-4.011 \cdot 10^{-3}$	$-9.223 \cdot 10^{-3}$	$-5.04 \cdot 10^{-4}$	$-9.43 \cdot 10^{-4}$	$-2.789 \cdot 10^{-3}$	$-1.996 \cdot 10^{-3}$	$-1.939 \cdot 10^{-3}$	$-4.781 \cdot 10^{-3}$	$-7.94 \cdot 10^{-4}$	$-7.769 \cdot 10^{-3}$	$3.6732 \cdot 10^{-2}$
21M	$-3.849 \cdot 10^{-3}$	$-3.207 \cdot 10^{-3}$	$-3.061 \cdot 10^{-3}$	$-7.587 \cdot 10^{-3}$	$2.33 \cdot 10^{-4}$	$-1.73 \cdot 10^{-4}$	$-1.830 \cdot 10^{-3}$	$-9.87 \cdot 10^{-4}$	$-1.069 \cdot 10^{-3}$	$-3.768 \cdot 10^{-3}$	$-3.0 \cdot 10^{-5}$	$-5.881 \cdot 10^{-3}$	$3.6674 \cdot 10^{-2}$
24M	$-2.871 \cdot 10^{-3}$	$-2.277 \cdot 10^{-3}$	$-2.143 \cdot 10^{-3}$	$-5.702 \cdot 10^{-3}$	$9.58 \cdot 10^{-4}$	$5.80 \cdot 10^{-4}$	$-9.13 \cdot 10^{-4}$	$4.2 \cdot 10^{-5}$	$-2.28 \cdot 10^{-4}$	$-2.795 \cdot 10^{-3}$	$7.20 \cdot 10^{-4}$	$-3.808 \cdot 10^{-3}$	$3.5870 \cdot 10^{-2}$
27M	$-2.602 \cdot 10^{-3}$	$-2.050 \cdot 10^{-3}$	$-1.926 \cdot 10^{-3}$	$-4.355 \cdot 10^{-3}$	$1.003 \cdot 10^{-3}$	$6.48 \cdot 10^{-4}$	$-7.00 \cdot 10^{-4}$	$4.06 \cdot 10^{-4}$	$-8.1 \cdot 10^{-5}$	$-2.529 \cdot 10^{-3}$	$7.85 \cdot 10^{-4}$	$-2.315 \cdot 10^{-3}$	$3.5338 \cdot 10^{-2}$
30M	$-2.201 \cdot 10^{-3}$	$-1.688 \cdot 10^{-3}$	$-1.572 \cdot 10^{-3}$	$-2.787 \cdot 10^{-3}$	$1.203 \cdot 10^{-3}$	$8.68 \cdot 10^{-4}$	$-3.53 \cdot 10^{-4}$	$9.31 \cdot 10^{-4}$	$2.09 \cdot 10^{-4}$	$-2.132 \cdot 10^{-3}$	$1.004 \cdot 10^{-3}$	$-6.32 \cdot 10^{-4}$	$3.3997 \cdot 10^{-2}$
33M	$-1.757 \cdot 10^{-3}$	$-1.278 \cdot 10^{-3}$	$-1.169 \cdot 10^{-3}$	$-1.142 \cdot 10^{-3}$	$1.468 \cdot 10^{-3}$	$1.152 \cdot 10^{-3}$	$4.3 \cdot 10^{-5}$	$1.518 \cdot 10^{-3}$	$5.55 \cdot 10^{-4}$	$-1.691 \cdot 10^{-3}$	$1.284 \cdot 10^{-3}$	$1.109 \cdot 10^{-3}$	$3.2926 \cdot 10^{-2}$
36M	$-1.525 \cdot 10^{-3}$	$-1.076 \cdot 10^{-3}$	$-9.74 \cdot 10^{-4}$	$2.89 \cdot 10^{-4}$	$1.541 \cdot 10^{-3}$	$1.242 \cdot 10^{-3}$	$2.31 \cdot 10^{-4}$	$1.905 \cdot 10^{-3}$	$7.00 \cdot 10^{-4}$	$-1.462 \cdot 10^{-3}$	$1.370 \cdot 10^{-3}$	$2.619 \cdot 10^{-3}$	$3.1747 \cdot 10^{-2}$
39M	$-1.622 \cdot 10^{-3}$	$-1.200 \cdot 10^{-3}$	$-1.104 \cdot 10^{-3}$	$1.363 \cdot 10^{-3}$	$1.302 \cdot 10^{-3}$	$1.017 \cdot 10^{-3}$	$9.6 \cdot 10^{-5}$	$1.968 \cdot 10^{-3}$	$5.26 \cdot 10^{-4}$	$-1.562 \cdot 10^{-3}$	$1.142 \cdot 10^{-3}$	$3.759 \cdot 10^{-3}$	$3.1183 \cdot 10^{-2}$
48M	$-1.778 \cdot 10^{-3}$	$-1.421 \cdot 10^{-3}$	$-1.341 \cdot 10^{-3}$	$4.428 \cdot 10^{-3}$	$8.05 \cdot 10^{-4}$	$5.58 \cdot 10^{-4}$	$-1.50 \cdot 10^{-4}$	$2.287 \cdot 10^{-3}$	$1.91 \cdot 10^{-4}$	$-1.724 \cdot 10^{-3}$	$6.64 \cdot 10^{-4}$	$6.975 \cdot 10^{-3}$	$2.8862 \cdot 10^{-2}$
60M	$-3.046 \cdot 10^{-3}$	$-2.750 \cdot 10^{-3}$	$-2.685 \cdot 10^{-3}$	$6.641 \cdot 10^{-3}$	$-7.77 \cdot 10^{-4}$	$-9.87 \cdot 10^{-4}$	$-1.494 \cdot 10^{-3}$	$1.575 \cdot 10^{-3}$	$-1.233 \cdot 10^{-3}$	$-2.999 \cdot 10^{-3}$	$-9.09 \cdot 10^{-4}$	$9.320 \cdot 10^{-3}$	$2.6963 \cdot 10^{-2}$
72M	$-4.013 \cdot 10^{-3}$	$-3.759 \cdot 10^{-3}$	$-3.706 \cdot 10^{-3}$	$8.321 \cdot 10^{-3}$	$-1.959 \cdot 10^{-3}$	$-2.139 \cdot 10^{-3}$	$-2.507 \cdot 10^{-3}$	$1.054 \cdot 10^{-3}$	$-2.299 \cdot 10^{-3}$	$-3.972 \cdot 10^{-3}$	$-2.091 \cdot 10^{-3}$	$1.1090 \cdot 10^{-2}$	$2.5544 \cdot 10^{-2}$

Table 3. *Cont.*

Root Mean Squared Error													
	MDNS	**MDNS-M**	**MDNS-P**	**MDNS-λ**	**MDNS-Macro**	**MDNS-MMacro**	**MDNS-PMacro**	**MDNS-MMacro End**	**MDNS-PMacro End**	**MDNS-S**	**MDNS-Smedia Macro**	**MDNS-Smacro**	**RW**
1M	$7.519 \cdot 10^{-3}$	$5.967 \cdot 10^{-3}$	$5.499 \cdot 10^{-3}$	$4.683 \cdot 10^{-3}$	$2.618 \cdot 10^{-3}$	$1.868 \cdot 10^{-3}$	$4.687 \cdot 10^{-3}$	$2.224 \cdot 10^{-3}$	$2.607 \cdot 10^{-3}$	$7.279 \cdot 10^{-3}$	$2.257 \cdot 10^{-3}$	$2.692 \cdot 10^{-3}$	$1.6458 \cdot 10^{-2}$
3M	$8.917 \cdot 10^{-3}$	$7.403 \cdot 10^{-3}$	$6.960 \cdot 10^{-3}$	$5.127 \cdot 10^{-3}$	$1.20 \cdot 10^{-4}$	$1.199 \cdot 10^{-3}$	$5.911 \cdot 10^{-3}$	$1.694 \cdot 10^{-3}$	$3.725 \cdot 10^{-3}$	$8.690 \cdot 10^{-3}$	$6.30 \cdot 10^{-4}$	$2.873 \cdot 10^{-3}$	$2.2750 \cdot 10^{-2}$
6M	$9.724 \cdot 10^{-3}$	$8.356 \cdot 10^{-3}$	$7.981 \cdot 10^{-3}$	$6.918 \cdot 10^{-3}$	$1.952 \cdot 10^{-3}$	$2.745 \cdot 10^{-3}$	$6.706 \cdot 10^{-3}$	$3.789 \cdot 10^{-3}$	$4.834 \cdot 10^{-3}$	$9.529 \cdot 10^{-3}$	$2.377 \cdot 10^{-3}$	$7.975 \cdot 10^{-3}$	$2.8095 \cdot 10^{-2}$
9M	$9.243 \cdot 10^{-3}$	$7.991 \cdot 10^{-3}$	$7.664 \cdot 10^{-3}$	$1.0098 \cdot 10^{-2}$	$1.973 \cdot 10^{-3}$	$2.703 \cdot 10^{-3}$	$6.224 \cdot 10^{-3}$	$3.936 \cdot 10^{-3}$	$4.572 \cdot 10^{-3}$	$9.071 \cdot 10^{-3}$	$2.399 \cdot 10^{-3}$	$9.929 \cdot 10^{-3}$	$3.0831 \cdot 10^{-2}$
12M	$8.546 \cdot 10^{-3}$	$7.395 \cdot 10^{-3}$	$7.107 \cdot 10^{-3}$	$1.1162 \cdot 10^{-2}$	$1.730 \cdot 10^{-3}$	$2.405 \cdot 10^{-3}$	$5.555 \cdot 10^{-3}$	$3.647 \cdot 10^{-3}$	$4.087 \cdot 10^{-3}$	$8.392 \cdot 10^{-3}$	$2.140 \cdot 10^{-3}$	$1.0209 \cdot 10^{-2}$	$3.4132 \cdot 10^{-2}$
15M	$7.622 \cdot 10^{-3}$	$6.562 \cdot 10^{-3}$	$6.304 \cdot 10^{-3}$	$1.0809 \cdot 10^{-2}$	$1.202 \cdot 10^{-3}$	$1.833 \cdot 10^{-3}$	$4.677 \cdot 10^{-3}$	$2.958 \cdot 10^{-3}$	$3.361 \cdot 10^{-3}$	$7.483 \cdot 10^{-3}$	$1.592 \cdot 10^{-3}$	$9.362 \cdot 10^{-3}$	$3.6183 \cdot 10^{-2}$
18M	$6.620 \cdot 10^{-3}$	$5.641 \cdot 10^{-3}$	$5.408 \cdot 10^{-3}$	$9.650 \cdot 10^{-3}$	$5.46 \cdot 10^{-4}$	$1.151 \cdot 10^{-3}$	$3.731 \cdot 10^{-3}$	$2.065 \cdot 10^{-3}$	$2.545 \cdot 10^{-3}$	$6.492 \cdot 10^{-3}$	$9.20 \cdot 10^{-4}$	$7.865 \cdot 10^{-3}$	$3.6732 \cdot 10^{-2}$
21M	$5.616 \cdot 10^{-3}$	$4.712 \cdot 10^{-3}$	$4.501 \cdot 10^{-3}$	$7.999 \cdot 10^{-3}$	$2.43 \cdot 10^{-4}$	$5.10 \cdot 10^{-4}$	$2.799 \cdot 10^{-3}$	$1.039 \cdot 10^{-3}$	$1.736 \cdot 10^{-3}$	$5.498 \cdot 10^{-3}$	$3.14 \cdot 10^{-4}$	$5.954 \cdot 10^{-3}$	$3.6674 \cdot 10^{-2}$
24M	$4.755 \cdot 10^{-3}$	$3.927 \cdot 10^{-3}$	$3.737 \cdot 10^{-3}$	$6.161 \cdot 10^{-3}$	$9.58 \cdot 10^{-4}$	$6.89 \cdot 10^{-4}$	$2.067 \cdot 10^{-3}$	$1.99 \cdot 10^{-4}$	$1.194 \cdot 10^{-3}$	$4.647 \cdot 10^{-3}$	$7.55 \cdot 10^{-4}$	$3.877 \cdot 10^{-3}$	$3.5870 \cdot 10^{-2}$
27M	$4.230 \cdot 10^{-3}$	$3.459 \cdot 10^{-3}$	$3.285 \cdot 10^{-3}$	$4.782 \cdot 10^{-3}$	$1.037 \cdot 10^{-3}$	$6.54 \cdot 10^{-4}$	$1.596 \cdot 10^{-3}$	$4.19 \cdot 10^{-4}$	$8.14 \cdot 10^{-4}$	$4.129 \cdot 10^{-3}$	$7.86 \cdot 10^{-4}$	$2.346 \cdot 10^{-3}$	$3.5338 \cdot 10^{-2}$
30M	$3.855 \cdot 10^{-3}$	$3.144 \cdot 10^{-3}$	$2.987 \cdot 10^{-3}$	$3.375 \cdot 10^{-3}$	$1.230 \cdot 10^{-3}$	$8.71 \cdot 10^{-4}$	$1.345 \cdot 10^{-3}$	$9.40 \cdot 10^{-4}$	$7.52 \cdot 10^{-4}$	$3.762 \cdot 10^{-3}$	$1.004 \cdot 10^{-3}$	$7.04 \cdot 10^{-4}$	$3.3997 \cdot 10^{-2}$
33M	$3.422 \cdot 10^{-3}$	$2.770 \cdot 10^{-3}$	$2.629 \cdot 10^{-3}$	$2.106 \cdot 10^{-3}$	$1.505 \cdot 10^{-3}$	$1.152 \cdot 10^{-3}$	$1.102 \cdot 10^{-3}$	$1.536 \cdot 10^{-3}$	$7.95 \cdot 10^{-4}$	$3.336 \cdot 10^{-3}$	$1.290 \cdot 10^{-3}$	$1.124 \cdot 10^{-3}$	$3.2926 \cdot 10^{-2}$
36M	$3.213 \cdot 10^{-3}$	$2.611 \cdot 10^{-3}$	$2.483 \cdot 10^{-3}$	$1.777 \cdot 10^{-3}$	$1.570 \cdot 10^{-3}$	$1.242 \cdot 10^{-3}$	$1.050 \cdot 10^{-3}$	$1.919 \cdot 10^{-3}$	$8.77 \cdot 10^{-4}$	$3.134 \cdot 10^{-3}$	$1.374 \cdot 10^{-3}$	$2.624 \cdot 10^{-3}$	$3.1747 \cdot 10^{-2}$
39M	$3.082 \cdot 10^{-3}$	$2.505 \cdot 10^{-3}$	$2.383 \cdot 10^{-3}$	$2.128 \cdot 10^{-3}$	$1.357 \cdot 10^{-3}$	$1.023 \cdot 10^{-3}$	$8.51 \cdot 10^{-4}$	$1.995 \cdot 10^{-3}$	$6.51 \cdot 10^{-4}$	$3.006 \cdot 10^{-3}$	$1.156 \cdot 10^{-3}$	$3.760 \cdot 10^{-3}$	$3.1183 \cdot 10^{-2}$
48M	$2.947 \cdot 10^{-3}$	$2.449 \cdot 10^{-3}$	$2.346 \cdot 10^{-3}$	$4.707 \cdot 10^{-3}$	$8.78 \cdot 10^{-4}$	$5.70 \cdot 10^{-4}$	$6.66 \cdot 10^{-4}$	$2.312 \cdot 10^{-3}$	$3.25 \cdot 10^{-4}$	$2.881 \cdot 10^{-3}$	$6.84 \cdot 10^{-4}$	$6.975 \cdot 10^{-3}$	$2.8862 \cdot 10^{-2}$
60M	$3.676 \cdot 10^{-3}$	$3.269 \cdot 10^{-3}$	$3.185 \cdot 10^{-3}$	$6.820 \cdot 10^{-3}$	$8.54 \cdot 10^{-4}$	$9.99 \cdot 10^{-4}$	$1.556 \cdot 10^{-3}$	$1.620 \cdot 10^{-3}$	$1.238 \cdot 10^{-3}$	$3.619 \cdot 10^{-3}$	$9.26 \cdot 10^{-4}$	$9.320 \cdot 10^{-3}$	$2.6963 \cdot 10^{-2}$
72M	$4.366 \cdot 10^{-3}$	$4.038 \cdot 10^{-3}$	$3.972 \cdot 10^{-3}$	$8.438 \cdot 10^{-3}$	$2.020 \cdot 10^{-3}$	$2.163 \cdot 10^{-3}$	$2.512 \cdot 10^{-3}$	$1.181 \cdot 10^{-3}$	$2.302 \cdot 10^{-3}$	$4.318 \cdot 10^{-3}$	$2.116 \cdot 10^{-3}$	$1.1091 \cdot 10^{-2}$	$2.5544 \cdot 10^{-2}$

Mean Absolute Error													
	MDNS	**MDNS-M**	**MDNS-P**	**MDNS-λ**	**MDNS-Macro**	**MDNS-MMacro**	**MDNS-PMacro**	**MDNS-MMacro End**	**MDNS-PMacro End**	**MDNS-S**	**MDNS-Smedia Macro**	**MDNS-Smacro**	**RW**
1M	$6.405 \cdot 10^{-3}$	$5.269 \cdot 10^{-3}$	$4.924 \cdot 10^{-3}$	$3.739 \cdot 10^{-3}$	$2.611 \cdot 10^{-3}$	$1.749 \cdot 10^{-3}$	$4.278 \cdot 10^{-3}$	$2.165 \cdot 10^{-3}$	$2.599 \cdot 10^{-3}$	$6.237 \cdot 10^{-3}$	$2.248 \cdot 10^{-3}$	$2.656 \cdot 10^{-3}$	$1.6458 \cdot 10^{-2}$
3M	$6.332 \cdot 10^{-3}$	$5.266 \cdot 10^{-3}$	$4.951 \cdot 10^{-3}$	$4.328 \cdot 10^{-3}$	$1.16 \cdot 10^{-4}$	$8.91 \cdot 10^{-4}$	$4.201 \cdot 10^{-3}$	$1.366 \cdot 10^{-3}$	$2.675 \cdot 10^{-3}$	$6.177 \cdot 10^{-3}$	$4.81 \cdot 10^{-4}$	$2.748 \cdot 10^{-3}$	$2.2750 \cdot 10^{-2}$
6M	$7.760 \cdot 10^{-3}$	$6.778 \cdot 10^{-3}$	$6.514 \cdot 10^{-3}$	$5.651 \cdot 10^{-3}$	$1.947 \cdot 10^{-3}$	$2.616 \cdot 10^{-3}$	$5.569 \cdot 10^{-3}$	$3.678 \cdot 10^{-3}$	$4.194 \cdot 10^{-3}$	$7.622 \cdot 10^{-3}$	$2.328 \cdot 10^{-3}$	$7.621 \cdot 10^{-3}$	$2.8095 \cdot 10^{-2}$
9M	$7.349 \cdot 10^{-3}$	$6.450 \cdot 10^{-3}$	$6.222 \cdot 10^{-3}$	$9.367 \cdot 10^{-3}$	$1.946 \cdot 10^{-3}$	$2.535 \cdot 10^{-3}$	$5.145 \cdot 10^{-3}$	$3.820 \cdot 10^{-3}$	$3.933 \cdot 10^{-3}$	$7.229 \cdot 10^{-3}$	$2.313 \cdot 10^{-3}$	$9.697 \cdot 10^{-3}$	$3.0831 \cdot 10^{-2}$
12M	$6.708 \cdot 10^{-3}$	$5.885 \cdot 10^{-3}$	$5.684 \cdot 10^{-3}$	$1.0598 \cdot 10^{-2}$	$1.684 \cdot 10^{-3}$	$2.212 \cdot 10^{-3}$	$4.528 \cdot 10^{-3}$	$3.535 \cdot 10^{-3}$	$3.455 \cdot 10^{-3}$	$6.602 \cdot 10^{-3}$	$2.028 \cdot 10^{-3}$	$1.0036 \cdot 10^{-2}$	$3.4132 \cdot 10^{-2}$
15M	$5.828 \cdot 10^{-3}$	$5.072 \cdot 10^{-3}$	$4.893 \cdot 10^{-3}$	$1.0329 \cdot 10^{-2}$	$1.151 \cdot 10^{-3}$	$1.629 \cdot 10^{-3}$	$3.693 \cdot 10^{-3}$	$2.862 \cdot 10^{-3}$	$2.740 \cdot 10^{-3}$	$5.732 \cdot 10^{-3}$	$1.467 \cdot 10^{-3}$	$9.230 \cdot 10^{-3}$	$3.6183 \cdot 10^{-2}$
18M	$4.868 \cdot 10^{-3}$	$4.172 \cdot 10^{-3}$	$4.011 \cdot 10^{-3}$	$9.223 \cdot 10^{-3}$	$5.04 \cdot 10^{-4}$	$9.43 \cdot 10^{-4}$	$2.789 \cdot 10^{-3}$	$1.996 \cdot 10^{-3}$	$1.939 \cdot 10^{-3}$	$4.781 \cdot 10^{-3}$	$7.94 \cdot 10^{-4}$	$7.769 \cdot 10^{-3}$	$3.6732 \cdot 10^{-2}$
21M	$4.089 \cdot 10^{-3}$	$3.452 \cdot 10^{-3}$	$3.300 \cdot 10^{-3}$	$7.587 \cdot 10^{-3}$	$2.33 \cdot 10^{-4}$	$4.80 \cdot 10^{-4}$	$2.118 \cdot 10^{-3}$	$9.87 \cdot 10^{-4}$	$1.368 \cdot 10^{-3}$	$4.004 \cdot 10^{-3}$	$3.12 \cdot 10^{-4}$	$5.881 \cdot 10^{-3}$	$3.6674 \cdot 10^{-2}$
24M	$3.790 \cdot 10^{-3}$	$3.199 \cdot 10^{-3}$	$3.062 \cdot 10^{-3}$	$5.702 \cdot 10^{-3}$	$9.58 \cdot 10^{-4}$	$5.80 \cdot 10^{-4}$	$1.855 \cdot 10^{-3}$	$1.95 \cdot 10^{-4}$	$1.172 \cdot 10^{-3}$	$3.712 \cdot 10^{-3}$	$7.20 \cdot 10^{-4}$	$3.808 \cdot 10^{-3}$	$3.5870 \cdot 10^{-2}$
27M	$3.335 \cdot 10^{-3}$	$2.786 \cdot 10^{-3}$	$2.662 \cdot 10^{-3}$	$4.355 \cdot 10^{-3}$	$1.003 \cdot 10^{-3}$	$6.48 \cdot 10^{-4}$	$1.435 \cdot 10^{-3}$	$4.06 \cdot 10^{-4}$	$8.10 \cdot 10^{-4}$	$3.264 \cdot 10^{-3}$	$7.85 \cdot 10^{-4}$	$2.315 \cdot 10^{-3}$	$3.5338 \cdot 10^{-2}$
30M	$3.165 \cdot 10^{-3}$	$2.653 \cdot 10^{-3}$	$2.540 \cdot 10^{-3}$	$2.787 \cdot 10^{-3}$	$1.203 \cdot 10^{-3}$	$8.68 \cdot 10^{-4}$	$1.298 \cdot 10^{-3}$	$9.31 \cdot 10^{-4}$	$7.23 \cdot 10^{-4}$	$3.099 \cdot 10^{-3}$	$1.004 \cdot 10^{-3}$	$6.32 \cdot 10^{-4}$	$3.3997 \cdot 10^{-2}$
33M	$2.936 \cdot 10^{-3}$	$2.458 \cdot 10^{-3}$	$2.354 \cdot 10^{-3}$	$1.770 \cdot 10^{-3}$	$1.468 \cdot 10^{-3}$	$1.152 \cdot 10^{-3}$	$1.102 \cdot 10^{-3}$	$1.518 \cdot 10^{-3}$	$5.69 \cdot 10^{-4}$	$2.876 \cdot 10^{-3}$	$1.284 \cdot 10^{-3}$	$1.109 \cdot 10^{-3}$	$3.2926 \cdot 10^{-2}$
36M	$2.828 \cdot 10^{-3}$	$2.379 \cdot 10^{-3}$	$2.284 \cdot 10^{-3}$	$1.754 \cdot 10^{-3}$	$1.541 \cdot 10^{-3}$	$1.242 \cdot 10^{-3}$	$1.024 \cdot 10^{-3}$	$1.905 \cdot 10^{-3}$	$7.00 \cdot 10^{-4}$	$2.772 \cdot 10^{-3}$	$1.370 \cdot 10^{-3}$	$2.619 \cdot 10^{-3}$	$3.1747 \cdot 10^{-2}$
39M	$2.621 \cdot 10^{-3}$	$2.199 \cdot 10^{-3}$	$2.112 \cdot 10^{-3}$	$1.634 \cdot 10^{-3}$	$1.302 \cdot 10^{-3}$	$1.017 \cdot 10^{-3}$	$8.45 \cdot 10^{-4}$	$1.968 \cdot 10^{-3}$	$5.26 \cdot 10^{-4}$	$2.569 \cdot 10^{-3}$	$1.142 \cdot 10^{-3}$	$3.759 \cdot 10^{-3}$	$3.1183 \cdot 10^{-2}$
48M	$2.350 \cdot 10^{-3}$	$1.995 \cdot 10^{-3}$	$1.925 \cdot 10^{-3}$	$4.428 \cdot 10^{-3}$	$8.05 \cdot 10^{-4}$	$5.58 \cdot 10^{-4}$	$6.49 \cdot 10^{-4}$	$2.287 \cdot 10^{-3}$	$2.63 \cdot 10^{-4}$	$2.308 \cdot 10^{-3}$	$6.64 \cdot 10^{-4}$	$6.975 \cdot 10^{-3}$	$2.8862 \cdot 10^{-2}$
60M	$3.046 \cdot 10^{-3}$	$2.750 \cdot 10^{-3}$	$2.685 \cdot 10^{-3}$	$6.641 \cdot 10^{-3}$	$7.77 \cdot 10^{-4}$	$9.87 \cdot 10^{-4}$	$1.494 \cdot 10^{-3}$	$1.575 \cdot 10^{-3}$	$1.233 \cdot 10^{-3}$	$2.999 \cdot 10^{-3}$	$9.09 \cdot 10^{-4}$	$9.320 \cdot 10^{-3}$	$2.6963 \cdot 10^{-2}$
72M	$4.013 \cdot 10^{-3}$	$3.759 \cdot 10^{-3}$	$3.706 \cdot 10^{-3}$	$8.321 \cdot 10^{-3}$	$1.959 \cdot 10^{-3}$	$2.139 \cdot 10^{-3}$	$2.507 \cdot 10^{-3}$	$1.054 \cdot 10^{-3}$	$2.299 \cdot 10^{-3}$	$3.972 \cdot 10^{-3}$	$2.091 \cdot 10^{-3}$	$1.1090 \cdot 10^{-2}$	$2.5544 \cdot 10^{-2}$

* Model MDNS-Sλ did not converge. The highlighted values have minimum mean error, root mean squared error or mean absolute error, according to the table, for each maturity.

Table 4. Out-of-Sample Error Measures—12-Month Forecast Horizon *.

	Mean Error												
	MDNS	**MDNS-M**	**MDNS-P**	**MDNS-λ**	**MDNS-Macro**	**MDNS-MMacro**	**MDNS-PMacro**	**MDNS-MMacro End**	**MDNS-PMacro End**	**MDNS-S**	**MDNS-Smedia Macro**	**MDNS-Smacro**	**RW**
1M	$-1.3453 \cdot 10^{-2}$	$-2.2373 \cdot 10^{-2}$	$-3.6912 \cdot 10^{-2}$	$-2.6520 \cdot 10^{-2}$	$-9.96 \cdot 10^{-4}$	$-2.0082 \cdot 10^{-2}$	$-4.663 \cdot 10^{-3}$	$-2.4004 \cdot 10^{-2}$	$-2.470 \cdot 10^{-3}$	$-1.3076 \cdot 10^{-2}$	$-3.8745 \cdot 10^{-2}$	$-1.4409 \cdot 10^{-2}$	$-1.00079 \cdot 10^{-1}$
3M	$-1.6449 \cdot 10^{-2}$	$-2.4534 \cdot 10^{-2}$	$-3.7980 \cdot 10^{-2}$	$-2.9988 \cdot 10^{-2}$	$-4.395 \cdot 10^{-3}$	$-2.2240 \cdot 10^{-2}$	$-8.359 \cdot 10^{-3}$	$-2.6055 \cdot 10^{-2}$	$-6.109 \cdot 10^{-3}$	$-1.6015 \cdot 10^{-2}$	$-4.0246 \cdot 10^{-2}$	$-1.7478 \cdot 10^{-2}$	$-9.7008 \cdot 10^{-2}$
6M	$-1.9861 \cdot 10^{-2}$	$-2.6869 \cdot 10^{-2}$	$-3.8882 \cdot 10^{-2}$	$-2.8273 \cdot 10^{-2}$	$-8.589 \cdot 10^{-3}$	$-2.4817 \cdot 10^{-2}$	$-1.2911 \cdot 10^{-2}$	$-2.8480 \cdot 10^{-2}$	$-1.0601 \cdot 10^{-2}$	$-1.9368 \cdot 10^{-2}$	$-4.1775 \cdot 10^{-2}$	$-2.1046 \cdot 10^{-2}$	$-9.1441 \cdot 10^{-2}$
9M	$-2.1746 \cdot 10^{-2}$	$-2.7854 \cdot 10^{-2}$	$-3.8640 \cdot 10^{-2}$	$-2.8314 \cdot 10^{-2}$	$-1.1388 \cdot 10^{-2}$	$-2.6248 \cdot 10^{-2}$	$-1.5984 \cdot 10^{-2}$	$-2.9769 \cdot 10^{-2}$	$-1.3637 \cdot 10^{-2}$	$-2.1218 \cdot 10^{-2}$	$-4.2204 \cdot 10^{-2}$	$-2.3215 \cdot 10^{-2}$	$-8.5034 \cdot 10^{-2}$
12M	$-2.2659 \cdot 10^{-2}$	$-2.8011 \cdot 10^{-2}$	$-3.7745 \cdot 10^{-2}$	$-2.8588 \cdot 10^{-2}$	$-1.3284 \cdot 10^{-2}$	$-2.6980 \cdot 10^{-2}$	$-1.8089 \cdot 10^{-2}$	$-3.0369 \cdot 10^{-2}$	$-1.5721 \cdot 10^{-2}$	$-2.2113 \cdot 10^{-2}$	$-4.1998 \cdot 10^{-2}$	$-2.4503 \cdot 10^{-2}$	$-7.9434 \cdot 10^{-2}$
15M	$-2.2860 \cdot 10^{-2}$	$-2.7576 \cdot 10^{-2}$	$-3.6402 \cdot 10^{-2}$	$-2.8497 \cdot 10^{-2}$	$-1.4491 \cdot 10^{-2}$	$-2.7195 \cdot 10^{-2}$	$-1.9455 \cdot 10^{-2}$	$-3.0462 \cdot 10^{-2}$	$-1.7077 \cdot 10^{-2}$	$-2.2309 \cdot 10^{-2}$	$-4.1342 \cdot 10^{-2}$	$-2.5134 \cdot 10^{-2}$	$-7.4483 \cdot 10^{-2}$
18M	$-2.2669 \cdot 10^{-2}$	$-2.6845 \cdot 10^{-2}$	$-3.4888 \cdot 10^{-2}$	$-2.8096 \cdot 10^{-2}$	$-1.5298 \cdot 10^{-2}$	$-2.7152 \cdot 10^{-2}$	$-2.0380 \cdot 10^{-2}$	$-3.0304 \cdot 10^{-2}$	$-1.8002 \cdot 10^{-2}$	$-2.2122 \cdot 10^{-2}$	$-4.0497 \cdot 10^{-2}$	$-2.5398 \cdot 10^{-2}$	$-7.0357 \cdot 10^{-2}$
21M	$-2.2139 \cdot 10^{-2}$	$-2.5856 \cdot 10^{-2}$	$-3.3219 \cdot 10^{-2}$	$-2.7353 \cdot 10^{-2}$	$-1.5738 \cdot 10^{-2}$	$-2.6860 \cdot 10^{-2}$	$-2.0906 \cdot 10^{-2}$	$-2.9906 \cdot 10^{-2}$	$-1.8534 \cdot 10^{-2}$	$-2.1602 \cdot 10^{-2}$	$-3.9468 \cdot 10^{-2}$	$-2.5331 \cdot 10^{-2}$	$-6.6635 \cdot 10^{-2}$
24M	$-2.1514 \cdot 10^{-2}$	$-2.4839 \cdot 10^{-2}$	$-3.1610 \cdot 10^{-2}$	$-2.6491 \cdot 10^{-2}$	$-1.6040 \cdot 10^{-2}$	$-2.6531 \cdot 10^{-2}$	$-2.1272 \cdot 10^{-2}$	$-2.9478 \cdot 10^{-2}$	$-1.8910 \cdot 10^{-2}$	$-2.0992 \cdot 10^{-2}$	$-3.8465 \cdot 10^{-2}$	$-2.5163 \cdot 10^{-2}$	$-6.3537 \cdot 10^{-2}$
27M	$-2.0863 \cdot 10^{-2}$	$-2.3851 \cdot 10^{-2}$	$-3.0104 \cdot 10^{-2}$	$-2.5576 \cdot 10^{-2}$	$-1.6267 \cdot 10^{-2}$	$-2.6209 \cdot 10^{-2}$	$-2.1544 \cdot 10^{-2}$	$-2.9064 \cdot 10^{-2}$	$-1.9194 \cdot 10^{-2}$	$-2.0359 \cdot 10^{-2}$	$-3.7530 \cdot 10^{-2}$	$-2.4955 \cdot 10^{-2}$	$-6.0774 \cdot 10^{-2}$
30M	$-2.0234 \cdot 10^{-2}$	$-2.2931 \cdot 10^{-2}$	$-2.8729 \cdot 10^{-2}$	$-2.4662 \cdot 10^{-2}$	$-1.6462 \cdot 10^{-2}$	$-2.5925 \cdot 10^{-2}$	$-2.1770 \cdot 10^{-2}$	$-2.8696 \cdot 10^{-2}$	$-1.9434 \cdot 10^{-2}$	$-1.9749 \cdot 10^{-2}$	$-3.6687 \cdot 10^{-2}$	$-2.4748 \cdot 10^{-2}$	$-5.9177 \cdot 10^{-2}$
33M	$-1.9472 \cdot 10^{-2}$	$-2.1916 \cdot 10^{-2}$	$-2.7313 \cdot 10^{-2}$	$-2.3603 \cdot 10^{-2}$	$-1.6469 \cdot 10^{-2}$	$-2.5514 \cdot 10^{-2}$	$-2.1796 \cdot 10^{-2}$	$-2.8206 \cdot 10^{-2}$	$-1.9475 \cdot 10^{-2}$	$-1.9006 \cdot 10^{-2}$	$-3.5766 \cdot 10^{-2}$	$-2.4385 \cdot 10^{-2}$	$-5.8134 \cdot 10^{-2}$
36M	$-1.8792 \cdot 10^{-2}$	$-2.1016 \cdot 10^{-2}$	$-2.6059 \cdot 10^{-2}$	$-2.2620 \cdot 10^{-2}$	$-1.6503 \cdot 10^{-2}$	$-2.5180 \cdot 10^{-2}$	$-2.1842 \cdot 10^{-2}$	$-2.7800 \cdot 10^{-2}$	$-1.9537 \cdot 10^{-2}$	$-1.8347 \cdot 10^{-2}$	$-3.4969 \cdot 10^{-2}$	$-2.4080 \cdot 10^{-2}$	$-5.7549 \cdot 10^{-2}$
39M	$-1.8252 \cdot 10^{-2}$	$-2.0283 \cdot 10^{-2}$	$-2.5011 \cdot 10^{-2}$	$-2.1778 \cdot 10^{-2}$	$-1.6625 \cdot 10^{-2}$	$-2.4977 \cdot 10^{-2}$	$-2.1970 \cdot 10^{-2}$	$-2.7529 \cdot 10^{-2}$	$-1.9680 \cdot 10^{-2}$	$-1.7826 \cdot 10^{-2}$	$-3.4344 \cdot 10^{-2}$	$-2.3891 \cdot 10^{-2}$	$-5.6574 \cdot 10^{-2}$
48M	$-1.7212 \cdot 10^{-2}$	$-1.8789 \cdot 10^{-2}$	$-2.2760 \cdot 10^{-2}$	$-1.9873 \cdot 10^{-2}$	$-1.7283 \cdot 10^{-2}$	$-2.4861 \cdot 10^{-2}$	$-2.2622 \cdot 10^{-2}$	$-2.7241 \cdot 10^{-2}$	$-2.0376 \cdot 10^{-2}$	$-1.6843 \cdot 10^{-2}$	$-3.3176 \cdot 10^{-2}$	$-2.3762 \cdot 10^{-2}$	$-5.4747 \cdot 10^{-2}$
60M	$-1.6479 \cdot 10^{-2}$	$-1.7644 \cdot 10^{-2}$	$-2.0911 \cdot 10^{-2}$	$-1.8151 \cdot 10^{-2}$	$-1.8262 \cdot 10^{-2}$	$-2.5127 \cdot 10^{-2}$	$-2.3569 \cdot 10^{-2}$	$-2.7330 \cdot 10^{-2}$	$-2.1372 \cdot 10^{-2}$	$-1.6171 \cdot 10^{-2}$	$-3.2414 \cdot 10^{-2}$	$-2.3960 \cdot 10^{-2}$	$-5.3842 \cdot 10^{-2}$
72M	$-1.5869 \cdot 10^{-2}$	$-1.6755 \cdot 10^{-2}$	$-1.9534 \cdot 10^{-2}$	$-1.6758 \cdot 10^{-2}$	$-1.8905 \cdot 10^{-2}$	$-2.5281 \cdot 10^{-2}$	$-2.4178 \cdot 10^{-2}$	$-2.7353 \cdot 10^{-2}$	$-2.2019 \cdot 10^{-2}$	$-1.5611 \cdot 10^{-2}$	$-3.1833 \cdot 10^{-2}$	$-2.4039 \cdot 10^{-2}$	$-5.3564 \cdot 10^{-2}$
	Root Mean Squared Error												
	MDNS	**MDNS-M**	**MDNS-P**	**MDNS-λ**	**MDNS-Macro**	**MDNS-MMacro**	**MDNS-PMacro**	**MDNS-MMacro End**	**MDNS-PMacro End**	**MDNS-S**	**MDNS-Smedia Macro**	**MDNS-Smacro**	**RW**
1M	$1.9581 \cdot 10^{-2}$	$2.6511 \cdot 10^{-2}$	$4.4188 \cdot 10^{-2}$	$3.2294 \cdot 10^{-2}$	$6.533 \cdot 10^{-3}$	$2.5864 \cdot 10^{-2}$	$1.0366 \cdot 10^{-2}$	$3.0501 \cdot 10^{-2}$	$9.044 \cdot 10^{-3}$	$1.9233 \cdot 10^{-2}$	$4.7274 \cdot 10^{-2}$	$2.0489 \cdot 10^{-2}$	$1.12325 \cdot 10^{-1}$
3M	$2.3064 \cdot 10^{-2}$	$2.9457 \cdot 10^{-2}$	$4.5828 \cdot 10^{-2}$	$3.5435 \cdot 10^{-2}$	$9.280 \cdot 10^{-3}$	$2.8638 \cdot 10^{-2}$	$1.4229 \cdot 10^{-2}$	$3.3147 \cdot 10^{-2}$	$1.2405 \cdot 10^{-2}$	$2.2658 \cdot 10^{-2}$	$4.9301 \cdot 10^{-2}$	$2.4097 \cdot 10^{-2}$	$1.09794 \cdot 10^{-1}$
6M	$2.6448 \cdot 10^{-2}$	$3.2168 \cdot 10^{-2}$	$4.6784 \cdot 10^{-2}$	$3.4985 \cdot 10^{-2}$	$1.2965 \cdot 10^{-2}$	$3.1297 \cdot 10^{-2}$	$1.8585 \cdot 10^{-2}$	$3.5626 \cdot 10^{-2}$	$1.6405 \cdot 10^{-2}$	$2.5979 \cdot 10^{-2}$	$5.0765 \cdot 10^{-2}$	$2.7576 \cdot 10^{-2}$	$1.05169 \cdot 10^{-1}$
9M	$2.7769 \cdot 10^{-2}$	$3.2904 \cdot 10^{-2}$	$4.6004 \cdot 10^{-2}$	$3.4793 \cdot 10^{-2}$	$1.5130 \cdot 10^{-2}$	$3.2216 \cdot 10^{-2}$	$2.1081 \cdot 10^{-2}$	$3.6376 \cdot 10^{-2}$	$1.8719 \cdot 10^{-2}$	$2.7260 \cdot 10^{-2}$	$5.0565 \cdot 10^{-2}$	$2.9156 \cdot 10^{-2}$	$9.9725 \cdot 10^{-2}$
12M	$2.8114 \cdot 10^{-2}$	$3.2697 \cdot 10^{-2}$	$4.4496 \cdot 10^{-2}$	$3.4426 \cdot 10^{-2}$	$1.6524 \cdot 10^{-2}$	$3.2397 \cdot 10^{-2}$	$2.2644 \cdot 10^{-2}$	$3.6399 \cdot 10^{-2}$	$2.0180 \cdot 10^{-2}$	$2.7585 \cdot 10^{-2}$	$4.9695 \cdot 10^{-2}$	$2.9878 \cdot 10^{-2}$	$9.5030 \cdot 10^{-2}$
15M	$2.7858 \cdot 10^{-2}$	$3.1937 \cdot 10^{-2}$	$4.2618 \cdot 10^{-2}$	$3.3729 \cdot 10^{-2}$	$1.7394 \cdot 10^{-2}$	$3.2171 \cdot 10^{-2}$	$2.3626 \cdot 10^{-2}$	$3.6022 \cdot 10^{-2}$	$2.1107 \cdot 10^{-2}$	$2.7321 \cdot 10^{-2}$	$4.8478 \cdot 10^{-2}$	$3.0062 \cdot 10^{-2}$	$9.0733 \cdot 10^{-2}$
18M	$2.7320 \cdot 10^{-2}$	$3.0957 \cdot 10^{-2}$	$4.0674 \cdot 10^{-2}$	$3.2849 \cdot 10^{-2}$	$1.7991 \cdot 10^{-2}$	$3.1806 \cdot 10^{-2}$	$2.4300 \cdot 10^{-2}$	$3.5515 \cdot 10^{-2}$	$2.1751 \cdot 10^{-2}$	$2.6785 \cdot 10^{-2}$	$4.7191 \cdot 10^{-2}$	$2.9996 \cdot 10^{-2}$	$8.7087 \cdot 10^{-2}$
21M	$2.6431 \cdot 10^{-2}$	$2.9686 \cdot 10^{-2}$	$3.8565 \cdot 10^{-2}$	$3.1647 \cdot 10^{-2}$	$1.8220 \cdot 10^{-2}$	$3.1186 \cdot 10^{-2}$	$2.4573 \cdot 10^{-2}$	$3.4763 \cdot 10^{-2}$	$2.2013 \cdot 10^{-2}$	$2.5905 \cdot 10^{-2}$	$4.5724 \cdot 10^{-2}$	$2.9597 \cdot 10^{-2}$	$8.4014 \cdot 10^{-2}$
24M	$2.5434 \cdot 10^{-2}$	$2.8362 \cdot 10^{-2}$	$3.6511 \cdot 10^{-2}$	$3.0348 \cdot 10^{-2}$	$1.8297 \cdot 10^{-2}$	$3.0524 \cdot 10^{-2}$	$2.4666 \cdot 10^{-2}$	$3.3979 \cdot 10^{-2}$	$2.2102 \cdot 10^{-2}$	$2.4922 \cdot 10^{-2}$	$4.4288 \cdot 10^{-2}$	$2.9093 \cdot 10^{-2}$	$8.1647 \cdot 10^{-2}$
27M	$2.4610 \cdot 10^{-2}$	$2.7235 \cdot 10^{-2}$	$3.4750 \cdot 10^{-2}$	$2.9205 \cdot 10^{-2}$	$1.8473 \cdot 10^{-2}$	$3.0055 \cdot 10^{-2}$	$2.4849 \cdot 10^{-2}$	$3.3397 \cdot 10^{-2}$	$2.2293 \cdot 10^{-2}$	$2.4116 \cdot 10^{-2}$	$4.3111 \cdot 10^{-2}$	$2.8736 \cdot 10^{-2}$	$7.9517 \cdot 10^{-2}$
30M	$2.3837 \cdot 10^{-2}$	$2.6202 \cdot 10^{-2}$	$3.3165 \cdot 10^{-2}$	$2.8116 \cdot 10^{-2}$	$1.8632 \cdot 10^{-2}$	$2.9660 \cdot 10^{-2}$	$2.5009 \cdot 10^{-2}$	$3.2900 \cdot 10^{-2}$	$2.2460 \cdot 10^{-2}$	$2.3362 \cdot 10^{-2}$	$4.2073 \cdot 10^{-2}$	$2.8409 \cdot 10^{-2}$	$7.8395 \cdot 10^{-2}$
33M	$2.2960 \cdot 10^{-2}$	$2.5099 \cdot 10^{-2}$	$3.1570 \cdot 10^{-2}$	$2.6925 \cdot 10^{-2}$	$1.8632 \cdot 10^{-2}$	$2.9164 \cdot 10^{-2}$	$2.5003 \cdot 10^{-2}$	$3.2307 \cdot 10^{-2}$	$2.2469 \cdot 10^{-2}$	$2.2504 \cdot 10^{-2}$	$4.0987 \cdot 10^{-2}$	$2.7953 \cdot 10^{-2}$	$7.7560 \cdot 10^{-2}$
36M	$2.2183 \cdot 10^{-2}$	$2.4122 \cdot 10^{-2}$	$3.0161 \cdot 10^{-2}$	$2.5839 \cdot 10^{-2}$	$1.8663 \cdot 10^{-2}$	$2.8761 \cdot 10^{-2}$	$2.5018 \cdot 10^{-2}$	$3.1817 \cdot 10^{-2}$	$2.2498 \cdot 10^{-2}$	$2.1748 \cdot 10^{-2}$	$4.0052 \cdot 10^{-2}$	$2.7573 \cdot 10^{-2}$	$7.7133 \cdot 10^{-2}$
39M	$2.1601 \cdot 10^{-2}$	$2.3362 \cdot 10^{-2}$	$2.9019 \cdot 10^{-2}$	$2.4955 \cdot 10^{-2}$	$1.8821 \cdot 10^{-2}$	$2.8539 \cdot 10^{-2}$	$2.5160 \cdot 10^{-2}$	$3.1515 \cdot 10^{-2}$	$2.2656 \cdot 10^{-2}$	$2.1187 \cdot 10^{-2}$	$3.9347 \cdot 10^{-2}$	$2.7359 \cdot 10^{-2}$	$7.6395 \cdot 10^{-2}$
48M	$2.0252 \cdot 10^{-2}$	$2.1616 \cdot 10^{-2}$	$2.6346 \cdot 10^{-2}$	$2.2804 \cdot 10^{-2}$	$1.9411 \cdot 10^{-2}$	$2.8208 \cdot 10^{-2}$	$2.5680 \cdot 10^{-2}$	$3.0976 \cdot 10^{-2}$	$2.3218 \cdot 10^{-2}$	$1.9891 \cdot 10^{-2}$	$3.7805 \cdot 10^{-2}$	$2.6992 \cdot 10^{-2}$	$7.4974 \cdot 10^{-2}$
60M	$1.9401 \cdot 10^{-2}$	$2.0395 \cdot 10^{-2}$	$2.4273 \cdot 10^{-2}$	$2.1064 \cdot 10^{-2}$	$2.0469 \cdot 10^{-2}$	$2.8440 \cdot 10^{-2}$	$2.6659 \cdot 10^{-2}$	$3.1004 \cdot 10^{-2}$	$2.4244 \cdot 10^{-2}$	$1.9102 \cdot 10^{-2}$	$3.6869 \cdot 10^{-2}$	$2.7133 \cdot 10^{-2}$	$7.4274 \cdot 10^{-2}$
72M	$1.8759 \cdot 10^{-2}$	$1.9504 \cdot 10^{-2}$	$2.2791 \cdot 10^{-2}$	$1.9744 \cdot 10^{-2}$	$2.1220 \cdot 10^{-2}$	$2.8624 \cdot 10^{-2}$	$2.7340 \cdot 10^{-2}$	$3.1037 \cdot 10^{-2}$	$2.4963 \cdot 10^{-2}$	$1.8508 \cdot 10^{-2}$	$3.6218 \cdot 10^{-2}$	$2.7225 \cdot 10^{-2}$	$7.3873 \cdot 10^{-2}$

Table 4. *Cont.*

Mean Absolute Error													
	MDNS	**MDNS-M**	**MDNS-P**	**MDNS-λ**	**MDNS-Macro**	**MDNS-MMacro**	**MDNS-PMacro**	**MDNS-MMacro End**	**MDNS-PMacro End**	**MDNS-S**	**MDNS-Smedia Macro**	**MDNS-Smacro**	**RW**
1M	$1.4231 \cdot 10^{-2}$	$2.2373 \cdot 10^{-2}$	$3.6912 \cdot 10^{-2}$	$2.6576 \cdot 10^{-2}$	$5.275 \cdot 10^{-3}$	$2.0122 \cdot 10^{-2}$	$9.031 \cdot 10^{-3}$	$9.031 \cdot 10^{-3}$	$7.775 \cdot 10^{-3}$	$1.3912 \cdot 10^{-2}$	$3.8745 \cdot 10^{-2}$	$1.4478 \cdot 10^{-2}$	$1.00079 \cdot 10^{-1}$
3M	$1.7324 \cdot 10^{-2}$	$2.4626 \cdot 10^{-2}$	$3.8074 \cdot 10^{-2}$	$2.9988 \cdot 10^{-2}$	$7.186 \cdot 10^{-3}$	$2.2531 \cdot 10^{-2}$	$1.2344 \cdot 10^{-2}$	$1.2344 \cdot 10^{-2}$	$1.0843 \cdot 10^{-2}$	$1.6921 \cdot 10^{-2}$	$4.0351 \cdot 10^{-2}$	$1.7939 \cdot 10^{-2}$	$9.7008 \cdot 10^{-2}$
6M	$2.0635 \cdot 10^{-2}$	$2.7091 \cdot 10^{-2}$	$3.9104 \cdot 10^{-2}$	$2.8753 \cdot 10^{-2}$	$1.0317 \cdot 10^{-2}$	$2.5214 \cdot 10^{-2}$	$1.5785 \cdot 10^{-2}$	$1.5785 \cdot 10^{-2}$	$1.4216 \cdot 10^{-2}$	$2.0198 \cdot 10^{-2}$	$4.2018 \cdot 10^{-2}$	$2.1451 \cdot 10^{-2}$	$9.1441 \cdot 10^{-2}$
9M	$2.2157 \cdot 10^{-2}$	$2.7996 \cdot 10^{-2}$	$3.8783 \cdot 10^{-2}$	$2.8871 \cdot 10^{-2}$	$1.2277 \cdot 10^{-2}$	$2.6430 \cdot 10^{-2}$	$1.7681 \cdot 10^{-2}$	$1.7681 \cdot 10^{-2}$	$1.5751 \cdot 10^{-2}$	$2.1651 \cdot 10^{-2}$	$4.2369 \cdot 10^{-2}$	$2.3402 \cdot 10^{-2}$	$8.5034 \cdot 10^{-2}$
12M	$2.2733 \cdot 10^{-2}$	$2.8044 \cdot 10^{-2}$	$3.7777 \cdot 10^{-2}$	$2.8928 \cdot 10^{-2}$	$1.3476 \cdot 10^{-2}$	$2.7032 \cdot 10^{-2}$	$1.8841 \cdot 10^{-2}$	$1.8841 \cdot 10^{-2}$	$1.6881 \cdot 10^{-2}$	$2.2214 \cdot 10^{-2}$	$4.2051 \cdot 10^{-2}$	$2.4568 \cdot 10^{-2}$	$7.9801 \cdot 10^{-2}$
15M	$2.2860 \cdot 10^{-2}$	$2.7576 \cdot 10^{-2}$	$3.6402 \cdot 10^{-2}$	$2.8622 \cdot 10^{-2}$	$1.4491 \cdot 10^{-2}$	$2.7195 \cdot 10^{-2}$	$1.9610 \cdot 10^{-2}$	$1.9610 \cdot 10^{-2}$	$1.7553 \cdot 10^{-2}$	$2.2309 \cdot 10^{-2}$	$4.1342 \cdot 10^{-2}$	$2.5134 \cdot 10^{-2}$	$7.5589 \cdot 10^{-2}$
18M	$2.2669 \cdot 10^{-2}$	$2.6845 \cdot 10^{-2}$	$3.4888 \cdot 10^{-2}$	$2.8100 \cdot 10^{-2}$	$1.5298 \cdot 10^{-2}$	$2.7152 \cdot 10^{-2}$	$2.0380 \cdot 10^{-2}$	$2.0380 \cdot 10^{-2}$	$1.8092 \cdot 10^{-2}$	$2.2122 \cdot 10^{-2}$	$4.0497 \cdot 10^{-2}$	$2.5398 \cdot 10^{-2}$	$7.2201 \cdot 10^{-2}$
21M	$2.2139 \cdot 10^{-2}$	$2.5856 \cdot 10^{-2}$	$3.3219 \cdot 10^{-2}$	$2.7353 \cdot 10^{-2}$	$1.5738 \cdot 10^{-2}$	$2.6860 \cdot 10^{-2}$	$2.0906 \cdot 10^{-2}$	$2.0906 \cdot 10^{-2}$	$1.8534 \cdot 10^{-2}$	$2.1602 \cdot 10^{-2}$	$3.9468 \cdot 10^{-2}$	$2.5331 \cdot 10^{-2}$	$6.9721 \cdot 10^{-2}$
24M	$2.1514 \cdot 10^{-2}$	$2.4839 \cdot 10^{-2}$	$3.1610 \cdot 10^{-2}$	$2.6491 \cdot 10^{-2}$	$1.6040 \cdot 10^{-2}$	$2.6531 \cdot 10^{-2}$	$2.1272 \cdot 10^{-2}$	$2.1272 \cdot 10^{-2}$	$1.8910 \cdot 10^{-2}$	$2.0992 \cdot 10^{-2}$	$3.8465 \cdot 10^{-2}$	$2.5163 \cdot 10^{-2}$	$6.7874 \cdot 10^{-2}$
27M	$2.0863 \cdot 10^{-2}$	$2.3851 \cdot 10^{-2}$	$3.0104 \cdot 10^{-2}$	$2.5576 \cdot 10^{-2}$	$1.6267 \cdot 10^{-2}$	$2.6209 \cdot 10^{-2}$	$2.1544 \cdot 10^{-2}$	$2.1544 \cdot 10^{-2}$	$1.9194 \cdot 10^{-2}$	$2.0359 \cdot 10^{-2}$	$3.7530 \cdot 10^{-2}$	$2.4955 \cdot 10^{-2}$	$6.6181 \cdot 10^{-2}$
30M	$2.0234 \cdot 10^{-2}$	$2.2931 \cdot 10^{-2}$	$2.8729 \cdot 10^{-2}$	$2.4662 \cdot 10^{-2}$	$1.6462 \cdot 10^{-2}$	$2.5925 \cdot 10^{-2}$	$2.1770 \cdot 10^{-2}$	$2.1770 \cdot 10^{-2}$	$1.9434 \cdot 10^{-2}$	$1.9749 \cdot 10^{-2}$	$3.6687 \cdot 10^{-2}$	$2.4748 \cdot 10^{-2}$	$6.5368 \cdot 10^{-2}$
33M	$1.9472 \cdot 10^{-2}$	$2.1916 \cdot 10^{-2}$	$2.7313 \cdot 10^{-2}$	$2.3603 \cdot 10^{-2}$	$1.6469 \cdot 10^{-2}$	$2.5514 \cdot 10^{-2}$	$2.1796 \cdot 10^{-2}$	$2.1796 \cdot 10^{-2}$	$1.9475 \cdot 10^{-2}$	$1.9006 \cdot 10^{-2}$	$3.5766 \cdot 10^{-2}$	$2.4385 \cdot 10^{-2}$	$6.4680 \cdot 10^{-2}$
36M	$1.8798 \cdot 10^{-2}$	$2.1026 \cdot 10^{-2}$	$2.6069 \cdot 10^{-2}$	$2.2620 \cdot 10^{-2}$	$1.6503 \cdot 10^{-2}$	$2.5180 \cdot 10^{-2}$	$2.1842 \cdot 10^{-2}$	$2.1842 \cdot 10^{-2}$	$1.9537 \cdot 10^{-2}$	$1.8352 \cdot 10^{-2}$	$3.4969 \cdot 10^{-2}$	$2.4080 \cdot 10^{-2}$	$6.4373 \cdot 10^{-2}$
39M	$1.8294 \cdot 10^{-2}$	$2.0331 \cdot 10^{-2}$	$2.5058 \cdot 10^{-2}$	$2.1778 \cdot 10^{-2}$	$1.6658 \cdot 10^{-2}$	$2.5011 \cdot 10^{-2}$	$2.2003 \cdot 10^{-2}$	$2.2003 \cdot 10^{-2}$	$1.9715 \cdot 10^{-2}$	$1.7868 \cdot 10^{-2}$	$3.4378 \cdot 10^{-2}$	$2.3910 \cdot 10^{-2}$	$6.3761 \cdot 10^{-2}$
48M	$1.7247 \cdot 10^{-2}$	$1.8832 \cdot 10^{-2}$	$2.2803 \cdot 10^{-2}$	$1.9873 \cdot 10^{-2}$	$1.7303 \cdot 10^{-2}$	$2.4881 \cdot 10^{-2}$	$2.2644 \cdot 10^{-2}$	$2.2644 \cdot 10^{-2}$	$2.0400 \cdot 10^{-2}$	$1.6875 \cdot 10^{-2}$	$3.3196 \cdot 10^{-2}$	$2.3769 \cdot 10^{-2}$	$6.2520 \cdot 10^{-2}$
60M	$1.6500 \cdot 10^{-2}$	$1.7677 \cdot 10^{-2}$	$2.0943 \cdot 10^{-2}$	$1.8187 \cdot 10^{-2}$	$1.8263 \cdot 10^{-2}$	$2.5129 \cdot 10^{-2}$	$2.3576 \cdot 10^{-2}$	$2.3576 \cdot 10^{-2}$	$2.1381 \cdot 10^{-2}$	$1.6188 \cdot 10^{-2}$	$3.2417 \cdot 10^{-2}$	$2.3960 \cdot 10^{-2}$	$6.1924 \cdot 10^{-2}$
72M	$1.5969 \cdot 10^{-2}$	$1.6869 \cdot 10^{-2}$	$1.9646 \cdot 10^{-2}$	$1.6974 \cdot 10^{-2}$	$1.8982 \cdot 10^{-2}$	$2.5358 \cdot 10^{-2}$	$2.4262 \cdot 10^{-2}$	$2.4262 \cdot 10^{-2}$	$2.2106 \cdot 10^{-2}$	$1.5704 \cdot 10^{-2}$	$3.1911 \cdot 10^{-2}$	$2.4107 \cdot 10^{-2}$	$6.1575 \cdot 10^{-2}$

* Model MDNS-Sλ did not converge. The highlighted values have minimum mean error, root mean squared error or mean absolute error, according to the table, for each maturity.

5. Conclusions

Regime changes resulting from the impact of monetary policy changes and other economic shocks such as economic cycles, financial crises, and changes in market sentiment can have a profound effect on the term structure of interest rates. These changes manifest through shifts in the shape, slope, and level of the yield curve. During periods of economic expansion or high inflation expectations, long-term interest rates tend to rise more rapidly than short-term rates, leading to a steepening of the yield curve. Conversely, during recessions or times of economic uncertainty, long-term rates may decline relative to short-term rates, resulting in a flattening or even an inverted yield curve. Furthermore, regime changes can influence the slope of the yield curve. A positive slope indicates higher long-term rates compared to short-term rates, reflecting expectations of economic growth. Conversely, a negative slope suggests expectations of an economic slowdown or recession. Regime changes can also cause an overall shift in the level of interest rates across all maturities. For instance, when central banks raise or lower policy rates, it leads to a parallel shift in the yield curve.

Monetary policy decisions made by central banks play a pivotal role in shaping interest rates and the yield curve. Changes in monetary policy, such as interest rate hikes or cuts, yield several effects on the term structure. These changes directly impact short-term interest rates. Tightening policies involving interest rate increases result in higher short-term rates, leading to an upward shift in the yield curve. Conversely, easing policies involving interest rate reductions cause short-term rates to decrease, resulting in a downward shift in the yield curve. Additionally, monetary policy changes can influence market expectations regarding future interest rates. If central banks indicate a bias towards tightening or loosening, it can impact longer-term interest rates and the slope of the yield curve. Anticipated policy changes can lead to adjustments in market expectations of future interest rates, thereby influencing the shape and slope of the yield curve. Monetary policy changes can also impact risk perceptions and investor sentiment. Unexpected policy actions can introduce uncertainty and volatility, which may affect the risk premium demanded by investors. Changes in the risk premium, in turn, can influence the yield spread between different maturities, thus shaping the yield curve accordingly.

This paper proposes several innovations for estimating the Interbank Deposit yield curve using different generalizations of the Nelson–Siegel dynamic model, comparing several specifications incorporating the effects of regime changes and time-varying parameters in the model structure, and thus make the modeling of the term structure of interest rates adaptable to the changes in the economic environment discussed above. These innovations increase the flexibility of the model, allowing it to overcome the limitations of simpler models such as in [20].

Term structure forecasts are utilized in a wide range of applications, including monetary policy analysis, investment decision-making, risk management, asset liability management, pricing and valuation, economic analysis, and financial planning. They provide critical insights into future interest rate movements and help market participants and policymakers make informed decisions in various financial and economic contexts. Our results indicate that the proposed modifications significantly improve yield curve forecasts.

Models with the best performance depend on the maturity, and for the forecast, they also depend on the forecast horizon. The incorporation of more flexibility in the decay factor through regime switch is only relevant in the estimation of a maturity of one month. For the other maturities, models with the incorporation of macroeconomic variables and at the same time with regime switches (MDNS-PMacro, MDNS-PMacroEnd and MDNS-SMacro) have a better fit. Incorporation of greater flexibility, such as in the models MDNS-PMacroEnd and MDNS-Smacro, is relevant for the adjustment in the estimation. Thus, the extensions proposed in this paper have a better fit for estimation with Brazilian ID data.

In conclusion, the models proposed in this paper, especially those with greater flexibility, are relevant for both the estimation and forecasting of the Brazilian yield curve. The incorporation of regime switches and/or macroeconomic variables is crucial for achieving better accuracy in the estimation and forecasting of the yield curve.

Our results are consistent with other works that indicate the importance of incorporating regime changes and other forms of temporal variation in parameters in forecasting procedures, as emphasized by [55–58]. The incorporation of these mechanisms avoids the need to use ad hoc methods such as estimations using rolling samples [59,60] and is an alternative to methods based on the combination of models for the construction of predictions [61]. Using regime switching structures, the yield curve forecasts are based on estimates of the persistence structure in effect during the forecast construction period, and thus avoid the impacts of using biased estimates of these parameters, reducing the associated measures of forecast error.

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Appendix A

Table A1. Model Confidence Set—1-Month Forecast Horizon *.

1-Month Maturity								**3-Month Maturity**							
Models	**Rank_M**	**v_M**	**MCS_M**	**Rank_R**	**v_R**	**MCS_R**	**Loss**	**Models**	**Rank_M**	**v_M**	**MCS_M**	**Rank_R**	**v_R**	**MCS_R**	**Loss**
MDNS	5	−0.0279	1.0000	8	3.00424	0.0188	2.45×10^{-5}	MDNS	9	0.64777	0.9596	5	2.2142	0.2952	4.29×10^{-5}
MDNS_M	6	0.32412	0.9996	11	5.53884	0.0000	2.55×10^{-5}	MDNS_M	6	0.42852	0.9924	8	2.70442	0.0540	3.89×10^{-5}
MDNS_P	11	1.99208	0.2054	10	4.57205	0.0000	3.85×10^{-5}	MDNS_P	10	0.76239	0.9194	2	1.89179	0.5342	4.85×10^{-5}
MDNS_Lambda	7	0.45004	0.9990	7	2.96519	0.0226	2.73×10^{-5}	MDNS_Lambda	8	0.5859	0.9720	4	2.05852	0.4044	4.28×10^{-5}
MDNS_Macro	1	−7.9353	1.0000	1	−1.2181	1.0000	6.69×10^{-6}	MDNS_Macro	1	−4.6531	1.0000	1	−1.8918	1.0000	1.01×10^{-5}
MDNS_MMacro	4	−0.6725	1.0000	5	2.63773	0.0944	2.11×10^{-5}	MDNS_MMacro	3	−0.7155	1.0000	10	4.20607	0.0000	3.02×10^{-5}
MDNS_MMacroEnd	3	−1.9544	1.0000	3	2.27073	0.2934	1.57×10^{-5}	MDNS_MMacroEnd	5	0.21586	1.0000	6	2.2512	0.2692	4.01×10^{-5}
MDNS_PMacroEnd	9	1.34095	0.6994	6	2.72089	0.0698	4.09×10^{-5}	MDNS_PMacroEnd	11	0.94801	0.8066	9	3.42541	0.0018	4.75×10^{-5}
MDNS_S	10	1.83819	0.3020	9	4.48854	0.0000	3.23×10^{-5}	MDNS_S	7	0.51269	0.9840	3	2.04071	0.5342	4.18×10^{-5}
MDNS_SmediaMacro	8	0.45333	0.9990	4	2.53475	0.2934	2.83×10^{-5}	MDNS_SmediaMacro	4	0.03341	1.0000	11	4.95557	0.0000	3.67×10^{-5}
MDNS_Smacro	2	−3.7331	1.0000	2	1.2181	1.0000	1.08×10^{-5}	MDNS_Smacro	2	−2.0043	1.0000	7	2.57727	0.0894	2.15×10^{-5}
6-Month Maturity								**9-Month Maturity**							
Models	**Rank_M**	**v_M**	**MCS_M**	**Rank_R**	**v_R**	**MCS_R**	**Loss**	**Models**	**Rank_M**	**v_M**	**MCS_M**	**Rank_R**	**v_R**	**MCS_R**	**Loss**
MDNS	7	0.85978	0.8768	4	2.33137	0.2522	5.85×10^{-5}	MDNS	7	0.66771	0.9448	4	1.85824	0.5474	6.29×10^{-5}
MDNS_M	6	0.11812	1.0000	6	2.40346	0.2098	5.00×10^{-5}	MDNS_M	6	0.06792	1.0000	3	1.84242	0.5650	5.51×10^{-5}
MDNS_P	11	1.19132	0.6826	7	2.68302	0.0954	6.16×10^{-5}	MDNS_P	11	1.43942	0.4788	8	2.49953	0.1198	7.16×10^{-5}
MDNS_Lambda	9	1.06723	0.7622	2	2.17637	0.3452	6.63×10^{-5}	MDNS_Lambda	10	1.40346	0.5058	7	2.24449	0.2478	8.44×10^{-5}
MDNS_Macro	1	−4.4096	1.0000	1	−2.1764	1.0000	1.72×10^{-5}	MDNS_Macro	1	−3.3865	1.0000	1	−1.702	1.0000	2.28×10^{-5}
MDNS_MMacro	3	−1.3663	1.0000	9	3.11958	0.0150	3.48×10^{-5}	MDNS_MMacro	5	−1.2743	1.0000	9	2.55274	0.0996	3.71×10^{-5}
MDNS_PMacro	12	1.56849	0.4262	12	4.32888	0.0000	7.31×10^{-5}	MDNS_PMacro	8	0.94151	0.8274	11	3.50472	0.0008	7.02×10^{-5}
MDNS_MMacroEnd	5	−0.9565	1.0000	8	2.80714	0.0614	3.93×10^{-5}	MDNS_MMacroEnd	3	−1.9111	1.0000	5	2.08635	0.3470	3.71×10^{-5}
MDNS_PMacroEnd	10	1.0929	0.7454	11	3.89891	0.0000	6.16×10^{-5}	MDNS_PMacroEnd	12	1.45297	0.4698	12	3.86713	0.0000	7.13×10^{-5}
MDNS_S	8	0.93574	0.8360	5	2.39913	0.2120	5.91×10^{-5}	MDNS_S	9	0.99463	0.7936	6	2.09673	0.3400	6.65×10^{-5}
MDNS_SmediaMacro	4	−1.293	1.0000	10	3.31265	0.0040	3.68×10^{-5}	MDNS_SmediaMacro	4	−1.2833	1.0000	10	2.57908	0.0920	3.89×10^{-5}
MDNS_Smacro	2	−3.2132	1.0000	3	2.32205	0.3452	3.10×10^{-5}	MDNS_Smacro	2	−3.1706	1.0000	2	1.70198	1.0000	3.50×10^{-5}
12-Month Maturity								**15-Month Maturity**							
Models	**Rank_M**	**v_M**	**MCS_M**	**Rank_R**	**v_R**	**MCS_R**	**Loss**	**Models**	**Rank_M**	**v_M**	**MCS_M**	**Rank_R**	**v_R**	**MCS_R**	**Loss**
MDNS	7	0.60735	0.9672	5	1.79937	0.5926	6.44×10^{-5}	MDNS	7	0.3533	0.9986	4	1.71101	0.6910	6.09×10^{-5}
MDNS_M	6	−0.0835	1.0000	4	1.71316	0.6684	5.61×10^{-5}	MDNS_M	6	−0.147	1.0000	3	1.66422	0.7356	5.57×10^{-5}
MDNS_P	11	1.62677	0.3530	10	2.67552	0.0642	7.46×10^{-5}	MDNS_P	11	1.73303	0.2936	10	2.87417	0.0330	7.36×10^{-5}
MDNS_Lambda	10	1.59534	0.3754	9	2.59478	0.0918	9.25×10^{-5}	MDNS_Lambda	10	1.70167	0.3126	9	2.84506	0.0364	9.41×10^{-5}
MDNS_Macro	2	−3.3358	1.0000	1	−1.5324	1.0000	2.69×10^{-5}	MDNS_Macro	1	−3.5065	1.0000	1	−1.323	1.0000	2.97×10^{-5}
MDNS_MMacro	5	−1.1823	1.0000	7	2.14857	0.3042	4.03×10^{-5}	MDNS_MMacro	5	−1.0697	1.0000	7	1.8872	0.5306	4.26×10^{-5}
MDNS_PMacro	8	0.66473	0.9548	11	3.1387	0.0072	6.74×10^{-5}	MDNS_PMacro	8	0.53337	0.9812	11	2.93064	0.0266	6.47×10^{-5}
MDNS_MMacroEnd	3	−2.5856	1.0000	2	1.53242	0.8064	3.74×10^{-5}	MDNS_MMacroEnd	3	−2.5899	1.0000	2	1.32297	0.9358	3.86×10^{-5}
MDNS_PMacroEnd	12	1.6969	0.3074	12	4.14133	0.0000	7.52×10^{-5}	MDNS_PMacroEnd	12	1.82442	0.2428	12	4.42232	0.0000	7.51×10^{-5}
MDNS_S	9	1.0432	0.7766	8	2.50883	0.1228	6.85×10^{-5}	MDNS_S	9	1.05408	0.7756	8	2.81312	0.0410	6.76×10^{-5}
MDNS_SmediaMacro	4	−1.2333	1.0000	6	1.94831	0.4592	4.11×10^{-5}	MDNS_SmediaMacro	4	−1.1595	1.0000	5	1.71129	0.6908	4.27×10^{-5}
MDNS_Smacro	1	−3.3736	1.0000	3	1.66968	0.8064	3.79×10^{-5}	MDNS_Smacro	2	−3.3446	1.0000	6	1.86292	0.5514	3.93×10^{-5}

Table A1. *Cont.*

18-Month Maturity

Models	Rank_M	v_M	MCS_M	Rank_R	v_R	MCS_R	Loss
MDNS	7	0.25297	0.9996	5	1.62167	0.7512	6.07×10^{-5}
MDNS_M	6	−0.0836	1.0000	4	1.59384	0.7724	5.73×10^{-5}
MDNS_P	12	1.7558	0.2808	9	2.88661	0.0304	7.31×10^{-5}
MDNS_Lambda	10	1.73582	0.2914	10	2.90469	0.0270	9.47×10^{-5}
MDNS_Macro	1	−3.5551	1.0000	1	−0.9834	1.0000	3.34×10^{-5}
MDNS_MMacro	5	−0.9195	1.0000	6	1.63389	0.7424	4.56×10^{-5}
MDNS_PMacro	8	0.42173	0.9950	8	2.86061	0.0354	6.33×10^{-5}
MDNS_MMacroEnd	3	−2.3371	1.0000	2	0.98336	0.9926	4.06×10^{-5}
MDNS_PMacroEnd	11	1.7499	0.2836	12	4.74032	0.0000	7.38×10^{-5}
MDNS_S	9	1.03862	0.7924	11	2.99909	0.0176	6.80×10^{-5}
MDNS_SmediaMacro	4	−1.0413	1.0000	3	1.46438	0.9926	4.52×10^{-5}
MDNS_Smacro	2	−3.2555	1.0000	7	2.01687	0.4166	4.11×10^{-5}

21-Month Maturity

Models	Rank_M	v_M	MCS_M	Rank_R	v_R	MCS_R	Loss
MDNS	7	0.17989	1.0000	5	1.83003	0.6134	5.89×10^{-5}
MDNS_M	6	0.08728	1.0000	6	1.84887	0.5994	5.82×10^{-5}
MDNS_P	12	1.97108	0.1750	11	3.32327	0.0022	7.04×10^{-5}
MDNS_Lambda	11	1.96672	0.1780	10	3.15423	0.0074	9.12×10^{-5}
MDNS_Macro	1	−4.2124	1.0000	1	−0.6151	1.0000	3.62×10^{-5}
MDNS_MMacro	5	−0.9824	1.0000	4	1.50824	0.8556	4.66×10^{-5}
MDNS_PMacro	8	0.44735	0.9974	9	2.94136	0.0268	6.17×10^{-5}
MDNS_MMacroEnd	3	−2.0601	1.0000	2	0.61506	1.0000	4.12×10^{-5}
MDNS_PMacroEnd	10	1.68599	0.3300	12	4.60752	0.0000	7.09×10^{-5}
MDNS_S	9	1.14954	0.7348	8	2.78756	0.0506	6.64×10^{-5}
MDNS_SmediaMacro	4	−1.0022	1.0000	3	1.20227	1.0000	4.63×10^{-5}
MDNS_Smacro	2	−3.4985	1.0000	7	1.97946	0.4888	4.17×10^{-5}

24-Month Maturity

Models	Rank_M	v_M	MCS_M	Rank_R	v_R	MCS_R	Loss
MDNS	6	0.05245	1.0000	6	1.63007	0.7640	5.60×10^{-5}
MDNS_M	7	0.13817	1.0000	7	1.68831	0.7248	5.67×10^{-5}
MDNS_P	11	1.83028	0.2506	9	3.12533	0.0084	6.68×10^{-5}
MDNS_Lambda	12	1.85888	0.2334	10	3.13628	0.0076	8.49×10^{-5}
MDNS_Macro	1	−4.1723	1.0000	1	−0.3021	1.0000	3.77×10^{-5}
MDNS_MMacro	4	−0.8228	1.0000	5	1.37746	0.9176	4.71×10^{-5}
MDNS_PMacro	8	0.45006	0.9966	8	2.81401	0.0440	5.94×10^{-5}
MDNS_MMacroEnd	3	−1.8553	1.0000	2	0.30211	1.0000	4.03×10^{-5}
MDNS_PMacroEnd	10	1.5208	0.4630	12	4.69169	0.0000	6.66×10^{-5}
MDNS_S	9	1.07849	0.7780	11	3.45706	0.0006	6.35×10^{-5}
MDNS_SmediaMacro	5	−0.8116	1.0000	3	1.11604	1.0000	4.68×10^{-5}
MDNS_Smacro	2	−3.0676	1.0000	4	1.3466	1.0000	4.14×10^{-5}

27-Month Maturity

Models	Rank_M	v_M	MCS_M	Rank_R	v_R	MCS_R	Loss
MDNS	6	0.00812	1.0000	6	1.17441	0.9622	5.59×10^{-5}
MDNS_M	7	0.21082	1.0000	7	1.31187	0.9124	5.77×10^{-5}
MDNS_P	11	1.45826	0.4962	8	2.39328	0.1694	6.56×10^{-5}
MDNS_Lambda	12	1.51843	0.4452	10	2.84148	0.0364	8.14×10^{-5}
MDNS_Macro	1	−3.2215	1.0000	1	−0.0618	1.0000	4.06×10^{-5}
MDNS_MMacro	5	−0.6084	1.0000	5	1.09093	0.9744	4.87×10^{-5}
MDNS_PMacro	8	0.39163	0.9982	9	2.46038	0.1430	5.93×10^{-5}
MDNS_MMacroEnd	3	−1.8088	1.0000	2	0.06182	1.0000	4.11×10^{-5}
MDNS_PMacroEnd	10	1.1345	0.7390	11	4.29245	0.0000	6.52×10^{-5}
MDNS_S	9	0.84426	0.9078	12	4.34076	0.0000	6.32×10^{-5}
MDNS_SmediaMacro	4	−0.6447	1.0000	4	1.06233	1.0000	4.86×10^{-5}
MDNS_Smacro	2	−2.3347	1.0000	3	0.74945	1.0000	4.28×10^{-5}

30-Month Maturity

Models	Rank_M	v_M	MCS_M	Rank_R	v_R	MCS_R	Loss
MDNS	6	0.01173	1.0000	6	1.10691	0.9792	5.56×10^{-5}
MDNS_M	7	0.22424	1.0000	7	1.28147	0.9294	5.76×10^{-5}
MDNS_P	12	1.30279	0.5836	8	2.02273	0.4012	6.42×10^{-5}
MDNS_Lambda	11	1.30243	0.5838	9	2.64112	0.0712	7.74×10^{-5}
MDNS_Macro	1	−2.5841	1.0000	2	0.2015	1.0000	4.26×10^{-5}
MDNS_MMacro	5	−0.4752	1.0000	4	0.90326	0.9954	4.97×10^{-5}
MDNS_PMacro	8	0.43438	0.9956	10	2.74412	0.0478	5.90×10^{-5}
MDNS_MMacroEnd	3	−1.9669	1.0000	1	−0.2015	1.0000	4.09×10^{-5}
MDNS_PMacroEnd	10	0.86386	0.8946	11	3.88652	0.0000	6.33×10^{-5}
MDNS_S	9	0.74669	0.9388	12	4.19321	0.0000	6.26×10^{-5}
MDNS_SmediaMacro	4	−0.5154	1.0000	5	0.9108	0.9954	4.94×10^{-5}
MDNS_Smacro	2	−1.9783	1.0000	3	0.42254	1.0000	4.38×10^{-5}

33-Month Maturity

Models	Rank_M	v_M	MCS_M	Rank_R	v_R	MCS_R	Loss
MDNS	6	0.02686	1.0000	6	1.10354	0.9796	5.40×10^{-5}
MDNS_M	7	0.32809	0.9996	7	1.75498	0.6100	5.68×10^{-5}
MDNS_P	11	1.09361	0.7746	8	1.87576	0.5068	6.12×10^{-5}
MDNS_Lambda	12	1.10617	0.7656	9	2.30731	0.2080	7.17×10^{-5}
MDNS_Macro	1	−2.1467	1.0000	2	0.42118	1.0000	4.32×10^{-5}
MDNS_MMacro	5	−0.4206	1.0000	4	0.78113	0.9996	4.87×10^{-5}
MDNS_PMacro	8	0.44769	0.9966	10	2.63552	0.0706	5.72×10^{-5}
MDNS_MMacroEnd	2	−1.9222	1.0000	1	−0.4212	1.0000	3.97×10^{-5}
MDNS_PMacroEnd	10	0.70296	0.9604	11	3.18613	0.0072	6.04×10^{-5}
MDNS_S	9	0.68317	0.9664	12	3.47507	0.0014	6.02×10^{-5}
MDNS_SmediaMacro	4	−0.4486	1.0000	5	0.80147	0.9996	4.87×10^{-5}
MDNS_Smacro	3	−1.7353	1.0000	3	0.51159	1.0000	4.32×10^{-5}

Table A1. *Cont.*

36-Month Maturity

Models	Rank_M	v_M	MCS_M	Rank_R	v_R	MCS_R	Loss
MDNS	6	0.06106	1.0000	6	1.16743	0.9716	5.30×10^{-5}
MDNS_M	7	0.36817	0.9996	8	1.94561	0.4446	5.58×10^{-5}
MDNS_P	12	0.97811	0.8496	7	1.84458	0.5270	5.89×10^{-5}
MDNS_Lambda	11	0.94502	0.8664	9	1.99763	0.4048	6.69×10^{-5}
MDNS_Macro	2	−1.7495	1.0000	2	0.60904	1.0000	4.38×10^{-5}
MDNS_MMacro	4	−0.3765	1.0000	4	0.74287	1.0000	4.83×10^{-5}
MDNS_PMacro	8	0.48968	0.9960	10	2.53876	0.1022	5.60×10^{-5}
MDNS_MMacroEnd	1	−1.9191	1.0000	1	−0.609	1.0000	3.86×10^{-5}
MDNS_PMacroEnd	9	0.61591	0.9848	11	2.69764	0.0596	5.84×10^{-5}
MDNS_S	10	0.65372	0.9796	12	2.8896	0.0270	5.85×10^{-5}
MDNS_SmediaMacro	5	−0.3756	1.0000	5	0.78051	1.0000	4.85×10^{-5}
MDNS_Smacro	3	−1.5695	1.0000	3	0.65965	1.0000	4.34×10^{-5}

39-Month Maturity

Models	Rank_M	v_M	MCS_M	Rank_R	v_R	MCS_R	Loss
MDNS	6	0.1179	1.0000	6	1.16897	0.9642	5.32×10^{-5}
MDNS_M	7	0.446	0.9984	9	1.99638	0.4070	5.60×10^{-5}
MDNS_P	12	0.90242	0.9160	7	1.67764	0.6754	5.81×10^{-5}
MDNS_Lambda	11	0.82214	0.9482	8	1.67933	0.6742	6.38×10^{-5}
MDNS_Macro	3	−1.4869	1.0000	2	0.71644	0.9996	4.51×10^{-5}
MDNS_MMacro	4	−0.3631	1.0000	4	0.7691	0.9992	4.84×10^{-5}
MDNS_PMacro	8	0.47894	0.9978	10	2.13358	0.3108	5.57×10^{-5}
MDNS_MMacroEnd	1	−1.6994	1.0000	1	−0.7164	1.0000	3.82×10^{-5}
MDNS_PMacroEnd	9	0.57932	0.9926	11	2.40062	0.1700	5.75×10^{-5}
MDNS_S	10	0.679	0.9820	12	2.60963	0.0828	5.81×10^{-5}
MDNS_SmediaMacro	5	−0.3294	1.0000	5	0.84124	0.9992	4.90×10^{-5}
MDNS_Smacro	2	−1.5082	1.0000	3	0.73274	0.9996	4.40×10^{-5}

48-Month Maturity

Models	Rank_M	v_M	MCS_M	Rank_R	v_R	MCS_R	Loss
MDNS	6	0.19904	1.0000	7	1.45847	0.8442	5.58×10^{-5}
MDNS_M	8	0.43532	0.9990	12	1.90556	0.4794	5.79×10^{-5}
MDNS_P	12	0.63521	0.9908	11	1.83728	0.5344	5.84×10^{-5}
MDNS_Lambda	7	0.25492	1.0000	4	1.11217	0.9778	5.80×10^{-5}
MDNS_Macro	3	−0.5204	1.0000	6	1.29032	0.9328	5.11×10^{-5}
MDNS_MMacro	4	−0.3123	1.0000	2	0.92993	0.9942	5.09×10^{-5}
MDNS_PMacro	10	0.5347	0.9970	9	1.79423	0.5668	5.80×10^{-5}
MDNS_MMacroEnd	1	−2.0954	1.0000	1	−0.9299	1.0000	3.87×10^{-5}
MDNS_PMacroEnd	11	0.58805	0.9952	10	1.83347	0.5368	6.05×10^{-5}
MDNS_S	9	0.49505	0.9974	8	1.56524	0.7656	5.87×10^{-5}
MDNS_SmediaMacro	5	−0.1067	1.0000	3	1.10358	0.9942	5.31×10^{-5}
MDNS_Smacro	2	−0.9395	1.0000	5	1.21148	0.9778	4.89×10^{-5}

60-Month Maturity

Models	Rank_M	v_M	MCS_M	Rank_R	v_R	MCS_R	Loss
MDNS	7	0.43578	0.9986	7	1.69983	0.6930	6.11×10^{-5}
MDNS_M	10	0.66062	0.9820	10	1.86755	0.5462	6.27×10^{-5}
MDNS_P	9	0.5347	0.9960	9	1.78431	0.6194	6.14×10^{-5}
MDNS_Lambda	4	−0.2943	1.0000	2	0.91112	0.9968	5.45×10^{-5}
MDNS_Macro	6	0.14766	1.0000	8	1.76542	0.6352	5.89×10^{-5}
MDNS_MMacro	3	−0.4399	1.0000	3	1.18697	0.9616	5.44×10^{-5}
MDNS_PMacro	11	0.82884	0.9412	12	2.05469	0.3930	6.35×10^{-5}
MDNS_MMacroEnd	1	−2.0441	1.0000	1	−0.9111	1.0000	4.01×10^{-5}
MDNS_PMacroEnd	12	0.83684	0.9378	11	2.00488	0.4296	6.57×10^{-5}
MDNS_S	8	0.51968	0.9962	6	1.68561	0.9616	6.21×10^{-5}
MDNS_SmediaMacro	5	0.00664	1.0000	4	1.34147	0.9616	5.82×10^{-5}
MDNS_Smacro	2	−0.6008	1.0000	5	1.57637	0.9616	5.52×10^{-5}

72-Month Maturity

Models	Rank_M	v_M	MCS_M	Rank_R	v_R	MCS_R	Loss
MDNS	9	0.64364	0.9848	8	1.90351	0.5390	6.50×10^{-5}
MDNS_M	11	0.76808	0.9640	9	2.00402	0.4546	6.60×10^{-5}
MDNS_P	6	0.46396	0.9986	7	1.83757	0.5952	6.34×10^{-5}
MDNS_Lambda	3	−0.5603	1.0000	2	0.73459	1.0000	5.31×10^{-5}
MDNS_Macro	7	0.52483	0.9968	11	2.11455	0.3654	6.37×10^{-5}
MDNS_MMacro	2	−0.5781	1.0000	3	1.27165	0.9430	5.57×10^{-5}
MDNS_PMacro	12	0.87837	0.9290	12	2.19753	0.3022	6.66×10^{-5}
MDNS_MMacroEnd	1	−2.2013	1.0000	1	−0.7346	1.0000	4.10×10^{-5}
MDNS_PMacroEnd	10	0.72953	0.9722	10	2.03571	0.4284	6.76×10^{-5}
MDNS_S	8	0.55179	0.9944	5	1.79459	0.9430	6.48×10^{-5}
MDNS_SmediaMacro	5	−0.0099	1.0000	4	1.44317	0.9430	6.05×10^{-5}
MDNS_Smacro	4	−0.237	1.0000	6	1.79775	0.9430	5.94×10^{-5}

* Models not reported in the tables were eliminated by the test.

Table A2. Model Confidence Set—12-Month Forecast Horizon *.

1-Month Maturity

Models	Rank_M	v_M	MCS_M	Rank_R	v_R	MCS_R	Loss
MDNS	1	−2.09542	1.0000	2	0.11496	1.0000	1.81×10^{-4}
MDNS_M	4	−1.82709	1.0000	7	1.243348	0.9482	2.17×10^{-4}
MDNS_P	8	−0.80079	1.0000	9	1.535254	0.8122	3.42×10^{-4}
MDNS_Lambda	2	−2.02119	1.0000	3	0.184344	1.0000	1.86×10^{-4}
MDNS_Macro	6	−1.52978	1.0000	4	0.617887	0.9998	2.30×10^{-4}
MDNS_MMacro	11	1.411762	0.3884	8	1.480174	0.8442	2.50×10^{-3}
MDNS_Pmacro	3	−1.90036	1.0000	1	−0.11496	1.0000	1.74×10^{-4}
MDNS_MMacroEnd	10	0.427403	0.9572	11	3.895032	0.0058	5.80×10^{-4}
MDNS_PmacroEnd	5	−1.67626	1.0000	6	1.157246	0.9666	3.01×10^{-4}
MDNS_S	9	−0.06094	1.0000	10	1.722818	0.6844	4.87×10^{-4}
MDNS_Smacro	7	−1.52029	1.0000	5	1.050876	0.9842	2.90×10^{-4}

3-Month Maturity

Models	Rank_M	v_M	MCS_M	Rank_R	v_R	MCS_R	Loss
MDNS	2	−1.93693	1.0000	3	0.68867	0.9988	2.12×10^{-4}
MDNS_M	7	−1.73384	1.0000	7	1.022512	0.9864	2.38×10^{-4}
MDNS_P	8	−0.66423	1.0000	8	1.443698	0.8474	3.58×10^{-4}
MDNS_Lambda	6	−1.78199	1.0000	6	0.82119	0.9970	2.31×10^{-4}
MDNS_Macro	5	−1.80775	1.0000	2	0.209961	1.0000	1.93×10^{-4}
MDNS_MMacro	11	1.386103	0.3908	9	1.47245	0.8312	2.25×10^{-3}
MDNS_Pmacro	3	−1.93654	1.0000	1	−0.20996	1.0000	1.79×10^{-4}
MDNS_MMacroEnd	10	0.66892	0.8260	11	3.678439	0.0112	5.97×10^{-4}
MDNS_PmacroEnd	1	−2.43056	1.0000	5	0.803842	0.9970	2.57×10^{-4}
MDNS_S	9	−0.02852	1.0000	10	1.698359	0.6888	4.71×10^{-4}
MDNS_Smacro	4	−1.85606	1.0000	4	0.693668	0.9988	2.56×10^{-4}

6-Month Maturity

Models	Rank_M	v_M	MCS_M	Rank_R	v_R	MCS_R	Loss
MDNS	2	−1.97901	1.0000	3	0.333927	1.0000	2.38×10^{-4}
MDNS_M	7	−1.66485	1.0000	7	0.901397	0.9944	2.71×10^{-4}
MDNS_P	8	−0.55236	1.0000	8	1.220485	0.9418	3.82×10^{-4}
MDNS_Lambda	4	−1.85359	1.0000	5	0.590744	1.0000	2.52×10^{-4}
MDNS_Macro	5	−1.76926	1.0000	1	−0.25195	1.0000	2.10×10^{-4}
MDNS_MMacro	11	1.38447	0.4022	9	1.470531	0.8328	2.08×10^{-3}
MDNS_Pmacro	6	−1.76189	1.0000	2	0.251949	1.0000	2.28×10^{-4}
MDNS_MMacroEnd	10	0.69738	0.8336	11	3.28348	0.0282	6.02×10^{-4}
MDNS_PmacroEnd	1	−2.14882	1.0000	6	0.682358	0.9998	2.82×10^{-4}
MDNS_S	9	−0.0405	1.0000	10	1.734513	0.6566	4.74×10^{-4}
MDNS_Smacro	3	−1.93387	1.0000	4	0.397505	1.0000	2.63×10^{-4}

9-Month Maturity

Models	Rank_M	v_M	MCS_M	Rank_R	v_R	MCS_R	Loss
MDNS	2	−2.12675	1.0000	3	0.039339	1.0000	2.63×10^{-4}
MDNS_M	5	−1.79207	1.0000	5	0.849475	0.9950	2.95×10^{-4}
MDNS_P	8	−0.5821	1.0000	8	1.215647	0.9438	4.01×10^{-4}
MDNS_Lambda	4	−1.90782	1.0000	7	1.044641	0.9794	2.87×10^{-4}
MDNS_Macro	7	−1.59841	1.0000	2	0.013681	1.0000	2.61×10^{-4}
MDNS_MMacro	11	1.4176	0.4038	9	1.546194	0.7880	1.95×10^{-3}
MDNS_Pmacro	6	−1.61999	1.0000	4	0.4089	1.0000	2.85×10^{-4}
MDNS_MMacroEnd	10	0.913616	0.7380	11	3.402395	0.0220	6.46×10^{-4}
MDNS_PmacroEnd	3	−1.94348	1.0000	6	0.922062	0.9912	3.31×10^{-4}
MDNS_S	9	−0.15163	1.0000	10	1.727605	0.6658	4.74×10^{-4}
MDNS_Smacro	1	−2.23798	1.0000	1	−0.01368	1.0000	2.60×10^{-4}

12-Month Maturity

Models	Rank_M	v_M	MCS_M	Rank_R	v_R	MCS_R	Loss
MDNS	1	−3.08864	1.0000	2	0.338116	1.0000	2.83×10^{-4}
MDNS_M	5	−2.58379	1.0000	4	0.639505	0.9998	3.10×10^{-4}
MDNS_P	8	−0.96263	1.0000	6	1.076002	0.9808	4.10×10^{-4}
MDNS_Lambda	3	−2.81683	1.0000	8	1.487684	0.8422	3.15×10^{-4}
MDNS_Macro	7	−1.86069	1.0000	3	0.372889	1.0000	3.12×10^{-4}
MDNS_MMacro	11	1.326622	0.5286	9	1.597777	0.7668	1.83×10^{-3}
MDNS_PMacro	6	−1.99638	1.0000	5	0.81128	0.9978	3.33×10^{-4}
MDNS_MMacroEnd	10	0.725167	0.9156	12	3.440038	0.0188	6.88×10^{-4}
MDNS_PMacroEnd	4	−2.65229	1.0000	7	1.395291	0.8894	3.73×10^{-4}
MDNS_S	9	−0.84015	1.0000	10	1.750132	0.6626	4.68×10^{-4}
MDNS_SmediaMacro	12	1.912836	0.2204	11	3.088935	0.0470	1.23×10^{-3}
MDNS_Smacro	2	−2.95961	1.0000	1	−0.33812	1.0000	2.49×10^{-4}

15-Month Maturity

Models	Rank_M	v_M	MCS_M	Rank_R	v_R	MCS_R	Loss
MDNS	1	−3.1685	1.0000	2	0.658091	0.9996	2.97×10^{-4}
MDNS_M	4	−2.70915	1.0000	4	0.773765	0.9988	3.20×10^{-4}
MDNS_P	8	−1.0517	1.0000	5	1.128233	0.9708	4.12×10^{-4}
MDNS_Lambda	3	−2.81291	1.0000	7	1.631774	0.7388	3.33×10^{-4}
MDNS_Macro	7	−1.57444	1.0000	3	0.716229	0.9992	3.55×10^{-4}
MDNS_MMacro	11	1.342537	0.5422	8	1.665066	0.7148	1.71×10^{-3}
MDNS_PMacro	6	−1.73912	1.0000	6	1.164638	0.9620	3.73×10^{-4}
MDNS_MMacroEnd	10	0.966792	0.8054	12	3.489898	0.0160	7.20×10^{-4}
MDNS_PMacroEnd	5	−2.34721	1.0000	10	1.880229	0.5564	4.05×10^{-4}
MDNS_S	9	−0.97114	1.0000	9	1.746972	0.6592	4.61×10^{-4}
MDNS_SmediaMacro	12	1.860318	0.2522	11	3.080579	0.0470	1.15×10^{-3}
MDNS_Smacro	2	−3.16811	1.0000	1	−0.65809	1.0000	2.34×10^{-4}

Table A2. *Cont.*

18-Month Maturity								21-Month Maturity							
Models	**Rank_M**	**v_M**	**MCS_M**	**Rank_R**	**v_R**	**MCS_R**	**Loss**	**Models**	**Rank_M**	**v_M**	**MCS_M**	**Rank_R**	**v_R**	**MCS_R**	**Loss**
MDNS	2	−3.23581	1.0000	2	0.947313	0.9916	3.14×10^{-4}	MDNS	2	−3.34278	1.0000	3	1.171206	0.9606	3.25×10^{-4}
MDNS_M	3	−2.89361	1.0000	3	0.980411	0.9894	3.31×10^{-4}	MDNS_M	3	−3.10634	1.0000	2	1.152211	1.0000	3.37×10^{-4}
MDNS_P	8	−1.17425	1.0000	5	1.122966	0.9728	4.12×10^{-4}	MDNS_P	6	−1.35347	1.0000	5	1.224761	0.9470	4.06×10^{-4}
MDNS_Lambda	4	−2.82989	1.0000	7	1.711951	0.6912	3.50×10^{-4}	MDNS_Lambda	4	−2.90892	1.0000	7	1.702375	0.6890	3.61×10^{-4}
MDNS_Macro	7	−1.36041	1.0000	4	1.004319	0.9888	3.93×10^{-4}	MDNS_Macro	9	−1.14949	1.0000	4	1.222861	0.9480	4.21×10^{-4}
MDNS_MMacro	11	1.393627	0.5232	8	1.789068	0.6354	1.61×10^{-3}	MDNS_MMacro	11	1.418247	0.5226	8	1.881044	0.5530	1.52×10^{-3}
MDNS_PMacro	6	−1.51778	1.0000	6	1.430509	0.8640	4.08×10^{-4}	MDNS_PMacro	7	−1.27457	1.0000	6	1.650868	0.7288	4.36×10^{-4}
MDNS_MMacroEnd	10	1.183034	0.6630	12	3.564789	0.0138	7.49×10^{-4}	MDNS_MMacroEnd	10	1.337288	0.5694	12	3.640633	0.0106	7.70×10^{-4}
MDNS_PMacroEnd	5	−2.02209	1.0000	10	2.315552	0.2582	4.34×10^{-4}	MDNS_PMacroEnd	5	−1.70298	1.0000	10	2.628212	0.1304	4.56×10^{-4}
MDNS_S	9	−1.06734	1.0000	9	1.964149	0.4980	4.58×10^{-4}	MDNS_S	8	−1.19837	1.0000	9	2.098129	0.3968	4.53×10^{-4}
MDNS_SmediaMacro	12	1.844255	0.2664	11	3.050744	0.0518	1.08×10^{-3}	MDNS_SmediaMacro	12	1.809928	0.2898	11	3.041782	0.0516	1.02×10^{-3}
MDNS_Smacro	1	−3.39564	1.0000	1	−0.94731	1.0000	2.23×10^{-4}	MDNS_Smacro	1	−3.57524	1.0000	1	−1.15221	1.0000	2.15×10^{-4}
24-Month Maturity								**27-Month Maturity**							
Models	**Rank_M**	**v_M**	**MCS_M**	**Rank_R**	**v_R**	**MCS_R**	**Loss**	**Models**	**Rank_M**	**v_M**	**MCS_M**	**Rank_R**	**v_R**	**MCS_R**	**Loss**
MDNS	2	−3.28806	1.0000	4	1.398374	0.8934	3.32×10^{-4}	MDNS	3	−3.38037	1.0000	4	1.687866	0.7198	3.40×10^{-4}
MDNS_M	3	−3.24605	1.0000	2	1.339432	1.0000	3.40×10^{-4}	MDNS_M	2	−3.45509	1.0000	3	1.59215	0.9922	3.44×10^{-4}
MDNS_P	5	−1.55738	1.0000	3	1.340975	0.9934	3.98×10^{-4}	MDNS_P	5	−1.88269	1.0000	2	1.546496	1.0000	3.92×10^{-4}
MDNS_Lambda	4	−2.8251	1.0000	6	1.827056	0.6040	3.66×10^{-4}	MDNS_Lambda	4	−2.84542	1.0000	7	2.151938	0.3874	3.72×10^{-4}
MDNS_Macro	9	−0.97585	1.0000	5	1.423941	0.8786	4.43×10^{-4}	MDNS_Macro	8	−0.8675	1.0000	5	1.725015	0.6938	4.63×10^{-4}
MDNS_MMacro	10	1.454896	0.5016	8	1.968348	0.4940	1.44×10^{-3}	MDNS_MMacro	10	1.469277	0.4912	6	2.032917	0.4712	1.37×10^{-3}
MDNS_PMacro	8	−1.04345	1.0000	7	1.870244	0.5718	4.59×10^{-4}	MDNS_PMacro	9	−0.85017	1.0000	8	2.209762	0.3508	4.79×10^{-4}
MDNS_MMacroEnd	11	1.503618	0.4660	12	3.684016	0.0064	7.86×10^{-4}	MDNS_MMacroEnd	11	1.657825	0.3750	12	3.495081	0.0156	8.01×10^{-4}
MDNS_PMacroEnd	6	−1.37272	1.0000	10	2.881674	0.0714	4.72×10^{-4}	MDNS_PMacroEnd	7	−1.09434	1.0000	11	3.283053	0.0270	4.88×10^{-4}
MDNS_S	7	−1.31288	1.0000	9	2.254201	0.3060	4.46×10^{-4}	MDNS_S	6	−1.37356	1.0000	9	2.439849	0.2176	4.42×10^{-4}
MDNS_SmediaMacro	12	1.785749	0.3062	11	3.023714	0.0480	9.69×10^{-4}	MDNS_SmediaMacro	12	1.85752	0.2710	10	3.027684	0.0600	9.28×10^{-4}
MDNS_Smacro	1	−3.67164	1.0000	1	−1.33943	1.0000	2.07×10^{-4}	MDNS_Smacro	1	−3.9776	1.0000	1	−1.5465	1.0000	2.01×10^{-4}
30-Month Maturity								**33-Month Maturity**							
Models	**Rank_M**	**v_M**	**MCS_M**	**Rank_R**	**v_R**	**MCS_R**	**Loss**	**Models**	**Rank_M**	**v_M**	**MCS_M**	**Rank_R**	**v_R**	**MCS_R**	**Loss**
MDNS	3	−3.35786	1.0000	4	1.838266	0.6024	3.44×10^{-4}	MDNS	3	−3.35606	1.0000	4	1.897881	0.5498	3.44×10^{-4}
MDNS_M	2	−3.56696	1.0000	3	1.713317	0.9954	3.46×10^{-4}	MDNS_M	2	−3.67591	1.0000	3	1.766344	0.9990	3.42×10^{-4}
MDNS_P	5	−2.12168	1.0000	2	1.627428	1.0000	3.85×10^{-4}	MDNS_P	5	−2.36386	1.0000	2	1.657812	1.0000	3.73×10^{-4}
MDNS_Lambda	4	−2.82515	1.0000	7	2.279956	0.2952	3.74×10^{-4}	MDNS_Lambda	4	−2.82126	1.0000	7	2.313783	0.2748	3.71×10^{-4}
MDNS_Macro	8	−0.70478	1.0000	5	1.878625	0.5742	4.78×10^{-4}	MDNS_Macro	8	−0.5536	1.0000	5	1.967992	0.5002	4.84×10^{-4}
MDNS_MMacro	10	1.507024	0.4744	6	2.13074	0.3900	1.30×10^{-3}	MDNS_MMacro	10	1.527718	0.4712	6	2.211019	0.3318	1.24×10^{-3}
MDNS_PMacro	9	−0.64284	1.0000	8	2.389063	0.2406	4.94×10^{-4}	MDNS_PMacro	9	−0.43656	1.0000	8	2.472108	0.2044	5.02×10^{-4}
MDNS_MMacroEnd	11	1.722125	0.3478	11	3.443819	0.0220	8.10×10^{-4}	MDNS_MMacroEnd	12	1.823285	0.2922	11	3.443765	0.0216	8.13×10^{-4}
MDNS_PMacroEnd	7	−0.81844	1.0000	12	3.469255	0.0214	4.99×10^{-4}	MDNS_PMacroEnd	7	−0.58139	1.0000	12	3.569407	0.0154	5.04×10^{-4}
MDNS_S	6	−1.4719	1.0000	9	2.552095	0.1704	4.36×10^{-4}	MDNS_S	6	−1.58269	1.0000	9	2.560904	0.1686	4.26×10^{-4}
MDNS_SmediaMacro	12	1.839814	0.2914	10	3.029884	0.0614	8.93×10^{-4}	MDNS_SmediaMacro	11	1.808835	0.3004	10	3.031517	0.0564	8.57×10^{-4}
MDNS_Smacro	1	−4.0373	1.0000	1	−1.62743	1.0000	1.96×10^{-4}	MDNS_Smacro	1	−4.07182	1.0000	1	−1.65781	1.0000	1.91×10^{-4}

Table A2. *Cont.*

36-Month Maturity

Models	Rank_M	v_M	MCS_M	Rank_R	v_R	MCS_R	Loss
MDNS	3	−3.34664	1.0000	4	1.990049	0.4878	3.42×10^{-4}
MDNS_M	2	−3.79023	1.0000	3	1.85518	0.9996	3.38×10^{-4}
MDNS_P	5	−2.63692	1.0000	2	1.726672	1.0000	3.63×10^{-4}
MDNS_Lambda	4	−2.81149	1.0000	7	2.386618	0.2436	3.66×10^{-4}
MDNS_Macro	7	−0.43048	1.0000	5	2.085006	0.4216	4.89×10^{-4}
MDNS_MMacro	10	1.530643	0.4554	6	2.244994	0.3228	1.18×10^{-3}
MDNS_PMacro	9	−0.26061	1.0000	9	2.643301	0.1400	5.08×10^{-4}
MDNS_MMacroEnd	12	1.905254	0.2506	11	3.477806	0.0162	8.14×10^{-4}
MDNS_PMacroEnd	8	−0.37439	1.0000	12	3.68643	0.0096	5.07×10^{-4}
MDNS_S	6	−1.68224	1.0000	8	2.57867	0.1618	4.16×10^{-4}
MDNS_SmediaMacro	11	1.783764	0.3086	10	3.05504	0.0524	8.27×10^{-4}
MDNS_Smacro	1	−4.11535	1.0000	1	−1.72667	1.0000	1.87×10^{-4}

39-Month Maturity

Models	Rank_M	v_M	MCS_M	Rank_R	v_R	MCS_R	Loss
MDNS	3	−3.40668	1.0000	4	2.050392	0.4402	3.42×10^{-4}
MDNS_M	2	−4.01817	1.0000	3	1.896727	1.0000	3.36×10^{-4}
MDNS_P	4	−2.96066	1.0000	2	1.744927	1.0000	3.55×10^{-4}
MDNS_Lambda	5	−2.88572	1.0000	7	2.428084	0.2218	3.64×10^{-4}
MDNS_Macro	7	−0.30515	1.0000	5	2.183782	0.3530	4.95×10^{-4}
MDNS_MMacro	10	1.602297	0.4300	6	2.406229	0.2318	1.13×10^{-3}
MDNS_PMacro	9	−0.09176	1.0000	9	2.738989	0.1124	5.15×10^{-4}
MDNS_MMacroEnd	12	1.906654	0.2658	11	3.33225	0.0244	8.17×10^{-4}
MDNS_PMacroEnd	8	−0.1899	1.0000	12	3.860319	0.0066	5.11×10^{-4}
MDNS_S	6	−1.79004	1.0000	8	2.651115	0.1352	4.08×10^{-4}
MDNS_SmediaMacro	11	1.81685	0.3102	10	3.137115	0.0420	8.03×10^{-4}
MDNS_Smacro	1	−4.1636	1.0000	1	−1.74493	1.0000	1.84×10^{-4}

48-Month Maturity

Models	Rank_M	v_M	MCS_M	Rank_R	v_R	MCS_R	Loss
MDNS	4	−3.4166	1.0000	4	2.168434	0.3564	3.36×10^{-4}
MDNS_M	1	−4.23941	1.0000	3	2.015817	1.0000	3.28×10^{-4}
MDNS_P	3	−3.76851	1.0000	2	1.869901	1.0000	3.38×10^{-4}
MDNS_Lambda	5	−2.90811	1.0000	6	2.461774	0.2090	3.51×10^{-4}
MDNS_Macro	7	0.073422	1.0000	5	2.389841	0.2414	5.11×10^{-4}
MDNS_MMacro	10	1.673142	0.3864	7	2.638079	0.1434	1.02×10^{-3}
MDNS_PMacro	9	0.348348	0.9990	9	3.068854	0.0528	5.29×10^{-4}
MDNS_MMacroEnd	12	2.005957	0.2178	11	3.271636	0.0308	8.17×10^{-4}
MDNS_PMacroEnd	8	0.22516	1.0000	12	4.04298	0.0038	5.18×10^{-4}
MDNS_S	6	−2.12451	1.0000	8	2.659018	0.1352	3.86×10^{-4}
MDNS_SmediaMacro	11	1.734047	0.3526	10	3.105811	0.0480	7.52×10^{-4}
MDNS_Smacro	2	−4.15811	1.0000	1	−1.8699	1.0000	1.73×10^{-4}

60-Month Maturity

Models	Rank_M	v_M	MCS_M	Rank_R	v_R	MCS_R	Loss
MDNS_Smacro	1	−2.3295	1.0000	1	−2.32946	1.0000	1.63×10^{-4}

72-Month Maturity

Models	Rank_M	v_M	MCS_M	Rank_R	v_R	MCS_R	Loss
MDNS_Smacro	1	−2.35514	1.0000	1	−2.35514	1.0000	1.61×10^{-4}

* Models not reported in the tables were eliminated by the test.

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