mathematics

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Exploration of New Solitons for the Fractional Perturbed Radhakrishnan–Kundu–Lakshmanan Model

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Abstract: The key objective of the current manuscript was to investigate the exact solutions of the fractional perturbed Radhakrishnan–Kundu–Lakshmanan model. For this purpose, we applied two reliable and efficient approaches; specifically, the modified simple equation (MSE) and exponential rational function (ERF) techniques. The methods considered in this paper offer solutions for problems in nonlinear theory and mathematical physics practice. We also present solutions obtained graphically with the Maple package program.

Keywords: exact solutions; fractional partial differential equation; symbolic computation; mathematical models; nonlinear equations

MSC: 35R11; 68W30; 83C15

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Citation: Kaplan, M.; Alqahtani, R.T. Exploration of New Solitons for the Fractional Perturbed Radhakrishnan–Kundu–Lakshmanan Model. *Mathematics* **2023**, *11*, 2562. https://doi.org/10.3390/math11112562

Academic Editor: Alberto Ferrero

Received: 2 May 2023
Revised: 17 May 2023
Accepted: 22 May 2023
Published: 3 June 2023

1. Introduction

In the field of science, researchers must deal with a broad class of fractional partial differential equations (FPDEs) [1–6]. Many systematic methods for finding exact solutions to FPDEs have been presented and used in recent decades, as a result of the development of symbolic packages. We can list some popular techniques, as follows: Kumar et al. applied the ansatz technique to the generalized Schrödinger–Boussinesq equations [7], Akinyemi et al. used the (G′/G)-expansion technique for the perturbed nonlinear Biswas–Milovic equation with Kudryashov's law of refractive index [8], and Hosseini et al. utilized a simplified Hirota technique for the Korteweg–de Vries–Caudrey–Dodd–Gibbon equation [9]. Akbulut et al. obtained some conservation laws of the Burgers–Fisher equation [10], Raza et al. used fractional order local M-derivative for the (2+1)-dimensional Kundu–Mukherjee–Naskar model [11], and Sadaf et al. employed the generalized projective Riccati equation technique for the modified nonlinear Schrödinger equation with a conformable fractional derivative [12]. Alharthi et al. utilized the (G′/G, 1/G)-expansion algorithm for the time-fractional BBM–Burger and Sharma–Tasso–Olver equations [13]. Osman et al. applied the Fan sub-equation [14].

The motivation for the current paper was to explore new exact solutions to the fractional perturbed Radhakrishnan–Kundu–Lakshmanan model with Kerr law nonlinearity, which can be given as:

$$iD_t^{\alpha}\varphi + m_1\varphi_{xx} + m_2|\varphi|^2\varphi - i\beta\varphi_x - i\gamma(|\varphi|^2\varphi)_x - i\delta(|\varphi|^2)_x\varphi - i\epsilon\varphi_{xxx} = 0, 0 < \alpha \le 1, \quad (1)$$

where $i = \sqrt{-1}$. Here, which term corresponds to which expression is given in the table below [15]:

$D_t^\alpha \varphi$	$\rightarrow$	Conformable fractional temporal evolution of the nonlinear wave.
φ	$\rightarrow$	complex-valued wave function x and t
x	$\rightarrow$	space
t	$\rightarrow$	time
m_1	$\rightarrow$	group-velocity dispersion
m_2	$\rightarrow$	coefficient of nonlinearity
β	$\rightarrow$	intermodal dispersion
γ	$\rightarrow$	self-steepening coefficient for short pulses
δ	$\rightarrow$	higher order dispersion coefficient
ϵ	$\rightarrow$	third order dispersion coefficient

Many different types of material, including semiconductors, exhibit power law nonlinearity. When $\alpha = 1$, the perturbed Radhakrishnan–Kundu–Lakshmanan equation in its original form is obtained, which is expressed as presented in [16–18].

2. The Conformable Derivative

Fractional derivatives play a crucial role in the literature, and several definitions of fractional derivatives have been discovered, including the Grunwald–Letnikov, Riemann–Liouville, Caputo, modified Riemann–Liouville, and Atangana–Baleanu derivatives [19,20]. In this study, we will use the conformable derivative, which was developed by Khalil et al. [21]. This derivative has an important feature that allows us to apply the chain rule, enabling us to reduce nonlinear differential equations to ordinary differential equations with the help of wave transforms.

Below are some basic terms that define the conformable derivative:

When $\psi : (0, \infty) \rightarrow \mathbb{R}$, the conformable derivative of ψ of order $\delta, 0 < \delta < 1$, is defined [22,23]:

$$T_\delta(\psi)(t) = \lim_{\varepsilon \rightarrow 0} \frac{\psi(t + \epsilon t^{1-\delta}) - \psi(t)}{\epsilon},$$

for all $t > 0$. Basic properties of the conformable derivative are given as follows [24–27]:

(1) $T_\delta(a\psi + b\varphi) = aT_\delta(\psi) + bT_\delta(\varphi)$, for all $a, b \in \mathbb{R}$,
(2) $T_\delta(t^\alpha) = \alpha t^{\alpha-\delta}$, for all $\alpha \in \mathbb{R}$,
(3) $T_\delta(\psi\varphi) = \psi T_\delta(\varphi) + \varphi T_\delta(\psi)$,
(4) $T_\delta(\psi/\varphi) = \dfrac{\varphi T_\delta(\psi) - \psi T_\delta(\varphi)}{\varphi^2}$,
(5) If ψ is differentiable, then $T_\delta(\psi)(t) = t^{1-\delta}\frac{d\psi}{dt}$,
(6) $\psi(t) = \lambda$, $T_\delta(\lambda) = 0$,for all constant functions
(7) Chain rule: Let $\psi, \varphi : (0, \infty) \rightarrow \mathbb{R}$ be a differentiable and $\delta-$differerentiable function then the chain rule is given by:

$$T_\delta(\psi \circ \varphi)(t) = t^{1-\alpha}\varphi'(t)\psi'(\varphi(t)).$$

The aim of this paper is to obtain the exact solutions of the conformable fractional perturbed Radhakrishnan–Kundu–Lakshmanan model with the Kerr law nonlinearity. For this, we give preliminary information about reducing the NPDEs to nonlinear ordinary differential equations (ODEs), then we reduce the given model to the ODE in Section 2. In Section 3, we give a brief description of the used methods, which are called the MSE and ERF techniques. In the subsequent section, we apply these methods to the given model and give graphical representations of the results. Finally, we give the conclusions.

3. Initial Information

In the current part of the manuscript, we first present some initial information that is useful to reduce an FPDE to an ordinary differential equation (ODE). The following NPDE can be taken into consideration for this:

$$F\left(\varphi, D_t^\alpha \varphi, D_x^\alpha \varphi, D_x^{2\alpha} \varphi, D_t^\alpha D_x^\alpha \varphi, D_t^{2\alpha} \varphi, \dots\right) = 0, \tag{2}$$

where F inherits φ and its partial derivatives, and φ is a complex-valued function.

The wave transformation is given by**:**

$$\varphi(x,t) = \phi(\varepsilon)e^{i\Phi},\ \varepsilon = k_1\left(x - \omega\frac{t^\alpha}{\alpha}\right),\ \Phi = -k_2 x + c\frac{t^\alpha}{\alpha} + \varrho, \tag{3}$$

where k_1, k_2, ω, and c are constants to be ascertained. We can create a system of equations by applying transformation Equation (3) to Equation (2), which sets the real and imaginary portions equal to zero. Subsequently, we resolve the resultant system to ascertain the conditions of the parameters and employ these outcomes. This yields the following ordinary differential equation (ODE), which we can integrate ε times.

$$Q(\varphi, \varphi', \varphi'', \varphi''', \dots) = 0, \tag{4}$$

Here, the partial derivative is expressed with respect to ε [28]. Namely,

$$\varphi^{'} = \frac{d\varphi}{d\varepsilon},\ \varphi^{''} = \frac{d^2\varphi}{d\varepsilon^2}, \dots$$

Applying Wave Transformation to the Given Model

From the implementation of the transformation Equation (3) to the Equation (1) and sorting the real and imaginary parts, the obtained ODE system is as follows:

$$k_1^2(m_1 + 3k_2\epsilon)\phi'' + (m_2 - k_2\gamma)\phi^3 - (c + m_1 k_2^2 + \beta k_2 + \epsilon k_2^3)\phi = 0, \tag{5}$$

and

$$k_1^2\epsilon\phi''' - (\omega + 2m_1k_2 + \beta + 3k_2^2\epsilon)\phi' - (3\gamma + 2\delta)\phi^2\phi' = 0. \tag{6}$$

Integrating (6), we obtain an ODE as follows:

$$3k_1^2\epsilon\phi'' - 3(\omega + 2m_1k_2 + \beta + 3k_2^2\epsilon)\phi - (3\gamma + 2\delta)\phi^3 = 0. \tag{7}$$

ϕ should satisfy Equations (5) and (7). Thus, we obtain a condition as follows:

$$\frac{m_1 + 3k_2\epsilon}{3\epsilon} = \frac{c + m_1k_2^2 + \beta k_2 + \epsilon k_2^3}{3(\omega + 2m_1k_2 + \beta + 3k_2^2\epsilon)} = -\frac{m_2 - k_2\gamma}{3\gamma + 2\delta}. \tag{8}$$

If we solve condition (8), we obtain the following values:

$$k_2 = -\frac{(3m_2\epsilon + 2m_1\delta + 3m_1\gamma)}{6\epsilon(\gamma + \delta)}, \tag{9}$$

and

$$\omega = \frac{\epsilon(c + m_1k_2^2 + \beta k_2 + \epsilon k_2^3)}{m_1 + 3k_2\epsilon} - (2m_1k_2 + \beta + 3\epsilon k_2^2). \tag{10}$$

If we balance ϕ'' and ϕ^3, we get $m = 1$.

4. Description of the Adopted Methods

We provide a description of the methods that have been employed.

4.1. Mse Technique

The MSE technique can be summarized as follows [29,30]:

First of all, the polynomial $\left(\frac{\varsigma'(\varepsilon)}{\varsigma(\varepsilon)}\right)$ can be used to express the exact solution of Equation (4)

$$\varphi(\varepsilon) = \sum_{j=0}^{m} \sigma_j \left[\frac{\varsigma'(\varepsilon)}{\varsigma(\varepsilon)}\right]^j, \ \sigma_j = \text{const.}, \sigma_m \neq 0. \tag{11}$$

Here, the homogeneous balance principle between the highest order derivative term and the highest order nonlinear term that appears in Equation (4) can be used to calculate the positive integer m, also known as the balancing number. In addition, j are the arbitrary real constants to be determined.

By substituting Equation (11) into Equation (4), we acquire a polynomial of $\varsigma^{-j}(\varepsilon)$ with the derivatives of $\varsigma(\varepsilon)$. An algebraic equation system is obtained that must be solved to determine σ_j $(j = 0, 1, 2, \ldots, m)$, $k_1, k_2, \omega, c, \varrho$ and $\varsigma(\varepsilon)$ by equating all the coefficients of $\varsigma^{-j}(\varepsilon)$ to zero $(j \geq 0)$. Ultimately, the values of $k_1, k_2, \omega, c, \varrho$ and $\varsigma(\varepsilon)$ are substituted into Equation (11) and the exact solutions of Equation (2) are obtained.

4.2. Erf Technique

Allowing for the solution of Equation (4) of the form [31,32]:

$$\varphi(\varepsilon) = \sum_{j=0}^{m} \frac{\sigma_j}{(1+e^{\varepsilon})^j}, \sigma_m \neq 0. \tag{12}$$

where σ_j $(j = 0, 1, 2, \ldots, m)$ are constants to be calculated and the positive integer m is the balancing number. If the substitution is made, of Equation (12) into Equation (4), a set of algebraic equations is obtained that involves σ_j $(j = 0, 1, 2, \ldots, m), k_1, k_2, \omega, c$, and ϱ. Following that, we can derive new exact solutions to equation Equation (2) from the solutions of this system.

5. Application of the Given Methods

5.1. Mse Technique

As per the incorporated technique, the exact solution of Equation (5) is given by:

$$\varphi(\varepsilon) = \sigma_0 + \sigma_1 \left[\frac{\varsigma'(\varepsilon)}{\varsigma(\varepsilon)}\right], \ \sigma_1 \neq 0. \tag{13}$$

Substituting Equation (13) into Equation (5), we obtain:

$$\varsigma^0(\varepsilon): \quad -k_2\gamma\sigma_0^3 - c\sigma_0 - k_2^2 m_1\sigma_0 - k_2\beta\sigma_0 + m_2\sigma_0^3 - \epsilon k_2^3\sigma_0 = 0, \tag{14}$$

$$\begin{aligned} \varsigma^1(\varepsilon): \quad & 3k_1^2k_2\epsilon\sigma_1\varsigma''' - k_2\beta\sigma_1\varsigma' + 3m_2\sigma_0^2\sigma_1\varsigma' - 3k_2\gamma\sigma_0^2\sigma_1\varsigma' \\ & + k_1^2 m_1\sigma_1\varsigma''' - c\sigma_1\varsigma' - k_2^2 m_1\sigma_1\varsigma' - \epsilon k_2^3\sigma_1\varsigma' = 0, \end{aligned} \tag{15}$$

$$\begin{aligned} \varsigma^2(\varepsilon): \quad & -9k_1^2k_2\epsilon\sigma_1\varsigma''\varsigma' + 3m_2\sigma_0\sigma_1^2(\varsigma')^2 \\ & -3k_1^2 m_1\sigma_1\varsigma''\varsigma' - 3k_2\gamma\sigma_0\sigma_1^2(\varsigma')^2 = 0, \end{aligned} \tag{16}$$

$$\varsigma^3(\varepsilon): \quad -k_2\gamma\sigma_1^3 + 6k_1^2k_2\epsilon\sigma_1 + 2k_1^2 m_1\sigma_1 + m_2\sigma_1^3 = 0. \tag{17}$$

If we solve Equations (14) and (17), we obtain

$$\sigma_0 = 0 \text{ and } \sigma_0 = \pm\frac{\sqrt{-(k_2\gamma - m_2)\left(c + k_2^2 m_1 + k_2\beta + \epsilon k_2^3\right)}}{k_2\gamma - m_2}, \tag{18}$$

and

$$\sigma_1 = 0 \text{ and } \sigma_1 = \pm\frac{k_1\sqrt{2}\sqrt{(k_2\gamma - m_2)(m_1 + 3k_2\epsilon)}}{k_2\gamma - m_2}. \tag{19}$$

By using these values and substituting them into the remaining system, namely Equations (15) and (16), we obtain a new equation system. If we solve this system, we find the following solution:

$$\varsigma(\varepsilon) = C_1 + C_2 \exp\left(-\frac{\varepsilon\sqrt{2}\sqrt{-k_2\gamma c - k_2^3\gamma m_1 - k_2^2\gamma\beta - k_2^4\gamma\epsilon + m_2 c + m_2 k_2^2 m_1 + m_2 k_2\beta + \beta\epsilon k_2^3}}{\sqrt{k_2\gamma m_1 + 3k_2^2\gamma\epsilon - m_2 m_1 - 3m_2 k_2\epsilon k_1}}\right)$$

Therefore, we find the exact solution of the fractional perturbed Radhakrishnan–Kundu–Lakshmanan model as follows:

$$\varphi(x,t) = e^{i\left(-k_2 x + c\frac{t^\alpha}{\alpha} + \varrho\right)}\left(\frac{\psi\left(C_2\exp\left(-\frac{\psi\sqrt{2}k_1\left(x - \omega\frac{t^\alpha}{\alpha}\right)}{\sqrt{(r\gamma-\beta)(3ra+\alpha)m}}\right) - C_1\right)}{(r\gamma - \beta)\left(C_1 + C_2\exp\left(-\frac{\psi\sqrt{2}k_1\left(x - \omega\frac{t^\alpha}{\alpha}\right)}{\sqrt{(r\gamma-\beta)(3ra+\alpha)m}}\right)\right)}\right) \tag{20}$$

where C_1 and C_2 are two arbitrary constants then $\psi = \sqrt{-(r\gamma - \beta)(ar^3 + r^2\alpha + rd + \omega)}$.

Note: If we substitute the values $\sigma_0 = 0$ and $\sigma_1 = \pm\frac{k_1\sqrt{2}\sqrt{(k_2\gamma - m_2)(m_1 + 3k_2\epsilon)}}{k_2\gamma - m_2}$, we find a trivial solution, which will be ignored. In addition, please note that $\sigma_1 \neq 0$. Therefore, the case for $\sigma_1 = 0$ will be ignored.

When we substitute the values $m_1 = 0.5$, $m_2 = 0.6$, $\beta = 0.2$, $\gamma = 0.4$, $\delta = 0.2$, $\epsilon = -0.7$, $c = 2$, $\varrho = 1.5$, $\alpha = 0.6$, $C_1 = 1$, $C_2 = 0.5$ into Equation (20), we can plot Figure 1. This solution is classified as a periodic type solution.

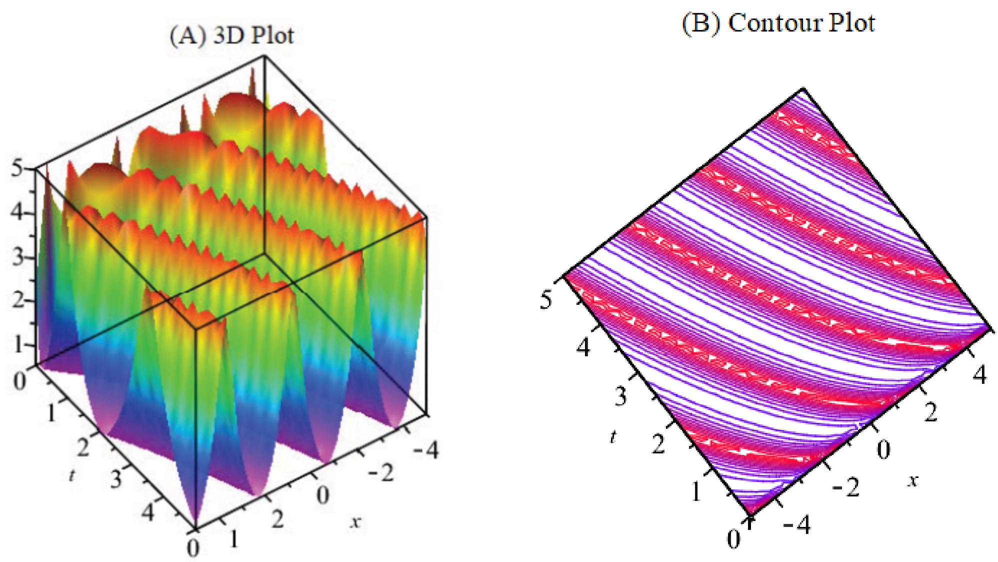

Figure 1. *Cont.*

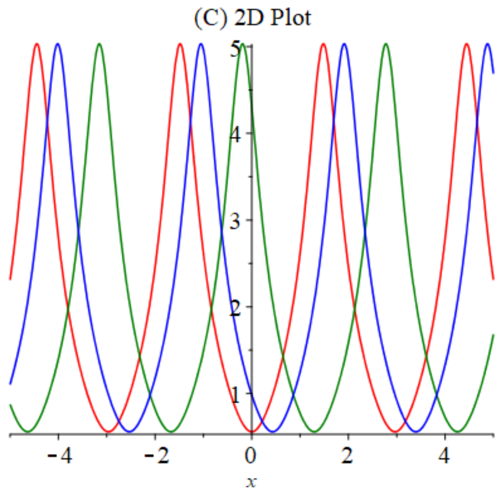

Figure 1. The 3D, contour, and 2D surfaces of a periodic type solution.

5.2. Erf Technique

Using this technique, we can consider the exact solution of Equation (5) as follows:

$$\phi(\varepsilon) = \sigma_0 + \frac{\sigma_1}{(1 + e^{\varepsilon})}. \tag{21}$$

If we substitute the solution Equation (21) into Equation (5) and collect all coefficients of $\phi(\varepsilon)$, and then equate them to zero, we obtain an equation system, which is called the determining equation system. The values of the σ_0, σ_1, and k_1 can be calculated as follows:

$$\begin{gathered} \sigma_0 = \pm\sqrt{-\frac{-\epsilon k_2^3 - m_1 k_2^2 - \beta k_2 - c}{-k_2\gamma + m_2}},\ \sigma_1 = \mp\frac{2\left(\epsilon k_2^3 + m_1 k_2^2 + \beta k_2 + c\right)}{(m_2 - k_2\gamma)\sqrt{-\frac{-\epsilon k_2^3 - m_1 k_2^2 - \beta k_2 - c}{-k_2\gamma + m_2}}}, \\ k_1 = \pm\sqrt{-\frac{2\left(\epsilon k_2^3 + m_1 k_2^2 + \beta k_2 + c\right)}{3k_2\epsilon + m_1}}. \end{gathered} \tag{22}$$

Finally, we obtain the exact solutions as follows:

$$\varphi(x,t) = \left(\frac{\left(\epsilon k_2^3 + m_1 k_2^2 + \beta k_2 + c\right)\left(e^{k_1\left(x - \omega\frac{t^\alpha}{\alpha}\right)} - 1\right)}{\sqrt{\frac{\epsilon k_2^3 + m_1 k_2^2 + \beta k_2 + c}{m_2 - k_2\gamma}}(m_2 - k_2\gamma)\left(1 + e^{k_1\left(x - \omega\frac{t^\alpha}{\alpha}\right)}\right)} \right) e^{i\left(-k_2 x + c\frac{t^\alpha}{\alpha} + \varrho\right)} \tag{23}$$

When we substitute the values $m_1 = 0.5$, $m_2 = 0.6$, $\beta = 0.2$, $\gamma = 0.4$, $\delta = 0.2$, $\epsilon = -0.7$, $c = 2$, $\varrho = 1.5$, $\alpha = 0.6$ into Equation (23), we can plot Figure 2. This solution is classified as a periodic singular soliton type solution.

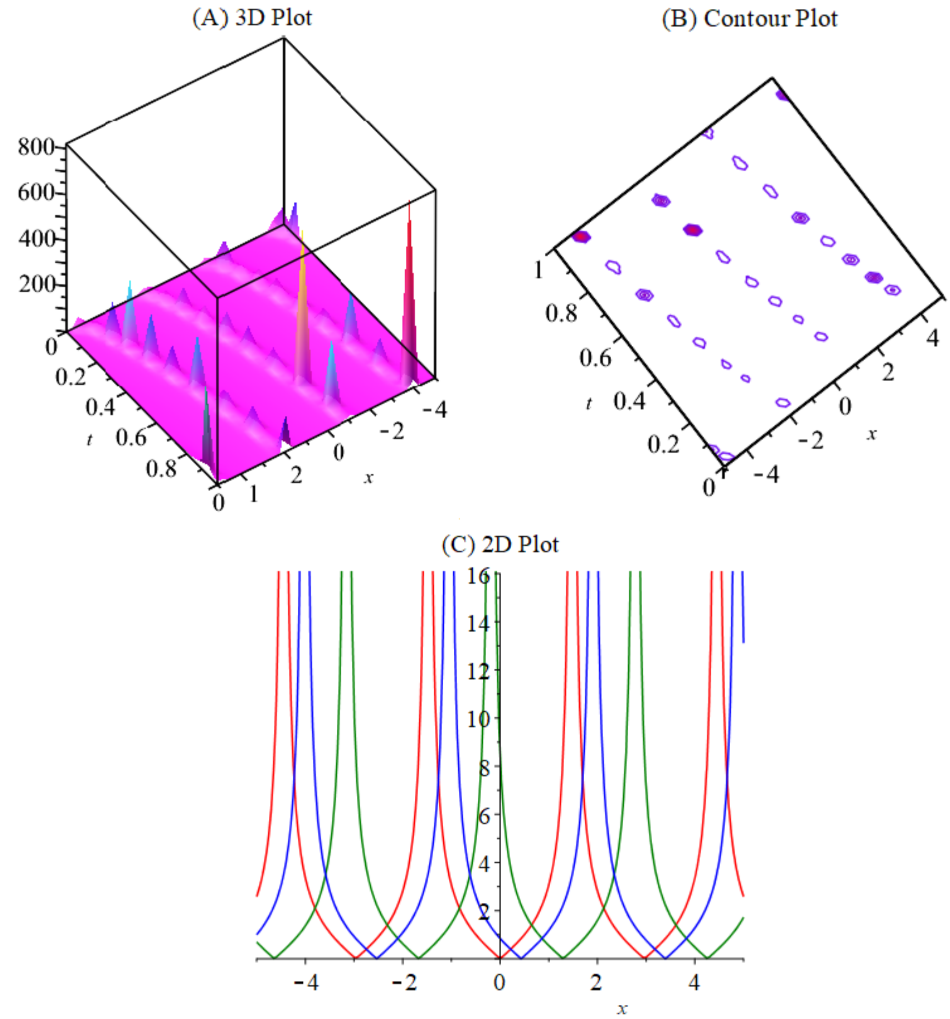

Figure 2. The 3D, contour, and 2D surfaces of a periodic singular soliton type solution.

6. Discussion

In this paper, two different types of solution were originated using the MSE and ERF techniques. The utilized solutions are different from the outcomes obtained using earlier techniques [33–35]. Equations (20) and (23) present a variety of different types of solution, by providing various parameter values. Arbitrary parameters are included in the solutions, and different solutions can be constructed by letting the parameters take different values. The obtained solutions are classified. Further, the depictions of two-dimensional and three-dimensional graphics are formed. The following details can be provided for these plots. Figures 1 and 2 depict solitary waves in different structures. Figure 1 was plotted for the values $m_1 = 0.5$, $m_2 = 0.6$, $\beta = 0.2$, $\gamma = 0.4$, $\delta = 0.2$, $\epsilon = -0.7$, $c = 2$, $\varrho = 1.5$, $\alpha = 0.6$, $C_1 = 1$, $C_2 = 0.5$ in Equation (20). This solution is classified as a periodic type solution. Figure 2 was plotted for the values $m_1 = 0.5$, $m_2 = 0.6$, $\beta = 0.2$, $\gamma = 0.4$, $\delta = 0.2$, $\epsilon = -0.7$, $c = 2$, $\varrho = 1.5$, $\alpha = 0.6$ in Equation (23). This solution is classified as a periodic singular soliton type solution. In Figures 1 and 2, the 3D and contour plots for the obtained solutions are shown. The suggested methodologies are feasible and efficacious. The Maple software program was utilized to conduct the simulations and analyze the outcomes. It is important to mention that the accuracy of the solutions was verified by substituting them into the equation.

7. Conclusions

In the current paper, we obtained several new results for the fractional perturbed Radhakrishnan–Kundu–Lakshmanan model. We used the MSE and ERF techniques, which are efficient and practical approaches to solving nonlinear FPDEs. Since we used a new auxiliary equation for the MSE technique, this approach is efficient for constructing novel solutions to the considered equation. Compared to other strategies, the ERF method is simpler to implement and the MSE method can be considered more powerful. Moreover, 2D and 3D graphs of the obtained results for particular cases of the parameters were given. These exact solutions are expected to be beneficial for understanding the physical meaning of the considered equation. We believe that the outcomes are useful for understanding the processes that the equation attempts to explain.

Author Contributions: M.K.: Methodology, Resources; R.T.A.: Validation, Funding acquisition; M.K. and R.T.A.: Data curation. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: The datasets utilized or examined during the present study can be obtained from the corresponding author upon reasonable request.

Acknowledgments: The authors extend their appreciation to the Deanship of Scientific Research at Imam Mohammad Ibn Saud Islamic University (IMSIU) for funding and supporting this work .

Conflicts of Interest: The authors declare no conflict of interest.

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