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Cost-Effective Imperfect Production-Inventory System under Variable Production Rate and Remanufacturing

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Abstract: Several industries are facing many challenges in their production systems due to increasing customer demand. Customer demand is growing for products with innovative features that are flexible, good quality, and appealing. This paper presents a flexible production-inventory system that produces multiple parts of a product. Defective products may be produced during the production process. Those defective products are remanufactured immediately after inspection. Limited budget and space constraints are considered, along with product assembly. Based on different distribution functions, non-linear equations are calculated using the Kuhn–Tucker optimization technique. Numerical examples, a graphical representation, and sensitivity analysis are presented in this paper. The solution procedure evaluates the minimization of the total investment based on the χ^2 distribution. This study examines electronic products those are more likely to be defective rather than perfect during production.

Keywords: flexible manufacturing; remanufacturing; product assembling; random defective rate; backorders; space and budget constraints

MSC: 90B05; 90C30

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1. Introduction

Initially, commercial entities are obliged to decrease costs and enhance their standards to ensure viability. In order to maintain their competitive position, they need to engage in such actions. Industries endeavor to decrease expenses while enhancing quality by directing their attention toward manufacturing activity (Taleizadeh et al. [1]). Hybrid remanufacturing is investigated for defective items to reduce only carbon emission costs. Taleizadeh et al. [1] enhanced quality improvement for a single-stage production. Several papers pertaining to production systems and supply chains are analyzed based on environmental issues. Collaboration between small and medium-sized industries was developed as a strategy for better feedback in the production and supply chain, and this approach was extended by the paper of Machado et al. [2]. They ignored possible extensions of assembly products. Mishra et al. [3] allowed backorders within the production-inventory model and ignored remanufacturing of defective products. They focused on controlling defective products. However, industries are always concerned for fulfilling customers' requirements by maintaining the delivery of products to customers without disputes. Flexible production systems aim to improve the production process and offer flexibility in machine production.

Flexibility refers to the extent to which a system can be changed to create the assembled product. This research aims to assist industries in progressing their businesses using the proposed strategy. Defective products are created during production, which is a major difficulty for industries. Remanufacturing defective products can be a solution to this. Hence, the proposed model reduces the waste represented by defective items by reworking the assembled product through flexible production.

The literature has the following research gaps:

- In the existing literature, several authors have focused on the issues of traditional production policies, where single products/multiple products are manufactured. Still, no strategies have been discussed to improve the economy of production systems. Almost every industry focuses on enhancing the manufacturing systems for innovative products, but the production economics must also be kept in mind.
- Previous models have focused on the effect of the total inventory cost when the defective rate is random for single-product manufacturing under single-stage production. Increasing customer expectations and producing assembled products through assembly processes are essential in competitive business markets. A research gap exists in considering that a single-product, manufactured under a traditional business plan, can increase the number of defective products returned by customers, which may decrease the sales and profit of the production system. Thus, the production economy may be hampered.
- Every industry may be concerned about making innovative, assembled products through flexible production systems to improve their business and economic conditions. The proposed model encompasses product assembly, finished item inspection, remanufacturing, backorders, and reworking at multiple stages to prevent any efficiently loss and improves the economic status of the production system.

1.1. Research Contribution

The research gaps filled herein are as follows:

- To fill the above-discussed research gap, the proposed model considers a flexible production system. Increasing production flexibility is a vital strategy for efficiently improving market responsibility under future demand uncertainty without any shortages. Production flexibility can allow the simultaneous building of different types of products in the same production facility. This model is designed for the assembly of products from multiple parts to fulfill customers' distinct requirements. Customers' expectations are always related to a variation of the product (this paper considers a specific electronic product) according to their own preferences. Only a product assembly technique based on a flexible production system can fulfill this requirement and increase the system profit.
- To avoid a defective final product, an inspection strategy is applied in the proposed model. This strategy ensures great monetary benefits for the production system. The model reduces backorders from end-users after the sale of a product. The inspection strategy for defective assembled products is conducted after production but before selling to the end-user, and defective products are sent to be remanufactured. The remanufacturing policy can recover defective products before sale and reduce the number of backorders from end-users. This research gap related to flexible production system is not studied earlier in literature. The production-inventory model analyzes numerical studies based on five separate probability distributions (uniform, triangular, double-triangular, beta, and χ^2 distribution), and finds the minimum cost with χ^2 distribution. This strategy can improve economical benefit of the system.
- Meeting end-users' demands and increasing sales are highly important for the economic growth of a production system. By applying flexible production, an industry's ability of quickly respond to end-users will increase. Assembly production for a single type of within a flexible production system is the contribution of this study.

- Unwanted increases in cost for maintaining production systems are a problem in almost every product (electronic products) as it indicates that works-in-process inventory are under control. This is because the production of any product can be predicted, although the product-supplying process depends on market demand. This study considers an inspection policy for products assembly through a just-in-time production policy. In this policy, all defective products are sent for remanufacturing, but not before selling the finished products to the end-users, which is more fruitful than the traditional approach of inspection. This can prevent unnecessary costs and aid in an increase in sales and overall profit. This method provides benefits to industries by reducing the costs of holding space, because defective items are sent to be remanufactured and then make them ready for sell. Thus, less waste is produced in the form of defective products under flexible production and cost-effective production. The remanufacturing of defective products under flexible production can be controlled to increase the quality of the final assembled products. Further, increasing the production flexibility will prevent shortages, arising under smart production. Another major financial benefit for industries is the reduction in returned products, achieved by remanufacturing products; otherwise, industries can be forced to invest substantial maintenance costs for huge numbers of products returned by end-users.

1.2. Organization of the Paper

The paper is organized according to the following sections: Section 2 reviews the literature. The problem definition, notation, and basic assumptions of this study are presented in Section 3. Section 4 describes the mathematical model for the five distribution functions. The methodology for obtaining the solution is discussed in Section 5. A few numerical examples are discussed in Section 6, and the sensitivity analysis of the numerical examples is presented in Section 7. The managerial insights are provided in Section 8. Finally, the concluding remarks of this study are summarized in Section 9.

2. Literature Review

2.1. Production-Inventory System under Flexible Production

Cárdenas-Barrón [4] and Sarkar et al. [5] developed an imperfect production system with fixed setup costs and manual inspections to identify defective product, which led to larger total system costs and could deliver defective items to the market due to inspection errors. The closed-loop supply chain are accounted for the product return rate for remanufacturing, which was focused on the competition and coordination of manufacturers. Similarly, Aydin et al. [6] considered new and remanufactured products. Hariga et al. [7] minimized the total supply chain cost by optimizing newly manufactured and remanufactured batches. Consecutive remanufacturing batches were produced by following M manufacturing batches. Heydari et al. [7] proposed the remanufacturing capacity as a variable, and the uncertainty of remanufacturing was associated with a few unprocessed inspected items. Qingdi et al. [8] analyzed a time-dependent decision making system in manufacturing. Furthermore, predecisional remanufacturing through the timing-matching method was explained in that model.

Taleizadeh et al. [1] investigated the remanufacturing technique using two channels: A and B. The remanufacturing processes were introduced to reduce carbon emissions. Sivashankari and Panayappan [9] proposed a production-inventory model for a single item that considered reworking. They considered shortages for their model. However, the concept of an assembled item, where each part of the item is manufactured and remanufactured in the same generation cycle with a flexible production rate and different distributed defect rates, was neglected by Sivashankari and Panayappan [9], which increased the system cost tremendously. Similarly, Sanjai and Periyasamy [10] proposed a production-inventory system with imperfect generation under different delivery policies. They neglected the concepts of variable production rates and assembled products. Machado et al. [2] proposed a production system considering environmental issues based on the collaboration of small and medium-sized companies. In their paper, Gao et al. [11] developed a green supply chain policy to uphold environmental regulations. The impact of the eco-label system was extended in this paper, as the production of some products was not always effective. Sarkar et al. [12] proposed the concepts of a variable lead time and variance under a smart supply chain system. This paper quantified the benefits but was not interested in improving the quality of products for the industry. Lagoudakis [13] dealt with cross-price elasticity-dependent demand. Mukherjee et al. [14] discussed carbon controlling policy for a fashion industry without consideration of backorder and lost sale. In a similar direction, Yang et al. [15] developed a smart manufacturing system under digital technologies for small, medium, and micro enterprises.

Bazan et al. [16] proposed an inventory model considering manufacturing and remanufacturing. They developed their study in the context of carbon policies. However, they considered a single product type and a constant production rate for their study. Bhatia and Elsayed [17] designed a smart manufacturing network using a robust, flexible programming approach. They used a fuzzy TOPSIS method for multi-criteria decision making problem. The effect of government subsidies on a remanufacturing model was studied by Chai et al. [18]. They considered the environmental effect for their study. However, the impact of flexible production was not considered in their study. Similarly, a manufacturing–remanufacturing production inventory model considering quality constraints was developed by Assid et al. [19]. They developed integrated control policies for the returns and replenishment of the inventory. Sarkar and Park [20] proposed a flexible production system with the consideration of lead time reduction. They considered a multi-stage complex production system but avoided budget and space constraints for an assembled product. Zheng et al. [21] introduced smart technologies to solve unreliability issues in a supply chain using a robust distribution approach. To optimize the total cost, they used two metaheuristic optimization techniques, genetic algorithms (GAs) and particle swarm optimization (PSO). However, they did not consider the uncertainty of the environment. Saxena and Sarkar [22] focused on randomly misplaced products and constructed an production model with two-stage production for remanufacturing products and overcame this issue. Taleizadeh et al. [23] developed a production inventory model by considering reworking. This paper proposed subsidy and advertisement policies. The authors showed that the demand depends on the product's selling price and green level. The model succeeded in maximizing the social impact and minimizing the environmental impact. Kanishka and Acherjee [24] reviewed 448 manuscripts based on additive manufacturing and remanufacturing. Kanishka and Acherjee's [24] study presented a flexible imperfect production-inventory system for assembled products. But, imperfect items remanufacturing within the same production cycle subject to budget and space constraints have not been studied in the literature previously. Thus, a pioneering attempt is made to fill this research gap in this study.

On the other hand, Dey et al. [25] and Amrouche et al. [26] developed dual-channel retailing systems for imperfect products. Some investments were incorporated into the model to decrease the lead time and improve the service quality. Different services were provided for the clients through the product's life cycle to maintain the company's brand image. However, the remanufacturing of the defective products was not considered there. Zhu et al. [27] developed a supply chain model under different carbon reduction policies. Saxena and Sarkar [28] visualized a replenishment and production coordination policy via mathematical modelling. They compared the traditional supply chain with misplacement and a blockchain-based supply chain with radio frequency identification. The study proved that RFID technology is highly profitable for the supply chain management. Since it is challenging for industries to meet the increasing demand, remanufacturing defective products can be significantly beneficial. They controlled the deterioration of products by considering preservation techniques, but no remanufacturing concepts for a sustainable production system were explored.

Several production inventory models have been developed by considering the remanufacturing of defective items. However, the existing literature has not yet considered a production-inventory model for single-stage assembled items under a variable production rate. In this study, a production-inventory system is considered for a single-stage assembled item whose different parts are manufactured and remanufactured within the same generation cycle. Defective items are generated randomly due to the imperfection of the system.

2.2. Remanufacturing of Defective Assembled Items

Cárdenas-Barrón [4] developed a model with a cleaner production approach. Wee et al. [29] extended Cárdenas-Barrón's [4] model with an alternative solution approach. Polotski et al. [30] evaluated the manufacturing model for raw materials and the remanufacturing model for returned products. The hypothesis of their model was developed based on a hybrid production process. M-Haidar and Nasr [31] discussed the reworking and screening processes at the end of production. Silva et al. [32] focused on the delivery performance of the TKS, GKS, and POLCA production systems and achieved short delivery times as well as on-time delivery. A make-to-stock strategy was adopted in the TKS system, directly related to the make-to-order policy. Kugele and Sarkar [33] extended Sarkar et al.'s [5] model by considering multi-item and multi-stage production processes. However, they considered a fixed production rate for their study. Mridha et al. [34] discussed the defective rate for a multi-item production model. The model proposed by Chen [35] implied that defective items affected supply chain performance. Thus, the impact of pricing, replenishment, and rework decisions were explained. Mtibaa and Erray [36] considered two types of non-defective products: first-rate products, which are high-quality items, and second-rate products, which are products of substandard quality. Reworking was provided for second-rate and non-conforming products, which improved the product quality, making it possible to sell the products at the best price. A simulation study of this model indicated that the performance of the POLCA system was inferior to that of the GKS system. Hage [37] proposed a dual-network model and explained the vocal flexibility that was observed. Chiu et al. [38] explored a multi-item stock by incorporating reworking along with a multiple-shipment plan. Li et al. [39] developed a dynamic intelligent manufacturing–remanufacturing system. After this, Polotski et al. [30] employed hybrid systems for manufacturing and remanufacturing while extending the feasibility conditions of deteriorating products. Their model yielded an optimal policy for maintenance and a joint production process. Asadi et al. [40] investigated the implications of realizing mix flexibility in an assembly system for the manufacturing of products. Their major finding was the low level of product modularity in mixed-product assembly lines (MPALs). Malik et al. [41] developed a multi-stage production system for defective items. The reworking process was designed according to a random defective rate in production, thereby reducing the waste generated by imperfect items. The purpose of their study was to retain the requisite standard quality and to reduce production costs.

Centobelli et al. [42] developed a model to explore the capabilities of green economic incentives and circular economy in maximizing the profitability of a production system. By remanufacturing, waste can be minimized along with costs. This paper applied confirmatory factor analysis (CFA) and structural equation modeling (SEM) to test the model. The findings showed the model's positive effect on the supply chain's economy. By considering remanufacturing and partial outsourcing, a smart manufacturing model was constructed by Li et al. [43]. Cifone et al. [44] developed a qualitative study based on a focus group design to explore how manufacturing and supply chain management professionals perceived the potential of digital technologies for increasing trade. The results showed that digital technologies could effectively improve the economy of production. Aslam et al. [45] examined the buyer–supplier relationship under a psychological contract breach. The inter-organizational and interpersonal impacts were examined in this paper to raise profits and enhance the economy of the production system. The findings of that study showed that the initial interpersonal and inter-organizational relationships influenced the buyer's response. That was one of the most important factors for improving the economic conditions of a production system or supply chain. Chaudhari et al. [46] discussed the effect of carbon ejection in a smart production system.

Since the circumstances of production are uncertain, authors such as Sarkar and Guchhait [47] have discussed the effect of asymmetric information sharing under a random demand process. Zhang et al. [48] developed a dual channel with the blockchain within a supply chain. Profits were optimized for various cases by an analytic investigation of different parameters. Promotional efforts and cost efficiency were discussed in the model. Shortages and backorders may occur in a supply chain due to defective products. Therefore, Bachar et al. [49] developed a model for reworking and outsourcing defective products to prevent backlogging. They showed that a random percentage of the defective products were reworkable and could be remanufactured to increase the efficiency of the production system. Various decision-making strategies were applied to increase the model's profitability, and classical optimization techniques achieved the optimal solutions.

Several manufacturing models have been developed by considering remanufacturing. Some have been developed by considering manufacturing and remanufacturing in the same production cycle. However, existing studies did not consider any single-stage production-inventory system for a single assembled item with a flexible production rate. Therefore, a pioneering attempt is made in this study to fill these gaps.

2.3. Minimum Backorders

The model proposed by Huang and Wu [50] considered the impact of backordering and batch demand on wholesalers. The paper minimized the total cost by deciding ordering and backordering quantities. Mittal et al. [51] presented an unreliable supply chain model, which indicated that inflation may influence the ordering policy. Xu et al. [52] proposed a stock inventory system with partial backlogging wherein the stochastic demand was utilized. In a similar direction, a cross-dock inventory system was proposed by Mukherjee et al. [53]. Bao et al. [54] studied an inventory model for single items with multiple demands and backorders. The corresponding demand classes were divided based on their unit backlogging costs. Kilic and Tunc [55] addressed the economic lot-sizing problem, and the total expected cost was optimized using the fixed production cost, holding cost, and backordering cost. Guo et al. [56] discussed a process for replenishing spare components by allowing dependent backorders. The isolated system of the model comprised backorders of distinct types of components. The dependent backorder Markov model was used for repairable isolated systems.

Bertazzi et al. [57] used a vendor-managed inventory (VMI) approach to design a periodic shipping policy that minimized the sum of transportation costs and inventory costs for a significant improvement in economy. Air freight shipments led to significant cost savings, both in the worst case and on average. Optimality was achieved using mixed-integer linear programming. Bi et al. [58] investigated the trade credit strategy and ordering policy for retailers. This paper aimed to find the optimal retailer decisions for the credit period, order quantity, and replenishment cycle to maximize the retailer's profit. Charpin et al. [59] identified the facilitators and barriers of mobile procurement platforms. The findings exhibited a decrease in cost and an increase in supply chain collaboration.

Bouzekri et al. [60] introduced a decision support system (DSS) for planning actions to achieve maximum economic benefits from the supply chain. They designed a new storage model to obtain optimal solutions for production and port models. Buisman and Rohmer [61] studied inventory decisions for stochastic problems in a production system for ameliorating products. The outcomes showed the influence of profit margins on the optimal strategy and the economic growth of the producer.

Ashraf et al. [62] developed a study based on third-party logistics and explored how production systems could effectively achieve the maximum benefits for their economy. The outcomes ensured a significant improvement in production economy. A crisp model was developed and extended to a fuzzy model to incorporate the imprecise nature of demand, which significantly minimized backorders. The optimal solution for the model was obtained using three newly developed algorithms. Zhou et al. [63] proposed game-theoretic policies and different business strategies based on the production rate, variable demand, and revenue-sharing contracts. Two models were developed based on coordination, non-coordination, and revenue-sharing contracts, each with three sub-cases. The models showed demand dependency on the green degree, average selling price, and product quality in a contemporary socio-economic situation. Balter et al. [64] investigated the implications of a finite lifetime for optimal firm investments considering investment decisions dependent on the timing and size of investments. Moreover, they considered strategic implications by implementing the finite product-life assumption in a duopoly framework. Astvansh and Jindal [65] proposed suitable policies for trade-credit financing. A multiple-shipment policy was utilized to deliver inventory and minimize backorders. Recently, Shajalal et al. [66] developed an algorithm through a deep neural network to predict backorders of a product for imbalanced data.

Several studies have focused on minimizing backorders. However, linear and average backorders in an imperfect production inventory model for assembled products with flexible production rates have rarely been addressed in the literature. Therefore, a pioneering attempt is made in this study to express the effect of backorders in a production-inventory model under a flexible production system. Please refer to Table 1 for the contributions of previous studies.

Table 1. Contributions of previous authors.

Author(s)	Production System	Production Rate	Remanufacturing	Backorders	Demand Type
Taleizadeh et al. [1]	SCM	Constant	NA	NA	Returned products
Machado et al. [2]	Single-stage	Constant	NA	NA	NA
Cárdenas-Barrón [4]	Single-stage	Constant	Single item	Planned	Constant
Sarkar et al. [5]	Single-stage	Constant	Single item	Planned	Constant
Aydin et al. [6]	Multi-stage	Constant	NA	NA	NA
Hariga et al. [7]	SCM	NA	NA	NA	NA
Qingdi et al. [8]	NA	NA	NA	NA	Predecisional
Gao et al. [11]	Traditional	Constant	NA	NA	NA
Alexopoulos et al. [12]	SCM	Constant	NA	NA	Selling-price-dependent
Wee et al. [29]	Single-stage	Constant	Single item	Planned	Constant
Polotski et al. [30]	Single-stage	Constant	NA	NA	NA
M-Haidar and Nasr [31]	NA	Constant	Fixed	NA	NA
Silva et al. [32]	Single-stage	Constant	NA	NA	NA
Chen [35]	NA	Imperfect deteriorating items	Constant	NA	NA
Mtibaa and Erray [36]	NA	Constant	Preventive maintenance	NA	NA
Hage [37]	Single-stage	Constnat	NA	NA	NA
Chiu et al. [38]	NA	Constant	Fixed	NA	NA
Asadi et al. [40]	Multi-stage	NA	NA	NA	
Malik et al. [41]	Multi-stage	Constant	Defective items	NA	NA
Huang and Wu [50]	NA	Constant	NA	Planned backorder	NA
Mittal et al. [51]	NA	Constant	NA	Influenced by inflation	NA
Xu et al. [52]	NA	Constant	NA	Partial	NA
Bao et al. [54]	NA	Constant	NA	Partial	NA
Kilic and Tunc [55]	NA	Constant	NA	Fixed	Fixed
Guo et al. [56]	NA	Constant	NA	Dependent	NA
Heydari et al. [67]	NA	Constant	NA	NA	Stocastic
This paper	Single-stage manufacturing and remanufacturing	Variable	Defective assembled items	Minimum	Constant

NA denotes 'not applicable'.

3. Problem Definition, Notation, and Assumptions

The problem definition, notation, and assumptions of the model are discussed in this section.

3.1. Problem Definition

This study analyzes a single assembled product that is produced using different parts with a flexible production process. An imperfect manufacturing system often features variability in the production processes, equipment breakdowns, and quality issues. The proposed system allocates resources effectively to address these imperfections by implementing a variable production rate. Defective items are reworked in the same production cycle. To manage waste and provide a cost-effective production inventory model, the remanufacturing of the defective items is performed in the same production cycle. Imperfect items are generated randomly, following certain distribution patterns. The process starts with backorders and produces perfect and imperfect items simultaneously. Imperfect items are identified through human inspection and remanufactured within time T_3. Due to the imperfection of production systems, shortages arise which are fully backlogged. In an imperfect manufacturing system, backorders can have several effects. A backorder occurs when a customer orders a product that is temporarily out of stock or not immediately available for delivery. Backorders in an imperfect manufacturing system can disrupt the customer experience, reduce sales, increase costs, and highlight underlying operational issues. Backorders can complicate inventory management. Manufacturers must balance the costs associated with carrying extra inventories to avoid backorders with the costs of lost sales of dissatisfied customers. Addressing the root causes of backorders and implementing strategies to minimize their occurrence are crucial for improving customer satisfaction and the overall efficiency of the manufacturing system. This study considers two backordering costs: the linear backordering cost and average backordering cost.

Budget and space constraints are common challenges when designing and implementing a production system. When working with a limited budget, focusing on cost-effective solutions is important. This may involve evaluating different equipment options, considering used or refurbished machinery, or exploring leasing or other financing arrangements. In this study, remanufacturing helped to minimize the wasting of material. One should optimize the production system layout to make the best use of the available space; consider factors such as workflow, material flow, and accessibility; and use techniques such as lean manufacturing principles to eliminate waste and improve space utilization. By considering budget and space constraints productively, engaging in thorough planning, and continuously monitoring and improving the production system, overcoming these challenges and achieving an effective and efficient production setup is possible. Our model is accounted for the restrictions of budget and space capacity. The total production-inventory cost is minimized by optimizing the production rate, the batch size of the item j used for assembly, and the quantity of backorders. A graphical diagram is presented in Figure 1.

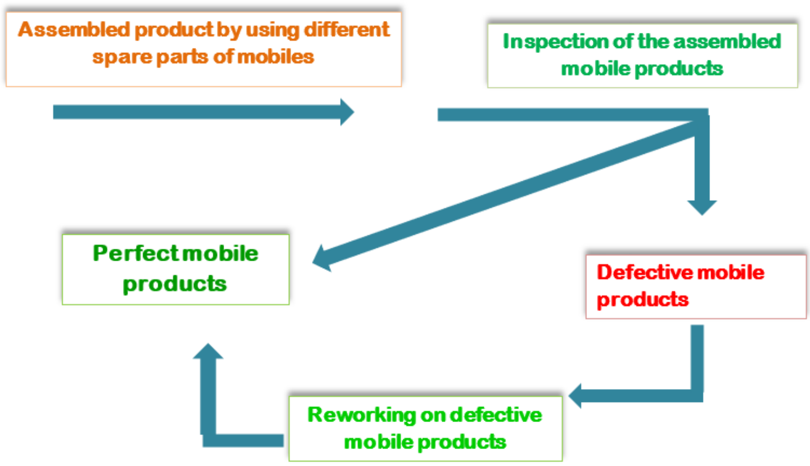

Figure 1. Diagram of the research problem.

3.2. Notation

Here, the notation used to explain the formulation of the model are described below.

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j	Number of parts required to manufacture a single assembled product, $j = 1, 2, 3, \ldots, m$

Decision variables

P_j	Production rate of item j (units/time)
Q_j	Production batch size of item j (units/cycle)
B_j	Quantity of backorders of item j (units)

Parameters

D_j	Demand rate of item j (units/time)
S_j	Setup cost of item j (USD/setup)
M_j	Manufacturing cost of product j (USD/unit)
H_j	Holding cost of item j per unit per unit time (USD/unit/time)
B_{l_j}	Linear backorder cost of item j per unit backorder per unit time (USD/unit/time)
m	Number of batches (integer)
B_{c_j}	Fixed backorder cost of item j per unit backorder (USD/backorder)
ξ_j	Backorder average of item j (units)
T	Cycle length (time units)
TC	Total cost per unit of time (units)
D_{r_j}	Cost of remanufacturing defective item j (USD/item)
V_j	Budget of item j (units)
f	Space capacity (units)
g	Total budget (USD)
$\bar{I}_j$	Average inventory of item j (units)
I_{max_j}	Maximum inventory, calculated as $I_{max_j} = I_{1_j} + I_{2_j}$ (units)
β_j	Random proportion rate of defective products j in each cycle, following a probability distribution (uniform, triangular, double-triangular, beta, and χ^2)
$E[\beta_j]$	Expected proportion of defective products j per cycle

3.3. Assumptions

The following assumptions for the model configuration are adopted in this study.

1. The model considers the manufacture of a single type of assembled product under a flexible production system, where the assembled product (specifically, an electronic product) is produced from several electronic parts j according to the customer's preference. The multiple parts j are sold separately from another warehouse. This study focuses on the effects of assembling products in a competitive market. Even though the demand remains fixed, a fixed demand under flexible production can be quickly fulfilled without any customer dissatisfaction.
2. In this model, the production rate is variable, and it is always greater than the demand. The whole inventory system of the model is assumed based on an infinite planning horizon under a flexible production, and it is possible to find the minimum production-inventory cost under an infinite time horizon (Alexopoulos et al. [12]).
3. Each assembled product is inspected after production, and defective products are quickly sent for remanufacturing within the same production cycle. The inspection cost for each product is fixed, even though this cost is adjusted according to the setup cost as it is significantly lower than the other costs. The rate of defective products under flexible production was allowed to be random (Dey et al. [25]). Thus, the optimum cost is evaluated separately based on five different distribution functions (uniform, triangular, double-triangular, beta, and χ^2).
4. The model initially faces shortages after selling the finished assembled products, but all shortages are fully backlogged. There is no lost sales. In this model, there are two types of backorder costs, i.e., the linear backorder cost (the backorder cost applies

to average backorders) and the fixed backorder cost (the backorder cost applies to the maximum backorder level). We proposes that the inventory holding costs are based on the average inventory and provided space and budget limitations as constraint types. The production rate is taken as a variable. Thus, a huge inventory during production, unexpected investment costs, and a large holding space will arise without space and budget constraints on the inventory (Sarkar et al. [5]).

4. Mathematical Model for the Innovative Production System

This paper considers the production of a single product assembled using different items. Under a flexible production system, the goal of this model is to minimize the damage to the assembled product and the inventory cost of the entire system while considering the restrictions of the space capacity and budget. The model is extended from the benchmark model of Cárdenas-Barrón [4] involving multi-item clean production and random defective rates.

Remark 1. *The present model will converge to Cárdenas-Barrón's [4] model if it considers only a single type of product with constant defective and production rates and without inspection.*

Remark 2. *If this model considers a random defective rate, a single type of item, a constant production rate without inspection, an assembled product, and budget & space constraints, then it will converge to the model of Sarkar et al. [5]*

Flexible production is achieved with initial backorders in the interval $[0, T_1]$ to reach the zero-inventory level, and inventory is acquired at the rate $P_j(1-\beta_j) - D_j$. After time $t = T_2$, the imperfect products start to remanufactured at the rate $(P_j - D_j)$ in the interval $[T_2, T_3]$. Furthermore, the inventory reaches its maximum capacity I_{max} at time T_3. After that, it begins to decrease in the interval $[T_3, T_4]$ at the rate D_j to the zero level. In the interval $[T_4, T_5]$, the backorder rate is B_j, and the demand is D_j. Thus, total cycle length T = production time + remanufacturing time + non-production time, i.e, $T = T_P + T_{RM} + T_{NP}$, where $T_P = T_1 + T_2, T_{RM} = T_3$, and $T_{NP} = T_4 + T_5$. Here, T_1 and T_5 are the times at which the system includes backorders, T_2 is the pure production time, T_3 is the pure remanufacturing time, and T_4 is the pure consumption time.

Therefore, as shown in Figure 2, the production time can be calculated as

$$\begin{aligned} T_1 + T_2 &= \frac{(I_{1_j} + B_j)}{(P_j(1 - E[\beta_j]) - D_j)} = \frac{Q_j}{P_j}, \\ \text{which gives,} \\ I_{1_j} &= Q_j\left((1 - E[\beta_j]) - \frac{D_j}{P_j}\right) - B_j. \end{aligned} \tag{1}$$

Additionally, from Figure 2, the remanufacturing time can be determined as

$$\begin{aligned} T_3 &= \frac{I_{2_j}}{(P_j - D_j)}, \\ \text{as} \quad T_3 &= \frac{E[\beta_j]Q_j}{P_j}. \\ \text{Therefore} \quad \frac{E[\beta_j]Q_j}{P_j} &= \frac{I_{2_j}}{(P_j - D_j)}, \\ I_{2_j} &= E[\beta_j]Q_j\left(1 - \frac{D_j}{P_j}\right). \end{aligned} \tag{2}$$

The maximum inventory can be determined by solving the following equation:

$$I_{max_j} = I_{1_j} + I_{2_j} = Q_j\left[1 - \frac{D_j}{P_j}(1 + E[\beta_j])\right] - B_j. \quad (3)$$

To determine the average inventory, it is necessary to calculate the entire inventory for the cycle time T and divide it by the cycle time. Thus, the average inventory is expressed as

$$I_{AVG_j} = \frac{1}{T}\left[\frac{1}{2}T_2 I_{1_j} + \frac{1}{2}T_3 I_{1_j} + \frac{1}{2}T_3 I_{max_j} + \frac{1}{2}T_4 I_{max_j}\right] \quad (4)$$

(for the values of T_1, T_2, T_3, T_4, and T_5, see Appendix A). Thus,

$$\begin{aligned} I_{AVG_j} &= \frac{1}{2Q_j\left[(1 - E[\beta_j]) - \frac{D_j}{P_j}\right]}\left[(Q_j^2 + B_j^2)(1 - E[\beta_j]) + \frac{Q_j^2 D_j^2}{P_j^2}(1 + E[\beta_j] + (E[\beta_j])^2) + \frac{Q_j^2 D_j}{P_j}((E[\beta_j])^3 - 2)\right. \\ &+ \left. 2B_j Q_j\left(\frac{D_j}{P_j} + E[\beta_j] - 1\right)\right]. \end{aligned} \quad (5)$$

The average backorder inventory is determined as

$$I_{BAVG_j} = \frac{1}{T}\left[\frac{1}{2}B_j T_1 + \frac{1}{2}\frac{B_j^2}{D_j}\right] = \frac{B_j^2(1 - E[\beta_j])}{2Q_j\left[(1 - E[\beta_j]) - \frac{D_j}{P_j}\right]}. \quad (6)$$

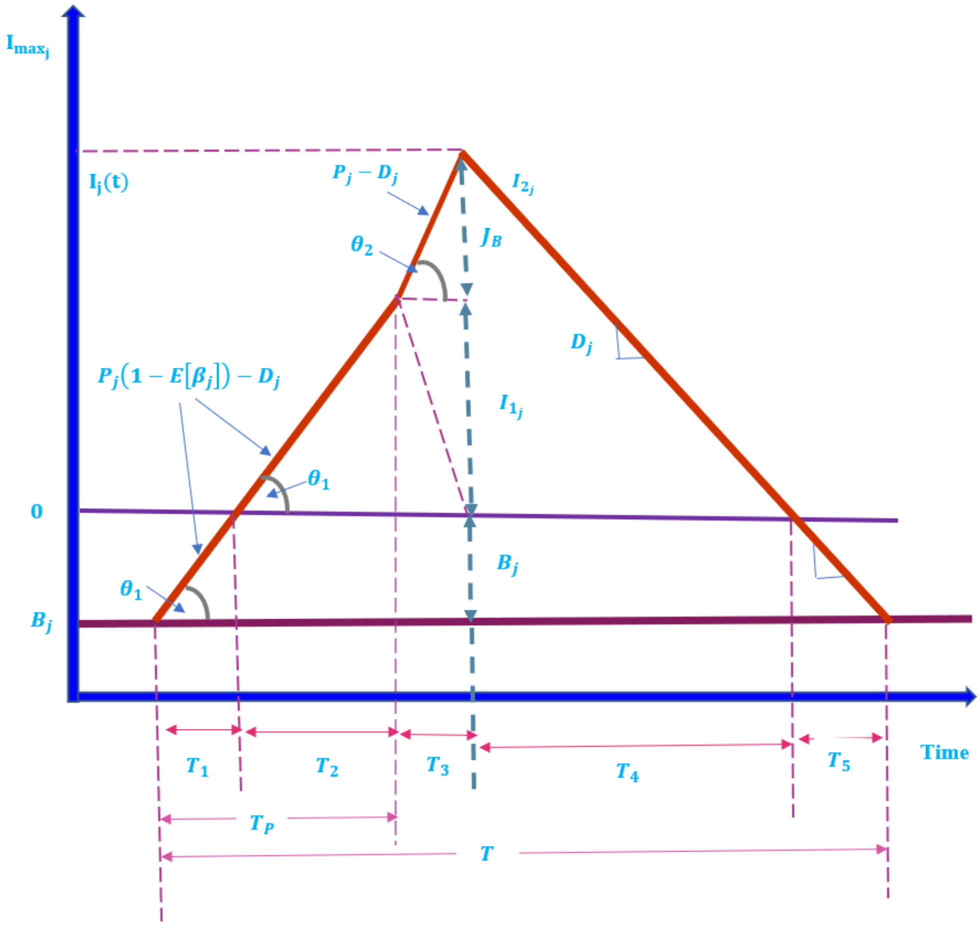

Figure 2. Inventory position of final products (see Sarkar et al. [5]).

All costs related to this single-stage manufacturing–remanufacturing production-inventory system are tabulated in Table 2. Detailed explanations of each cost are provided under the Table.

Table 2. Different cost components for this single-stage production-inventory model.

Cost	Reason for the Cost	Time Period	Expression
Setup with inspection cost	To set up the production and inspection processes	0	$\frac{S_j D_j}{Q_j}.$
Manufacturing cost	To manufacture the parts of the assembled products in a single-stage within the time interval	$T_1 + T_2$	$\frac{M_j P_j (1 + E[\beta_j])}{\frac{P_j}{D_j}}.$
Remanufacturing cost	To remanufacture the imperfect items within the time interval	T_3	$D_{r_j} P_j E[\beta_j] \frac{D_j}{Q_j}.$
Holding cost	To hold the parts of the assembled product for the entire cycle	T	$H_j I_{AVG_j}$
Backorder cost	To account for backorders due to imperfect manufacture through the entire cycle	T	$\frac{B_{cj} B_j D_j}{Q_j} + B_{lj} \frac{B_j^2 (1 - E[\beta_j])}{2Q_j \left[(1 - E[\beta_j]) - \frac{D_j}{P_j} \right]}.$

4.1. Setup with Inspection Cost

After the production of a single type of assembled product by making j parts, the fixed setup cost is S_j. The inspection strategy is applied just after production to separate out any defective products. Therefore, the model includes the inspection cost in the setup cost, as this inspection cost is much lower than any other cost. The cycle time is $T = \frac{Q_j}{D_j}$. Thus, the per-cycle setup cost is calculated using the following formula:

$$\frac{S_j D_j}{Q_j}. \tag{7}$$

4.2. Manufacturing Cost

Throughout the production time, the manufacturing cost is evaluated as the per-unit manufacturing cost M_j multiplied by the production rate P_j at the early inventory level. In this inventory model, the expected value of defective products $E[\beta_j]$ is considered separately. Therefore, the per-cycle manufacturing cost is calculated as follows:

$$\frac{M_j P_j (1 + E[\beta_j])}{\frac{P_j}{D_j}}. \tag{8}$$

4.3. Inventory Holding Cost

The model calculates the inventory holding cost based on the average inventory level, which is previously described. Thus, the per-cycle holding cost is determined as follows:

$$= \frac{H_j}{T} \left[\frac{\left\{ Q_j \left[(1 - E[\beta_j]) - \frac{D_j}{P_j} \right] - B_j \right\}^2}{2[P_j (1 - E[\beta_j]) - D_j]} + \frac{E[\beta_j] Q_j \left\{ Q_j \left[1 - \frac{E[\beta_j]}{2} - (1 + \frac{E[\beta_j]}{2}) \frac{D_j}{P_j} \right] - B_j \right\}}{P_j} + \frac{\left\{ Q_j \left[1 - (1 + E[\beta_j]) \frac{D_j}{P_j} \right] - B_j^2 \right\}}{2D_j} \right]. \tag{9}$$

4.4. Backorder Cost

To determine the average backorder, we calculate the linear backorder cost as the per-unit linear backorder cost B_{lj} multiplied by the average backorder inventory I_{BAVG_j}. Thus, the linear backorder cost is expressed as $B_{lj}I_{BAVG_j}$. On the other hand, at the maximum backorder level, the fixed backorder cost $B_{cj}B_j$ is calculated based on the backorder quantity B_j. Therefore, the total backorder cost is expressed as

$$= \frac{B_{cj}B_jD_j}{Q_j} + B_{lj}I_{BAVG_j} = \frac{B_{cj}B_jD_j}{Q_j} + B_{lj}\frac{B_j^2(1-E[\beta_j])}{2Q_j\left[(1-E[\beta_j]) - \frac{D_j}{P_j}\right]}. \tag{10}$$

4.5. **Remanufacturing Cost**

Defective products occur randomly in a flexible production system. Thus, to remanufacture these products, the model includes some additional cost for just-in-time remanufacturing. Hence, the average remanufacturing cost for the defective products is expressed as

$$D_{r_j}P_jE[\beta_j]\frac{D_j}{Q_j}. \tag{11}$$

Therefore, the total cost of the production-inventory system consists of the setup with inspection cost, inventory holding cost, backorder cost, manufacturing cost, and remanufacturing cost. This can be expressed as

$$TC = \sum_{j=1}^{m}\left[\frac{S_jD_j}{Q_j} + H_jI_{AVG_j} + \frac{B_{cj}B_jD_j}{Q_j} + B_{lj}I_{BAVG_j} + M_jD_j(1+E[\beta_j]) + P_jE[\beta_j]D_{r_j}\frac{D_j}{Q_j}\right]. \tag{12}$$

Substituting the values of I_{AVG_j}, B_{cj}, B_{lj}, and I_{BAVG_j} into the total cost function, the resulting equation takes the form

$$\begin{aligned} TC(P_j,Q_j,B_j) &= \sum_{j=1}^{m}\left[\frac{S_jD_j}{Q_j} + H_j\left[\frac{1}{2Q_j\left[(1-E[\beta_j]) - \frac{D_j}{P_j}\right]}\left[(Q_j^2+B_j^2)(1-E[\beta_j]) + \frac{Q_j^2D_j^2}{P_j^2}\left(1+E[\beta_j]+(E[\beta_j])^2\right)\right.\right.\right. \\ &+ \left.\left.\frac{Q_j^2D_j}{P_j}\left((E[\beta_j])^3-2\right) + 2B_jQ_j\left(\frac{D_j}{P_j}+E[\beta_j]-1\right)\right]\right] + \frac{B_{cj}B_jD_j}{Q_j} + B_{lj}\left[\frac{B_j^2(1-E[\beta_j])}{2Q_j\left[(1-E[\beta_j]) - \frac{D_j}{P_j}\right]}\right] \\ &+ \left. M_jD_j(1+E[\beta_j]) + P_jE[\beta_j]D_{r_j}\frac{D_j}{Q_j}\right]. \end{aligned} \tag{13}$$

4.6. Budget and Space Constraints

Under flexible production, the production rate is not fixed. Therefore, to impose limitations on the inventory and to reduce the investments, budget and space capacity are proposed as boundary conditions. These are obtained as follows:

$$\sum_{j=1}^{m} Q_jV_j \leq g \tag{14}$$

$$\sum_{j=1}^{m} Q_jS_{c_j} \leq f. \tag{15}$$

Equation (13) represents the total cost of the model, which is minimized based on three decision variables: P_j, Q_j, and B_j.

5. Solution Methodology

There are three unknown variables P_j, Q_j, and B_j in the cost function, and these variables needed to be optimized. The cost equation including the constraints presented in (13)–(15) is as follows:

$$\begin{aligned}
Min\ TC(P_j, Q_j, B_j) \quad = \quad & \sum_{j=1}^{m} \left[\frac{S_j D_j}{Q_j} + H_j \left[\frac{1}{2Q_j \left[(1-E[\beta_j]) - \frac{D_j}{P_j} \right]} \left[(Q_j^2 + B_j^2)(1 - E[\beta_j]) + \frac{Q_j^2 D_j^2}{P_j^2} \left(1 + E[\beta_j] + (E[\beta_j])^2 \right) \right. \right. \right. \\
+ \quad & \left. \left. \frac{Q_j^2 D_j}{P_j} \left((E[\beta_j])^3 - 2 \right) + 2B_j Q_j \left(\frac{D_j}{P_j} + E[\beta_j] - 1 \right) \right] \right] + \frac{B_{cj} B_j D_j}{Q_j} + B_{lj} \left[\frac{B_j^2 (1 - E[\beta_j])}{2Q_j \left[(1-E[\beta_j]) - \frac{D_j}{P_j} \right]} \right] \\
+ \quad & \left. M_j D_j (1 + E[\beta_j]) + P_j E[\beta_j] D_{r_j} \frac{D_j}{Q_j} \right]
\end{aligned} \tag{16}$$

subject to
$\sum_{j=1}^{m} Q_j V_j \leq g$
$\sum_{j=1}^{m} Q_j S_{c_j} \leq f$
and $P_j, Q_j, B_j > 0,\ j = 1, 2, \ldots, m.$

This equation is a non-linear expression. To solve this non-linear expression, the Kuhn–Tucker condition is applied by determining the Lagrange multiplier equation for this expression. Two constraints are added to this expression by considering two Lagrange multiplier coefficients: λ_1 and λ_2.

$$\begin{aligned}
L(P_j, Q_j, B_j, \lambda_1, \lambda_2) \quad = \quad & \sum_{j=1}^{m} \left[\frac{S_j D_j}{Q_j} + H_j \left[\frac{1}{2Q_j \left[(1 - E[\beta_j]) - \frac{D_j}{P_j} \right]} \left[(Q_j^2 + B_j^2)(1 - E[\beta_j]) + \frac{Q_j^2 D_j^2}{P_j^2} \left(1 + E[\beta_j] + (E[\beta_j])^2 \right) \right. \right. \right. \\
+ \quad & \left. \left. \frac{Q_j^2 D_j}{P_j} \left((E[\beta_j])^3 - 2 \right) + 2B_j Q_j \left(\frac{D_j}{P_j} + E[\beta_j] - 1 \right) \right] \right] + \frac{B_{cj} B_j D_j}{Q_j} + B_{lj} \left[\frac{B_j^2 (1 - E[\beta_j])}{2Q_j \left[(1 - E[\beta_j]) - \frac{D_j}{P_j} \right]} \right] \\
+ \quad & \left. M_j D_j (1 + E[\beta_j]) + P_j E[\beta_j] D_{r_j} \frac{D_j}{Q_j} \right] + \lambda_1 \left(\sum_{j=1}^{m} Q_j V_j - g \right) + \lambda_2 \left(\sum_{j=1}^{m} Q_j S_{c_j} - f \right).
\end{aligned} \tag{17}$$

After determining the Lagrange equation, four cases are considered along with this equation. For the case $\lambda_1 = 0$, $\lambda_2 = 0$, Q_j has different values; therefore, this case is neglected. The value of Q_j is differed for the other three cases: $\lambda_1 \neq 0$, $\lambda_2 = 0$; $\lambda_2 \neq 0$, $\lambda_1 = 0$; and $\lambda_1 \neq 0$, $\lambda_2 \neq 0$. We differentiate $L(P_j, Q_j, B_j, \lambda_1, \lambda_2)$ with respect to P_j, Q_j, B_j, λ_1, and λ_2.

$$\frac{\partial L(P_j, Q_j, B_j, \lambda_1, \lambda_2)}{\partial P_j} = E[\beta_j] D_{r_j} \frac{D_j}{Q_j} - \frac{H_j}{2Q_j} \left[R_1 \frac{D_j}{R_*^2 P_j^2} + R_2 \left(\frac{2}{R_* P_j^3} + \frac{D_j}{R_*^2 P_j^4} \right) + R_3 \left(\frac{1}{R_* P_j^2} + \frac{D_j}{R_*^2 P_j^3} \right) \right] \tag{18}$$

$$\frac{\partial L(P_j, Q_j, B_j, \lambda_1, \lambda_2)}{\partial B_j} = \frac{2B_j H_j (1 - E[\beta_j])}{2Q_j R_*} - H_j + \frac{B_{cj} D_j}{Q_j} + \frac{B_j B_{lj} (1 - E[\beta_j])}{2Q_j R_*}. \tag{19}$$

Equating $\frac{\partial L(P_j,Q_j,B_j,\lambda_1,\lambda_2)}{\partial P_j}$ and $\frac{\partial L(P_j,Q_j,B_j,\lambda_1,\lambda_2)}{\partial B_j}$ to zero,

$$P_j = \frac{\left(R_2 E[\beta_j] - R_2 \right) + \sqrt{\left(R_2 E[\beta_j] - R_2 \right)^2 + R_2 D_j \left(R_3 - E[\beta_j] R_3 + R_1 \right)}}{\left(R_3 - E[\beta_j] R_3 + R_1 \right)} \tag{20}$$

$$B_j = \left[H_j - \frac{B_{cj} D_j}{Q_j} \right] \frac{Q_j \left[1 - E[\beta_j] - \frac{D_j}{P_j} \right]}{\left(1 - E[\beta_j] \right) [H_j + B_{lj}]}. \tag{21}$$

Case I: $\lambda_1 \neq 0$, $\lambda_2 = 0$

$$\frac{\partial L(P_j, Q_j, B_j, \lambda_1, \lambda_2)}{\partial Q_j} = -\frac{R_4}{Q_j^2} + H_j R_5 + \lambda_1 V_j \Rightarrow Q_j = \left[\frac{R_4}{H_j R_5 + \lambda_1 V_j}\right]^{\frac{1}{2}} \tag{22}$$

$$\frac{\partial L(P_j, Q_j, B_j, \lambda_1, \lambda_2)}{\partial \lambda_1} = \left(\sum_{j=1}^{m} Q_j V_j - g\right) \Rightarrow \lambda_1 = \frac{R_4 V_j^2 - H_j R_5 g^2}{V_j g^2}. \tag{23}$$

Case II: $\lambda_2 \neq 0$, $\lambda_1 = 0$

$$\frac{\partial L(P_j, Q_j, B_j, \lambda_1, \lambda_2)}{\partial Q_j} = -\frac{R_4}{Q_j^2} + H_j R_5 + \lambda_2 S_{c_j} \Rightarrow Q_j = \left[\frac{R_4}{H_j R_5 + \lambda_2 S_{c_j}}\right]^{\frac{1}{2}} \tag{24}$$

$$\frac{\partial L(P_j, Q_j, B_j, \lambda_1, \lambda_2)}{\partial \lambda_2} = \left(\sum_{j=1}^{m} Q_j S_{c_j} - f\right) \Rightarrow \lambda_2 = \frac{R_4 S_{c_j}^2 - H_j R_5 f^2}{S_{c_j} f^2}. \tag{25}$$

Case III: $\lambda_1 \neq 0$, $\lambda_2 \neq 0$

$$\frac{\partial L(P_j, Q_j, B_j, \lambda_1, \lambda_2)}{\partial Q_j} = -\frac{R_4}{Q_j^2} + H_j R_5 + \lambda_1 V_j + \lambda_2 S_{c_j}; \Rightarrow Q_j = \left[\frac{R_4}{H_j R_5 + \lambda_1 V_j + \lambda_2 S_{c_j}}\right]^{\frac{1}{2}}. \tag{26}$$

Sufficient condition: The analytical solution is verified as the global solution by applying the Hessian matrix method, and it is observed that all minors of the Hessian matrix are positive. See Appendix B for this calculation.

6. Numerical Example

In this section, based on the model formulation, all the constant parameters for assembling items $j = 2$ and $j = 3$ are summarized in Tables 3 and 4, respectively. Next, Table 5 presents the different formulas of the distribution functions. In each row, the expected values of the constant coefficients are provided for each distribution pattern in Table 6, and the last row of Table 6 presents the values of the coefficients that are applied to calculate the numerical solution along with these distribution functions.

Table 3. Parametric values for assembled products with two spare parts.

Parameters ($j = 2$)	Values	Parameters	Values
m	2	D_j (units/time)	300, 350
λ_2	18	B_{lj} (USD/unit/unit time)	10, 11
λ_2	18	B_{lj} (USD/unit/unit time)	10, 11
H_j (USD/unit time)	50, 55	B_{cj} (USD/unit)	1, 2
S_j (USD/setup)	50.22, 55.22	M_j (USD/unit)	7, 8
V_j (USD/unit)	55, 65	S_{cj} (square meters)	3, 4
D_{rj} (USD/item)	5, 7	g (USD)	33,000
f	400	λ_1	15

Table 4. Parametric values.

Parameters ($j = 3$)	**Values**	**Parameters**	**Values**
m	3	D_j (USD/unit/unit time)	$350, 320, 300$
λ_2	0.008	B_{lj} (USD/unit/unit time)	$10, 10, 10$
H_j (USD/unit time)	$50, 50, 50$	B_{cj} (USD/unit)	$1, 1, 1$
S_j (USD/setup)	$50.22, 50.22, 50.22$	M_j (USD/unit)	$7, 7, 7$
V_j (USD/unit)	$60, 60, 60$	S_{cj} (square meters)	$2, 2, 2$
D_{rj} (USD/item)	$6, 6, 6$	g (USD)	58,000
f	400	λ_1	0.009

Table 5. Formulas for determining the expected value with different distribution patterns.

Distribution	**Formula for Calculating the Expected Value $E[\beta_j]$**
Uniform distribution	$\frac{a_j+b_j}{2}$
Triangular distribution	$\frac{a_j+b_j+c_j}{3}$
Double-triangular distribution	$\frac{a_j+4b_j+c_j}{6}$
Beta (β) distribution	$\frac{\alpha_j}{\alpha_j+\beta_j}$
χ^2 distribution	κ_j

In these formulas, a_j, b_j, c_j, α_j, β_j, and κ_j are the scaling parameters of different distribution patterns.

Table 6. Formulas for determining the expected value with different distribution patterns.

Distribution	**Uniform**	**Triangular**	**Double-Triangular**	**Beta**	χ^2
	(a_j, b_j)	(a_j, b_j, c_j)	(a_j, b_j, c_j)	(α_j, β_j)	(κ_j)
	$(0.03, 0.07)$ $(0.03, 0.07)$ $(0.04, 0.08)$	$(0.03, 0.04, 0.07)$ $(0.03, 0.04, 0.07)$ $(0.04, 0.04, 0.07)$	$(0.03, 0.04, 0.07)$ $(0.03, 0.04, 0.07)$ $(0.04, 0.04, 0.08)$	$(0.03, 0.07)$ $(0.03, 0.07)$ $(0.04, 0.08)$	0.03 0.03 0.04
	$0.03, 0.04, 0.05$	$0.047, 0.047, 0.05$	$0.043, 0.043, 0.047$	$0.3, 0.3, 0.33$	$0.03, 0.03, 0.04$

A comparison between the optimal solutions for both $j = 2$ and $j = 3$ is presented in Tables 7 and 8, respectively. Each row of both tables considers variable parameters P_j, Q_j, and B_j and their optimal values along with each distribution function. Further, we separately obtain the total inventory costs for these five distribution functions. In the following tables, it can be seen that the total cost is minimized under χ^2 distribution for both assembled items $j = 2$ and $j = 3$.

In this model, budget and space are assumed to be limited, meaning that the setup cost is increased. Therefore, the production batch size tends to increase with the larger setup cost. Further, the increased production batch size causes an increase in the production rate per production run. Thus, the analysis of these graphics determines that when both the production rate and production batch size increase, the total inventory cost minimizes.

Table 7. Optimal results for an assembled item with two spare parts.

Distribution	Production Rate (P_j) (Units/Time)	Production Batch Size Q_j (Units/Year)	Backorder Quantity B_j (Units/Year)	Total Cost (USD/Year)
Uniform distribution	90, 23	770, 678	68, 57	236,309
Triangular distribution	91, 43	800, 178	80, 108	32,652
Double-triangular distribution	92, 199	449, 319	60.81, 30.62	27,180
Beta distribution	90.31, 43.9	650, 418	89, 112	335,531
χ^2-distribution	39.28, 23.3	164.44, 117.48	55.62, 43.03	**22,641**

Table 8. Optimal results for an assembled item with three spare parts.

Distribution	Production Rate (P_j) (Units/Time)	Production Batch Size Q_j (Units/Year)	Backorder Quantity B_j (Units/Year)	Total Cost (USD/Year)
Uniform distribution	517.57, 572.91, 445.39	36.105, 51.32, 68.57	8.74, 22.32, 26.98	28,435
Triangular distribution	663.76, 343.29, 373.39	56.32, 61.95, 128.16	18.32, 2.01, 27.63	27,300
Double-triangular distribution	685.7, 612.44, 628.6	43.48, 70.83, 68.17	14.16, 21.46, 28.10	30,377
Beta distribution	545.12, 700.2, 708	25.23, 91, 82	98, 99, 99	45,306
χ^2-distribution	485.99, 386.43, 404.34	51.03, 26.44, 51.73	24.06, 20.65, 21.93	**26,938**

Comparison with Existing Literature

In this section, a numerical and theoretical comparison with the existing literature is conducted. A detailed comparison is presented in Table 9. Due to the use of a flexible production system, an assembled product, and single-stage manufacturing and remanufacturing, the present study reduces costs more than twenty times compared to the models of Sivashankari and Panayappan [9] and Sanjai and Periyasamy [10]. On the other hand, the cost of this study is more than seven times higher than that of Cárdenas-Barrón's [4] and Sarkar et al.'s [5] models. However, Cárdenas-Barrón's [4] and Sarkar et al.'s [5] models considered only a single type of item, whereas the present study considers an assembled item with different parts. It is obvious that the setups of these models are different. However, the present study provides more realistic results due to the use of variable production rates in a single-stage manufacturing–remanufacturing process with budget and space constraints.

Table 9. Numerical and theoretical comparison with previous studies.

Existing Literature	Production Rate	Product Type	Defective Rate	Total Cost (USD/Year)
Cárdenas-Barrón [4]	Fixed	Single	Fixed rate	3430
Sarkar et al. [5]	Fixed	Single	Random	2629
Sivashankari and Panayappan [9]	Fixed	Single	Shortages	455,185
Sanjai and Periyasamy [10]	Fixed	Single	Not considered	450,915
This study	Variable	Single assembled	Random	22,641

7. Sensitivity Analysis

A sensitivity analysis is performed for all parameters, including the two constraints, as shown in Table 10. The sensitivity values corresponding to the five different distribution patterns are calculated separately.

Table 10. Sensitivity analysis.

Parameter	Changes	BD	TD	DTD	UD	CD
a_j	−50	−35.77	−47.35	−45.87	−47.15	−50.72
	−25	−17.88	−23.68	−22.94	−23.57	−25.36
	+25	17.88	23.68	22.94	23.57	25.36
	+50	35.77	47.35	45.87	47.15	50.72
b_j	−50	−14.85	−2.52	−1.11	−3.76	−10.33
	−25	−6.10	−1.25	−0.55	−1.85	−4.90
	+25	4.53	1.22	0.55	1.79	4.50
	+50	–	2.41	1.09	3.54	8.69
c_j	−50	−8.65	−5.48	−10.21	−8.30	−8.55
	−25	−4.71	−2.68	−4.87	−3.99	−0.99
	+25	5.80	2.56	4.51	3.74	1.87
	+50	–	5.02	8.71	7.28	−1.34
κ_j	−50	−0.08	−0.66	−0.61	–	–
	−25	–	−0.068	−0.80	–	–
	+25	–	0.075	0.79	–	–
	+50	–	0.045	0.050	–	0.004
f	−50	+13.72	+29.31	+27.88	+22.25	+24.75
	−25	+6.86	+14.66	+13.94	+10.96	+12.38
	+25	−6.86	−14.66	−13.94	−10.96	−12.38
	+50	−13.72	+29.31	−27.88	−22.25	−24.75
g	−50	+0.21	+0.41	+0.39	+0.35	+0.38
	−25	+0.10	+0.21	+0.20	+0.17	+0.19
	+25	−0.10	−0.21	−0.20	−0.17	−0.38
	+50	−0.21	−0.41	−0.39	−0.35	−0.19
M_j *	−50	−11.45	–	−22.33	–	–
	−25	–	−21.28	−31.41	−11.65	−15.77
	+25	14.34	90.55	60.02	39.63	18.50
	+50	17.80	27.45	68.33	57.59	65.48
S_j	−50	−10.64	−10.74	−12.21	−12.96	−13.85
	−25	−10.33	−11.35	−12.20	−12.32	−11.74
	+25	10.31	10.32	11.17	12.29	10.67
	+50	20.63	12.58	12.30	13.55	33.30
H_j	−50	−7.51	−1.73	−2.70	–	–
	−25	−2.74	−3.34	−3.37	−3.34	–
	+25	2.58	3.04	3.08	3.05	2.92
	+50	5.03	5.88	5.96	–	5.64
B_{cj}	−50	−0.034	−0.32	−0.25	−1.30	−1.44
	−25	−0.17	−0.064	−0.61	−0.64	−0.70
	+25	0.16	0.061	0.59	0.61	0.66
	+50	0.33	0.09	1.14	0.18	0.27
B_{lj}	−50	−0.49	−0.62	−0.65	−0.63	−0.55
	−25	−0.24	−0.29	−0.31	−0.30	−0.26
	+25	0.21	0.27	0.28	0.27	0.24
	+50	0.41	0.51	0.54	0.51	0.46

BD: β distribution, TD: triangular distribution, DTD: double-triangular distribution, UD: uniform distribution, CD: χ^2 distribution, "−" means 'not feasible'.

- The values of each column for the parameter a_j are equal but present opposite signs for each distribution pattern. This indicates that the total cost is proportionally affected by the change in the parameter a_j. The total cost is slightly affected by the change in the value of parameter b_j. c_j is more effective under β and χ^2 distribution. Moreover, κ_j has a smaller impact on the total cost than the other parameters.
- The space and budget constraints with equivalent changes are more sensitive in this model. Based on Table 10, both constraints have a consistent effect on the total cost.

- The changes in the two parameters, i.e., manufacturing cost and holding cost, are substantial for both negative and positive changes. Furthermore, a negative sign exists for each distribution, which implies that the total cost is not substantially affected by the changes in the two parameters.
- In the −50% to +50% range, minor changes in the setup cost, linear backorder cost, and fixed backorder cost parameters occurs at every stage of the distributions. Thus, these minor changes affects the total cost, indicating that the total costs changes with distributions.

8. Managerial Implications

This paper focuses on assembling products under variable production rates. Through the flexible production, production rates increases, and the number of products increases too. As a result, organizations need to manage large setup costs. A variable production rate can provide several benefits for an imperfect manufacturing system. An imperfect manufacturing system may experience unexpected disruptions. With a variable production rate, the system can quickly adapt to changes in demand by increasing or decreasing production levels accordingly. This flexibility helps to avoid overproduction or underproduction and optimizes the utilization of resources. Variable production rates allow a system to align the production level with demand by reducing inventory costs. By producing at a rate that matches demand, a system can minimize excess inventory holding costs, obsolescence risks, and storage expenses. This cost efficiency contributes to a better financial performance and profitability. Variable production rates can minimize wastes in an imperfect manufacturing system. By producing smaller batches or adjusting production levels based on the demand, a system can reduce defective or obsolete inventory accumulation. This leads to less waste generation, lower reworking costs, and improved overall process efficiency. A variable production rate can enhance the performance and resilience of an imperfect manufacturing system. Based on this study, managers can make the right decisions regarding production rates by making their systems cost-effective.

If a system consists of various components or sub-systems those are not well-integrated or coordinated, it can result in inefficiencies, misalignment, and operational challenges. A lack of synchronization between different parts of the manufacturing process can lead to delays, errors, and wasted resources. In this study, the manufacturing process of electronic goods such as mobile phone is considered, which involves less maintenance complexity. However, for a complex system such as car production assembly, some maintenance costs is needed to be included within the total cost functions. Thus, it is important for organizations to identify and address the imperfections in their manufacturing systems to improve the overall performance, productivity, and product quality. This may involve conducting a thorough analysis of the system, identifying the root causes of the issues, implementing corrective measures, and continuously monitoring & optimizing the manufacturing processes.

Many manufacturing organizations tend to increase production batch sizes to avoid unnecessary shortages. This means that the total investment increases. This model minimizes the total investment cost by introducing space and budget constraints. Numerical experiments are proposed to optimize the batch size of the assembled items that the manufacturing organization can store for future stock. The total cost is minimized separately under χ^2 distribution for two assembled items, $j = 2$ and $j = 3$. By calculating a proper backorder size, managers can control shortages and earn more revenue. This study focuses on a planned backorder system, wherein linear and fixed backorders are considered for cost-effectiveness.

For an imperfect production inventory system, the generation of defective items is crucial, as these items cannot be sold on the market in order to preserve the company's brand image. The number of defective items produced in a single stage is unpredictable. Therefore, in this study, five different types of distribution are considered and compared to determine the most cost-effective defect rate. The findings clearly shows that a χ^2 -distributed defective rate is more cost-effective for a single-stage manufacturing–remanufacturing system, whereas triangular distribution provides fruitful results for three-part items. Therefore, this study can be used to make several major decisions regarding single-stage manufacturing–remanufacturing production inventory systems with a single assembled item.

9. Conclusions

An inventory model for innovative production was proposed in this paper. In continuously changing business environments, flexible production must be sufficiently capable of meeting customer requirements. In the production process, defective items might be produced randomly, and this model considered a reworking strategy to be implemented after surveying such defective items. Thus, the assembled product could be remanufactured without any shortages. To reduce the manufacturing cost, the productivity variation was increased to improve the quality of the assembled products. The budget and space capacity were considered as constraints. The total inventory cost was calculated based on five different distribution functions, using the defined variable parameters of production rate, production batch size, and backorder quantity. A numerical study proved that χ^2 distribution yielded the global minimum total cost compared to all other distribution functions. Flexible production systems are becoming more and more essential to customers due to their innovative product requirements. Because these flexible production systems can handle variations in the assembled product, process sequence, and production volume, they provide several advantages for an industry by reworking defective items to reduce the size of the part inventory and the lead time. Indeed, this approach improves product quality and system reliability, leading to a more competitive business than under a traditional production system. On the other hand, there are some limitations to flexible production systems. As the model focused on reworking defective items, it mainly considered electronic parts as the assembled items, which could accurately be identified as defective during the production process. Since space and budget were limited, difficulties could arise due to increasing customer demand, even though flexible production systems should fulfill customers' requirements.

In this study, some electronic goods like mobile phones were considered as assembled products. However, the system may change if one considers another complex production system like car production. For a car production system's reliability, maintenance is essential. Future researcher may consider this type of complex system to extend this study. Human inspection of defective items is one of the limitations of this study, and autonomation policies can be adopted within the multiple-stage production process for the inspection. This study considered a single-stage production for all parts of the assembled items, which is another limitation of this study. This model can be considered for the multi-assembly production system, wherein the autonomation policy can be adopted for the trade-off between autonomation and inspection error costs. Different intelligent technology can make the manufacturing system smart. This model can be further extended by considering the random breakdowns within each production system. By considering deteriorating items, one can extend this study in the future (Mishra et al. [68]).

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Appendix A. Values of T_1, T_2, T_3, T_4, and T_5

The values of the time $T_1 - T_5$ are provided as follows:

$$T_1 = \frac{B_j}{(P_j(1-E[\beta_j]) - D_j)};\ T_2 = \frac{Q_j}{P_j} - \frac{B_j}{(P_j(1-E[\beta_j]) - D_j)};\ T_3 = \frac{Q_j}{P_j}E[\beta_j];\ T_4 = \frac{Q_j\left[1-(1+E[\beta_j])\frac{D_j}{P_j}\right] - B_j}{D_j};\ T_5 = \frac{B_j}{D_j}$$

Appendix B. Different Values for Optimality Calculation

$$\begin{aligned}
R_* &= \sum_{j=1}^{m}\left[(1-E[\beta_j]) - \frac{D_j}{P_j}\right];\ R_1 = \sum_{j=1}^{m}\left[(Q_j^2 + B_j^2)(1-E[\beta_j])\right];\ R_2 = \sum_{j=1}^{m} Q_j^2 D_j^2\left(1+E[\beta_j]+E[\beta_j]^2\right);\\
R_3 &= \sum_{j=1}^{m} Q_j^2 D_j\left(E[\beta_j]^3 - 2\right);\ R_4 = \sum_{j=1}^{m}\left[S_j D_j + B_{cj}B_j D_j + P_j E[\beta_j] D_{r_j} D_j + B_{lj}\left(\frac{B_j^2(1-E[\beta_j])}{2R_*}\right) + \frac{B_j^2(1-E[\beta_j])H_j}{2R_*}\right]\\
R_5 &= \sum_{j=1}^{m}\left[\frac{1}{2R_*}\left[(1-E[\beta_j]) + \frac{D_j^2}{P_j^2}\left(1+E[\beta_j]+(E[\beta_j])^2\right) + \frac{D_j}{P_j}\left((E[\beta_j])^3 - 2\right)\right]\right];\\
R_6 &= \left[(1-E[\beta_j]) + \frac{D_j^2}{P_j^2}\left(1+E[\beta_j]+(E[\beta_j])^2\right) + \frac{D_j}{P_j}\left((E[\beta_j])^3 - 2\right)\right]\\
R_7 &= \left[\frac{2R_1 D_j^2}{P_j^3 R_*^3} + \frac{2R_1 D_j}{P_j^3 R_*^2} + \frac{2R_2 D_j}{P_j^4 R_*^2} + \frac{6R_2}{P_j^4 R_*} + \frac{4R_2 D_j}{P_j^5 R_*^2} + \frac{2R_2 D_j^2}{P_j^5 R_*^3} + \frac{R_3 D_j}{P_j^3 R_*^2} + \frac{2R_3}{P_j^3 R_*} + \frac{3R_3 D_j}{P_j^4 R_*^2}\right]\\
R_8 &= \frac{1}{Q_j^2}\left[-E[\beta_j]D_{r_j}D_j + B_{lj}\left(\frac{B_j^2(1-E[\beta_j])D_j}{2R_*^2 P_j^2}\right) + \frac{B_j^2(1-E[\beta_j])H_j D_j}{22R_*^2 P_j^2}\right];\ R_9 = \left[-\frac{D_j}{2R_*^2 P_j^2}\right]R_6;\\
R_{10} &= \frac{D_j}{P_j^2}\left(2-(E[\beta_j])^3\right) - \frac{2D_j^2(1+E[\beta_j]+(E[\beta_j])^2)}{P_j^3}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 L(P_j,Q_j,B_j,\lambda_1,\lambda_2)}{\partial P_j^2} &= \frac{H_j R_7}{2Q_j};\ \frac{\partial^2 L(P_j,Q_j,B_j,\lambda_1,\lambda_2)}{\partial Q_j^2} = \frac{2R_4}{Q_j^3};\ \frac{\partial^2 L(P_j,Q_j,B_j,\lambda_1,\lambda_2)}{\partial B_j^2} = \frac{(B_{lj}+H_j)(1-E[\beta_j])}{Q_j R_*}\\
\frac{\partial^2 L(P_j,Q_j,B_j,\lambda_1,\lambda_2)}{\partial Q_j B_j} &= \frac{\partial^2 L(P_j,Q_j,B_j,\lambda_1,\lambda_2)}{\partial B_j Q_j} = \frac{B_j(B_{lj}+H_j)(E[\beta_j]-1)}{Q_j^2 R_*} - \frac{B_{cj}}{Q_j^2}\\
\frac{\partial^2 L(P_j,Q_j,B_j,\lambda_1,\lambda_2)}{\partial Q_j P_j} &= \frac{\partial^2 L(P_j,Q_j,B_j,\lambda_1,\lambda_2)}{\partial P_j Q_j} = \frac{R_8}{Q_j^2} + H_j R_9 + \frac{R_{10}H_j}{2R_*}\\
\frac{\partial^2 L(P_j,Q_j,B_j,\lambda_1,\lambda_2)}{\partial P_j B_j} &= \frac{\partial^2 L(P_j,Q_j,B_j,\lambda_1,\lambda_2)}{\partial B_j P_j} = \frac{D_j B_j(B_{lj}+H_j)(E[\beta_j]-1)}{Q_j R_*^2 P_j^2}\\
|H_{22}|_{(P_j,Q_j)} &= \frac{\partial^2 L(\cdot)}{\partial Q_j^2}\frac{\partial^2 L(\cdot)}{\partial P_j^2} - \left(\frac{\partial^2 L(\cdot)}{\partial P_j Q_j}\right)^2 = \left[\frac{2R_4}{Q_j^3}\right]\left[\frac{H_j R_7}{2Q_j}\right] - \left[\frac{R_8}{Q_j^2} + H_j R_9 + \frac{R_{10}H_j}{2R_*}\right]^2 > 0
\end{aligned}$$

According to the calculation, the first term is greater than the square term; thus, it is the positive part. The third-order principal minor of $|H|$ is

$$
\begin{aligned}
|H_{33}|_{(P_j,Q_j,B_j)} &= \begin{vmatrix} \frac{\partial^2 L(\cdot)}{\partial (Q_j)^2} & \frac{\partial^2 L(\cdot)}{\partial Q_j \partial P_j} & \frac{\partial^2 L(\cdot)}{\partial Q_j \partial B_j} \\ \frac{\partial^2 L(\cdot)}{\partial P_j \partial Q_j} & \frac{\partial^2 L(\cdot)}{\partial (P_j)^2} & \frac{\partial^2 L(\cdot)}{\partial P_j \partial B_j} \\ \frac{\partial^2 L(\cdot)}{\partial B_j \partial Q_j} & \frac{\partial^2 L(\cdot)}{\partial B_j \partial P_j} & \frac{\partial^2 L(\cdot)}{\partial B_j^2} \end{vmatrix} = \frac{\partial^2 L(\cdot)}{\partial B_j \partial Q_j}\left[\frac{\partial^2 L(\cdot)}{\partial Q_j \partial P_j}\frac{\partial^2 L(\cdot)}{\partial P_j \partial B_j} - \frac{\partial^2 L(\cdot)}{\partial Q_j \partial B_j}\frac{\partial^2 L(\cdot)}{\partial (P_j)^2}\right] - \frac{\partial^2 L(\cdot)}{\partial B_j \partial P_j}\left[\frac{\partial^2 L(\cdot)}{\partial (Q_j)^2}\frac{\partial^2 L(\cdot)}{\partial P_j \partial B_j}\right. \\
&- \left.\frac{\partial^2 L(\cdot)}{\partial Q_j \partial B_j}\frac{\partial^2 L(\cdot)}{\partial P_j \partial Q_j}\right] + \frac{\partial^2 L(\cdot)}{\partial (B_j)^2}|H_{22}| \\
&= \left[\frac{B_j(B_{lj}+H_j)(E[\beta_j]-1)}{Q_j^2 R_*} - \frac{B_{cj}}{Q_j^2}\right]\left[\frac{R_8}{Q_j^2} + H_j R_9 + \frac{R_{10} H_j}{2R_*}\right]\left[\frac{D_j B_j(B_{lj}+H_j)(E[\beta_j]-1)}{Q_j R_*^2 P_j^2}\right] \\
&- \left[\frac{B_j(B_{lj}+H_j)(E[\beta_j]-1)}{Q_j^2 R_*} - \frac{B_{cj}}{Q_j^2}\right]^2\left[\frac{H_j R_7}{2Q_j}\right] - \left[\frac{D_j B_j(B_{lj}+H_j)(E[\beta_j]-1)}{Q_j R_*^2 P_j^2}\right]^2\left[\frac{2R_4}{Q_j^3}\right] \\
&+ \left[\frac{D_j B_j(B_{lj}+H_j)(E[\beta_j]-1)}{Q_j R_*^2 P_j^2}\right]\left[\frac{B_j(B_{lj}+H_j)(E[\beta_j]-1)}{Q_j^2 R_*} - \frac{B_{cj}}{Q_j^2}\right]\left[\frac{R_8}{Q_j^2} + H_j R_9 + \frac{R_{10} H_j}{2R_*}\right] \\
&+ \left[\frac{(B_{lj}+H_j)(1-E[\beta_j])}{Q_j R_*}\right]|H_{22}| > 0
\end{aligned}
$$

$[L(\cdot) = L(P_j, Q_j, B_j, \lambda_1, \lambda_2)]$. Similarly, using these values, the third-order is greater than zero.

Hence, the optimum solutions of the decision variables are global solutions.

References

1. Taleizadeh, A.A.; Alizadeh-Basban, N.; Niaki, S. A closed-loop supply chain considering carbon reduction, quality improvement effort, and return policy under two remanufacturing scenarios. *J. Clean. Prod.* **2019**, *232*, 1230–1250. [CrossRef]
2. Machado, M.-C.; Vivaldini, M.; Oliveira, O.J. Production and supply-chain as the basis for SMEs' environmental management development: A systematic literature review. *J. Clean. Prod.* **2020**, *273*, 123141. [CrossRef]
3. Mishra, U.; Wu, J.Z.; Sarkar, B. Optimum sustainable inventory management with backorder and deterioration under controllable carbon emissions. *J. Clean. Prod.* **2021**, *123699*, 136539. [CrossRef]
4. Cárdenas-Barrón, L.E. Economic production quantity with rework process at a single-stage manufacturing system with planned backorders. *Comp. Indus. Eng.* **2009**, *57*, 1105–1113.
5. Sarkar, B.; Cárdenas-Barrón, L.E.; Sarkar, M.; Singgih, M.L. An economic production quantity model with random defective rate, rework process and backorders for a single stage production system. *J. Manuf. Syst.* **2014**, *33*, 423–435. [CrossRef]
6. Aydin, R.; Kwong, C.K.; Ji, P. Coordination of the closed-loop supply chain for product line design with consideration of remanufactured products. *J. Clean. Prod.* **2016**, *114*, 286–298. [CrossRef]
7. Hariga, M.; As′ad, R.; Khan, Z. Manufacturing-remanufacturing policies for a centralized two stage supply chain under consignment stock partnership. *Int. J. Prod. Econ.* **2017**, *183*, 362–374. [CrossRef]
8. Qingdi, K.; Jie, L.; Song, S.; Wang, H. Timing Matching Method of Decision-Making in Predecisional Remanufacturing for Mechanical Products. *Procedia CIRP* **2019**, *80*, 566–571. [CrossRef]
9. Sivashankari, C.K.; Panayappan, S. Production inventory model with reworking of imperfect production, scrap and shortages. *Int. J. Manag. Sci. Eng. Manag.* **2014**, *1*, 9–20. [CrossRef]
10. Sanjai, M.; Periyasamy, S. Production inventory model with reworking of imperfect items and integrates cost reduction delivery policy. *Int. J. Oper. Res.* **2018**, *32*, 329–349. [CrossRef]
11. Gao, J.; Xiao, Z.; Wei, H.; Zhou, G. Dual-channel green supply chain management with eco-label policy: A perspective of two types of green products. *Comp. Indus. Eng.* **2020**, *146*, 106613. [CrossRef]
12. Alexopoulos, K.; Anagiannis, L.; Nikolakis, N.; Chryssolouris, G. A quantitative approach to resilience in manufacturing systems. *Int. J. Prod. Res.* **2022**, *60*, 7178–7193. [CrossRef]
13. Lagoudakis, K.H. The effect of online shopping channels on brand choice, product exploration and price elasticities. *Int. J. Ind. Organ.* **2023**, *87*, 102918. [CrossRef]
14. Mukherjee, T.; Sangal, I.; Sarkar, B.; Almaamari, Q.; Alkadash, T.M. How effective is reverse cross-docking and carbon policies in controlling carbon emission from the fashion industry? *Mathematics* **2023**, *11*, 2880. [CrossRef]
15. Yang, L.; Zou, H.; Shang, C.; Ye, X.; Rani, P. Adoption of information and digital technologies for sustainable smart manufacturing systems for industry 4.0 in small, medium, and micro enterprises (SMMEs). *Technol. Forecast. Soc. Chang.* **2023**, *188*, 122308. [CrossRef]
16. Bazan, E.; Jaber, M.Y.; El Saadany, A.M.A. Carbon emissions and energy effects on manufacturing–remanufacturing inventory models. *Comp. Indus. Eng.* **2015**, *178*, 109126. [CrossRef]
17. Bhatia, P.; Elsayed, N.D. Facilitating decision-making for the adoption of smart manufacturing technologies by SMEs via fuzzy TOPSIS. *Int. J. Prod. Econ.* **2023**, *257*, 108762. [CrossRef]
18. Chai, Q.; Sun, M.; Lai, K.; Xiao, Z. The effects of government subsidies and environmental regulation on remanufacturing. *Comp. Indus. Eng.* **2023**, *178*, 109126. [CrossRef]

19. Assid, M.; Gharbi, A.; Hajji, A. Integrated control policies of production, returns' replenishment and inspection for unreliable hybrid manufacturing-remanufacturing systems with a quality constraint. *Comp. Indus. Eng.* **2023**, *176*, 109000. [CrossRef]
20. Sarkar, M.; Park, K.S. Reduction of makespan through flexible production and remanufacturing to maintain the multi-stage automated complex production system. *Comp. Indus. Eng.* **2023**, *177*, 108993. [CrossRef]
21. Zheng, M.; Dong, S.; Zhou, Y.; Choi, T.M. Sourcing decisions with uncertain time-dependent supply from an unreliable supplier. *Eur. J. Oper. Res.* **2022**, *308*, 1365–1379. [CrossRef]
22. Saxena, N.; Sarkar, B. Random misplacement and production process reliability: A sustainable industrial approach to deal with the discrepancy and deficiency. *J. Indus. Manag. Optim.* **2023**, *19*, 4844–4873. [CrossRef]
23. Taleizadeh, A.A.; Moshtagh, M.S.; Vahedi-Nouri, B.; Sarkar, B. New products or remanufactured products: Which is consumer-friendly under a closed-loop multi-level supply chain? *J. Retail. Consum. Serv.* **2023**, *73*, 103295. [CrossRef]
24. Kanishka, K.; Acherjee, B. A systematic review of additive manufacturing-based remanufacturing techniques for component repair and restoration. *J. Manuf. Process.* **2023**, *89*, 220–283. [CrossRef]
25. Dey, B.K.; Sarkar, M.; Chaudhuri, K.; Sarkar, B. Do you think that the home delivery is good for retailing? *J. Retail. Consum. Serv.* **2023**, *72*, 103237. [CrossRef]
26. Amrouche, N.; Pei, Z.; Yan, R. Service strategies and channel coordination in the age of E-commerce. *Expert Syst. Appl.* **2023**, *214*, 119135. [CrossRef]
27. Zhu, S.; Li, J.; Wang, S.; Xia, Y.; Wang, Y. The role of blockchain technology in the dual-channel supply chain dominated by a brand owner. *Int. J. Prod. Econ.* **2023**, *258*, 108791. [CrossRef]
28. Saxena, N.; Sarkar, B. How does the retailing industry decide the best replenishment strategy by utilizing technological support through blockchain? *J. Retail. Consum. Serv.* **2023**, *71*, 103151. [CrossRef]
29. Wee, H.M.; Wang, W.T.; Cárdenas-Barrón, L.E. An alternative analysis and solution procedure for the EPQ model with rework process at a single-stage manufacturing system with planned backorders. *Comp. Indus. Eng.* **2014**, *33*, 423–435. [CrossRef]
30. Polotski, V.; Kenne, J.-P.; Gharbi, A. Joint production and maintenance optimization in flexible hybrid Manufacturing-Remanufacturing systems under age-dependent deterioration. *Int. J. Prod. Econ.* **2019**, *216*, 239–254. [CrossRef]
31. M-Haidar, L.; Nasr, W. Production lot sizing with quality screening and rework. *App. Math. Model.* **2016**, *40*, 3242–3256. [CrossRef]
32. Silva, C.; Reis, V.; C-Silva, S. A comparision of production control systems in a flexible flow shop. *Procedia Manuf.* **2017**, *13*, 1090–1095. [CrossRef]
33. Kugele, A.S.H.; Sarkar, B. Reducing carbon emissions of a multi-stage smart production for biofuel towards sustainable development. *Comp. Indus. Eng.* **2023**, *70*, 93–113. [CrossRef]
34. Mridha, B.; Ramana, G.V.; Pareek, S.; Sarkar, B. An efficient sustainable smart approach to biofuel production with emphasizing the environmental and energy aspects. *Fuel* **2023**, *336*, 126896. [CrossRef]
35. Chen, T.-H. Optimizing pricing, replenishment and rework decision for imperfect and deteriorating items in a manufacturer-retailer channel. *Int. J. Prod. Econ.* **2017**, *183*, 539–550. [CrossRef]
36. Mtibaa, G.; Erray, W. Integrated Maintenance-Quality policy with rework process under improved imperfect preventive maintenance. *Rel. Eng. Sys. Saf.* **2018**, *173*, 1–11. [CrossRef]
37. Hage, S.-R. Dual neural network model of speech and language evolution: New insights on flexibility of vocal production systems and involvement of frontal cortex. *Curr. Opin. Behav. Sci.* **2018**, *21*, 80–87. [CrossRef]
38. Chiu, S.W.; Wu, C.-S.; Tseng, C.-T. Incorporating an expedited rate, rework, and a multi-shipment policy into a multi-item stock refilling system. *Oper. Res. Perspect.* **2019**, *6*, 100–115. [CrossRef]
39. Li, K.; Zhang, L.; Fu, H.; Liu, B. The effect of intelligent manufacturing on remanufacturing decisions. *Comp. Indus. Eng.* **2023**, *178*, 109114. [CrossRef]
40. Asadi, N.; Jackson, M.; Fundin, A. Implications of realizing mix flexibility in assembly systems for product modularity-A case study. *J. Manu. Sys.* **2019**, *52*, 13–22. [CrossRef]
41. Malik, A.I.; Sarkar, B.; Iqbal, M.W.; Ullah, M.; Khan, I.; Ramzan, M.B. Coordination supply chain management in flexible production system and service level constraint: A Nash bargaining model. *Comp. Indus. Eng.* **2023**, *177*, 109002. [CrossRef]
42. Centobelli, P.; Cerchione, R.; Esposito, E.; Passaro, R. Determinants of the transition towards circular economy in SMEs: A sustainable supply chain management perspective. *Int. J. Prod. Econ.* **2021**, *242*, 108297. [CrossRef]
43. Li, J.; Ou, J.; Cao, B. The roles of cooperative advertising and endogenous online price discount in a dual-channel supply chain. *Comp. Indus. Eng.* **2023**, *176*, 108980. [CrossRef]
44. Cifone, F.D.; Hoberg, K.; Holweg, M.; Staudacher, A.P. 'Lean 4.0': How can digital technologies support lean practices? *Int. J. Prod. Econ.* **2021**, *241*, 108258. [CrossRef]
45. Aslam, H.; Wanke, P.; Khalid, A.; Roubaud, D.; Waseem, M.; Jabbour, C.J.C.; Grebinevych, O.; de Sousa, Jabbour, A.B.L. A scenario-based experimental study of buyer supplier relationship commitment in the context of a psychological contract breach: Implications for supply chain management. *Int. J. Prod. Econ.* **2022**, *249*, 108503. [CrossRef]
46. Chaudhari, U.; Bhadoriya, A.; Jani, M.Y.; Sarkar, B. A generalized payment policy for deteriorating items when demand depends on price, stock, and advertisement under carbon tax regulations. *Math. Comput. Simul.* **2023**, *207*, 556–574. [CrossRef]
47. Sarkar, B.; Guchhait, R. Ramification of information asymmetry on a green supply chain management with the cap-trade, service, and vendor-managed inventory strategies. *Electron. Commer. Res. Appl.* **2023**, *60*, 101274. [CrossRef]

48. Zhang, T.; Dong, P.; Chen, X.; Gong, Y. The impacts of blockchain adoption on a dual-channel supply chain with risk-averse members. *Omega* **2023**, *114*, 102747. [CrossRef]
49. Bachar, R.K.; Bhuniya, S.; Ghosh, S.K.; AlArjani, A.; Attia, E.; Uddin, M.S.; Sarkar, B. Product outsourcing policy for a sustainable flexible manufacturing system with reworking and green investment. *Math. Biosci. Eng.* **2023**, *20*, 1376–1401. [CrossRef]
50. Huang, B.; Wu, A. EOQ model with batch demand and planned backorders. *App. Math. Model.* **2016**, *40*, 5482–5496. [CrossRef]
51. Mittal, M.; Khanna, A.; Jaggi, C.K. Retailer's ordering policy for deteriorating imperfect quality items when demand and price are time-dependent under inflationary conditions and permissible delay in payments. *Int. J. Proc. Manag.* **2017**, *10*, 142–149. [CrossRef]
52. Xu, Y.; Bisi, A.; Dada, M. A finite-horizon inventory system with partial backorders and inventory holdback. *Oper. Res. Lett.* **2017**, *45*, 315–322. [CrossRef]
53. Mukherjee, T.; Sangal, I.; Sarkar, B.; Almaamari, Q.A. Logistic models to minimize the material handling cost within a cross-dock. *Math. Biosci. Eng.* **2023**, *20*, 3099–3119. [CrossRef]
54. Bao, L.; Liu, Z.; Yu, Y. On the decomposition property for a dynamic inventory rationing problem with multiple demand classes and backorder. *Euro. J. Oper. Res.* **2018**, *265*, 99–106. [CrossRef]
55. Kilic, O.A.; Tunc, H. Heuristics for the stochastic economic lotsizing problem with remanufacturing under backordering costs. *Euro. J. Oper. Res.* **2019**, *276*, 880–892. [CrossRef]
56. Guo, L.; Wang, Y.; Kong, D.; Zhang, Z.; Yang, Y.T. Decisions on spare parts allocation for repaiable isolated system with dependent backorders. *Comp. Indus. Eng.* **2019**, *127*, 8–20. [CrossRef]
57. Bertazzi, L.; Moezi, S.D.; Maggioni, F. The value of integration of full container load, less than container load and air freight shipments in vendor–managed inventory systems. *Int. J. Prod. Econ.* **2021**, *241*, 108260. [CrossRef]
58. Bi, G.; Wang, P.; Wang, D.; Yin, Y. Optimal credit period and ordering policy with credit-dependent demand under two-level trade credit. *Int. J. Prod. Econ.* **2021**, *242*, 108311. [CrossRef]
59. Charpin, R.; Lee, M.K.; Wu, T. Mobile procurement platforms: Bridging the online and offline worlds in China's restaurant industry. *Int. J. Prod. Econ.* **2021**, *241*, 108256. [CrossRef]
60. Bouzekri, H.; Bara, N.; Alpan, G.; Giard, V. An integrated Decision Support System for planning production, storage and bulk port operations in a fertilizer supply chain. *Int. J. Prod. Econ.* **2022**, *252*, 108561. [CrossRef]
61. Buisman, M.E.; Rohmer, S.U. Inventory decisions for ameliorating products under consideration of stochastic demand. *Int. J. Prod. Econ.* **2022**, *252*, 108595. [CrossRef]
62. Ashraf, M.H.; Chen, Y.; Yalcin, M.G. Minding Braess Paradox amid third-party logistics hub capacity expansion triggered by demand surge. *Int. J. Prod. Econ.* **2022**, *248*, 108454. [CrossRef]
63. Zhou, Q.; Meng, C.; Sheu, J.-B.; Yuen, K.F. Remanufacturing mode and strategic decision: A game-theoretic approach. *Int. J. Prod. Econ.* **2023**, *260*, 108841. [CrossRef]
64. Balter, A.G.; Huisman, K.J.; Kort, P.M. Effects of creative destruction on the size and timing of an investment. *Int. J. Prod. Econ.* **2022**, *252*, 108572. [CrossRef]
65. Astvansh, V.; Jindal, N. Differential Effects of Received Trade Credit and Provided Trade Credit on Firm Value. *Prod. Oper. Manag.* **2023**, *31*, 781–798. [CrossRef]
66. Shajalal, M.; Hajek, P.; Abedin, M.Z. Product backorder prediction using deep neural network on imbalanced data. *Int. J. Prod. Res.* **2023**, *61*, 302–319. [CrossRef]
67. Heydari, J.; Govindan, K.; Sadeghi, R. Reverse supply chain coordination under stochastic remanufacturing capacity. *Int. J. Prod. Econ.* **2018**, *202*, 1–11. [CrossRef]
68. Mishra, U.; Mashud, A.H.M; Roy, S.K.; Uddin, M.S. The effect of rebate value and selling price-dependent demand for a four-level production manufacturing system. *J. Indus. Manag. Optim.* **2023**, *19*, 1367–1394. [CrossRef]