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Review

A Survey of Combined Heat and Power-Based Unit Commitment Problem: Optimization Algorithms, Case Studies, Challenges, and Future Directions

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Abstract: Combined generation units of heat and power, known as CHP units, are one of the most prominent applications of distributed generations in modern power systems. This concept refers to the simultaneous operation of two or more forms of energy from a simple primary source. Due to the numerous environmental, economic, and technical advantages, the use of this technology in modern power systems is highly emphasized. As a result, various issues of interest in the control, operation, and planning of power networks have experienced significant changes and faced important challenges. In this way, the unit commitment problem (UCP) as one of the fundamental studies in the operation of integrated power, and heat systems have experienced some major conceptual and methodological changes. This work, as a complementary review, details the CHP-based UCP (CHPbUCP) in terms of objective functions, constraints, simulation tools, and applied hardwares. Furthermore, some useful data on case studies are provided for researchers and operators. Finally, the work addresses some challenges and opens new perspectives for future research.

Keywords: classical methods; combined heat and power (CHP) units; CHP-based UCP (CHPbUCP); constraints; deterministic; heat-only units (HOUs); hybrid methods; non-deterministic; objective function (OF); optimization algorithms; power-only units (POUs); total operation cost (TOC); total pollutant emission (TPE); uncertainty

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1. Introduction

In today's power plants, due to the combustion of fossil fuels and the resulting heat, the energy produced is converted into electrical energy. The most common form of these systems is thermal power plants, which play a major role in providing the electricity needs of different communities. In these power plants, on average, only one-third of the input fuel energy is converted into useful electrical energy; in other words, the efficiency of these power plants is about 30 to 35 percent. In this type of electrical energy generation, a large amount of thermal energy is wasted through various equipment such as condensers, boilers, cooling towers, pumps, and piping systems.

The use of waste heat in the combustion process increases energy efficiency, reduces fuel consumption, and thus reduces the cost of energy supply. The waste heat of these systems can be used for heating, cooling, and many other industrial processes. Combined heat and power generation, in addition to increasing efficiency and reducing fuel consumption, can generally reduce greenhouse gas emissions. The Federal Energy Regulatory Commission (FERC) defined the cogeneration unit as the simultaneous production of electrical power and useful thermal energy by sequentially using energy from a single fuel source [1].

Large combined heat and power (CHP) units are often used to feed district heating grids in Europe, China, and post-Soviet countries [2]. In addition, the integrated energy grids in Europe are moving towards a smart energy grid by connecting electricity, gas, and heat grids to increase their flexibility and efficiency [3]. Employing the flexibility of district heating systems (DHSs), by integrating electricity and heat systems, improves the overall network flexibility, reduces the operating costs of the entire system, and reduces renewable energy source (RES) curtailment [4]. In CHP units, the heat energy produced is used as an energy source in the electricity generation process. Consumers who need a lot of heat energy during the day (manufacturing industries, hospitals, buildings, large offices, laundries, etc.) can use CHP to reduce their total costs. These units are environmentally friendly technology due to their high energy efficiency compared to conventional thermal power units [5]. A CHP unit can significantly increase fuel efficiency from about 30% [6] to 90% [7], reduce production costs by about 10 to 40% [8], and reduce environmental emissions by 13–18% [9,10]. In addition, losses in CHP units can be reduced even by 10–20% of fuel consumption [11].

Generally, the CHP units are designed into two categories, coupled and decoupled. Coupled CHP generation implies a linear dependence between power generation, heat generation, and fuel consumption, resulting in one degree of freedom (DOF). The DOF means that only one of the mentioned parameters should be mentioned as an optimization variable and the rest are derived from the optimized variable. Decoupled CHP includes two DOF in both combinations of power generation, heat generation, and fuel consumption and requires two optimization variables. The remaining parameter can be searched from two optimized parameters [3].

The use of this new technology has become increasingly popular due to its numerous environmental benefits (EnBs), the increased energy efficiency, and, of course, the many new challenges that various areas of modern power systems, including planning and operation have brought. Solving these challenges has led to new problems and studies in power systems, especially in the field of operation. Examples of these new problems are heat and power economic dispatch (CHPED) [12], combined heat and power economic emission dispatch (CHPEED) [13], and the CHP-based unit commitment problem (CHPbUCP), or cogeneration-based unit commitment problem (CbUCP). As the literature confirms, the CHPbUCP is a problem of very large dimensions, which, in some cases, uses decomposition methods for the time horizon of 2020–2030 in the Berlin CHP system; by dividing the mentioned time into 2 half-years and applying a time interval of 4 h, the size of the problem equals to 480,668 variables, 828,630 constraints, and 10,458 binary variables [14].

UCP is a sub-problem in energy scheduling that determines the shutdown and start-up times of power plants (or generation units) to meet the forecasted demand in a cost-effective manner by considering various constraints [5]. UCP decisions are usually evaluated only once or twice a day [15]. On the other hand, the CHPbUCP tries to find the optimal scheduling of CHP plants in a heat and power system within a specified time interval of one week or more to minimize the total operating costs. The vast majority of the literature review research deals exclusively with conventional or power-only units (POUs) [16–19], and a very limited number of studies (e.g., ref. [20]) have addressed it as a general rather than a detailed note.

1.1. A Brief Review of Relevant Previous Research Works

This section refers to the most important reviews in the field of CHP units, which specifically examine the optimization of the operation of these units, along with different aspects of each study. Providing this general background helps highlight the neglected aspects of the under-studied topic and the importance of investigating them.

In one of the first studies from 2014, Cho et al. [21] reviewed the methods applied to different concepts of combined cooling, heat, and power (CCHP) systems. They classified the proposed references in this field into four main categories of analyses: energy savings or energy efficiency, economics, exergy, and emissions. In addition, two types of strategies, including system design and optimal operating conditions, are detailed for CCHP system optimization. Furthermore, some mathematical methods, including genetic algorithm (GA), stochastic optimization (SO), mixed integer non-linear programming (MINLP), particle swarm optimization (PSO), and multi-objective optimization (MOO) to find the optimal operating strategies are addressed. The interesting note about this reference is that, despite its comprehensiveness in optimization and performance improvement, there is no direct and efficient reference to the CHPbUCP.

Ref. [22] reviewed the planning, modeling, and energy management (EM) strategies proposed for the CCHP microgrid. For this purpose, the authors first described four constituent elements of the CCHP microgrid (MG) as storage system, prime mover, renewable energy source, and load. In addition, based on what is described in [23], the important parameters in the planning of the CCHP MG were addressed and detailed as the economic benefits (EcBs), the EnBs, and the energy utilization efficiency benefits (EUEBs). Furthermore, the maximum rectangle (MR) method, non-linear programming (NLP), linear programming (LP), mixed-integer non-linear programming (MINLP), and GA fuzzy logic (FL) are addressed as the suitable methods for EM in the CCHP MG. Similar to the previous review article (i.e., [21]), this reference did not specifically address the issue of the CHPbUCP.

Mohammadi et al. [24] reviewed the concepts and models of energy hub (EH) by focusing on the multi-energy systems (MESs). They defined an EH as a system in which generation, conversion, energy storage, and consumption of different energy carriers are possible and practical. The components of an EH are defined as resources (including renewable energy sources (RESs), natural gas (NG), electricity, and district heating (DH)), a conversion and transmission sub-system, energy storage systems (ESSs), and demand and consumption (electrical, heat, . . .). Moreover, different types related to each part are discussed in the literature. In addition, the authors have provided a general conceptual framework for EHs. This reference, similar to the other cited studies, did not focus significantly on the CHPbUCP.

In another study [25], and as a supplementary work in line with ref. [24], the optimal management (OM) of EHs and smart EHs (SEHS) has been comprehensively investigated. To achieve this goal, the application of micro energy hubs (mEHs) including the industrial, residential, agricultural, and commercial types is detailed with special focus on different demand-side management programs (DSMPs). Despite the outstanding efforts of the authors of this work on OM, the issue of the CHPbUCP is not specifically mentioned.

A comprehensive review of the application of heuristic algorithms to search for the optimal solution of combined heat and power economic dispatch (CHPED) from economic and environmental aspects has been investigated in [26]. After introducing the general structure of the CHPED problem, including the objective function for minimizing the total fuel cost of the committed CHP units, heat-only units (HOUs), and power-only units (POUs), the problem constraints are addressed as the valve point loading effects (VPLE) consideration, power balance, heat balance, capacity limits of POUs, CHP units, and HOUs, and power transmission loss of the network. The authors described the used heuristic algorithms to solve the CHPED problem as the bee colony optimization (BCO), artificial immune system (AIS), civilized swarm optimization (CSO), crisscross optimization algorithm (COA), cuckoo search (CS), differential evolution (DE), exchange market algorithm (EMA), firefly algorithm (FA), gravitational search algorithm (GSA), GA, grey wolf optimization (GWO), group search optimization (GSO), harmony search (HS), invasive weed optimization (IWO), krill herd (KH), line-up competition algorithm (LCA), PSO, and teaching learning-based optimization (TLBO). The paper was supported by some simulation case studies for various operating conditions. This paper basically shows the CHPED problem, and there is no background regarding the CHPbUCP.

Salgado and Pedrero [1] have investigated the short-term operation scheduling in CHP, or cogeneration systems from 1983 to 2006. The authors, at the beginning, have introduced a general background of some research and developments on these systems. The authors have then focused on short-term models, considering the search for one-day planning periods with hourly periods, which cover some operation issues such as ED and UC. They categorized different applied methods for short-term operation planning into artificial intelligence (AI), classical methods, and stochastic modeling. They also mentioned topics such as the environmental aspects, time of uses rate (TOU), the multi-hour economic dispatch (ED) problem, and multi-objective optimization problems.

In another study, the technologies and operation strategies (OSs) for the flexibility of CHP units are investigated by Wang et al. [27]. The main focuses of this research are on CHP flexibility applications in integrated energy systems (IESs), technical flexibility of CHP technologies, and operation strategies of CHPs based on two types of strategies: cost-based operation strategy (CBOS) and price-based operation strategy (PBOS). There are three basic missions for flexibility in IESs, including renewable energy plant owner (real-time power balancing, reduce renewable energy curtailment, and arbitrage in energy market), system operator (reserves, system inertia, voltage control), and DSM. In addition, the general structures of the optimization problems for the CBOS and PBOS are detailed by addressing some optimization programs and algorithms and addressing their application to ED and the UCP. What is interesting about this reference is that, unlike previous references that did not mention the CHPbUCP, this reference (i.e., [27]) stated that a base UCP and ED are analyzed in some research ignoring uncertainty, which is solved by using some commercial solvers and simulation tools such as CPLEX, GUROBI, and DICOPT. However, no further explanations on the CHPbUCP are provided.

Sadeghi et al. [28] presented a survey on the planning strategies proposed for EHs accomplished by the important factors affecting the correlation of energy systems (ESs). They categorized the contents of EHs as direct connections, converters (adapting and changing types), and ESSs. In addition, from the perspective of the planning strategies proposed in EH, two main categories are mentioned as expansion planning (long-term consisting of integrated expansion planning of NG and power networks, co-planning expansion of NG pipelines and transmission lines neglecting the EH approach, augmented expansion planning of G&ES in the EH framework) and operation planning (to access low-carbon economy, considering competitive energy markets and in the paradigm of smart grids, buildings' energy systems, simultaneously monitoring energy systems' conditions and reliability). The authors addressed the relevant research related to each content in detail. Again, similar to previous reviews, the CHPbUCP is not detailed in the described strategies.

Ref. [29] presented a survey on optimization and modeling of the CHPD problem. Unlike the previous literature, this reference discussed the topic of the CHPbUCP in a very short section. Moreover, the main differences and challenges of the CHPbUCP, such as the dynamic nature compared to CHPED and CHPEED problems, are presented. Moreover, MLIP and linearization methods are proposed as the most common techniques to solve the CHPbUCP. Furthermore, some issues such as the difficulty of handling constraints, premature convergence, and MOO are addressed in relation to the concept of optimization in CHP units. The paper ends with some simulation results related to various case study tests.

Zhang et al. [30] presented a survey on modeling and solution techniques for optimal operation of IEHS. Perhaps one of the most important differences of this reference compared to previous ones is that it has examined the uncertainties related to the IEHS issues in a more comprehensive and detailed manner. In this regard, stochastic programming (SP) and robust optimization (RO) are described in detail. The authors classified the characteristics and structure of IEHS as an electric power system (EPS), including generation, transmission, and distribution, or district heating system (DHS). In addition, the general model for optimal operation of an IEHS is addressed as the objective function (economic efficiency, social welfare, or accommodation of RESs) and the constraints (EPS constraints, DHS constraints, EPS and DHS coupled constraints) for satisfying the security issues. In addition, some general explanations of different operating modes, including steady-state, quasi-dynamic, and dynamic categories, are detailed. The solution methods for optimal operation of IEHS are classified and detailed in four significant categories of integrated energy flow, decentralized optimization, relaxation and convexification, and intelligent algorithms. A very important point about this reference is that despite its comprehensiveness in studying the methods and strategies related to the field of IEHS compared to previous research, the CHPbUCP has not yet been investigated separately and comprehensively.

Bagherian et al. [31] presented a classification of optimization methods applied to integrated energy systems (IES), considering the RESs, using the CHP and CCHP systems. They classified the optimization techniques into two main groups of constrained and unconstrained methods. Furthermore, they discussed some applications of GA, PSO, and mixed integer linear programming (MILP) to different problems, consisting of the UCP.

In ref. [32], a comprehensive review of UC methods since 2015 is presented without any exclusive classification or description of the CHPbUCP. In addition, ref. [33] presented some detailed statistics on economic load dispatch (ELD) and CHP scheduling (CHPS) without any classification.

Moreover, Alsagri and Alrobaian [34] have presented a complete and updated overview of meta-heuristic optimization algorithms used in CHP systems, including the CHPED and CHPEED problems, in two categories of single objective and multi-objective algorithms. They divided the suggested algorithms in single objective for CHP optimization into evolutionary algorithms (EAs) (including GAs, differential evolution (DE) algorithm, hyper-spherical search (HSS), artificial immune system (AIS), and the stochastic fractal search (SFS) algorithms), swarm intelligence-based (SI-based) algorithms (including different variants of PSO, whale optimization algorithm (WOA), cuckoo search algorithm (CSA), group search optimization (GSO), FA, bee colony optimization (BCO), ant colony search algorithm (ACSA), squirrel search algorithm (SSA), and grey wolf optimization (GWO)), human-based algorithms (including harmony search (HS), teaching learning-based optimization (TLBO), exchange market algorithm (EMA), and social cognitive optimization (SCO)), physics-based algorithms (including gravitational search algorithm (GSA), charged system search algorithm (CSSA), and heat transfer search algorithm (HTS)), and hybrid meta-heuristic methods (including combining the meta-heuristics methods such as combinatorial time-varying acceleration coefficients-gravitational search algorithm-particle swarm optimization (TVAC-GSA-PSO), combining the bat algorithm (BA) and artificial bee colony (ABC) algorithm based on the chaotic-based self-adaptive (CbSA) (CbSA-BAABC), combining the meta-heuristics and the machine learning programming, and combining the meta-heuristics and the mathematical programming methods). Moreover, the MO algorithms for CHP optimization are divided into EAs (including GA versions (NSGA, NSGAII) and DE), SI-based algorithms (including PSO, GWO, FA, multi-objective bacterial colony chemotaxis algorithm (MOBCC), and the technique for order preference by similarity to an ideal solution (TOPSIS) method), and hybrid meta-heuristics (include modified cuckoo search algorithm and differential evolution (MCSA-DE) and the self-adaptive charged system search algorithm (SACSS)). Despite the comprehensive classifications presented on the optimization problems mentioned in CHPED and CHPEED concepts, this article does not provide any discussion of the CHPbUCP.

As the last example of a series of related studies in this field, ref. [20] should be addressed because the author has provided a comprehensive review of the profit-based UCP (PBUCP), dealing with the problem formulation, including various objective functions and constraints. Moreover, a complete classification of proposed methods and algorithms for the problem has been discussed. The main point in this study is that CHP units have been removed, and it has been stated that this field is outside the scope of the study.

1.2. The Reasons for This Study

Due to the importance of using CHP units in integrated electricity and heat systems after the 1990s when the initial ideas for using these units were raised, several works of research with an emphasis on the manufacturing, operation, planning, and updating of these units have been carried out or are being carried out. Different investigations have been done in terms of the application of these units in different time scales of operation and planning of modern energy systems, including different carriers.

Considering the existence of some published research in the field of the CHPbUCP, this research does not seek to repeat or duplicate the same topics and strategies. According to the author's knowledge, no comprehensive study has yet been conducted to address the frameworks, formulations, solution methods, used algorithms, and challenges posed by the CHPbUCP.

This study is precisely to meet this basic need of researchers, planners, and operators and, as one of the most serious efforts to fill this gap, tries to review and evaluate the related studies with a comprehensive view. Based on this, the most important innovations of the present article are divided and separated as follows:

- Providing a comprehensive CHPbUCP formulation consisting of different objective functions and constraints.
- Classification of different solution methods proposed to solve the CHPbUCP.
- Segmentation of various proposed algorithms for solving the under-study problem including mathematical and heuristic methods.
- Introducing the test systems used in simulation case studies as a useful guidance for researchers and planners.
- Addressing some beneficial information about the used software, simulation tools, and time-interval studies for the problem.

It should be noted that this work is mainly focused on the CHPbUCP; at the same time, there are important and significant studies that insisted on the operational issues of the UCP (i.e., [2]), analyzing the joint effects of centralized CHP plants and thermal storage on power systems (i.e., [35]), the CHPbUCP in the presence of hydropower generation (i.e., [36]), optimization of industrial microgrids considering RES, energy sources, CHP, thermal, and ESS [37], the impacts of thermal inertia as energy storage [38], integration of flow temperatures in the problem models [39], application of information theoretical evaluation of aggregation methods in the CHPbUCP [40], the electricity-aware heat UCP in a heat market [41], the UCP in multi-energy systems (MES) and microgrids (MGs) [42], and the role of CHP units as the spinning reserve in the UCP considering RES, pumped storage units, and coal/oil-based generators in a Taiwan power system [43]. Therefore, the author prefers to address these subjects in other relevant surveys.

1.3. Paper Organization

The remainder of this paper is structured as follows: Section 2 details the CHPbUCP formulation. At first, the objective functions, including the single objective, and multi-objective addressed. Then, the problem constraints, consisting of system constraints, unit constraints, and security constraints, are addressed. In Section 3, applied methods/algorithms, including deterministic methods and non-deterministic methods, are briefly introduced. The first type is detailed in classical or conventional methods and evolutionary or hybrid algorithms. The non-deterministic methods are also addressed in this section. Then, the single objective versus multi-objective and deterministic versus non-deterministic are detailed. In the next sub-sections, modeling types for the CHPbUCP, strategies, equipment, and facilities modeled, and used case studies are mentioned. Further concepts such as study periods, simulation tools and software, and PC data used are also addressed. Section 4 details some proposed methods/algorithms, including the multi-objective CHPbUCP, deterministic mathematical-based methods, deterministic heuristic-based methods, and non-deterministic-based methods. Section 5 details some open contexts and challenging issues. Finally, Section 6 concludes some important concepts.

2. CHPbUCP Formulation

The CHPbUCP is a complex, non-linear, mixed-variable, and high-dimensional problem including both integer and non-integer variables. As an important issue, the feasible operation region (FOR) of CHP units should be defined. Generally, the FOR describes the region in which a CHP unit can operate.

In this section, the general formulation of the CHPbUCP, including OF and different constraints, is introduced.

2.1. Objective Functions

2.1.1. Single Objective

The mentioned problem searches for the minimum total operation cost, including the fuel cost in a given time interval, in the single-objective form, as presented in Equation (1). Some references (e.g., [44]) have considered only the first three terms, related to the operating costs of POUs, HOUs, and CHP units in the objective function. Many references, such as [45], added the fourth term (total start-up costs) to the objective function. There are few references that consider the total shut-down costs in the formulation of the problem (e.g., [46]). Here, the full model, considering all five terms, is considered as follows:

$$\begin{aligned}&\min TOC(p,c,h,t)\\&=\min\{\sum_{t=1}^{T}(\sum_{p=1}^{N_{POU}}C_{POU,p}(P_p(t))U_p(t)+\sum_{c=1}^{N_{CHP}}C_{CHP,c}(P_c(t),H_c(t))U_c(t)+\sum_{h=1}^{N_{HOU}}C_{HOU,h}(H_h(t))U_h(t)\\&+TSUC(t)+TSDC(t))\}\end{aligned} \tag{1}$$

where

$$C_{POU.p}(P_p(t))=a_p+b_pP_p(t)+c_pP_p(t)^2+\left|d_p sin\left\{f_p\left(P_p^{min}-P_p(t)\right)\right\}\right| \tag{2}$$

$$\begin{aligned}&C_{CHP.c}(P_c(t).H_c(t))\\&\quad=\alpha_c+\beta_cP_c(t)+\gamma_cP_c(t)^2+\delta_cH_c(t)+\varepsilon_cH_c(t)^2\\&\quad+\epsilon_cP_c(t)H_c(t)\end{aligned} \tag{3}$$

$$C_{HOU.h}(H_h(t))=\zeta_h+\theta_hH_h(t)+\vartheta_hH_h(t)^2 \tag{4}$$

It should be noted that the final term of Equation (2) defines the valve point loading effects (VPLE), which are modeled in the form of a non-convex item.

In addition, in some references (i.e., [43]), the operation cost functions of POUs and CHP units are estimated by a linear function for simplicity.

The total start-up and shut-down costs (TSUC and TSDC) of all generating units are calculated as follows:

$$TSUC(t)=\sum_{p=1}^{N_{POU}}ST_p(t)+\sum_{c=1}^{N_{CHP}}ST_c(t)+\sum_{h=1}^{N_{HOU}}ST_h(t) \tag{5}$$

$$ST_n(t)=\begin{cases}HS_n & T_n^{off}(t)\le T_n^{dw}+T_n^{cold}\\ CS_n & T_n^{off}(t)>T_n^{dw}+T_n^{cold}\end{cases}\quad t\in T;n\in N_T \tag{6}$$

$$TSDC(t)=\sum_{p=1}^{N_{POU}}SD_p(t)+\sum_{c=1}^{N_{CHP}}SD_c(t)+\sum_{h=1}^{N_{HOU}}SD_h(t) \tag{7}$$

where $N_T=N_{POU}+N_{CHP}+N_{HOU}$.

2.1.2. Multi-Objective

In the multi-objective form, the CHPbUCP is defined as minimizing total operation or fuel cost and total pollutant emission (TPE) simultaneously over the study period, as follows:

$$\begin{aligned} & min\{TOC(p,c,h,t)+TPE(p,c,h,t)\} \\ & = minTOC(p,c,h,t) + min \sum_{t=1}^{T} \Big(\sum_{p=1}^{N_{POU}} E_{POU,p}\big(P_p(t)U_p(t)\big) + \sum_{c=1}^{N_{CHP}} E_{CHP,c}(P_c(t))U_c(t) \\ & + \sum_{h=1}^{N_{HOU}} E_{HOU,h}(H_h(t))U_h(t)\Big) \end{aligned} \tag{8}$$

$$E_{POU,p}\big(P_p(t)\big) = \sum_{p=1}^{N_{POU}} \big(\tau_p + \upsilon_p P_p(t) + \varphi_p P_p(t)^2 + \phi_p \exp\left(\chi_p P_p(t)\right)\big) \quad t = 1, \ldots, T \tag{9}$$

$$E_{CHP,c}(P_c(t)) = \sum_{c=1}^{N_{CHP}} (o_c + \pi_c)P_c(t) \quad t = 1, \ldots, T \tag{10}$$

$$E_{HOU,h}(H_h(t)) = \sum_{h=1}^{N_{HOU}} (\nu_h + \xi_h)H_h(t) \quad t = 1, \ldots, T \tag{11}$$

2.2. Problem Constraints

The CHPbUCP is a constrained framework consisting of system and unit constraints. The details of these constraints are explained below.

2.2.1. System Constraints

- Power spinning reserve (PSR): this constraint obligates the maximum available active power being more than or equal to the estimated electrical demand plus the spinning reserve power in all sub-intervals [47] by considering power losses as:

$$\sum_{p=1}^{N_{POU}} P_p^{MAX} U_p(t) + \sum_{c=1}^{N_{CHP}} P_c^{MAX}(H_c(t))U_c(t) \geq P_D(t) + P_{Loss}(t) + SRP(t) \qquad t \in T \tag{12}$$

- Heat spinning reserve (HSR): the maximum available heat should be greater than or equal to the forecasted heat demand plus the spinning reserve heat at each sub-interval [47], considering heat losses as:

$$\sum_{h=1}^{N_{HOU}} H_h^{MAX} U_h(t) + \sum_{c=1}^{N_{CHP}} H_c^{MAX}(P_c(t))U_c(t) \geq H_D(t) + H_{Loss}(t) + SRH(t) \qquad t \in T \tag{13}$$

- Generated power constraint (GPC): the total active power generated must meet the power demand plus active losses in each sub-interval, as:

$$\sum_{p=1}^{N_{POU}} P_p(t)U_p(t) + \sum_{c=1}^{N_{CHP}} P_c(t)U_c(t) = P_D(t) + P_{Loss}(t) \qquad t = 1, \ldots, T \tag{14}$$

$$\begin{aligned} P_{Loss}(t) = & \sum_{p=1}^{N_{POU}} \sum_{p=1}^{N_{POU}} P_p(t)B_{p.p}\, P_p(t) \\ & + \sum_{p=1}^{N_{POU}} \sum_{c=1}^{N_{CHP}} P_p(t)B_{p.c}\, P_c(t) + \sum_{c=1}^{N_{CHP}} \sum_{c=1}^{N_{CHP}} P_c(t)B_{c.c}\, P_c(t) \quad t = 1, \ldots, T \end{aligned} \tag{15}$$

- Generated heat constraint (GHC): the total generated heat must satisfy heat demand at each sub-interval, as:

$$\sum_{c=1}^{N_{CHP}} H_c(t)U_c(t) + \sum_{h=1}^{N_{HOU}} H_h(t)U_h(t) = H_D(t) + H_{Loss}(t) \qquad t = 1, \ldots, T \tag{16}$$

- The heat loss (H_L) consists of two parts, head (friction) and convection losses, as [48]:

$$H_{Loss}(t) = H_{loss,head}(t) + H_{loss,conv}(t) \quad t = 1, \ldots, T \tag{17}$$

The first occurs when heat is transferred between places of different temperatures. It mainly stands for loss from inside the pipeline with high temperature to outside the pipeline with low temperature, as follows [48]:

$$H_{loss,conv}(t) = \frac{T_1(t) - T_2(t)}{\frac{1}{2\pi L h_1} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k L} + \frac{1}{2\pi L h_2}} \qquad t = 1, \ldots, T \tag{18}$$

where r_2 and r_1 are the outer and inner radius of the pipeline, respectively, $T_1(t)$ and $T_2(t)$ are the inner temperature of the pipeline and ambient temperature in hour t, respectively, h_1 and h_2 are the convection coefficients of the stream inside the pipeline and in the air, respectively, L is the length of the heat transfer pipeline, and k is the conduction coefficient of the pipeline.

In addition, head (friction) loss is calculated by [48]:

$$H_{loss,head}(t) = 0.25\left[\log\left(\frac{\frac{e}{D}}{3.7} + \frac{5.74}{\left(\frac{\rho(t)\overline{V(t)}D}{\mu}\right)^{0.9}}\right)\right]^{-2} \cdot\frac{L}{D}\cdot\frac{\overline{V(t)}^2}{2} \qquad t = 1, \ldots, T \tag{19}$$

where D is the pipeline diameter, $\overline{V(t)}$ demonstrates the average flow velocity in hour t, $\rho(t)$ describes the fluid density in hour t, e is the roughness wall, and $\mu(t)$ is the fluid velocity in hour t.

The heat loss is modeled by the following relation in ref. [49]:

$$\Delta H_{loss}(t) = \frac{2\pi\lambda\left(T_p(t) - T^{outside}(t)\right)L_p}{\ln\left(D_p/d_p\right)} \tag{20}$$

where $\Delta H_{loss}(t)$ is heat loss of pipeline p at instant t, λ is the thermal conductivity of the pipeline thermal insulation layer, $T_p(t)$ is the hot water temperature in the pipeline at instant t, $T^{outside}(t)$ is the ambient temperature at instant t, L_p is pipeline length (m), and D_p and d_p are the outer and inner diameters of the pipeline, respectively.

It should be noted that some references such as [50] have used the concept of quasi-dynamics temperature for modeling the heat loss by using the node model in two steps. First, regardless of the heat loss, the outlet temperature is forecasted using the total time delays about mass flow and the historic inlet temperatures. In the second step, the temperature drop by heat losses is used to calculate the outlet temperature.

2.2.2. Unit Constraints

- Minimum up and down times (MUDTs): each generation unit needs a minimum time to be committed or de-committed, as:

$$U_n(t) = \begin{cases} 1; & T_n^{on}(t-1) \le T_n^{up} \\ 0; & T_n^{off}(t-1) \le T_n^{dw} \\ 1 \ or \ 0; & otherwise \end{cases} \quad t \in T; n \in N_T \tag{21}$$

- Capacity limits for POUs (CLPOUs): each POU in the committed state should operate in the range of minimum and maximum limits, as:

$$P_p^{min} \le P_p(t) \le P_p^{MAX} \quad t = 1, \ldots, T; p = 1, \ldots, N_{POU} \tag{22}$$

- Capacity limits for CHPs (CLCHPs) or feasible operating region (FOR): each CHP unit in the committed state should operate in the FOR, as:

$$\begin{aligned} &\{P_c(t), H_c(t)\} \in FOR_c \quad t = 1, \ldots, T; c = 1, \ldots, N_{CHP} \\ &P_c^{min}(H_c(t)) \le P_c(t) \le P_c^{MAX}(H_c(t)) \quad t = 1, \ldots, T; c = 1, \ldots, N_{CHP} \\ &H_c^{min}(P_c(t)) \le H_c(t) \le H_c^{MAX}(P_c(t)) \quad t = 1, \ldots, T; c = 1, \ldots, N_{CHP} \end{aligned} \tag{23}$$

One of the most challenging issues in the CHPbUCP is the modeling of the FOR, which describes the interdependence between heat and power of CHP units. Generally, the FOR characteristic can be convex or non-convex [13,51,52]. In the convex form, the increase in the generated electric power leads to a decrease in the produced heat, and the increase in the generated heat also leads to a decrease in the electric power [53]. It can be concluded that the angles of a convex FOR are all less than 180°, while this concept is not true for a non-convex FOR [53]. The convexity of a FOR means that if a CHP unit works at two separate points, that unit can also work at any point of the connecting line between these two separate points [54]. The convex form of the FOR is described by the LP model [55], while the non-convex curve is represented by the MILP model [56]. A CHP unit generates power and heat based on its respective FOR (see Figure 1).

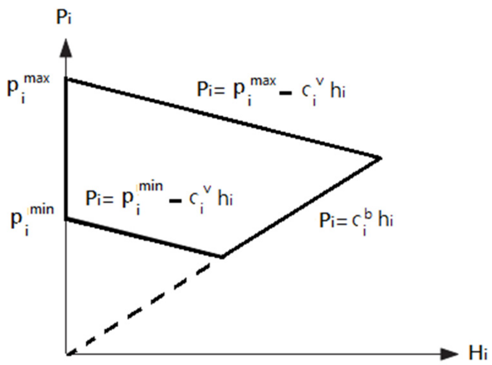

Figure 1. A sample FOR of a CHP unit. Reproduced with permission from [57]. Copyright 2023 Elsevier.

When the unit generates only electricity, p_i should be limited in the range of $\left[p_i^{min}, p_i^{max}\right]$. The generated heat, h_i, depends on the generated electricity and will be affected by the back pressure (BP) limit, i.e., $p_i \ge c_i^b h_i$, where the c_i^b is the BP coefficient of the i-th unit and is typically in the range of [0.35, 0.75] [57]. Moreover, the c_i^v is the conversion rate between heating and electricity, which should be satisfied by $p_i + c_i^v h_i \ge p_i^{min}$ and $p_i + c_i^v h_i \le p_i^{max}$ and is in the range of [0.1, 0.2]. CHP plants that generate only electricity ($h_i = 0$) are known as condensing (CON) units, with $c_i^v = 0$. To maximize efficiency, other plants can only generate heat and electricity in a fixed relationship that works at $p_i = c_i^b h_i$. These types of CHP plants are known as BP units, with $c_i^v = 1$ [57].

In the BP mode, the energy produced by the CHP plant depends on a fixed ratio of heat and power, while in the extraction mode, this ratio is relatively relaxed and enables the system to be flexible compared to others [47]. The extraction and BP modes in a CHP generation plant allow the facility to adopt immediate change in production depending on the ratio of the power and heat generation [45]. Detailed modeling of these two operating modes in the CHPbUCP is addressed in [45].

- Capacity limits for HOUs (CLHOUs): each HOU in the committed state should operate in the range of minimum and maximum limits, as:

$$H_h^{min} \le H_h(t) \le H_h^{MAX} \quad t = 1, \ldots, T; h = 1, \ldots, N_{HOU} \tag{24}$$

- Ramp-rate limits of POUs (RRLPOUs):

$$\begin{array}{ll} P_p(t) - P_p(t-1) \le RUP_p & t = 2, \ldots, T; p = 1, \ldots, N_{POU} \\ P_p(t-1) - P_p(t) \le RDP_p & t = 2, \ldots, T; p = 1, \ldots, N_{POU} \end{array} \tag{25}$$

- Ramp-rate limits of CHP units (RRLCHPs):

$$\begin{array}{ll} P_c(t) - P_c(t-1) \le RUP_c & t = 2, \ldots, T; c = 1, \ldots, N_{CHP} \\ P_c(t-1) - P_c(t) \le RDP_c & t = 2, \ldots, T; c = 1, \ldots, N_{CHP} \\ H_c(t) - H_c(t-1) \le RUH_c & t = 2, \ldots, T; c = 1, \ldots, N_{CHP} \\ H_c(t-1) - H_c(t) \le RDH_c & t = 2, \ldots, T; c = 1, \ldots, N_{CHP} \end{array} \tag{26}$$

- Ramp-rate limits of HOUs (RRLHOUs):

$$\begin{array}{ll} H_h(t) - H_h(t-1) \le RUH_h & t = 2, \ldots, T; h = 1, \ldots, N_{HOU} \\ H_h(t-1) - H_h(t) \le RDH_h & t = 2, \ldots, T; h = 1, \ldots, N_{HOU} \end{array} \tag{27}$$

- Prohibited operating zones for POUs (POZPOUs)

$$\begin{cases} P_p^{min} \le P_p(t) \le P_{p,l}^{L} & (p = 1, \ldots, N_{POU}) \\ P_{p,j}^{U} \le P_p(t) \le P_{p,j}^{L} & (j = 1, \ldots, N_{zp}; p = 1, \ldots, N_{POU}) \\ P_{p.N_{zp}}^{min} \le P_p(t) \le P_p^{MAX} & (p = 1, \ldots, N_{POU}) \end{cases} \tag{28}$$

- Unit status limits (USLs): due to reliability issues, operational limitations, or economic reasons, some units may need to be committed (must run) at certain time intervals or de-committed due to mandatory outages or repairs and maintenance (must not run).
- Forced OFF-states, or must OFF (MOF): defines the forced OFF-states for generation units, e.g., because of maintenance periods [5].

$$U_k(t) = 0 \quad t \in T; k \in N_T \tag{29}$$

- Forced ON-states, or must ON (MON): defines forced ON-states for generation units, e.g., due to economic considerations and/or operating reliability [5].

$$U_r(t) = 1 \quad t \in T; r \in N_T \tag{30}$$

2.2.3. Security Constraints

Despite the fact that the security-constraint unit commitment problem (SCUCP) has received major attention in power systems and many studies have been conducted on it (e.g., [58,59]), limited studies have addressed it in the presence of CHP units [60,61].

The most important security constraints of the system are the power flow equations, lines capacity, and bus magnitude voltages, as follows:

- Power flow equations [60]:

$$\begin{array}{l} V_i(t) \sum\limits_{j=1}^{N} V_j(t).\left(G_{ij}\cos\theta_{ij}(t) + B_{ij}\sin\theta_{ij}\ (t)\right) - P_{p,i}(t) - P_{c,i}(t) + P_{D,i}(t) = 0 \\ V_i(t) \sum\limits_{j=1}^{N} V_j(t)\left(G_{ij}\sin\theta_{ij}(t) - B_{ij}\cos\theta_{ij}(t)\right) - Q_{p,i}(t) - Q_{c,i}(t) + Q_{D,i}(t) = 0 \end{array} \tag{31}$$

where $P^D_{i,h} = P^F_{i,h} + P^{IL}_{i,h}$, $Q^D_{i,h} = Q^F_{i,h} + Q^{IL}_{i,h}$.

Some references (i.e., [61]) proposed a linearized distribution power flow branch model, which includes voltage magnitudes and reactive power and leads to an almost indistinguishable result compared to other models, as:

$$V_j(t) = V_i(t) - \left(P_{ij}(t)r_{ij} + Q_{ij}(t)x_{ij}\right)/V_0 \quad (32)$$

- Lines capacity [60]:

$$\left|S_{ij}(t)\right| \leq S^{MAX}_{ij} \quad (33)$$

It should be noted that some references (i.e., [50]) modeled the line capacity or line flow by using the DC power flow.

- Bus magnitude voltages [60]:

$$\left|V^{min}_i\right| \leq V_i(t) \leq \left|V^{MAX}_i\right| \quad (34)$$

3. A Taxonomy of the Proposed Methods for the CHPbUCP

Despite the existence of numerous references on CHP units in the power system, there are few that focus precisely on the subject of the CHPbUCP. Table 1 provides general information on relevant references as of 25 December 2022.

Table 1. General information on CHPbUCP-related references as of 20 December 2022.

Ref.	Year	Journal/Conference
[62]	1993	Desalination
[63]	2000	IFAC Power Plants and Power Systems Control, Brussels, Belgium
[64]	2000	IFAC Power Plants and Power Systems Con
[65]	2005	Applied Energy
[66]	2005	15th power systems computation conference (PSCC), Liege, 22–26 August
[15]	2007	IEEE Transactions on Energy Conversion
[5]	2008	European Journal of Operational Research
[67]	2009	European Journal of Operational Research
[54]	2009	Energy Conversion and Management
[68]	2009	Applied Energy
[57]	2012	Computers & Operations Research
[14]	2012	Energy
[69]	2013	Mathematical Problems in Engineering
[48]	2013	IEEE, 2013 10th International Conference on Electrical Engineering/Electronics, Computer, Telecommunications and Information Technology
[70]	2014	Energy
[71]	2014	ENERGYCON 2014, May 13–16, Dubrovnik, Croatia
[50]	2015	IEEE Transactions on Sustainable Energy
[46]	2016	Energy
[44]	2016	IEEE Transactions on Smart Grid
[72]	2016	International Journal of Computer and Systems Engineering
[73]	2017	Energy
[74]	2017	North American Power Symposium (NAPS). IEEE
[75]	2018	Energy
[47]	2018	Energy Conversion and Management
[76]	2018	IEEE Transactions on Power Systems
[45]	2019	Energy
[61]	2019	IEEE Transactions on Power Systems
[4]	2019	IEEE Transactions on Sustainable Energy

Table 1. *Cont.*

Ref.	Year	Journal/Conference
[77]	2019	IOP Conference Series, Earth and Environmental Science
[78]	2020	IEEE Transactions on Sustainable Energy
[79]	2021	IEEE Transactions on Power Systems
[80]	2021	Environmental progress and sustainable energy
[81]	2021	Journal of cleaner production
[82]	2021	IEEE systems journal
[83]	2021	International Journal of Electrical Power and Energy Systems
[84]	2021	Sustainable Energy Technologies and Assessments
[85]	2022	Available at SSRN: https://ssrn.com/abstract=4149517 or http://dx.doi.org/10.2139/ssrn.4149517 (accessed on 25 December 2022)

In addition, Table 2 provides a comprehensive overview of the addressed references in terms of the objective function (s) and related constraints.

This table confirms that:

- There is no reference that models the POZPOUs, as these constraints add complexity, non-linearity, and non-convexity to the problem. As a result, some powerful tools and algorithms are needed to handle this issue.
- There are few references, such as [48], that model losses of electrical and thermal networks, despite the existence of references that only model the heat losses, namely, [4,50,61,77,78,81–83].
- The three constraints of USLs, MON, and MOF are rarely modeled in reported studies. All these constraints add to the complexity of the CHPbUCP due to their non-linear nature.

3.1. Applied Methods/Algorithms

Generally, the proposed approaches and techniques to solve the complex, non-linear, non-continuous, and non-convex CHPbUCP are divided into two main categories of deterministic and non-deterministic approaches. The first one is also classified into two types: classical, or conventional methods, and evolutionary, heuristic, or hybrid approaches. In the following, the detailed descriptions of each suggested method in the mentioned classification are presented. It should be noted that the classification presented here may be applicable to many complex optimization problems in the field of engineering, especially power system planning and operation. However, the details may change depending on the problem under study. In addition, presenting all the needed details of the suggested algorithms/methods is beyond the scope of this article, and it is necessary to know the details and to understand their essence by reading the cited literature.

Table 2. A taxonomy of addressed references in terms of the objective function (s) and corresponding constraints on the CHPbUCP.

Ref.	OF * (Minimizing)	TSUC (Equations (5) and (6))	TSDC (Equation (7))	P_{loss} (Equation (15))	H_{loss} (Equations (17)–(19))	PSR (Equation (12))	HSR (Equation (13))	GPC (Equation (14))	GHC (Equation (16))	MUDTs (Equation (21))	CLPOUs (Equation (22))	CLCHPs, FOR (Equation (23))	CLHOUs (Equation (24))	RRLPOUs (Equation (25))	RRLCHPs (Equation (26))	RRLHOUs (Equation (27))	POZPOUs (Equation (28))	MON (Equation (30))	M OF (Equation (29))
[63]	TOC (Equations (1)–(4))							•	•		•	•	•						
[64]	TOC (Equations (1)–(4))	•	•					•	•	•	•	•	•						
[69]	TOC, TPE (Equations (1)–(4), (8)–(11))			•				•	•			•		•	•				
[65]	Maximize the profit of the CHP system					•	•	•	•	•	•	•	•						
[15]	Cost, reliability, and environmental							•	•	•	•		•	•					
[5]	TOC (Equations (1)–(4))	•	•					•	•	•	•	•	•					•	•
[67]	TOC (Equations (1)–(4))	•	•					•	•	•	•	•	•					•	•
[54]	TOC (Equations (1)–(4))	•	•					•	•	•	•	•	•					•	•
[68]	Equivalent annual cost, first order							•	•		•	•	•	•					
[57]	TOC (Equations (1)–(4))	•						•	•		•	•	•						
[48]	TOC (Equations (1)–(4))	•	•	•	•									•					

Table 2. *Cont.*

Ref.	OF * (Minimizing)	TSUC (Equations (5) and (6))	TSDC (Equation (7))	P_{loss} (Equation (15))	H_{loss} (Equations (17)–(19))	PSR (Equation (12))	HSR (Equation (13))	GPC (Equation (14))	GHC (Equation (16))	MUDTs (Equation (21))	CLPOUs (Equation (22))	CLCHPs, FOR (Equation (23))	CLHOUs (Equation (24))	RRLPOUs (Equation (25))	RRLCHPs (Equation (26))	RRLHOUs (Equation (27))	POZPOUs (Equation (28))	MON (Equation (30))	M OF (Equation (29))
[70]	Net income of a CHP unit, includes the revenue of selling power to the grid, total fuel cost, and the cost of additional cooling from chillers	•	•					•	•	•	•	•	•	•	•	•			
[62]	TOC (Equations (1)–(4))	•	•			•	•	•	•	•	•	•	•						
[66]	TOC (Equations (1)–(4))	•	•					•	•	•									
[71]	TOC (Equations (1)–(4))	•	•					•	•	•	•	•	•	•	•	•			
[46]	TOC (Equations (1)–(4)), Emission	•	•			•	•	•	•	•	•	•	•						
[44]	TOC (Equations (1)–(4))	•	•					•	•	•	•	•	•	•	•	•			
[72]	TOC (Equations (1)–(4))	•	•					•	•	•	•	•	•					•	•
[50]	TOC (Equations (1)–(4))	•	•		•	•	•	•	•	•	•	•	•	•	•	•			

Table 2. *Cont.*

Ref.	OF * (Minimizing)	TSUC (Equations (5) and (6))	TSDC (Equation (7))	P_{loss} (Equation (15))	H_{loss} (Equations (17)–(19))	PSR (Equation (12))	HSR (Equation (13))	GPC (Equation (14))	GHC (Equation (16))	MUDTs (Equation (21))	CLPOUs (Equation (22))	CLCHPs, FOR (Equation (23))	CLHOUs (Equation (24))	RRLPOUs (Equation (25))	RRLCHPs (Equation (26))	RRLHOUs (Equation (27))	POZPOUs (Equation (28))	MON (Equation (30))	M OF (Equation (29))
[73]	TOC (Equations (1)–(4)), and CO2 emission cost	•				•	•	•	•	•	•	•	•	•					
[74]	TOC (Equations (1)–(4))	•	•			•	•	•	•	•	•	•	•						
[75]	TOC (Equations (1)–(4)), and environmental emission cost	•				•	•	•	•	•	•	•	•	•					
[47]	TOC (Equations (1)–(4))	•				•	•	•	•	•	•	•	•	•	•	•			
[76]	TOC (Equations (1)–(4))	•				•	•	•	•	•	•	•	•						
[45]	TOC (Equations (1)–(4)), profit, and environmental emission cost	•	•			•	•	•	•	•	•	•	•	•	•	•			
[61]	TOC (Equations (1)–(4))	•	•		•	•	•	•	•	•	•	•	•	•	•	•			
[4]	TOC (Equations (1)–(4))	•	•		•	•	•	•	•	•	•	•	•	•	•	•			

Table 2. *Cont.*

Ref.	OF * (Minimizing)	TSUC (Equations (5) and (6))	TSDC (Equation (7))	P_{loss} (Equation (15))	H_{loss} (Equations (17)–(19))	PSR (Equation (12))	HSR (Equation (13))	GPC (Equation (14))	GHC (Equation (16))	MUDTs (Equation (21))	CLPOUs (Equation (22))	CLCHPs, FOR (Equation (23))	CLHOUs (Equation (24))	RRLPOUs (Equation (25))	RRLCHPs (Equation (26))	RRLHOUs (Equation (27))	POZPOUs (Equation (28))	MON (Equation (30))	M OF (Equation (29))
[77]	TOC (Equations (1)–(4))	•	•		•	•	•	•	•	•	•	•	•	•					
[78]	The total cost of worst cases over the ambiguity set, including some probability distributions	•	•		•			•	•	•	•	•	•	•	•	•			
[79]	TOC (Equations (1)–(4))	•	•					•	•	•		•	•		•				
[80]	TOC (Equations (1)–(4)) and TPE (Equations (8)–(11))	•				•	•	•	•	•	•	•	•	•	•	•			
[81]	TOC (Equations (1)–(4))	•	•		•	•	•	•	•	•	•	•	•	•	•	•			
[82]	TOC (Equations (1)–(4))	•			•	•	•	•	•	•	•	•	•	•	•	•			
[83]	TOC (Equations (1)–(4))	•	•		•	•	•	•	•	•	•	•	•	•	•	•			
[84]	TOC (Equations (1)–(4))	•	•			•	•	•	•	•	•	•	•	•	•	•			
[85]	TOC (Equations (1)–(4))	•	•			•	•	•	•	•	•	•	•	•	•	•			

* Important note: the TOC of some references may be different from what was defined in this work (i.e., Equation (1)).

3.1.1. Deterministic Methods

A. Classical or Conventional Methods

Conventional, or classical, techniques are robust and fast and they provide almost similar cost values, but their runtimes may be different [86]. Moreover, they suffer from high dimensionality. These methods are derivative-based approaches and are highly sensitive to the objective function nature, type of constraints, and initial point. Therefore, the obtained results may not be global or even close to the global optimal solution [20]. The main proposed methods in this category are as follows:

- Lagrangian function: [63].
- Multi-stage optimization method based on the decomposition of the complex optimization problem in smaller sub-problems: [64].
- MILP and Lagrangian relaxation (LR): [65].
- Linear programming: [68].
- Large MIP problem: [57].
- MIP: [14,70].
- B&B using LP: [66].
- Double benders decomposition (DBD): [46].
- Combined sequential DP techniques and linear relaxation of ON/OFF states: [72].
- Modified double Benders decomposition method: [74].
- Mixed integer quadratic programming (MIQP): [4,77,83].
- DP-based sequential DP based on LR of the unit states, sequential commitment of plants in small groups: [54].

B. Evolutionary, Heuristic, or Hybrid Algorithms

These algorithms have been suggested and used to overcome the shortcomings of conventional or classical methods. These types of optimization methods can solve the complex problems randomly. They have low sensitivity to the nature of objective function (s) and have no need to calculate gradients or derivatives. Despite the advantages of these algorithms, they suffer from premature convergence, less robustness, and becoming stuck in the local optimal points. To overcome these challenges, hybrid algorithms based on combining two or more types of heuristic or mathematical algorithms were suggested. It increases the accuracy and strength of the suggested algorithms. In addition, there are some other heuristic methods known as hyperheuristics, which have been suggested to deal with more complex optimization problems. They comprise a set of methods that are motivated to automate the design of heuristic methods to solve the hard computational search. More details on this subject can be found in [87–89]. The algorithms applied in this category for the CHPbUCP are as follows:

- DP-relaxation and sequential commitment (DP-RSC) algorithm based on LR of the unit states and sequential commitment of generation units one by one [5].
- DP optimization technique combined with a heuristic de-commitment plan [15].
- Heuristic improved unit de-commitment (IUD) algorithm [67].
- Hybrid differential evolution (DE) and sequential quadratic programming (SQP) (DE-SQP) [69].
- Full load average cost method combined with some analysis of incremental cost [62].
- Heuristic optimization algorithm [73,75].

3.1.2. Non-Deterministic Methods

Solving an optimization problem under uncertainty is characterized by the necessity of making decisions without knowing their full effects. The problem uncertainties should be molded in such a manner that their effects on decision-making can properly be taken into account. There are many techniques to do this in the CHPbUCP, as follows:

- Robust optimization formulation with linear decision rules [71].
- Information gap decision theory [44].

- Combined deterministic and robust models [50].
- The combined optimization method: combinatorial binary successive approximation strategy and CSO [47].
- The combination of the simplicial decomposition method (SDM), the augmented LM (ALM), and the non-linear Gauss–Seidel (GS), known as the SDM-GS-ALM [76].
- The mixed integer model of the problem combined with binary PSO (BPSO) (to consider binary variables) and PSO technique (to consider real variables) [45].
- Multistage distributionally RO (DRO) [78].
- Recast DRO [79]: considering an ambiguity set incorporating both Wasserstein and moment metric information of uncertain contingencies.
- An enhanced optimization approach (EOA) combining conventional and global optimization techniques [80].
- Three-stage multi-time scale stochastic UCE (SUCE) dispatch model [81].
- A two-stage weather-driven risk-constrained robust UC (WRRUC) model [82].
- Binary differential evolution (BDE) along with priority list (PL) [85].
- Decomposition of the main problem into several sub-problems, finding a separate solution for each of the sub-problems and considering couplings between sub-problems with variations of individual sub-problems [64].
- Resembling Newton's method and using the Hessian of the Lagrangian function using the Broyden–Fletcher–Goldfarb–Shanno (BFGS) quasi-Newton updating method at each step [69].

3.2. Single Objective Versus Multi-Objective

As the literature confirms, most of the reported references minimized total operation costs. Among these references, we can refer to [4,5,44,47,48,50,54,57,61–64,66,67,71,72,74,76,77,79,81–85]. The mentioned research focuses exclusively on the CHPbUCP. However, there are some references that addressed single-objective problems such as maximizing the profit of the CHP system [65], equivalent annual cost [68], net income of a CHP plant, including the cost of the additional cooling from electrical chillers, the revenue of selling power to the grid, and the total fuel cost [70], and the expected total cost of the worst case over the ambiguity set, using some probability distributions [78], in a CHPbUCP framework.

In contrast to single-objective problems, there are some references that focus on multi-objective. Most of the references that have paid attention to two-objective functions in solving the CHPbUCP have considered TOC and TPE. Among this category, references [46,69,73,75,80] can be addressed. Furthermore, there are some research that focused on other objective functions, such as cost, reliability, and environmental issues [15] and TOC, profit, and environmental emission costs [45].

3.3. Deterministic Versus Non-Deterministic

Many reviewed references solved the CHPbUCP in a deterministic structure, such as [5,45–47,54,57,62,64–66,69,70,73,75,80,85]. However, there are limited references that modeled the uncertain CHPbUCP by considering some uncertainties; however, finally, they made the model deterministic for simplicity. Some samples are modeling the wind [50], three different scenarios [74], and wind and solar photovoltaic (PV) [83], which consider different uncertainties in a deterministic model.

In non-deterministic classification methods, we can mention the use of the Monte Carlo simulation (MCS), stochastic techniques, and robust optimization methods. Refs. [15,68] have applied the MCS.

In the stochastics category, we can address the following issues that model different sources of uncertainty in the CHPbUCP:

- Fuel costs, carbon dioxide emission prices, and electricity prices [14].
- Wind turbine, photovoltaic, ambient temperature, wind velocity, and solar irradiation are forecasted [48].
- Uncertainty in the heat demand and the electricity prices [71].

- The pool price [44].
- Among the robust optimization methods, the following samples can be mentioned:
- Wind power [76].
- Contingencies due to hurricane [79].
- Weather parameter-driven uncertainty [81].
- Solar power [84].

3.4. Modeling Types for the CHPbUCP

The literature confirms that there are some frameworks to deal with the CHPbUCP. The main propoed structures to deal with this problem are:

- Market framework: [65,67].
- Deregulated power market: [5,54].
- Profit-based UC problem (PBUCP): [46,73].
- Generation company's (GenCo's) profit: [44].
- Transmission-constrained CHPbUCP: [50,72]: For the first time, the transmission-constrained multi-site CHPbUCP is addressed in [50,72]. Ref. [50] presented the concept of the district heating system (DHS) for this subject, which consists of pipeline networks, heat sources, and thermal loads. It is generally classified in heat transmission and distribution systems. Radial transmission systems are connected to thermal energy produced through thermal stations using circulation pumps and heat exchangers, connecting transmission and distribution systems indirectly.
- The multi-agent system (MAS): [76].

3.5. Strategies, Equipment, and Facilities Modeled

Detailed studies of relevant references show that different strategies, equipment, and facilities are modeled in the CHPbUCP. Some of the most prominent examples in this field are as follows:

- Modeling heat storage: [71].
- Considering plug-in electric vehicles (PEVs): [74].
- Modeling the CCHP units (including CHP and absorption chiller): [75].
- Modeling the dual-mode CHPs (DM-CHP): [45,47].
- Modeling the multi-agent system (MAS) [76].
- Considering the heat exchange stations (HESs), thermal storage, inertia of pipelines, and transmission line capacities [4].
- Modeling the extreme weather events: [79].
- Integrated electricity and heat energy distribution systems (IEHEDP) considering the concentrated solar power plant hybridized with the CHP plant [81].
- Interruptible loads (IL), demand response (DR), solar heat exchanger (SHE), electric boiler (EB), and electrical storage system: [84].
- Wind power plants, solar PV plants (SPVPs), and PEVs, fuel storage: [85].
- Modeling the forecasted wind power [15].
- Water demand constraints: [62].
- Annual electricity, maintenance, and fuel cost considering the annualized capital cost of MG in a specified discount rate [68].
- Newton–Raphson power flow using lead acid batteries to deal with the renewable energy uncertainties [48].

3.6. Used Case Studies

The test systems used in the references are so that it can be said that every investigtion used a unique system. In a general view, the systems used can be divided into three general types: unit-based case studies, integrated case studies, and modified case studies based on real-world cases. The first type did not model the configurations of the electrical or gas networks or model their operation conditions. The second type combines or integrates two cases of EPS and DHS, known as IEHEDP. Some of the EPS networks used in the second category are the modified versions of IEEE test cases. The third type is basiclly the real-world networks.

An important point to note is that due to the fact that the studied systems are not similar, it is practically impossible to compare the reported solutions resulting from the application of different methods or algorithms for solving the CHPbUCP.

3.6.1. The First Type of Case Studies: Unit-Based Cases

- 2 production units and a heat storage: [71].
- 2 CHP thermal units, 2 non-CHP thermal units, 1 wind farm, 3 HESs: [4].
- 2 POUs, 1 wind farm, 3 electrical loads, 2 CHP units, 3 thermal loads: [76].
- 5 CHP units: [54].
- 4 POUs, 3 CHP, 1 HOU: [44].
- 10 CHP units: [54].
- 8 POUs, 2 CHPs, 1 HOU: [69].
- 8 POUs, 2 boilers, 2 district-heating systems: [65].
- 10 POUs, 1 CHP unit, 1 HOU: [45,80].
- 10 POUs, 2 CHP units, 1 HOU: [46,73,74].
- 10 POUs, 2 CCHPs, 1 HOU: [75].
- 10 POUs, 5 CHP units, 1 HOU: [45,47].
- 5 POUs, 10 CHP units, 1 POU: [47].
- 17 CHP units, 1 POU, 2 HOUs: [5,67].
- 17 CHP units, 2 POUs, 1 HOU: [54].
- 5 POUs, 10 CHP units, 5 HOUs: [47].
- 10 POUs, 5 CHP units, 5 HOUs: [45,47].
- 10 POUs, 6 CHP units, 3 HOUs, 1 wind turbine generator (WTG), 1 SPVPs, and 100,000 PEVs: [85]
- 6 bus power networks considering wind integration, the supplied heat by CHPs in 6 nodes: [50].

3.6.2. The Second Type of Case Studies: Integrated Cases

- IEEE 14 buses, 2 CHP units, 1 gas engine: [48].
- IEEE 14 node, includes 2 POUs (each equipped with 3 units), 2 CHP units (equipped with 3 and 4 CHP units), 1 wind farm (220 MW): [77].
- An 8 node DHS and the IEEE 33 bus: [84].
- Modified IEEE RTS-79 test system, includes 6 CHP units, 3 heat storage tanks (HSTs), 3 electric boilers (EBs), 2 solar PV plants, 2 wind farms, and 1 condensing unit: [83].
- IEHEDP, includes 4 bus, 5 node (consists of 2 CHP units and 2 HOUs): [61,79].
- IEHEDP, includes 33 bus, 32 node (consists of 3 CHP units and 3 HOUs): [61,79] (includes RES).
- IEHEDP, includes 123 bus, 32 node (consists of 3 CHP units and 3 HOUs): [79].
- 3 zone small-scale case: 6 bus power network and a 6 node heating network: [82].
- 4 zone large-scale case: the improved IEEE 118 bus system with 5 heating systems (each contains 6 nodes): [82].

3.6.3. The Third Type of Case Studies: Modified Based on Real-World Cases

- Specific microgrid in the UK, 3 CHP units, wind turbines, photovoltaic arrays, 1 boiler, electricity storage, thermal energy storage: [68].
- The benchmark system by 'Electricité de France', including 1 CHP, 6 turbo alternators, and 4 steam boilers: [66].
- A 5 site test system, based on Finnish Energy companies' data, each site includes power and heat production units, electrical and thermal demand, and the number of units changes between 13 and 18: [72].
- 19 generation power units and 6 suggested heat units to supply the heating system in Berlin: [14].

- 20 units (duplicates of 10 plants of DONG Energy, the main Danish energy company), 3 units for the interconnections with Sweden, Germany, and Norway are considered, 4 condensing units, 14 cogeneration units, 5 BP units: [57].
- The extended 40 bus power electrical grid of Gansu Province, includes hybrid CSP-CHP plants, POUs, wind farms, and PV plants: [81].
- Large-scale heat and power systems Berlin (West and East), 8 POUs, 21 CHP units, 27 HOUs: [64].
- Dutch power system, 70 units: [15].
- The Barry Island MCES, includes a 9 bus electrical grid, a 32 node DHN,3 EHs, 2 RES units: [78].
- Barry Island, includes a 9 bus power grid, a 32 node heating system, 3 energy hubs, wind farms, and photovoltaic arrays: [61].
- A real power grid, located in Northeastern China, includes 14.8 GW of wind power generation and 13.2 GW CHP units: [50].
- Day-ahead energy market in the ERCOT for EM of CHP unit located at UT Austin: [70].

3.7. Study Periods

Many references investigated the CHPbUCP on a daily basis. However, there are some other time intervals in this context, such as weekly (168 h), monthly (672 h), or other special cases. The used study periods for the CHPbUCP are as follows:

- 24 h: [44,46,47,50,61,64,66,72–75,80,82,83,85], (winter and summer load cases): [69].
- Next 36 h: [15].
- Special cases: [65] (31 July to 3 September, 25 September to 5 November, and 27 November to 31 December 2000).
- Monthly (672 h), weekly (168 h), and daily (24 h): [5,54,67].
- Yearly analysis: [68].
- One year partitioning into one-hour intervals: [57].
- Other horizons: [14] (2 half-year periods and time intervals of 4 h (2 × 1095 time intervals)), ref. [70] (hourly data over 4 months (from June to September, 2012)), ref. [4] (a typical winter-day scenario with abundant wind resources at night hours), ref. [81] (electricity and heat loads in 4 successive days, and 8 scenarios in 4 successive days considering some weather impacts).

3.8. Simulation Tools and Software

This section introduces simulation tools and software that have been used in various references. The most important software used include C++, FORTRAN (77, 90), MATLAB (different versions), and GAMS in different versions. Interested readers are referred to the relevant references for more details. The used software and simulation tools are as follows:

- C++: [72] (Microsoft Visual Studio, version 2013, and ILOG CPLEX 12.5), ref. [54] (Power Simplex (PS) algorithm and ILOG CPLEX 10.2 solver), ref. [57] (the ZIB optimization suite including solving constraint integer programs (SCIP) 1.1.0 and SoPlex 1.4.0.).
- CPLEX: [81] (CPLEX 13), ref. [84] (for solving MILP).
- FORTRAN 90: [45,47,80].
- FORTRAN 77: [62] (using a program written by authors, known as "STOP")
- GAMS: [61,65] (solver CPLEX 7.5), ref. [14] (CPLEX), ref. [44] (CONOPT solver (to solve NLP sub-problem)), CPLEX solver (to solve the MIP sub-problem)), ref. [79] (CPLEX), ref. [78](CPLEX, uses the cut and branch approach to solve the MILP formulation).
- MATLAB: [4,69,70,73,85] (Gurobi Solver), ref. [77] (Gurobi Solver), ref. [83] (YALMIP toolkit and GUROBI), ref. [82] (YALMIP toolbox, the mathematical programs were tautly done in Gurobi 9.0.), ref. [76] (Gurobi 7.5.2), ref. [50] (all LPs and MILPs were programmed in the Gurobi 6.0 solvers).
- Special software: [15] (provided by the Dutch TSO TenneT).

3.9. PC Data Used

The literature confirms the application of different PC data to the CHPbUCP. Here, based on the reported research, this issue is addressed in Table 3.

Table 3. General data on PC used for solving the CHPbUCP.

Ref.	PC Data Used
[63]	Watcom, Fortran 10.6, and C languages, FoxPro
[64]	Pentium 11,400 MHz processor
[65]	Intel Celeron processor, 497 MHz, 128 MB RAM
[5]	Windows XP, 86 GHz Pentium 4 PC (0.99 GB RAM)
[67]	Windows XP, Pentium 4 PC, 2.2 GHz (512 MB RAM)
[54]	Windows XP, 2.2 GHz Pentium 4 PC (RAM 512Mb)
[57]	Intel(R) Core (TM) 2 CPU 6300 1.86 GHz, 2 GB RAM, the Ubuntu
[70]	Windows (32-bit), Intel Core™2 Duo processor 2.54 GHz, 4.00 GB of RAM
[46]	Pentium P4, Core 2 Duo 2.4 GHz, PC, 1 GB RAM
[44]	Windows Vista OS, PC, 2.66 GHz Core-Duo processor, 3 GB RAM
[72]	Windows 7, 2.67 GHz Core CPU with 4 GB RAM
[50]	PC, 4 processors, 3.40 GHz, 8 GB
[73]	3.1 GHz, 4 GB RAM
[47]	1.66 GHz, Pentium-IV, 2 GB RAM
[76]	Quad-core processor, 2.40 GHz, 4 GB
[45]	PC, 1.66 GHz, Pentium-IV, 2 GB RAM
[61]	PC, 3.2 GHz Intel Core processor, 8 GB
[78]	PC, 3.2 GHz Intel Core processor, 8 GB
[79]	PC, 3.2 GHz IntelCore i7CPU, 8GB
[80]	Intel(R) Core (TM) 7500 CPU @2.70 GHz 2.90 GHz
[82]	16 GB, 2.20 GHz
[83]	Dell P5820x (CPU: i9-9900X, RAM: 64 GB)
[85]	Intel(R) Core (TM) i7-4790 CPU 3.66 GHz and 16 GB RAM, 64-bit

4. Details of Some Proposed Methods/Algorithms

In this section, the detailed specifications of some proposed methods in different categories are discussed.

4.1. Multi-Objective CHPbUCP

Ref. [45] used the fuzzy membership function to evaluate each OF for solving the MO problem based on the relevant maximum and minimum limits [45]. To find the best non-dominated solutions, the priority rankings are optimized, satisfying all the constraints by applying some optimization techniques and the penalty method approach.

In ref. [80], the EOA was proposed to the MO CHPbUCP based on the combination of global and conventional optimization methods. The nature-inspired binary and continuous PSO (BPSO-PSO) methods are used to solve the problem in the presence of binary (ON/OFF status) and real variables, respectively. Furthermore, the successive approximation approach (SAA) is implemented to model binary variables of the problem, and global search is used for handling the continuous variables or finding the optimal heat/power generation schedule, known as the hybrid PSO and society civilization technique (hPSO-SCA). Moreover, a fuzzy membership function with the cardinal priority method (CPM) determines the various objective functions of the problem. Furthermore, the decision-making technique finds the best-satisfied non-dominated solution by using the Pareto optimal front. The proposed strategy for constraint handling with SAA hybridization of SCA and PSO is as follows:

- Data entry: problem specification and optimization parameters of the suggested algorithm.
- Constraint handling: commit sufficient units and power generation are randomly initialized within the bounds, satisfying the MUP/MDT constraint [46], unit decommitment of the excess units [46], and RRLPOUs, RRLCHPs, and RRLHOUs.

- Evaluate the fitness function (FF): implementing the SAA to search for the optimum states of the CHPbUCP and using the hPSO-SCA to determine the optimal power of committed plans.

4.2. Deterministic Mathematical-Based Methods

- Using the Lagrangian function [63]: the optimality conditions of the Lagrangian function are checked by using the λ-iteration method in three sub-problems, including:
  - the optimization of the power and heat demands if the CHP units do not contribute to the power system optimization,
  - solving the optimal problem if the CHP units do not contribute to the heat system optimization,
  - the optimization of the CHP units if they operate as some independent producers in the power and heat markets.
- The multi-stage method [64]: this method is based on decomposing the main problem into several sub-problems, finding separate solutions for each sub-problem, and considering the couplings between different sub-problems with separate sub-problem variations.
- MILP: Logic-based techniques, decomposition, cutting plane, and branch and bound are the techniques that have been used [90]. Some solvers apply the B&B technique, using bounding and branching and relaxation [3,90]. To tackle the problem with an LP-relaxation, in the MILP framework, integrality constraints are initially ignored. At the step of branching, the LP-relaxation is repartitioned into two branches of choosing a relaxed integer variable and setting the upper integer value of the LP-relaxed solution as the lower bound for one branch and the lower integer value as the upper for the other one [3,90]. By satisfying all integrality constraints by specific solutions, any other branches with higher OF values will be bounded [90,91]. The best method for the LP-relaxation is known as convex or sharp hull formulation [90,92]. The MILP is implemented in the GAMS programming language by using the CPLEX 7.5 solver [65]. In large-scale power systems with many time steps and/or units, the run times are very long, and it is sometimes impossible to achieve near-optimal solutions A solution method to solve the optimization problem faster is using LR [65]. In addition, in [14], the CHPbUCP is solved using the MIP formulation GAMS/CPLEX. Gurobi and CPLEX are two notable solvers used for MILP [3].
- Linear programming [68]: the UCP is formulated in the LP structure but imposes some modified constraints to model the MG.
- MIQP: Ref. [4] suggested the HES model based on a linear approximation considering the thermal storages of pipelines and buildings. It also mentioned wind power integration. The proposed model is solved by using the GUROBI solver interfaced through MATLAB [4,77] or by using MATLAB, YALMIP toolkit, and GUROBI [83].
- Large mixed integer programming problem (LMIP) [57]: the CHPbUCP in the long-term periods of one year is converted into hourly time intervals, which are more suitable in long-term power system planning, especially in scenario analysis. Using LMIP, there are two types of suitable but computationally intractable solutions: the solution obtained from market simulation and the optimal solution. For this purpose, some heuristic methods, including local search methods and mixed integer programming heuristics, have been implemented. In the heuristic technique, an MIP decides on the unit status (UC schedule) using an LP post-processing to specify the generation outputs. The other approach is using a simple greedy construction heuristic implementing the stochastic local search and ED.
- MINLP [70]: In the first phase, the CHP plant scheduling problem is modeled by using an NLP formulation. The scheduling problem is then formulated in an MINLP framework by considering both binary decisions and continuous variables to find the optimal usage of generation plants over a study period. The SQP (sequential quadratic programming) method was solved by using NLP. In addition, the combined SQP

algorithm—SCIP (to tackle with constraint integer problems) solver is used to solve the MINLP problem.

- B & B: To solve the power and steam generations, an exact MIP method, B&B, is used in [66]. For this purpose, at first, the optimization problems and relevant constraints are converted into a linear form. Moreover, some extra binary optimization variables are utilized. To implement the different phases of the CHPbUCP, estimated objective functions, optimization of electricity production, optimization of heat production, ED, and overall optimization are implemented. Furthermore, ref. [50] handled the B&B in three phases of the master UC problem, network security check sub-problems, and feasibility check of the DHN sub-problem. In the first phase, the commitment and dispatch schedules of production units, which minimize the total costs of operation, satisfying the relevant constraints as well as feasibility cuts, are provided by using the LR or MILP solvers. The second phase is responsible for checking the feasibility of network constraints based on the given schedules of units. Finally, the sub-problem is handled to analyze the feasibility of DHN operation constraints based on the given heat schedules. An extended version of ref. [50], known as dynamic regrouping-based DP, is addressed in ref. [93].
- DBD method [46]: Benders decomposition is a technique applied to solve large-scale problems. It decomposes the main problem into one master and several sub-problems. By solving sub-problems, a set of dual variables are found, which generate Benders cuts of the master problem. The suggested BD consists of two BD algorithms, known as the outer and the inner BDs. The outer BD (i.e., the master problem) specifies the ON/OFF state of production units, while the sub-problem solves the ED problem. The inner BD is used to solve the EDP (the outer sub-problem). For the BD algorithm, the variables related to heat generation are solved in the master problem, while the ones representing the power generation are kept in the sub-problem. It should be noted that in ref. [74], the DBD with some minor changes is applied to solve the CHPbUCP in the presence of PEVs, known as the CHPUC-PEV problem in smart grids. In that ref., the master problem determines the integer variables (ON/OFF state of each generating unit) for the outer BD, and the sub-problem solves the ED by modeling the charge/discharge scheduling problem.
- Combined linear relaxation and sequential DP techniques [72]: The relaxed states are applied to reduce the dimension of the CHPbUCP. The DP is suggested to improve the solution quality, and it is mainly used to convert the CHP systems from the form of single site into multi-site, which is called the MDP-RSC.

4.3. Deterministic Heuristic-Based Methods

- Priority list (PL) [85]: PL is prepared based on each unit's parameter, where the cost per generated unit of a power plant at its maximum output is generally the least among all output levels. The PL is calculated based on the average full-load cost of a generation unit as cost per unit of output while the unit is at full capacity.
- Full load average cost principle and some incremental cost analysis [62]: the objective function is solved approximately for each hour by three steps, including calculating the least heat system additions at full load until satisfying the relevant constraints, fixing the distiller loads, and calculating the least heat power increments starting with the minimum operating capacities until satisfying the relevant constraints.
- Decoupling method [94]: in this technique, the hydro and CHP subsystems are decoupled then solved independently. In the first level of optimization, the solution of the thermal system is found for any one week and any relevant (fixed) value of imports from the hydro system. Then, by using these data, the solution of the hydro system (the second level of optimization) is searched.
- DP optimization algorithm combined with a heuristic de-commitment scheme [95], [15]: This method initially applies the LR technique to solve the commitment schedule by load relaxing, unit ramp, and rate spinning reserve limitations. Then, a generation

schedule is obtained by applying the forward dispatch, backward dispatch, and dispatch modification. It satisfies the ramp rate and spinning reserve requirements. In the next stage, the probabilistic reserve assessment is applied to update the mentioned schedule and to meet a predefined risk. The tradeoff between the expected cost of energy not served and the total cost of the UC schedule specifies the risk index. At last, a de-commitment unit method to find the optimal solution to the reserve over-commitment problem is applied in an LR-based UCP.

- DP algorithm, based on the DP-RSC1 [5]: This technique is an updated version of the DP considering the LR of the ON/OFF states of generation units and sequential commitment of units. First, all generation units are relaxed over all planning horizon hours. Then, the ON/OFF states of each plant over the whole horizon are determined using the DP based on a predetermined plant sequence, while the other production units remain at their already determined ON/OFF or relaxed states. The run-time of the DP-RSC1 technique is a function of the number of periods, a number of generating units, and the run-time needed for solving a single-period EDP.
- Improved unit de-commitment (IUD) algorithm [67]: It starts with an improved initial solution by using a heuristic procedure with less heat surplus, in which the relative cost-efficiency of the generation units can be specified more accurately. Then, the subsequent de-commitment procedures can off the least cost-efficient plants properly. The heuristic procedure utilizes both the Lagrangian relaxation and linear relaxation principles. The first one relaxes the heat and power demand constraints and the other one is related to the ON/OFF states of the plants.
- Dynamic regrouping dynamic programming using relaxation and sequential commitment (DRDP-RSC) algorithm [54]: This technique, which is applied to the CHPbUCP in multi-period deregulated power markets, is a dynamic regrouping-based DP algorithm. It utilizes linear relaxation of unit states and sequential commitment of units in different groups. The dimension of the UCP is reduced by the relaxed states of the plants, and the DRO is utilized to improve the solution quality.
- The heuristic optimization algorithm [73]: In this algorithm, profit improvement and impressive low run-time are achieved based on the suggested FF to find the optimal output power of units. The units are ordered based on their best FFs. The units with higher FF have the priority to be scheduled first. Then, different combinations of units are defined, satisfying all relevant constraints. Due to variable hourly energy prices, power plants with negative profits are shut down. The significant steps of the proposed method, in an iterative procedure, are as follows:
    - The committed units in each hour are determined using FF value calculation. According to calculated FF values, the units with higher priorities are found. The FF is updated when the selling reserve is not fulfilled.
    - Calculate the initial optimal outputs and spinning reserve of units based on their FFs by applying the GA.
    - Sort the units according to their FF.
    - Consider a sorted unit's index; a lower value translates into a higher priority of units for committing.
    - De-commit the units with negative economic profit.
    - If the total generation is more than demand, the outputs of committed units must be reduced to satisfy the system balance constraints.
    - Check the MUT and MDT constraints.
    - Consider start-up costs.
    - Calculate economic profit.

It should be noted that similar work has been addressed by the authors in [96]. In addition, this method is applied to solve the economic environmental unit commitment for integrated combined cooling, heat, and power (CCHP) units in ref. [75].

- The combination of binary successive approximation (BSA) and civilized swarm optimization (CSO) [47]: The binary successive approximation, as a local search method, is applied to update the unit status, iteratively and CSO, as the global search technique is used to search the optimal production schedule of the committed units. In the suggested method, the commitment of HOUs and CHP units is performed by the priority list (PL). The power spinning reserve is satisfied by POUs and CHP units. The MUT/MDT of generating units must be met during the procedure, which may lead the system to higher operating costs and excessive spinning reserves. In this way, it may be necessary to de-commit some units by satisfying the limits of power storage and thermal spinning.
- The SDM-GS-ALM [76]: It consists of two loops. The main loop is based on the framework of the augmented Lagrangian method (ALM), while the inner loop is a GS iteration. The parallelism of the proposed algorithm is based on the decomposability of the dual function and the augmented Lagrangian (AL) function. The suggested method was addressed in the decomposition of UCP, a method for approximating the convex hull, the procedure of the SDM-GS-ALM algorithm, and convergence and optimality. As it was stated, the numbers of binary variables, continuous variables, and constraints for small-scale, large-scale, and 118 bus power systems are 188, 744, 1726; 3525, 9216, 25,425; and 2538, 6768, 19,390; respectively.
- The MIP combined with BPSO and PSO [45]: To deal with discrete decision variables, the BPSO is suggested, using the sigmoid function for scaling the velocity in the range of (0–1) [97]. In the suggested method by [45], at the first step, the generating unit statuses are controlled by using binary decision variables, while the power or heat production is a continuous variable. Therefore, the initial heat and power are randomly generated for POUs, HOUs, and CHP units. Then, in the commitment step, a generating UC strategy to deal with discrete variables is stabilized by satisfying the MUT/MDT, PSR, and HSR constraints. The next step updates the status of generating units by using the BPSO technique. The last step finds the optimum values of power and heat of the committed units by using the PSO.

4.4. Non-Deterministic-Based Methods

RES, and especially wind power generation, will be limited by the flexible operation of CHP units, mainly in the winter, due to the strong dependence of supply heat and power generation [50,72].

Ref. [4] deals with the joint commitment of HES and generation plants for CHP units in the presence of wind power plants using the MIQP. In addition, the impacts of wind power on the UCP and EDP of Dutch thermal generation were investigated in [15].

Generally, the uncertainties in the UCP can be modeled using three methods, including interval optimization (IO) [98], RO [99], and SO [100] techniques.

The limitations of SO and RO have been solved by applying an intermediate approach known as DRO [78,101]. The mentioned technique integrates the existing distribution data with an ambiguity set to describe the probability distributions that can eliminate the inherent dependence of the SO method on exact probability distributions and provide less conservative results than the results of the RO method [78].

In the following, some non-deterministic methods applied to the CHPbUCP are addressed.

A single-level reformulation of robust optimization with linear decision rules is introduced in [71]. In this method, recourse decisions (storage operation and heat and power production) are approximated by affine functions of the uncertain sets. Then, using linear decision rules, equality constraints, and inequality constraints, objective functions are reformulated. The proposed schedule is robust against heat consumption deviations.

The impacts of large-scale wind power plants on power system operation includes environmental, reliability, and cost issues, which are described in [15]. It uses a time series of predicted and observed 15 min average wind speeds at wind farm locations. The method uses the MCS technique for frequent revisions of conventional production unit schedules by applying the data on present wind energy output and estimates it for the future 36 h.

To overcome the fluctuation of RES in an MG, a battery storage system, consisting of a lead acid battery, is proposed in ref. [48]. The correlation between temperature and solar irradiation is considered. Moreover, the intermittent wind velocity is separated into two parts of an average hourly wind velocity and uncertainty of wind velocity. The second item is modeled as a normal distribution with zero mean and time-dependent variance. In addition, the heat loads are assumed to be uncertain and are produced by a normal random distribution.

The uncertainty in the heat demand and electricity prices that CHP unit owners receive for the power they sell in the market is molded in [71]. Furthermore, heat storage is considered. The robust optimization model focused on day-ahead heat and power dispatch and a real-time re-dispatch variable.

In ref. [44], the information gap decision theory (IGDT) is used to solve the CHPbUCP. The basis of this method is considering the uncertainty as an unlimited gap based on available information, and the area between what should be known and what is known is identified with uncertainty for optimal decision-making. When few data are available, IGDT provides relatively acceptable solutions. It is also expected that the system will have pivotal changes and ignore the current conditions. In this theory, robustness is defined as the largest amount of uncertainty of a predicted value allowed such that the decision never leads to failure. In addition, the system model is defined as the reward of the decision-maker for selected values of the decision variable by expressing the uncertain parameter. The uncertainty of different parameters is defined using different models, such as the envelope-bound [102]. This method helps the decision-maker to ensure the adjustment of decision variables from the risk of reaching the minimum requirements in the presence of uncertainties in uncontrollable parameters. A risk-averse model is proposed for GenCo to evaluate the robustness of its decisions against low pool prices as well as the opportunities associated with high pool prices, based on which GenCo can determine the sales price strategy with UC and ED.

In ref. [50], a two-stage RO for handling the UCHPbUCP is used. The first-stage decision variables include the commitment states of POUs, and the second-stage recourse variables are the dispatch of generation and the output heat after modeling the available wind power. Mentioning these decision variables, the feasible regions of the first- and the second-stage decisions are defined. The proposed model was solved by using the global optimality using the column-and-constraint generation (C&CG) method [103].

In [74], the impacts of parking lots penetration, as small portable power plants, on the CHPbUCP, known as CHPUC-PEV in smart grids, is investigated. The deterministic problem was solved in three different scenarios.

In [76], the robust SDM-GS-ALM is addressed in the presence of the wind power uncertainty of each agent. The solution includes a permitted output interval for wind farms, a base point for generation units, and a base point for wind farms. The forecasted value of wind power is mentioned as an interval. Before solving the problem, each wind farm uploads the forecasted available wind power intervals. After solving the problem, the permitted output intervals are sent back to the wind farms. Then, the relevant constraints are checked. In addition, some penalty costs of potential wind power spillage, aimed at maximizing the wind power utilization, are mentioned. In this method, the robust constraints are equivalently transformed into deterministic constraints.

Distributed robust optimization (DRO) including the RES is presented in [61]. The optimal day-ahead CHPbUCP considering the variable RES (the variable output power is expressed as the expected values) is solved by applying a two-stage DRO model in which UCP and EDP solutions are determined by the first- and second-stage decisions. The second-stage ED decisions are estimated by using the linear decision rules and the second-order cone duality. The proposed formulation is a tractable mixed-integer second-order cone programming problem and is efficiently solved by off-the-shelf optimization packages.

The dimensionality of constraints and variables defined by linear decision rules are reduced by using the simplified affine policies. Finally, the proposed formulation minimizes the worst-case expected total cost over an ambiguity set characterized by the pre-determined support and moments (including expected values and variances) of variable RES power output. Moreover, the multistage DRO look-ahead unit commitment is proposed in [78]. The main differences between multi-stage and two-stage DRO are [78]: in the multistage model, the non-anticipativity of the remaining stage decision variables is respected, but the two-stage model violates the non-anticipativity; and in the two-stage model, the generation capacity and CHP unit ramp-rate constraints for the remaining periods are applied in the second stage, however, such constants were moved to the first stage in the multi-stage model. Furthermore, for modeling the uncertainties in [78], non-parametric probabilistic forecasting is considered based on quantile forecasts. However, interval and density forecasts are two other methods for this purpose.

In addition, DRO considering the extreme weather events is suggested in [79]. A DRO model for the resilient operation in an IEHEDP considering the extreme weather events as the primary reasons for extensive damage to power systems is proposed. Moreover, ref. [104] provides some useful data on this concept.

Ref. [82] suggested a two-stage WRRUC model. In the proposed model, for the UCP in IEHEDP, the conservatism reduces the robust decision-making. In addition, the uncertainty set boundaries are set as adaptive variables, and the aftermath of un-modeled uncertainties was quantified by additional operational risk terms in the OF. The suggested model is solved by the C&CG algorithm. Mathematically, the suggested method includes master and slave stages. The first one minimizes the base case costs of the UCP, while the other one is the sub-problem that checks the feasibility of the operation strategy against the uncertainties. The suggested model considers the uncertainties of demand-side, generation-side, and network-side. In addition, the suggested method mentions the correlation between all of the uncertainties, which all originated from the inherent forecast error of weather parameters. In demand-side uncertainty modeling, the heat balance of the building windows, the ventilation dissipation of the building, the heat balance and thermal comfort, and the uncertainty set of the demand-side are considered. Furthermore, the generation-side uncertainty molded the output of renewable generation (i.e., wind farms and PV plants). The network side uncertainty includes the transmission capacity of the overhead lines as a function of weather parameters, mainly due to the influence of wind velocity. For this purpose, the dynamic line rating (DLR) concept is introduced.

A three-stage multi-time scale SUCE dispatch technique is introduced in [81]. The thermal energy storage (TES) system of the hybrid concentrating solar power CHP (CSP-CHP) unit has a large energy storage capacity. Therefore, the remaining state of charge (SOC) of TES in an operating day can significantly affect the system scheduling. The suggested model initially optimizes the remaining SOC of TES in the operating day. Then, day-ahead dispatch and real-time dispatch are implemented based on the optimized SOC. At the second stage, known as the day-ahead dispatch stage, the UC dispatch decisions are optimized. The decisions include both the output power and regulating reserves for all committed units in a day-ahead framework. In the third stage, known as real-time dispatch, the imbalance of power between the day-ahead and real-time is eliminated based on the ED problem. The suggested model considers the operational conditions of future days in an MILP-ED model.

The hybrid RO-stochastic programming (SP) optimization is addressed in [84]. High penetration effects of PV units and flexible thermal and electrical loads are considered. To model solar radiation uncertainty, SP is applied, while the RO is implemented to model price uncertainties. Furthermore, the electrical storage system is used to manage uncertainty. Uncertainty in electricity market price is an erratic parameter because it cannot be accurately predicted. This makes RO a convenient way to handle it. In the suggested technique, the variation interval of the uncertain parameters is defined as uncertainty sets, which can be presented by a percentage of the forested value.

5. Challenges and Future Challenges

The literature review confirms that there are still many issues in CHPbUCP modeling. The following are the main issues:

- There is no work that provides all the constraints governing the problem. POZPOUs, USLs, MON, and MOF are some of these constraints that are not fully considered as a complete package in modeling the problem.
- Modeling heat and electrical losses in an integrated electricity gas network is another challenging issue in the CHPbUCP. Although different approximate formulas have been suggested in a few sources, more work is needed to provide a practical method to realize this issue.
- There is a need to provide complementary modeling of the CHPbUCP, including different equipment, such as energy storage (electrical and thermal) devices, EV charging stations, heat exchangers, hydropower generation, and pumped storage units.
- There is a need to present some standard case studies to verify the ability, convergence priority, and speed of different proposed methods/algorithms. A fair comparison is practical if all assumptions, constraints, hardware, and software data are similar.
- The dynamic convergence of heuristic algorithms used for the mentioned problem can be considered as an interesting subject for application.
- Using some comprehensive optimal power–gas flow models to deal with the complete operational modes of integrated modern energy systems is of considerable importance [105–107].
- Using some hybrid algorithms to solve the problem and some powerful methods, such as game theory, to address the full competition in the power system.
- Classification of the importance of different scenarios, social welfare modeling, and different emissions in the problem.
- Considering the CHPbUCP in a multi-zone framework is a challenging issue that should be significantly focused on in future research.
- Considering the risk parameter in the CHPbUCP, or forming a risk version of the problem, known as the RBCHPbUCP, mainly in competitive power–gas markets.
- Solving the CHPbUCP problem is of great importance in modern power systems, include 100% RES, containing solar and wind power plants, and some technologies such as plug-in hybrid electric vehicles/electric vehicles (PHEVs/EVs).
- Finding the optimal solution for the problem in the shortest time and with the highest robustness is another important task in solving the mentioned problem.
- Reliability is the main concern of the power system, and ignoring it will lead to power system instability. Operators strive to maintain reliability at a reasonable level. Obviously, it is necessary to pay attention to reliability concepts in integrated heat and power systems.

6. Conclusions

This paper examined the CHPbUCP from different aspects as an important issue in the planning of the modern combined heat and power systems. For this purpose, at first, similar review works were investigated and some important features were highlighted. In addition to dealing with objective functions, single-objective and multi-objective frameworks and deterministic and non-deterministic methods are introduced. Furthermore, some detailed specifications on modeled strategies, facilities, case studies, study periods, time intervals, simulation tools and software, and PC data are tabulated. What remains open for researchers is to focus on robust optimization methods and model new facilities of modern integrated energy systems. In addition, it is more important to use more powerful optimization algorithms to handle this complex problem in the shortest time and with the highest robustness. Increasing the reliability and resiliency of smart integrated electricity and heat power systems are the other interesting issues in this field.

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Variables/Parameters

p	Index for power-only units (POUs)
c	Index for combined heat and power (CHP) units
h	Index for HOUs
t	Time instant, hour
T	Time horizon for commitment, i.e., 24 h
N_T	Total number of all production units
N_{POU}	Number of POUs
N_{CHP}	Number of CHP units
N_{HOU}	Number of heat-only units (HOUs)
N_T	Total number of all production units
L	Length of the heat transfer pipeline
h_1	Convection coefficient of stream inside the heat transfer pipeline
h_2	Convection coefficient of air
r_1	Inner radius of the pipeline
r_2	Outer radius of the pipeline
k	Conduction coefficient of pipeline
D	Pipeline diameter
e	Roughness wall
λ	Thermal conductivity of pipeline thermal insulation layer (GJ/m·K)
L_p	Pipeline length (m)
D_p	Inner diameter of pipeline p (m)
d_p	Outer diameter of pipe p (m)
HS_n	The hot start-up cost for unit n
CS_n	The hot shut-down cost for unit n
T_n^{dw}, T_n^{up}	The minimum OFF/ON time durations of unit n
T_n^{on}, T_n^{off}	The ON/OFF time of the unit n at hour t from its start-up and shutdown time
T_n^{cold}	The time taken for the cooling state of unit n
N_{zp}	Total number of prohibited operating zones (POZ) for POU p
r_{ij}	The resistance of line *i*–*j*
x_{ij}	The reactance of line *i*–*j*
V_0	The base voltage
S_{ij}^{MAX}	The maximum capacity of line *i*–*j*
$B_{p,p}$, $B_{p,c}$, $B_{c,c}$	The coefficients of power losses in matrix B for POUs and CHP units
P_p^{MAX}	The maximum generation of POU p
P_c^{MAX}	The maximum generation of CHP c
H_h^{MAX}	The maximum heat of HOU h
H_c^{MAX}	The maximum heat of CHP c
P_p^{min}	The minimum generation of POU p
P_c^{min}	The minimum generation of CHP c
H_h^{min}	The minimum heat of HOU h
H_c^{min}	The minimum heat of CHP c
FOR_c	The feasibility operation region for CHP c
$U_c(t)$	The unit status (0–1) of CHP c at hour t
$U_p(t)$	The unit status (0–1) of POU p at hour t
$U_h(t)$	The unit status (0–1) of HOU h at hour t

RUP_p, RDP_p	Ramp-rate limits of power (up-down) for POU p
RUP_c , RDP_c	Ramp-rate limits of power (up-down) for CHP c
RUH_c, RDH_c	Ramp-rate limits of heat (up-down) for CHP c
RUH_h, RDH_h	Ramp-rate limits of heat (up-down) for HPU h
$P_{p,j}^{U}$, $P_{p,l}^{L}$	The upper and lower limits of the j-th POZ for POU p
$P_{p,N_{zp}}^{min}$	The minimum power for POZ p regarding POU p
$C_{POU,p}(P_p(t))$	The fuel cost function of POU p, at hour t, for produced power P_p
$C_{CHP,c}(P_c(t), H_c(t))$	The fuel cost function of CHP c, at hour t, for produced power P_c and heat H_c
$C_{HOU,h}(H_h(t))$	The fuel cost function of HOU h, at hour t, for produced heat H_h
$TPE(p, c, h, t)$	The total operation cost for generated P_p by POU p, generated P_c, H_c by CHP c, and generated H_h by HOU h
$E_{POU,p}(P_p(t))$	The emission of POU p, at hour t, for generated power P_p
$E_{CHP,c}(P_c(t))$	The emission of CHP c, at hour t, for generated power P_c and heat H_c
$E_{HOU,h}(H_h(t))$	The emission of HOU h, at hour t, for generated heat H_h
$ST_p(t)$	Start-up cost for POU p at hour t
$ST_c(t)$	Start-up cost for CHP c at hour t
$ST_h(t)$	Start-up cost for HOU h at hour t
$ST_n(t)$	Start-up cost for unit n at hour t
$SD_p(t)$	Shut-down cost for POU p at hour t
$SD_c(t)$	Shut-down cost for CHP c at hour t
$SD_h(t)$	Shut-down cost for HOU h at hour t
$SRP(t)$	Spinning reserve power at hour t
$SRH(t)$	Spinning reserve heat at hour t
$\overline{V(t)}$	Average flow velocity in hour t
$\rho(t)$	Fluid density in hour t
$\mu(t)$	Fluid velocity in hour t
$P_p(t)$	Produced power by POU p at hour t
$P_c(t)$	Produced power by CHP c at hour t
$H_c(t)$	Produced heat by CHP c at hour t
$H_h(t)$	Produced heat by HOU h at hour t
$T_1(t)$	Inner temperature of pipeline in hour t
$T_2(t)$	Ambient temperature of pipeline in hour t
$\Delta H_{loss}(t)$	Heat loss of pipeline p (GJ/m·h) at instant t
$T_p(t)$	Temperature of hot water in pipeline (K) at instant t
$T^{outside}(t)$	Ambient temperature (K) at instant t
$P_D(t)$	Total active power demand at hour t
$P_{Loss}(t)$	Total power loss at hour t
$H_D(t)$	Total heat demand at hour t
$H_{loss,conv}(t)$	Convection heat loss
$H_{loss,head}(t)$	Head (friction) heat loss
$H_{Loss}(t)$	Total heat loss at hour t
$TSUC(t)$	Total start-up cost at hour t
$TSDC(t)$	Total shut-down cost at hour t
$P_{ij}(t)$	The active power of line i–j at instant t
$Q_{ij}(t)$	The reactive power of line i–j at instant t
$\left\lvert S_{ij}(t) \right\rvert$	The net capacity of line i–j at instant t
$\left\lvert V_i^{min} \right\rvert$	The minimum voltage magnitude at bus i at instant t
$V_i(t)$	The voltage at bus i
$\lvert V_i^{MAX} \rvert$	The maximum voltage magnitude at bus i
$d_p sin\left\{ f_p \left(P_p^{min} - P_p(t) \right) \right\}$	The valve point loading effects of POU p, at hour t, for generated power P_p
a_p, b_p, c_p	The cost coefficients for POU p
α_c , β_c , γ_c , δ_c, ε_c, ϵ_c	The cost coefficients for CHP c
$\zeta_h, \theta_h, \vartheta_h$	The cost coefficients for HOU h
τ_p, υ_p, φ_p, ϕ_p, χ_p	The emission coefficients for POU p
o_c, π_c	The emission coefficients for CHP c
ν_h, ξ_h	The emission coefficients for HOU h

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