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# From the Steam Engine to STEAM Education: An Experience with Pre-Service Mathematics Teachers

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**Abstract:** In this paper, we describe an educational experience in the context of the Master's degree that is compulsory in Spain to become a secondary education mathematics teacher. Master's students from two universities in Madrid (Spain) attended lectures that addressed—emphasizing the concourse of a dynamic geometry software package—some historical, didactic and mathematical issues related to linkage mechanisms, such as those arising in the 18th and 19th centuries during the development of the steam engine. Afterwards, participants were asked to provide three different kinds of feedback: (i) working on an assigned group task, (ii) individually answering a questionnaire, and (iii) proposing some classroom activity, imagining it would be addressed to their prospective pupils. All three issues focused on the specific topic of the attended lectures. In the framework of Mason's reflective discourse analysis, the information supplied by the participants has been analyzed. The objective was to explore what they have learned from the experience and what their perception is of the potential interest in linkages as a methodological instrument for their future professional activity as teachers. This analysis is then the basis upon which to reflect on the opportunities (and problems) that this particular bar-joint linkages methodological approach could bring towards providing future mathematics teachers with attractive tools that would contribute to enhancing a STEAM-oriented education. Finally, the students' answers allow us to conclude that the experience was beneficial for these pre-service teachers, both in improving their knowledge on linkages history, mathematics, industrial, technological and artistic applications, and in enhancing the use in the classroom of this very suitable STEAM context.

**Keywords:** STEAM education; bar-joint mechanisms; linkages; geometry; dynamic geometry; computer algebra; algebra; pre-service teacher training; secondary education; mathematics

**MSC:** 97U70; 97G99

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## 1. Introduction

Approaching scientific education through a problem (or inquiry)-based STEAM (Science, Technology, Engineering, Arts, and Mathematics) trans-disciplinary approach (see Section 2) is a current trend that is present in the educational curricula of many different countries. See, for example, Refs. [1,2] describing the impact of some international tests (e.g., PISA) on the development of STEAM-focused syllabuses in such diverse cultural and geographic contexts as European or Gulf Cooperation Council (GCC) States. We understand that a natural consequence of the contemporary predominance and ubiquity of this tendency should be the consideration of such methodology in the initial training of teachers.

Therefore, the aim of this paper is the following: the description (see Section 3) and analysis of an experience with students attending a compulsory (in Spain, for getting a teaching license) Master's degree to become secondary education mathematics teachers. Students deal with the introduction, proposal, and development of activities with linkages, regarding this (both mathematical and physical) context as a relevant environment that could help to foster a STEAM classroom methodology.

The final research goal of the experience was to collect and analyze (see Section 4), with the help of the *discourse analysis/reflection on practice* methodology (see Section 3), students' reactions to the lectures on the proposed topic, as well as their performance in some assigned activities. We considered that such data could bring relevant information about the potential advantages, possible difficulties, and requirements, of the inclusion of linkage mechanisms activities in mathematics teachers' initial training courses. See Section 5 for the discussion and final conclusions in view of the obtained results.

## 2. Theoretical Context

In this section, we briefly introduce (mentioning different relevant references for further details) the two basic theoretical ingredients of our research experience: the mathematics of mechanical linkages and the concept of STEAM education and its relevance during the initial teacher training period.

### 2.1. STEAM Education

As declared in the title of this article, the experience we are going to describe and analyze throughout these pages is clearly related to the current educational trend labeled as STEM (or STEAM) education. Thus, let us include here a summary introduction to this terminology and concept, emphasizing those issues we consider more relevant to our work.

It is usually accepted that the acronym STEM (Science, Technology, Engineering, and Mathematics) was first employed in 2001 by the director of the Education and Humanities division of the National Science Foundation (NSF), Judith A. Ramaley, in reference to the curricula of the disciplines involved. Since then, it has been gaining importance both at political and educational levels (see [3] for a relevant initial reference in the European context), while adopting slightly different meanings, depending on the perspective we look at it from, cf. [4–6].

More recently, STEM became STEAM, an extended acronym that reflects a further integrative approach, where the A (from Art) is added to the original STEM label, meaning that the contents of science and humanities should be merged, and highlighting the relevance of creativity in practicing science and engineering [7].

Nevertheless, the STEM/STEAM education proposal does not just claim the merging of contents from diverse disciplines, but also the pursuit of a specific methodology that focuses on creativity and realistic learning of different disciplines from problems and projects [8]. It is a methodology that we can trace back to Jean Piaget's theory of constructivism [9], emphasizing higher thinking skills in performance-based assessment. Constructivism enhances creative thinking, fostering the creation of enthusiastic, independent learners capable of combining experience, knowledge, sensation, and logic. In classes where constructive study is applied, students are supposed to find a better place for learning and to be able to think about knowledge, instead of memorizing facts, thus training in the planning of practical projects to apply them to real situations [10].

As quoted by [11], supporting the same perception: "The curriculum integration can be characterized broadly in four terms: (a) organization of the curriculum around real-world issues and problems relevant to students; (b) planning of learning experiences to integrate pertinent subject knowledge; (c) use of knowledge to address the central issue rather than learning in subjects; and (d) application of knowledge through substantive problem-solving activities and projects" [12].

Indeed, there are official documents from different educational systems (see, for example, the references in the Spanish national (Disposición adicional 25.4, diciembre

2020: *En todo caso, las Administraciones educativas impulsarán el incremento de la presencia de alumnas en estudios del ámbito de las ciencias, tecnología, ingeniería, artes y matemáticas, así como en las enseñanzas de formación profesional con menor demanda femenina.* Translation: Additional Provision 25.4, December 2020: *In any case, the Educational Administrations will promote the increase in the presence of female students in studies in the field of science, technologyía, engineering, arts and mathematics, as well as in vocational training courses with less female demand*) [13] or regional (Currículo ESO/Bachillerato Cantabria, orientaciones metodológicas: *. . . se promoverá el aprendizaje interdisciplinar de investigación basado en la solución de problemas, los métodos de trabajo cooperativo y los grupos interactivos.* Translation: Currículo ESO/Bachillerato Cantabria, methodological orientations: *. . . Interdisciplinary research learning based on problem solving, cooperative work methods and interactive groups will be promoted*) [14] curricula that reflect the importance of integrating Science, Technology, Engineering, Arts and Mathematics, and that consider Project-Based Learning (PBL) as the preferred way to integrate these disciplines in the classroom [15]. In the previous paragraphs, we have already expressed the relevance for the STEAM approach of focusing on problems and projects [7,8,10]. Indeed, working on realistic, open projects, can be considered as the only unifying context in which the diverse STEAM disciplines are ready to co-operate and contribute to the learning process. As analyzed in a recent article [15], the pair STEAM-PBL has been characterized and evaluated in different ways and, for example, certain studies have shown that mathematics teachers who work with the STEAM-PBL approach promoted high cognitive demands and positive perceptions about mathematics in projects where training environments were generated through discussion and a significant feedback loop.

Finally, we would like to highlight another feature of the STEAM-PBL methodology: the focus on problems posed in *real world* contexts, addressed through the integration of content and skills from the different disciplines through an inquiry-based, collaborative learning approach (see, for example, the overview of instructional practices described in some selected articles in Table 1 in [16], arranged in different categories: Integration of STEM content, Focus on problems, Inquiry, Design, Teamwork, Student-centered, Hands-on, Assessment).

In this way, STEAM methodology connects with another, quite significant, current educational trend: the relevance of a competency-based learning design that requires the "... breaking with the structure that traditionally deals with academic disciplines and opting for an interdisciplinary approach to the teaching-learning process." (Original text: *. . . "una ruptura con la estructura tradicional de las disciplinas académicas y una apuesta por un enfoque interdisciplinar del proceso enseñanza-aprendizaje"*) [17]. These are competences that the Council of the European Union [18] breaks-down as follows: (1) Literacy; (2) Multilingual; (3) Mathematics, science, technology and engineering; (4) Digital; (5) Personal, social and learning to learn; (6) Citizenship; (7) Entrepreneurship; (8) Cultural awareness and expressions.

Most European curricula have echoed these skills with the aim of achieving lifelong learning, which enables citizens to meet the professional needs of today's society [19]. See [20] for a detailed presentation of the STEAM-PBL impact concerning the development of key competences.

In what follows, we will shortly describe how the chosen topic (linkages) for our didactic experience is particularly suitable to address the above-mentioned four key characteristics of STEAM education:

- Trans-disciplinary STEAM content, including humanities (art, history);
- Knowledge construction through a project-based methodology;
- Related to real world problems;
- Contributing to the advancement of key competences, as required in the official curriculum.

*2.2. Linkages*

According to [21], a linkage is *a system of interconnected machine elements, such as rods, springs, and pivots, used to transmit power or motion* (Definition available online at https://www.thefreedictionary.com/linkage (accessed on 29 September 2022)). Similar definitions, emphasizing the key role of connection when describing the meaning of the word linkage, appear in other different popular sources such as [22].

Since linkages are quite basic objects, involved in many different contexts, they can be subject to many different approaches; for instance, through mechanical engineering, robotics, kinematics, dynamics, or control theory, considering the linkages involved in robot arms or robotics platforms and trying to find out how to move the linkages (the arms, the legs) to achieve a desired position, orientation or trajectory of one of them (the hand, the platform). See the classic book of [23] for a detailed exposition. In this same context, linkages can also be connected to an important computer algebra problem, by modeling the placement of the linkage bodies through polynomial equations with parameters, and attempting to solve such a system in a symbolic way, see [24,25].

Or, in a very different framework, that of the history of technology, linkages can be regarded as relevant objects of research throughout the 19th century, towards the design of linkages with some specific performance, a challenge that is behind the advancement of the steam machine technology, or is related to the foundations of computers, as detailed in [26–28] and the references therein, which include mentions of some illustrious scientific characters such as Babbage, Kempe, Chebyshev, Sylvester, Cayley or Poincare.

Let us remark that, beyond the—perhaps now outdated—technological applications related to the construction of linkages able to display whatever given geometric object (curve, variety), the involved mathematical problem has been formulated as a precise conjecture (the so called "universality of linkages") by the eminent Fields Medal mathematician W. Thurston, which has been affirmatively solved by Kapovich and Millson just in the current 21st century [29].

. . . Additionally, of course, linkages have already been approached and profited by many in the educative context. To mention just a couple of classic examples, we could refer to the monograph [26], to the recent PhD [30], or to web pages such as https://www.macchinematematiche.org or https://digital.library.cornell.edu/collections/kmoddl (accessed 13 January 2023), devoted to "mathematics machines" and to their potential use in the classroom.

We should remark that, although there is a quite recent and strong trend for exploiting the possibilities of robotics in a STEAM-driven education, e.g., [31–33], it focuses more on robots as a programmed artifact, e.g., on programming issues involving Arduino, BBC micro:bits [34], etc. and not on the geometric issues associated with robot arms built with linkages, which are the kind of problems that underlie the experience we are dealing with.

Indeed, the approach to linkage theory that we have developed in the presentation for mathematics teacher-training students considers the interaction of four different perspectives simultaneously:

- Mechanisms (e.g., historical origins, problems, and applications involved in the development of linkage theory, etc.);
- Mathematics (issues related to the geometric locus achieved by a certain point on a given linkage). We have already included some references dealing with this and the previous item;
- Dynamic geometry (e.g., construction of linkages with dynamic geometry programs, observation of their behavior when dragging some points on the construction);
- STEAM-driven education, with special emphasis on the development of geometric reasoning competences, including the modeling of dynamic geometry artwork (e.g., Theo Jansen's Strandbeest, see https://www.youtube.com/watch?v=C97kMKwZ2-g, and its GeoGebra implementation at https://www.geogebra.org/m/mNheeHTS (accessed 13 January 2023)).

This interaction is already more or less explicitly stated in [35]:

> "Mechanical linkages which occur in many common household items, as well as in 'mathematical machines' from the past, offer a wealth of geometry appropriate for secondary school mathematics. Dynamic geometry models of these linkages form an interface between the concrete and the theoretical, and create a visually rich environment for students to explore, conjecture and construct geometric proofs,"

where we can find, albeit in the context of Secondary Education, an early (recalling the acronym STEM was practically starting to exist at that time, see [5]) antecedent of our pilot study.

For details about the specific approach, content and examples concerning linkages and their educational role that have been presented during the lectures that took place in our experience, we refer the reader to the references [27,28], which develop and illustrate with images the above-mentioned interactions.

## 3. Research Goals, Context and Methodology

As already announced, our STEAM experience was developed in the context of the Master's degree that is mandatory in Spain to become a secondary school mathematics teacher. Students of this Master's degree from two universities in Madrid (Spain) attended, first, a short lecture in which historical, computational, didactic and mathematical issues related to linkages, such as those arising in the 18th and 19th centuries through the development of the steam engine, were addressed (see more details and references, specifically developing the content of this lecture, in Section 2.2). Subsequently, participants were asked to provide different types of feedback (by completing an assigned group task, individually answering a questionnaire, and creating a didactic proposal) on what they had learned in the course and on their perception of the potential interest of linkages as a methodological tool for their future professional activity as mathematics teachers.

### *3.1. Research Goals*

The general research goal of this experience was to outline the possible potential (advantages, difficulties and requirements) of including linkage mechanisms activities in mathematics teachers' initial training courses, with a view towards their future professional development.

Specifically, our aim is to answer the following research questions:

- Did the participants benefit, in general terms, from the experience?
- Did the participants increase, for their professional development, their network of knowledge concerning the technological/pedagogical/content dimensions (TPACK)?

Both questions lead us to synthesize the final aim of this research: to demonstrate the richness of linkage mechanisms to propose activities within the framework of a STEAM methodology in the mathematics classroom.

### *3.2. Our Experience: Context*

To become a secondary school mathematics teacher in Spain, it is mandatory to complete a Master's degree, the Secondary School Teacher Training Master (SSTTM), after having completed a four year scientific or technological undergraduate (mathematics, physics, computer science, engineering, economics, etc.). This Master's degree consists of 60 ECTS, distributed as around 25% for generic subjects, 45% for specialization subjects (such as mathematics) and the remaining 30% for practicum in schools and a Master's thesis. It is developed over one academic year.

To address the previously stated research goals of STEAM-oriented education, the authors performed a research experiment involving all such prospective mathematics teacher students during the 2020–2021 academic year undertaking the SSTTM in the mathematics specialization at their universities, Universidad Antonio de Nebrija (UAN) (73 students in total) and Universidad Rey Juan Carlos (URJC) (29 students in total). Indeed, for the UAN, the total number of students attending these Master's studies to become a mathematics teacher was 73, but they were split into two groups that attended the same Master's courses but with different classrooms, hours, lecturers, etc. All of them were involved in the experience. Likewise, we involved all 29 students at URJC that undertook the same Master's degree that academic year, where they were together in a single class group. Notice that in the UAN the maximum number of students per group in that Master's Speciality is 40, while the maximum is 30 for the URJC.

In both cases, the experiment took place on a subject that involved some 150 working hours for the student (including attending lectures, homework, etc.)—that is about 10% of the whole SSTTM-Mathematics Master's degree. The duration of the experience (as detailed below) can be estimated to be about eight to ten hours of student work.

URJC is a public university with more than 46,000 students, which makes it the second largest university in the Madrid region in Spain, while UAN is a private university with around 12,000 students, also located in the Madrid region. The SSTTM program at URJC is face-to-face, but during the 2020–2021 academic year it was taught online, via videoconferencing, due to pandemic restrictions. On the other hand, the same degree at the UAN is designed to be delivered online with weekly synchronous lectures, given by videoconferencing. Both universities use the Blackboard virtual campus platform. For videoconferencing, UAN uses Collaborate Ultra (a tool from Blackboard), while URJC used Teams (a tool from Microsoft).

In the case of UAN, the 73 students following the SSTTM program in Mathematics were distributed into two different groups (33 in UAN-Group 1 and 40 in UAN-Group 2) and the experience was developed within the framework of the subject "Didactic of Geometry, Measurement of Magnitudes and Statistics". The number of students from URJC was 29 (URJC-Group); they were in one group and the experience was included in the subject "Mathematics of everyday life". The students were informed that they were part of an experiment of which the aim was to show them some basic facts about linkages, and to gauge their possible didactic interest for their future teaching profession. Moreover, the outcomes of the different tasks assigned during the experiment were evaluated by the professor of the subject and the obtained mark was included as an additional item in the subject final grade.

### *3.3. Our Experience: Design and Development*

The students participating in our experience, after receiving some lectures on the topic, were asked:

1. To fill-in a questionnaire about their perception (in different contexts: personal, professional, etc.) of the taught subject;
2. To design and justify some didactic activity using linkages as if they were going to use it in the future as teachers in a secondary education classroom;
3. To work in a team with other students, addressing some open-ended activity proposed by the lecturer (a different activity for each group), submitting or presenting the proposed solution.

In more detail, the methodology followed in the design and implementation of the experiment was developed according to the following steps:

- Design of the materials for the activity to be developed (authors):
    - Lecture preparation including some GeoGebra (www.geogebra.org) and Maple (www.maplesoft.com) (accessed 13 January 2023) files;
    - Proposal of seven different open-ended activities related to the lecture to be completed by students in groups; and
    - Design of the final questionnaire about some personal data and the evaluation of their experience with the training.

- Synchronous training session (two hours per group) with students via videoconferencing (authors and students):
  - One hour lecture; and
  - One hour of supervised work on an open-ended activity assigned to each of the four to six student groups.
- Homework (students):
  - Completion of the open-ended activity in the group;
  - Proposal of a teaching activity inspired in the session (individual);
  - Individual feedback by completing the questionnaire.
- Analysis of results (authors):
- The analysis included the student's profile, feedback during the training session, individual feedback given through the questionnaire, and surveys of the open-ended activities;
- Conclusions and open questions for future work.

The initial lecture in the training session was conducted online by one of the authors of this paper, who has experience in the field of computational algebra, mathematics education and dynamic geometry, and different previous publications on linkages (see orcid.org/0000-0002-1011-295X (accessed 13 January 2023) for a detailed list of publications). He acted as an invited speaker in each of the involved classrooms. The other authors and usual lecturers of the different students' groups attended the initial session, helping the speaker with moderating the class.

The one hour lecture started with the introduction and antecedents (historical, technological, engineering, mathematical, artistic) of articulated models of mechanisms, with special emphasis on those that favor the drawing of curves, as well as a reminder of the main STEAM features, as described in Section 2.2.

Then, among other didactic resources, the exploration of such mechanisms through a Dynamic Geometry system and their visualization by various digital means was proposed. Finally, the experience required the realization, by different groups of students, of some tasks dealing with the construction of certain mechanisms or by answering some queries concerning their performance, following different queries described in the enclosed references.

All throughout the experience, the consideration of this methodological approach as an instrument for the generation, discussion and establishment of some key concepts in geometry and algebra was emphasized. In order to fix some of these goals, students were challenged to work in groups on different open-ended tasks. Some examples of the types of tasks can be seen in Figures 1 and 2.

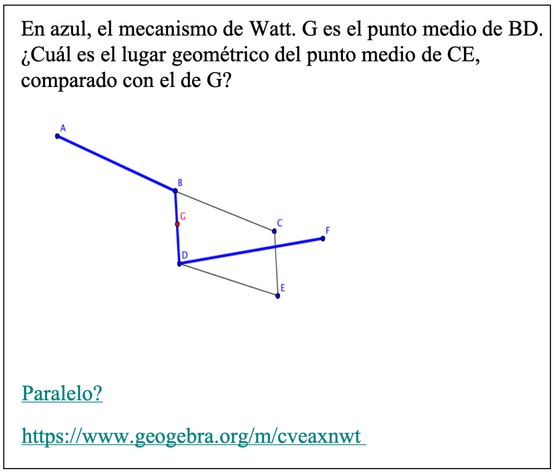


**Figure 1.** One of the tasks proposed to the students: *In blue, the Watt mechanism. G is the midpoint of BD. What is the locus of the midpoint of CE, compared to that of G?*

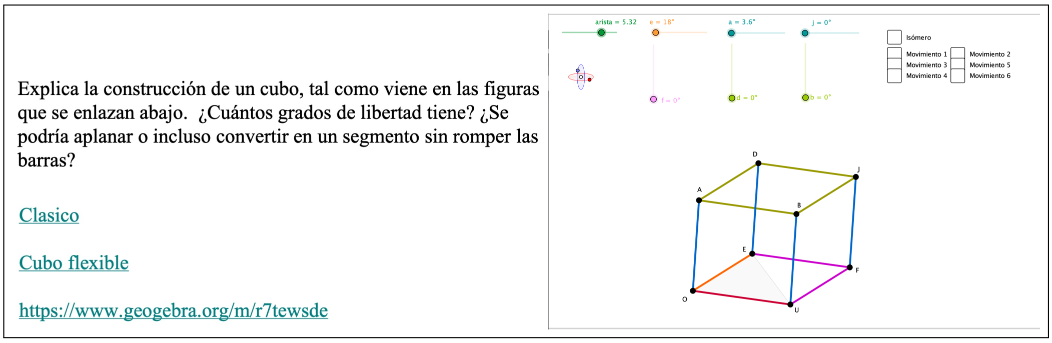


**Figure 2.** Another task proposed to the students: *Explain the construction of a cube as shown in the figures linked below. How many degrees of freedom does it have? Could it be flattened or even made into a segment without being flattened or even made into a segment without breaking the bards?*

Below (Figure 3) you can see the work developed by one of the student groups concerning the Watt's mechanism assignment.

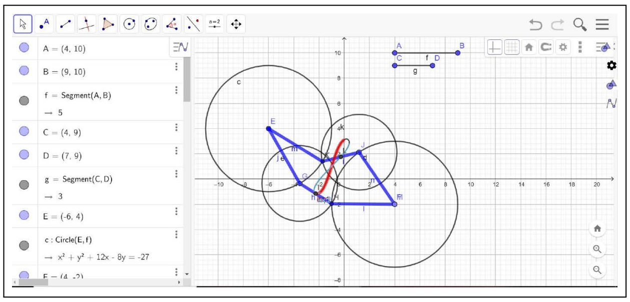


**Figure 3.** A reply, using GeoGebra, to the following task: *In blue, Watt's mechanism. G is the midpoint of BD. What is the locus of the midpoint of CE, compared to that of G?*

Then, during the hour of supervised work, the students were separated into virtual rooms, where they could solve the tasks with the aid of the lecturers and the specialist; they entered the rooms when they were asked to do so. Students worked in groups for half an hour and then the session continued, sharing the results obtained for each group and discussing impressions gained during the experience.

Assigned homework was carried out in different ways at each university. In the URJC-Group, as it had associated a certain grading weight to the assessment of the outcome of the experience, one week was dedicated to the activity. The students submitted, on the one hand, the results of the assigned task through a Google Form, and on the other, the didactic proposal, through the URJC's virtual space. In the case of the UN-Groups, both the questionnaire and a brief proposal for a teaching activity were carried out in Google Forms, dedicating three days of individual work to the activity.

Finally, the students filled out a questionnaire regarding their evaluation of the experience and its potential interest for their professional future. This questionnaire (see Section 4.2) has been the tool for the analysis of the experience and for attempting to answer the research questions.

### *3.4. Methodology for the Qualitative Data and Discourse Analysis*

In this paper we will rely, for the analysis of the initial teacher trainees' own reflective narratives regarding the developed experience, on the theoretical framework of Sfard's Discourse Analysis [36], the Reflection on Practice and the "Discipline of Noticing" ([37–39]). See [40] for a recent article describing, with this methodology, an experience involving the same type of students but concerning automated reasoning tools in dynamic geometry. Other examples of works using reflective narrative analysis within the same theoretical framework, but related to practicing teachers, can be found in [41] (dealing with university teachers), or in [42] (for Primary School teachers).

From our experience, we estimate that the study of the narrative of students' answers to the proposed questionnaire will help us to assess the contribution of our experience towards improving students' acquisition of competences for their professional development in a STEAM mathematical environment, as well as their understanding and appreciation concerning the mathematics issues considered throughout the experience, i.e., linkages. All of this will be measured through the level of depth and detail shown by the students in their answers.

In fact, Mason [37] distinguishes two kinds or levels that can be present in some collection of narratives: "account-of" and "account-for". Mason's original text says:

> "An account-of describes as objectively as possible by minimizing emotive terms, evaluation, judgments and explanation). (...) By contrast, an account-for introduces explanation, theorising and perhaps judgment and evaluation" [37] (p. 40)

and follows:

> "To account-for something is to offer interpretation, explanation, value-judgment, justification or criticism. To give an account-of is to describe or define something in terms that others who were present (or who might have been present) can recognize" [37] (p. 41).

Following Mason's [37] work, an "account-for" level of narrative reflection explains or interprets its own experience from a more personal, complex or subjective point of view, recognizing key ideas, establishing relationships in a critical and prospective way; whereas an "account-of" level of narrative reflection describes ideas or identifies causes from a more aseptic or impersonal position that is only descriptive and even anecdotal.

For the purposes of our experience with mathematics initial teachers' training, and mimicking—almost literally translating from Spanish to English—the arguments in [40], in what follows we will consider three degrees of narrative reflection degree, based on the ideas and terminology of Mason [37]:

- The narrative with a low reflection degree or "account-of", in which the student only describes or refers to anecdotal events of the training sessions and that we could relate, in some way, to a low state of Mason's "account-of" dimension;
- The narrative with a intermediate reflection degree or "account", in which the teaching disposition is naive and slightly conscious. Thus, in this case, we consider that their degree of narrative reflection is valued at a midpoint, between a high "account-of" and a low "account-for", for expressing attempts of interpretation of crucial ideas from training sessions;
- The narrative with a high reflection degree or "account-for", which already indicates a reflective and personal teaching disposition, with an attempt to conceptualize the key ideas of the sessions and a teaching expectation for the professional future.

On the other hand, following [43] and the references therein, the analysis of the narratives from the students in our experience will be considered, as well the framework of the TPACK (Technological Pedagogical Content Knowledge) perspective [44] (see Figure 4), focusing on the identification of potential areas of professional development in future teachers, by increasing teachers' knowledge of mathematics content and by showing them new approaches and methods that could be applicable in their future teaching.

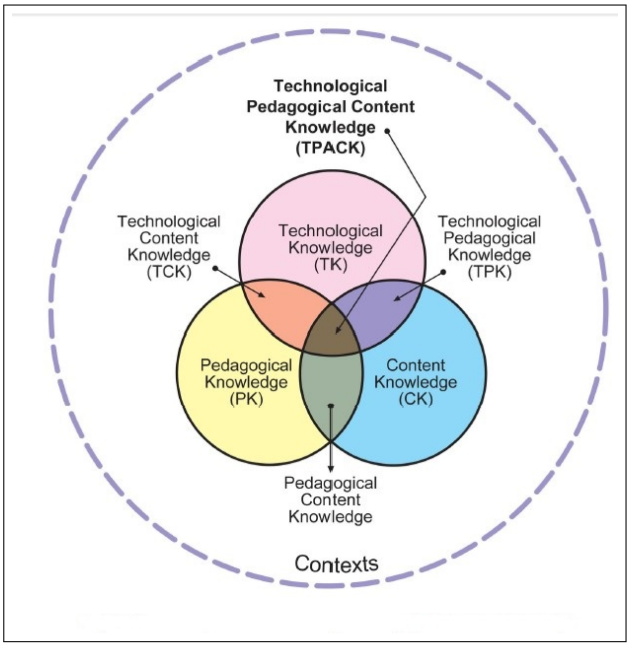


**Figure 4.** The TPACK framework, following [29] (2009, p. 63).

## 4. Analysis of Results

### *4.1. Students Participation*

The participation of the students in the training session and in the homework tasks has been unequal. The reason for this is, in some sense, the different scope of the courses in the two Master's programs and the different evaluation criteria.

All 73 students from the two class-groups of UAN participated in the lecture and in the working groups to discuss and complete the proposed open-ended activity, but only 79% of UAN-Group 1 and 68% of UAN-Group 2 answered the questionnaire and proposed a teaching activity. In the planning of the course "Didactics of Geometry, Measurement of magnitudes and Statistics" of UAN, the evaluation of the open-ended activity was part of the evaluation in terms of deliverable activities. However, the response to the questionnaire and the didactic proposal was proposed on a voluntary basis, having 2 days as a deadline and it contributed only to improving a participation mark.

In the case of URJC-Group, three students were excused from class attendance, thus they did not participate in the synchronous training session, but they visualized it afterwards and carried out the rest of the activities. Therefore, 26 students followed and participated in the training session and all 29 completed the three proposed homework activities. In this case, the open-ended activity and the didactic proposal were important evaluation items and the students had 10 days to submit them.

Summarizing, a total of 82 SSTTM students completed the questionnaire, 53 of them from UAN (26 from UAN-Group 1 and 27 from UAN-Group 2) and the remaining 29 from URJC-Group. Their initial degree studies were substantially different: mathematics, industrial, aeronautical, civil or other engineering, physics, computer sciences, architecture, .etc. Due to the mode of delivery of the Master's degree, the average student profile and age varied between the two universities. Figure 5 contains a table giving an overview of the number and percentage of involved students in terms of university and previous training.

At URJC, where the program is defined as face-to-face, young students who have recently completed their undergraduate studies predominate and, among them, 34% come from undergraduate studies in mathematics. At UAN, where the program is offered online, the students are older and, for the most part, have had professional experience outside the field of education; here, the predominant background is in industrial engineering which represents 34% of the total.

| | UAN | | URJC | | TOTAL | |
|---|---|---|---|---|---|---|
| **Previous training** | **no. students** | **%** | **no. students** | **%** | **no. students** | **%** |
| Aeronautical/Aerospace Engineering | 2 | 4% | 3 | 10% | 5 | 6% |
| Architecture | 8 | 15% | 3 | 10% | 11 | 13% |
| Mathematics/Statistic | 1 | 2% | 10 | 34% | 11 | 13% |
| Physics/Chemistry | 3 | 6% | 1 | 3% | 4 | 5% |
| Civil Engineering | 8 | 15% | 0 | 0% | 8 | 10% |
| Industrial Engineering | 18 | 34% | 4 | 14% | 22 | 27% |
| Computer Sciences/Telco | 6 | 11% | 3 | 10% | 9 | 11% |
| Others | 7 | 13% | 5 | 17% | 12 | 15% |
| TOTAL | **53** | | **29** | | **82** | |

**Figure 5.** Distribution of students that completed the questionnaire according to their previous undergraduate studies. Data are given by university (UAN, URJC) and total, then the number of students (no.students) and their percentage (%) of the total in their column.

### 4.2. *Analysis of the Questionnaire*

In order to answer the research questions we stated in Section 3.1, the students had to fill out at the end of the term a questionnaire regarding their evaluation of the experience and its potential interest for their professional future. These questions (closely related to those of [40], although in a different context) were the following:

1. Report two brief-but-vivid moments in the lecture session;
2. Formulate a question for you, related to the session;
3. Answer your question in a specific way;
4. What is not clear to you?
5. What do you think you have learned?

Next, we will analyse the data obtained from the students' responses to the questionnaire, attending only those of Questions 1, 4 and 5. Indeed, Questions 2 and 3 are in some sense redundant, as they are somehow already present in the other three questions (what is not clear for the student or what the student relates as vivid moments or as something the student has learned could be considered as important questions/answers that the student should formulate to him/herself). Note that this is a qualitative questionnaire, and that reproducing all the responses will not be possible. We have used two approaches:

(1) Classifying the results attending the referred instant of the lecture where the students have set the moments they emphasised in the session (Question 1), or the part of the lecture that it was not clear for him/her (Question 4);
(2) Analyzing the narrative reflection of the responses, attending the three degrees of the TPACK methodology.

#### 4.2.1. Qualitative and Quantitative Analysis by Topics Interest

In order to analyze the responses to Question 1: "Report two brief-but-vivid moments in the lecture session", we divided the contents of the lecture into different stages, following the development of the talk:

I. Introduction to the STEAM methodology
II. Linkages: concept. Introduction (Torres Quevedo);
III. Linkages in the context of industrial revolution (end of XVIII century), the search for linkages drawing a straight-line;
IV. Mathematical issues: algebraic formulation, degrees of freedom/dimension of space of solutions, locus, Thurston's conjecture, Kapovich–Millson solution (2002) on the universality of linkages, etc.;
V. Linkages as conceptual and methodological antecedents to computers and to computer algebra, in particular;
VI. Linkages in some artistic works (Lopez Binder, Theo Jansen);
VII. Resources for working online with linkages (virtual museums, dynamic geometry tools, automated reasoning. . . ).

Subsequently, the responses were ranked and matched to each of these stages. Figure 6 shows the percentage of responses that refer to each moment of the talk, according to this classification.

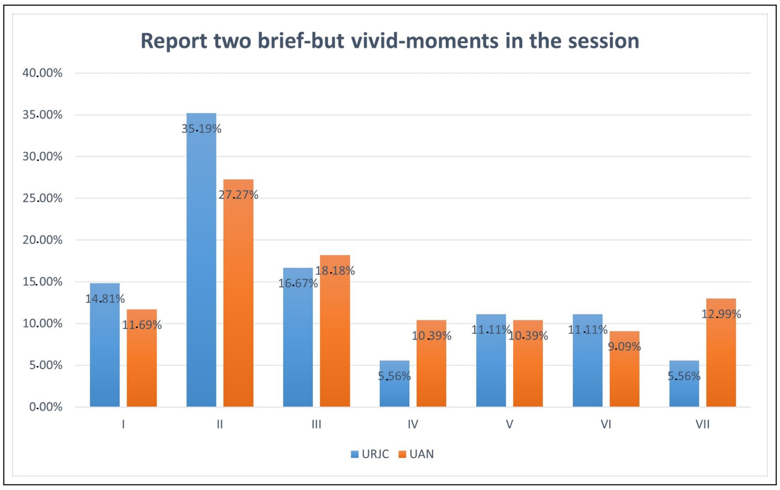


**Figure 6.** Answers to question 1: *Report two brief-but vivid-moments in the session*.

As can easily be appreciated, students highlight the introduction of the historical elements in the lecture, with special reference to the figure of Leonardo Torres Quevedo (items II and III). This Spanish scientist was not well known among students, especially among those of them that were not engineers; they were impressed by his scientific goals and they were proud of him as a Spanish scientist.

Item II also considered the introduction of the linkages concept. Approximately 60% of the students in the groups under analysis had studied linkages in their previous career, but only slightly. All of them were delighted with the contents. The STEM methodology was also highly valued, and so was the last part of the session dedicated to resources for working with linkages. Those items are directly involved with applications in the classroom, in terms of methodology, elements treasured in pre-service teachers training.

Analyzing the data of Question 4 required a slightly different division of the structure of the lecture:

I. Introduction to the STEAM methodology;
II. Concept, uses of linkages in the history;
III. Mathematical issues: algebraic formulation, degrees of freedom/dimension of space of solutions, locus, Thurston's conjecture, Kapovich–Millson solution (2002) on the universality of linkages, etc.;
IV. Modelization, computer algebra (GeoGebra);
V. Maple computation;
VI. Linkages in some artistic works (Lopez Binder, Theo Jansen);
VII. Resources for working on-line with linkages (virtual museums, dynamic geometry tools, automated reasoning. . . );
VIII. "I understood the whole lecture";
IX. Problems during the session (classroom management, solving the task in groups, duration of the class,. . . );
X. "I didn't understand anything".

The results of the analysis are shown in the graphics below (Figures 7 and 8), discriminating on this occasion by group:

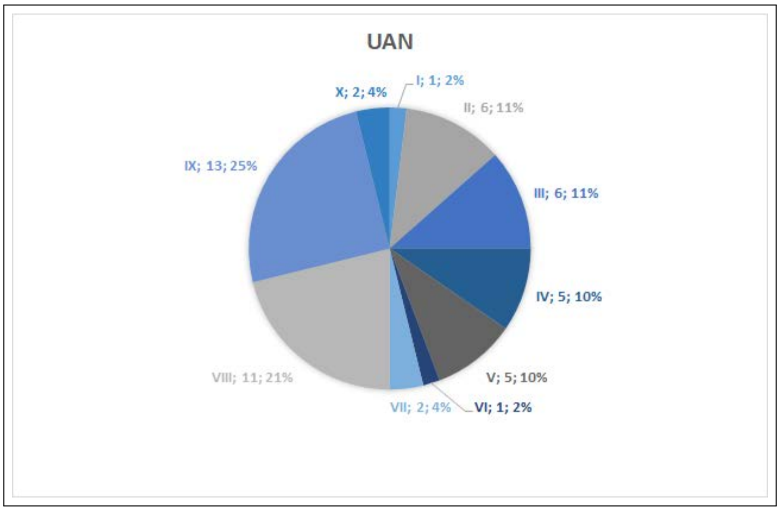


**Figure 7.** Answers to question 4 *What is not clear to you?* given by UAN group.

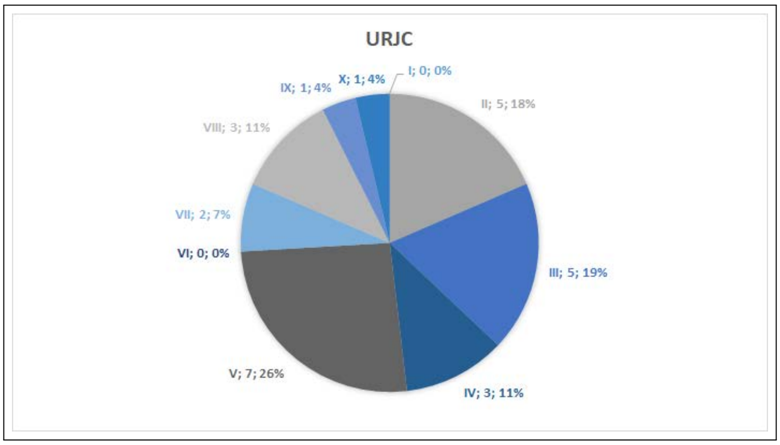


**Figure 8.** Answers to question 4 *What was not clear to you?* given by URJC group.

In this analysis, data obtained in each group are slightly different. While at URJC, students encountered more difficulties with items in V (related to MAPLE's computation) and III (some issues about Kempe's Theorem, and its algebraic formulation), students of group UAN where confused with III, and a number of them described problems solving the associated tasks.

4.2.2. Discursive Analysis

We now consider the second methodological approach, analyzing the answers of the students following the discursive analysis methodology (see Section 3.4), and identifying the three grades (low, intermediate and high) according to Section 3.4 of the answers to Questions 1, 4 and 5.

Their classification in these three levels has been performed, in view of the discursive analysis of the responses, as follows: responses that just express some anecdotal event during the session, or some very limited, personal appreciation, are rated as low grade ("account-of"), while responses that reflect some level of understanding of the didactic potential of the session, or of some concrete items in the session but without going much further, are classified as intermediate grade ("account"). Finally, responses that denote a prospective reflection, with a long term, professional perspective well beyond the particular session, are classified as high grade ("account-for").

Question 1: *Report two brief-but-vivid moments in the lecture session.*

Students' reactions to Question 1 showed 33% with a low grade of reflection ("account-of"), 49% with an intermediate grade of reflection ("account") and 18% with a high grade of reflection ("account-for"). See more detailed results in Figure 9.

| | Question 1 | | | | | |
|---|---|---|---|---|---|---|
| | low | | Intermediate | | high | |
| | no. Students | % | no. Students | % | no. Students | % |
| UAN | 19 | 36% | 28 | 53% | 6 | 11% |
| URJC | 8 | 28% | 12 | 41% | 9 | 31% |
| TOTALES | 27 | 33% | 40 | 49% | 15 | 18% |

**Figure 9.** Distribution of responses to Question 1 according to the three grades of reflection (low, intermediate, high) by giving the number of students (no.students) classified and their percentage (%) of the total in their row (UAN, URJC and total).

Some responses to Question 1 according to the grade of reflection:

- Account-of: "When trying to make the groups that was a bit chaotic" or "The teacher's grand handling in GeoGebra" are classified as "account-of";
- Account: "One of the things that caught my attention that for the speaker of the talk is that, under his criteria, he added to the term STEAM the concept of History and under his explanation, because the relationship was quite logical", or "Usefulness of the mechanisms developed by Torres Quevedo and their application to models of everyday life";
- Account-for: "Discovering the topic of algebraic machines, unknown to me until now, which can be perfectly exploited in the creation of projects based on real contexts, involving other disciplines (including Engineering, Physics, Technology, History, Art, and of course Mathematics), allowing collaborative work and requiring research by students". "The example of the pendulum that was used to find the square roots, seemed to me a very simple example and easy to apply in the classroom, which has motivated me to investigate it on my own", or "It has been of special interest to know about Jansen's resources in GeoGebra, which we can take to the mathematics classroom in Secondary Ed".

Question 4: *What was not clear to you?*

The distribution of rates for responses to Question 4 are close to Question 1: 29% "account-of", 54% "account" and 17% "account-for". See Figure 10.

| | Question 4 | | | | | |
|---|---|---|---|---|---|---|
| | low | | Intermediate | | high | |
| | no. Students | % | no. Students | % | no. Students | % |
| UAN | 17 | 32% | 30 | 57% | 6 | 11% |
| URJC | 7 | 24% | 14 | 48% | 8 | 28% |
| TOTALES | 24 | 29% | 44 | 54% | 14 | 17% |

**Figure 10.** Distribution of responses to Question 4 according to the three grades of reflection (low, intermediate, high) by giving the number of students (no.students) classified and their percentage (%) of the total in their row (UAN, URJC and total).

Examples of responses classified in each grade are:

- Account-of: "There was very little time to execute with GeoGebra the requested activities since a high knowledge of this tool was assumed", "The activities are a bit difficult or at least with a lot of time loss to connect in group";
- Account: "It has been difficult for me to observe Kempes' theorem, since it is a mixture between Euler's theorem and conics, so I have searched for more information to understand it better", and "For which courses are these resources used?";

- Account-for: "It is clear that, in order to be able to teach my students (and even more so in the times we live in, when information is accessible on the internet to anyone who knows how to look for it) I must become an expert in the subject", or "It has not been very clear to me how I could explain well to the students the relationship between algebraic machines and computer algebra".

Question 5: *What do you think you have learned?*

Finally, we studied from this perspective the answers to Question 5, which can also enlighten our research question 2, "Did the participants increase, for their professional development, their network of knowledge concerning the technological/pedagogical/content dimensions (TPACK)?". In this case, there were fewer responses, with 19% classified as "account of", while for the number of responses "account-for", 44% increase their rate with an "account" response rate of 37%. See Figure 11.

| | Question 5 | | | | | |
|---|---|---|---|---|---|---|
| | low | | Intermediate | | high | |
| | no. Students | % | no. Students | % | no. Students | % |
| UAN | 10 | 19% | 20 | 38% | 23 | 43% |
| URJC | 5 | 18% | 10 | 35% | 14 | 47% |
| TOTALES | 15 | 19% | 30 | 37% | 36 | 44% |

**Figure 11.** Distribution of responses to Question 5 according to the three grades of reflection (low, intermediate, high) by giving the number of students (no.students) classified and their percentage (%) of the total in their row (UAN, URJC and total).

Examples of responses according to the three grades are:

- Account-of: "Group work, GeoGebra, more about TEAMS", "That there is a lot to learn from these mechanisms, which were designed without the need for modern programs";
- Account: "A very simple way to model mechanisms that can be found in everyday objects or situations. One more tool to give a multidisciplinary (trans-disciplinary) approach to classroom sessions", "To have another vision of mathematics from the point of view of STEAM methodology";
- Account-for: "That the resources of mechanisms related to geometry can be a good resource to use in the teaching of mathematics. Additionally, making use of GeoGebra through incomplete exercises can be very interesting, making mathematics a trans-disciplinary subject", "The use of articulated mechanical models in mathematics class for the generation of ideas and learning of complex concepts is a very effective resource to foster a STEAM educational environment as well as to introduce historical scientific contexts to students. The creation of simulations in environments such as GeoGebra allow the student to better internalize dynamic geometry with tangible and variable objects, becoming a very powerful tool to show complicated geometric concepts and relationships in a simple and intuitive way", "Another vision of mathematics and how to teach it in a more creative and practical way to call students' interests".

Conclusions about these results derived from the whole experience will be included in the next section.

## 5. Conclusions

In the previous sections, we have described and analyzed an educational experience in the framework of STEAM education and in the context of the Master's degree that is compulsory in Spain to become a secondary education mathematics teacher. The experience emphasized the concourse of a dynamic geometry software package to present and discuss some historical, didactic and mathematical issues related to linkage mechanisms, such as those arising in the 18th and 19th centuries along the development of the steam engine (see Section 2.2).

As documented in Section 2.1, STEAM education now plays a relevant role in the education curricula of many countries. Thus, providing teachers with rich contexts with which to approach, globally and jointly, the different disciplines (including mathematics, engineering, history, arts, technology) involved in this methodology is both urgent and badly needed, since there are not so many global contexts. As we have summarily argued in Section 2.2, linkage theory, history and applications, could provide one such rare, suitable framework with which to develop a STEAM-PBL methodology. As an obvious consequence, analyzing the impact of presenting such proposal to future teachers is also relevant.

Taking as a starting point the description of the content and development of a lecture on linkages addressed to the future mathematics teachers, we have analyzed, using different quantitative and qualitative approaches, the information supplied by the participants through the presentation of the assigned activities and the answers to some questions.

This analysis is then the basis for answering our research questions (see Section 3.1) about the perception by these students of the potential interest in linkages as a methodological instrument for their future professional activity as teachers, and to reflect on the opportunities (and problems) that this particular bar-joint linkages methodological approach could bring, to enhance a STEAM-oriented education.

We think that a reasonable answer to the research questions follows from the responses of the students to the proposed questionnaire. Students' answers to Question 1 ("Report two brief-but-vivid moments in the lecture session"), the majority with a reasonable professional perspective ("account"), clearly drive us to the conclusion that the students were delighted with the experience. They valued the lecture as very positiive, in a global sense, with special attention paid to the introduction of historical elements, the concept of linkages, and the application of linkages and their resources, in the teaching of STEM subjects. Of course, some doubts and difficulties arise, as described when analyzing answers to Question 4. It is interesting to remark that the different groups of students behaved quite homogeneously, both in the choice of the more interesting moments and in the distribution of percentages concerning the level of reflections.

Likewise, the description of the answers to Question 5, with the highest percentage of "account-for" discourses in all groups, clearly show that students have learned relevant technological/pedagogical/content knowledge. Moreover, as a complementary support of this conclusion, let us mention that, on the other hand, every student group solved the proposed open-ended activity successfully, and managed to express and clarify the doubts that arose during the experience. Although we have not fully analyzed the didactic activity individually proposed by each of the students, since the conditions for carrying it out were not uniform in the two groups of study (URJC and UAN), the already-analyzed activities clearly show that the students have seen, and express in their proposals, the didactic relevance of the use of linkages in the classroom.

Finally, we can conclude, supported by all these results, that the experience was positive for the students, pre-service teachers, both in terms of improving their knowledge of a singular topic, simultaneously traditional and very suitable for the novel STEAM methodology, as well as enhancing the possibilities of considering its application in their future role as mathematics teachers. This is consistent with the findings of other, relatively similar, studies on the use of mechanisms in the STEAM methodology, which we have referred to in Section 2.2.

Obviously, the experience we have described here is just a starting point for studying more deeply the possibilities of the use of linkages in a STEAM-PBL classroom. It should be extended to other experiments with in-service teachers, who could bring new perspectives from their professional experiences. Additionally and more importantly, it should be implemented in a classroom with young students, to evaluate the real possibilities of this approach. As mentioned already in Section 2.2, current experiences in this context are biased towards the use of robotics, and do not put an emphasis on the geometric issues we emphasize in our proposal: from the steam machine to STEAM education!

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