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Solitary Wave Solutions of the Fractional-Stochastic Quantum Zakharov–Kuznetsov Equation Arises in Quantum Magneto Plasma

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Abstract: In this paper, we consider the (3 + 1)-dimensional fractional-stochastic quantum Zakharov–Kuznetsov equation (FSQZKE) with M-truncated derivative. To find novel trigonometric, hyperbolic, elliptic, and rational fractional solutions, two techniques are used: the Jacobi elliptic function approach and the modified F-expansion method. We also expand on a few earlier findings. The extended quantum Zakharov–Kuznetsov has practical applications in dealing with quantum electronpositron–ion magnetoplasmas, warm ions, and hot isothermal electrons in the presence of uniform magnetic fields, which makes the solutions obtained useful in analyzing a number of intriguing physical phenomena. We plot our data in MATLAB and display various 3D and 2D graphical representations to explain how the stochastic term and fractional derivative influence the exact solutions of the FSEQZKE.

Keywords: Stochastic Zakharov–Kuznetsov equation; truncated M-fractional derivative; Jacobi elliptic function method; modified F-expansion method

MSC: 60H10; 35A20; 35C05; 35C08

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Citation: Mohammed, W.W.; Al-Askar, F.M.; Cesarano, C.; El-Morshedy, M. Solitary Wave Solutions of the Fractional-Stochastic Quantum Zakharov-Kuznetsov Equation Arises in Quantum Magneto Plasma. *Mathematics* **2023**, *11*, 488. https://doi.org/10.3390/math11020488

Academic Editors: Francisco Chiclana, Sergei Petrovskii, Matjaz Perc, Antonio Di Crescenzo and Marjan Mernik

Received: 17 December 2022
Revised: 11 January 2023
Accepted: 13 January 2023
Published: 16 January 2023

1. Introduction

The stochastic model is evaluated based on several criteria to guarantee accuracy in likely outcomes. Therefore, the stochastic model must account for all potential sources of uncertainty in order to present all viable scenarios and produce the most accurate probability distribution. Additionally, each probability is connected to the others within the model, which helps determine how random the inputs are as a whole. Further predictions and information forecasting are performed using these probabilities. As a result, the significance of fluctuations or randomness in many phenomena has now been demonstrated. Because of this, random effects have become more crucial for modeling a variety of physical processes that take place in disciplines including oceanography, finance, physics, biology, meteorology, environmental sciences, and others [1–3]. Stochastic partial differential equations are the best mathematical representations of complex systems when noise or random effects are involved.

In contrast, fractional differential equations (FDEs) are used to explain a wide variety of physical phenomena in electromagnetic theory, engineering fields, mathematical biology, signal processing, and other fields of science. Additionally, the fractional-order derivative can be used to represent a variety of physical phenomena, such as sound electrostatics, heat, elasticity, gravity, diffusion, and many others [4–6]. Numerous definitions have been proposed due to the significance of the fractional-order derivative, including He's fractional derivative, conformable fractional definitions, the Riemann–Liouville derivative, the Riesz derivative, the Grunwald–Letnikov derivative, the new truncated M-fractional derivative, and Beta derivative [7–14].

A novel fractional derivative known as the truncated M-fractional derivative has just been proposed by Sousa et al. [14]. From here, let us define the truncated M-fractional derivative for the function $u : [0, \infty) \to \mathbb{R}$ of order $\alpha \in (0, 1)$ as follows:

$$\mathbb{T}^{i,\alpha,\beta}_{M,z} u(z) = \lim_{h \to 0} \frac{u(z +_i E_\beta(hz^{-\alpha})) - u(z)}{h}, \tag{1}$$

where the truncated Mittag–Leffler function ${}_iE_\beta(x)$, for $\beta > 0$ and $x \in \mathbb{C}$, [15] with one parameter is defined as

$${}_iE_\beta(z) = \sum_{k=0}^{i} \frac{z^k}{\Gamma(\beta k + 1)}.$$

The following features of the truncated M-fractional derivative are met for any constants a and b [14,16]:

$$\begin{aligned}
(1)\ \mathbb{T}^{i,\alpha,\beta}_{M,z}(au + bv) &= a\mathbb{T}^{i,\alpha,\beta}_{M,z}(u) + b\mathbb{T}^{i,\alpha,\beta}_{M,z}(v),\ (2)\ \mathbb{T}^{i,\alpha,\beta}_{M,z}(z^\nu) = \frac{\nu}{\Gamma(\beta+1)} z^{\nu-\alpha},\\
(3)\ \mathbb{T}^{i,\alpha,\beta}_{M,z}(uv) &= u\mathbb{T}^{i,\alpha,\beta}_{M,z} v + v\mathbb{T}^{i,\alpha,\beta}_{M,z} u,\quad (4)\ \mathbb{T}^{i,\alpha,\beta}_{M,z}(u)(z) = \frac{z^{1-\alpha}}{\Gamma(\beta+1)} \frac{du}{dz},\\
(5)\ \mathbb{T}^{i,\alpha,\beta}_{M,z}(u \circ v)(z) &= u'(v(z))\mathbb{T}^{i,\alpha,\beta}_{M,z} v(z).
\end{aligned}$$

The implications of FDEs has resulted in the development of various powerful and efficient methods for determining the exact solutions to these equations. Several of these techniques are the Riccati–Bernoulli sub-ODE [17], the tanh-sech method [18,19], the Jacobi elliptic function [20], Hirota's method [21], $exp(-\varphi(\varsigma))$-expansion [22], the perturbation method [23,24], Lie symmetry analysis [25], the invariant subspace method [26], the sine-cosine method [27,28], modified extended mapping [29], the (G'/G)-expansion method [30,31], etc.

Recent study on fractional differential equations with stochastic terms has been examined, for example, [32–38] and the references therein. As a result, we consider the (3 + 1)-dimensional fractional-stochastic quantum Zakharov–Kuznetsov equation (FSQZKE) with M-truncated derivative as follows:

$$\varphi_t + A\varphi\mathbb{T}^{0,\alpha,\beta}_{M,z}\varphi + B\mathbb{T}^{0,\alpha,\beta}_{M,zzz}\varphi + C\mathbb{T}^{0,\alpha,\beta}_{M,zxx}\varphi + C\mathbb{T}^{0,\alpha,\beta}_{M,zyy}\varphi = \sigma\varphi W_t, \tag{2}$$

where $\varphi = \varphi(x, y, z, t)$ is the electrostatic potential. A, B, and C are well-known constants expressing dispersive and nonlinear coefficients. $\mathbb{T}^{0,\alpha,\beta}_{M,z}$ is the M-truncated derivative defined in Equation (1) with $i = 1$. $W(t)$ is the Wiener process, and φW_t is a multiplicative noise in the Itô sense. σ is the intensity of noise.

In the case of $\alpha \to 1$ and $\sigma = \beta = 0$, then Equation (2) tends to the classical form:

$$\varphi_t + A\varphi\varphi_z + B\varphi_{zzz} + C\varphi_{zxx} + C\varphi_{zyy} = 0. \tag{3}$$

In Moslem et al. [39] and Washimi and Taniuti [40], respectively, the reductive perturbation approach and a sequence of transformations were used to derive Equation (3). The Zakharov–Kuznetsov equation is used frequently in engineering, applied mathematics, and physics. It particularly appears in the field of plasma physics [41]. This Equation (3) could be employed to illustrate how low-frequency ion-acoustic waves propagate in a dense quantum magneto-plasma [42]. Different methods for obtaining the exact solutions of quantum Zakharov–Kuznetsov Equation (3), such as the Jacobi elliptic equation and generalized (G'/G)-expansion [43,44], Hirota bilinear and auxiliary equation [45], exp-function, modified F-expansion methods [46], the generalized unified method [47], and the extended F-expansion method [48], while Equation (3) with a stochastic term and fractional derivative with the M-truncated derivative has not been addressed previously before.

Achieving the analytical solutions of FSQZKE (2) is the motivation and main objective of this article. The Jacobi elliptic function approach and the modified F-expansion method are used to obtain these solutions. In addition, we generalize some earlier results, such as the solutions presented in [43,48]. In describing some significant physical occurrences, the solutions provided would be of great benefit to physicists. With the aid of the MATLAB program, we also introduce a number of graphical representations to examine the effects of the stochastic term and fractional derivative on the analytical solution of the FSQZKE (2).

This article is organized as follows: In Section 2, we use the wave transformation to get the wave equation for the FSQZKE (2). In Section 3, we have the exact fractional solutions of the FSQZKE (2), while in Section 4, we present some graphical representations to see the effect of fractional derivative on the attained solutions of the FSQZKE. Finally, the conclusions of the paper are stated.

2. Wave Equation for the FSQZKE

The next wave transformation is used to generate the wave equation for the FSQZKE (2):

$$\varphi(x,y,z,t) = u(\eta)e^{(\sigma W(t)-\frac{1}{2}\sigma^2 t)}, \quad \eta = \frac{\Gamma(\beta+1)}{\alpha}(\eta_1 x^\alpha + \eta_2 y^\alpha + \eta_3 z^\alpha) + \lambda t, \tag{4}$$

where u is deterministic function, $\eta_1,\ \eta_2,\ \eta_3$ and λ are unknown constants. As we observe

$$\varphi_t = (\lambda u' + \sigma u W_t)e^{(\sigma W(t)-\frac{1}{2}\sigma^2 t)},\ \mathbb{T}^{0,\alpha,\beta}_{M,z}\varphi = \eta_3 u' e^{(\sigma W(t)-\frac{1}{2}\sigma^2 t)}, \tag{5}$$

and

$$\begin{aligned} \mathbb{T}^{0,\alpha,\beta}_{M,zzz}\varphi &= \eta_3^3 u''' e^{(\sigma W(t)-\frac{1}{2}\sigma^2 t)},\ \mathbb{T}^{0,3\alpha,\beta}_{M,zxx}\varphi = \eta_3\eta_1^2 u''' e^{(\sigma W(t)-\frac{1}{2}\sigma^2 t)}, \\ \mathbb{T}^{0,\alpha,\beta}_{M,zyy}\varphi &= \eta_3\eta_2^2 u''' e^{(\sigma W(t)-\frac{1}{2}\sigma^2 t)}. \end{aligned} \tag{6}$$

Plugging Equation (4) into Equation (2) and using (5) and (6), we have

$$\lambda u' + A\eta_3 u u' e^{(\sigma W(t)-\frac{1}{2}\sigma^2 t)} + [B\eta_3^3 + C\eta_3\eta_1^2 + C\eta_3\eta_2^2]u''' = 0. \tag{7}$$

We take the expectation $\mathbb{E}(\cdot)$, which satisfies (i) $\mathbb{E}(XY) = \mathbb{E}(X)\mathbb{E}((Y)$ if X and Y are independent random variable (ii) $\mathbb{E}(X) = X$ if X is deterministic, on both sides

$$\lambda u' + A\eta_3 u u' e^{-\frac{1}{2}\sigma^2 t}\mathbb{E}(e^{(\sigma W(t))}) + [B\eta_3^3 + C\eta_3\eta_1^2 + C\eta_3\eta_2^2]u''' = 0.$$

Since $W(t)$ is a Normal process, $\mathbb{E}(e^{\sigma W(t)}) = e^{\frac{1}{2}\sigma^2 t}$. Hence, the above equation becomes

$$\lambda u' + A\eta_3 u u' + [B\eta_3^3 + C\eta_3\eta_1^2 + C\eta_3\eta_2^2]u''' = 0. \tag{8}$$

Integrating Equation (8) once and putting the integration constants equal to zero, we get

$$u'' + \ell_1 u + \ell_2 u^2 = 0, \tag{9}$$

where

$$\ell_1 = \frac{\lambda}{B\eta_3^3 + C\eta_3\eta_1^2 + C\eta_3\eta_2^2} \text{ and } \ell_2 = \frac{A\eta_3}{2(B\eta_3^2 + C\eta_1^2 + C\eta_2^2)}.$$

3. Exact Solutions of FSQZKE

In order to obtain solutions for FSQZKE (2), we apply two various approaches: the Jacobi elliptic function (JEF) approach and the modified F-expansion method.

3.1. JEF Method

Here, we use the JEF approach (see [49]). With regard to Equation (9), the solutions have the next form:

$$u(\eta) = \sum_{j=0}^{M} \hbar_j [\Omega(\eta)]^j, \tag{10}$$

where $\hbar_0$, $\hbar_1$, ..., $\hbar_M$ are unknown constants and $\hbar_M \neq 0$, $\Omega(\eta) = sn(\eta, m)$ is the Jacobi elliptic sine function for $0 < m < 1$. To calculate M, we balance u^2 with u'' in Equation (9) to have

$$2M = M + 2,$$

therefore

$$M = 2. \tag{11}$$

Rewriting Equation (10) by utilizing Equation (11) as

$$u(\eta) = \hbar_0 + \hbar_1 \Omega(\eta) + \hbar_2 \Omega^2(\eta), \tag{12}$$

Differentiating Equation (12) twice

$$u''(\eta) = 2\hbar_2 - \hbar_1(m^2+1)\Omega - 4\hbar_2(m^2+1)\Omega^2 + 2\hbar_1 m^2 \Omega^3 + 6\hbar_2 m^2 \Omega^4. \tag{13}$$

substituting Equations (12) and (13) into Equation (9), we attain

$$(6m^2\hbar_2 + \ell_2\hbar_2^2)\Omega^4 + (2m^2\hbar_1 + 2\ell_2\hbar_1\hbar_2)\Omega^3 + (2\hbar_0\ell_2\hbar_2 - 4\hbar_2(m^2+1) + \ell_1\hbar_2 + \ell_2\hbar_1^2)\Omega^2$$
$$-[(m^2+1)\hbar_1 - \ell_1\hbar_1 - 2\ell_2\hbar_0\hbar_1]\Omega + (2\hbar_2 + \ell_1\hbar_0 + \ell_2\hbar_0^2) = 0.$$

Balancing each coefficient of Ω^n to zero, yields

$$6m^2\hbar_2 + \ell_2\hbar_2^2 = 0,$$
$$2m^2\hbar_1 + 2\ell_2\hbar_1\hbar_2 = 0,$$
$$2\hbar_0\ell_2\hbar_2 - 4\hbar_2(m^2+1) + \ell_1\hbar_2 + \ell_2\hbar_1^2 = 0,$$
$$(m^2+1)\hbar_1 - \ell_1\hbar_1 - 2\ell_2\hbar_0\hbar_1 = 0,$$

and

$$2\hbar_2 + \ell_1\hbar_0 + \ell_2\hbar_0^2 = 0.$$

We obtain the next two sets after solving these equations:

First set:

$$\begin{cases} \hbar_0 = \frac{2(m^2+1) - 2\sqrt{m^4 - m^2 + 1}}{\ell_2}, \\ \hbar_1 = 0, \\ \hbar_2 = \frac{-6m^2}{\ell_2}, \\ \lambda = 4(B\eta_3^3 + C\eta_3\eta_1^2 + C\eta_3\eta_2^2)\sqrt{m^4 - m^2 + 1}. \end{cases}$$

Second set:

$$\begin{cases} \hbar_0 = \frac{2(m^2+1) + 2\sqrt{m^4 - m^2 + 1}}{\ell_2}, \\ \hbar_1 = 0, \\ \hbar_2 = \frac{-6m^2}{\ell_2}, \\ \lambda = -4(B\eta_3^3 + C\eta_3\eta_1^2 + C\eta_3\eta_2^2)\sqrt{m^4 - m^2 + 1}. \end{cases}$$

For the first set, the solutions of FSQZKE (2), using (12), are

$$\varphi(x, y, z, t) = [\frac{2(m^2+1) - 2\sqrt{m^4 - m^2 + 1}}{\ell_2} - \frac{6m^2}{\ell_2} sn^2(\eta, m)] e^{(\sigma W(t) - \frac{1}{2}\sigma^2 t)}, \tag{14}$$

where $\eta = \frac{\Gamma(\beta+1)}{\alpha}(\eta_1 x^\alpha + \eta_2 y^\alpha + \eta_3 z^\alpha) + 4\sqrt{m^4 - m^2 + 1}(B\eta_3^3 + C\eta_3\eta_1^2 + C\eta_3\eta_2^2)t$. If $m \to 1$, then Equation (14) takes the form

$$\varphi(x,y,z,t) = [\frac{2}{\ell_2} - \frac{6}{\ell_2}\tanh^2(\eta)]e^{(\sigma W(t) - \frac{1}{2}\sigma^2 t)}, \tag{15}$$

where $\eta = \frac{\Gamma(\beta+1)}{\alpha}(\eta_1 x^\alpha + \eta_2 y^\alpha + \eta_3 z^\alpha) + 4(B\eta_3^3 + C\eta_3\eta_1^2 + C\eta_3\eta_2^2)t$.

For the second set, the solutions of FSQZKE (2), using (12), are

$$\varphi(x,y,z,t) = [\frac{2(m^2+1) + 2\sqrt{m^4 - m^2 + 1}}{\ell_2} - \frac{6m^2}{\ell_2}sn^2(\eta, m)]e^{(\sigma W(t) - \frac{1}{2}\sigma^2 t)}, \tag{16}$$

where $\eta = \frac{\Gamma(\beta+1)}{\alpha}(\eta_1 x^\alpha + \eta_2 y^\alpha + \eta_3 z^\alpha) - 4\sqrt{m^4 - m^2 + 1}(B\eta_3^3 + C\eta_3\eta_1^2 + C\eta_3\eta_2^2)t$.

If $m \to 1$, then Equation (16) becomes

$$\varphi(x,y,z,t) = [\frac{6}{\ell_2} - \frac{6}{\ell_2}\tanh^2(\eta)]e^{(\sigma W(t) - \frac{1}{2}\sigma^2 t)} = \frac{6}{\ell_2}\text{sech}^2(\eta)e^{(\sigma W(t) - \frac{1}{2}\sigma^2 t)}, \tag{17}$$

where $\eta = \frac{\Gamma(\beta+1)}{\alpha}(\eta_1 x^\alpha + \eta_2 y^\alpha + \eta_3 z^\alpha) - 4(B\eta_3^3 + C\eta_3\eta_1^2 + C\eta_3\eta_2^2)t$.

Analogously, we can replace sn in (12) by cn to have the FSQZKE (2) as follows:

$$\varphi(x,y,z,t) = [\frac{(2-4m^2) - 2\sqrt{m^4 - m^2 + 1}}{\ell_2} + \frac{6m^2}{\ell_2}cn^2(\eta, m)]e^{(\sigma W(t) - \frac{1}{2}\sigma^2 t)}, \tag{18}$$

or

$$\varphi(x,y,z,t) = [\frac{(2-4m^2) + 2\sqrt{m^4 - m^2 + 1}}{\ell_2} + \frac{6m^2}{\ell_2}cn^2(\eta, m)]e^{(\sigma W(t) - \frac{1}{2}\sigma^2 t)}. \tag{19}$$

If $m \to 1$, then the solutions (18) become

$$\varphi(x,y,z,t) = [\frac{-4}{\ell_2} + \frac{6}{\ell_2}\text{sech}^2(\eta)]e^{(\sigma W(t) - \frac{1}{2}\sigma^2 t)}, \tag{20}$$

or

$$\varphi(x,y,z,t) = \frac{6}{\ell_2}\text{sech}^2(\eta)e^{(\sigma W(t) - \frac{1}{2}\sigma^2 t)}, \tag{21}$$

where $\eta = \frac{\Gamma(\beta+1)}{\alpha}(\eta_1 x^\alpha + \eta_2 y^\alpha + \eta_3 z^\alpha) - 4(B\eta_3^3 + C\eta_3\eta_1^2 + C\eta_3\eta_2^2)t$.

Remark 1. *Setting $\alpha = 1$, $\beta = \sigma = 0$ in Equations (16) and (19), we obtain the same solutions (30) and (32), respectively, stated in [48].*

Remark 2. *Putting $\alpha = 1$, $\beta = \sigma = 0$ in Equations (16), (15) and (21), we obtain the similar solutions (3.30), (3.16), and (3.15) reported in [43] via extended generalized (G'/G)-expansion.*

3.2. Modified F-Expansion Method

Here, we use the modified F-expansion method (see [50]). Let us suppose the solution u of Equation (9) has the type (with $M = 2$):

$$u(\eta) = a_0 + a_1 F + a_2 F^2 + \frac{b_1}{F} + \frac{b_1}{F^2}, \tag{22}$$

where F solves

$$F' = F^2 + k, \tag{23}$$

where k is a real constant. The Equation (23) has the solutions:

$$\phi(\mu) = \sqrt{k}\tan(\sqrt{k}\mu) \text{ or } \phi(\mu) = -\sqrt{k}\cot(\sqrt{k}\mu), \tag{24}$$

If $k > 0$, or

$$\phi(\mu) = -\sqrt{-k}\tanh(\sqrt{-k}\mu) \ \text{ or } \phi(\mu) = -\sqrt{-k}\coth(\sqrt{-k}\mu), \tag{25}$$

If $k < 0$, or

$$\phi(\mu) = \frac{-1}{\mu}, \tag{26}$$

If $k = 0$.

Now, substituting Equation (22) into Equation (9) we obtain

$$\begin{aligned}
&(6a_2 + \ell_2 a_2^2)F^4 + (2a_1 + 2\ell_2 a_1 a_2)F^3 + (8ka_2 + 2a_0 a_2 \ell_2 + a_1^2 \ell_2 + \ell_1 a_2)F^2 \\
&(2ka_1 + \ell_1 a_1 + 2\ell_2 a_0 a_1 + 2a_2 b_1)F + (2k^2 a_2 + 2b_2 + \ell_1 a_0 + \ell_2 a_0^2 + 2\ell_2 a_1 b_1 \\
&+2\ell_2 a_2 b_2) + (2kb_1 + 2\ell_2 a_0 b_1 + 2\ell_2 a_1 b_2 + \ell_1 b_1)F^{-1} + (8kb_2 + 2a_0 b_2 \ell_2 \\
&+b_1^2 \ell_2 + \ell_1 b_2)F^{-2} + (2b_1 k^2 + 2\ell_2 b_1 b_2)F^{-3} + (6k^2 b_2 + \ell_2 b_2^2)F^{-4} = 0
\end{aligned}$$

Setting each power of F's coefficients to zero as follows:

$$6a_2 + \ell_2 a_2^2 = 0,$$

$$\begin{gathered}
2a_1 + 2\ell_2 a_1 a_2 = 0, \\
8ka_2 + 2a_0 a_2 \ell_2 + a_1^2 \ell_2 + \ell_1 a_2 = 0, \\
2ka_1 + \ell_1 a_1 + 2\ell_2 a_0 a_1 + 2a_2 b_1 = 0, \\
2k^2 a_2 + 2b_2 + \ell_1 a_0 + \ell_2 a_0^2 + 2\ell_2 a_1 b_1 + 2\ell_2 a_2 b_2 = 0, \\
2kb_1 + 2\ell_2 a_0 b_1 + 2\ell_2 a_1 b_2 + \ell_1 b_1 = 0, \\
8kb_2 + 2a_0 b_2 \ell_2 + b_1^2 \ell_2 + \ell_1 b_2 = 0, \\
2b_1 k^2 + 2\ell_2 b_1 b_2 = 0
\end{gathered}$$

and

$$6k^2 b_2 + \ell_2 b_2^2 = 0.$$

Solving these equations yields the four distinct sets as follows:

First set:

$$a_0 = \frac{-6k}{\ell_2},\ a_1 = 0,\ a_2 = \frac{-6}{\ell_2},\ b_1 = 0,\ b_2 = 0,\ \lambda = 4k(B\eta_3^3 + C\eta_3\eta_1^2 + C\eta_3\eta_2^2), \tag{27}$$

Second set:

$$a_0 = \frac{-2k}{\ell_2},\ a_1 = 0,\ a_2 = \frac{-6}{\ell_2},\ b_1 = 0,\ b_2 = 0,\ \lambda = -4k(B\eta_3^3 + C\eta_3\eta_1^2 + C\eta_3\eta_2^2). \tag{28}$$

Third set:

$$a_0 = \frac{-12k}{\ell_2},\ a_1 = 0,\ a_2 = \frac{-6}{\ell_2},\ b_1 = 0,\ b_2 = \frac{-6k^2}{\ell_2},\ \lambda = 16k(B\eta_3^3 + C\eta_3\eta_1^2 + C\eta_3\eta_2^2), \tag{29}$$

Fourth set:

$$a_0 = \frac{8k}{\ell_2},\ a_1 = 0,\ a_2 = \frac{-6}{\ell_2},\ b_1 = 0,\ b_2 = \frac{-6k^2}{\ell_2},\ \lambda = -14k(B\eta_3^3 + C\eta_3\eta_1^2 + C\eta_3\eta_2^2). \tag{30}$$

First set: The solution of Equation (9) in this case is

$$u(\eta) = \frac{-6k}{\ell_2} - \frac{6}{\ell_2}F^2(\eta).$$

For $F(\eta)$, there are three cases:
Case1: If $k > 0$, then by using (24) we obtain

$$u(\eta) = \frac{-6k}{\ell_2} - \frac{6k}{\ell_2}\tan^2(\sqrt{k}\eta) = -\frac{6k}{\ell_2}\sec^2(\sqrt{k}\eta),$$

and

$$u(\eta) = \frac{-6k}{\ell_2} - \frac{6k}{\ell_2}\cot^2(\sqrt{k}\eta) = \frac{-6k}{\ell_2}\csc^2(\sqrt{k}\eta).$$

Thus, the FSQZKE (2) has the solutions

$$\varphi_{1,1}(x,y,z,t) = -\frac{6k}{\ell_2}\sec^2(\sqrt{k}\eta)e^{(\sigma W(t) - \frac{1}{2}\sigma^2 t)}, \tag{31}$$

and

$$\varphi_{1,2}(x,y,z,t) = \frac{-6k}{\ell_2}\csc^2(\sqrt{k}\eta)e^{(\sigma W(t) - \frac{1}{2}\sigma^2 t)}, \tag{32}$$

where $\eta = \frac{\Gamma(\beta+1)}{\alpha}(\eta_1 x^\alpha + \eta_2 y^\alpha + \eta_3 z^\alpha) + 4k(B\eta_3^3 + C\eta_3\eta_1^2 + C\eta_3\eta_2^2)t$.
Case2: If $k < 0$, then by using (25) we obtain

$$u(\eta) = \frac{-6k}{\ell_2} + \frac{6k}{\ell_2}\tanh^2(\sqrt{-k}\eta) = \frac{-6k}{\ell_2}\mathrm{sech}^2(\sqrt{-k}\eta),$$

and

$$u(\eta) = \frac{-6k}{\ell_2} + \frac{6k}{\ell_2}\coth^2(\sqrt{-k}\eta) = \frac{6k}{\ell_2}\mathrm{csch}^2(\sqrt{-k}\eta).$$

Thus, the FSQZKE (2) has the solutions

$$\varphi_{1,3}(x,y,z,t) = \frac{-6k}{\ell_2}\mathrm{sech}^2(\sqrt{-k}\eta)e^{(\sigma W(t) - \frac{1}{2}\sigma^2 t)}, \tag{33}$$

and

$$\varphi_{1,4}(x,y,z,t) = \frac{6k}{\ell_2}\mathrm{csch}^2(\sqrt{-k}\eta)e^{(\sigma W(t) - \frac{1}{2}\sigma^2 t)}, \tag{34}$$

where $\eta = \frac{\Gamma(\beta+1)}{\alpha}(\eta_1 x^\alpha + \eta_2 y^\alpha + \eta_3 z^\alpha) + 4k(B\eta_3^3 + C\eta_3\eta_1^2 + C\eta_3\eta_2^2)t$.
Case3: If $k = 0$, then by using (26) we obtain

$$u(\eta) = \frac{6}{\ell_2}\frac{1}{\eta^2}.$$

Thus, the FSQZKE (2) has the solution

$$\varphi_{1,5}(x,y,z,t) = [-\frac{6}{\ell_2}\frac{1}{\eta^2}]e^{(\sigma W(t) - \frac{1}{2}\sigma^2 t)}, \tag{35}$$

where $\eta = \frac{\Gamma(\beta+1)}{\alpha}(\eta_1 x^\alpha + \eta_2 y^\alpha + \eta_3 z^\alpha) + 4k(B\eta_3^3 + C\eta_3\eta_1^2 + C\eta_3\eta_2^2)t$.
Second set: The Equation (9) has the solution

$$u(\eta) = \frac{-2k}{\ell_2} - \frac{6}{\ell_2}F^2(\eta)$$

For $F(\eta)$, there are three cases:
Case1: If $k > 0$, then by using (24) we obtain

$$u(\eta) = \frac{-2k}{\ell_2} - \frac{6k}{\ell_2}\tan^2(\sqrt{k}\eta),$$

and

$$u(\eta) = \frac{-2k}{\ell_2} - \frac{6k}{\ell_2}\cot^2(\sqrt{k}\eta).$$

Thus, the FSQZKE (2) has the solutions

$$\varphi_{2,1}(x,y,z,t) = [\frac{-2k}{\ell_2} - \frac{6k}{\ell_2}\tan^2(\sqrt{k}\eta)]e^{(\sigma W(t)-\frac{1}{2}\sigma^2 t)}, \quad (36)$$

and

$$\varphi_{2,2}(x,y,z,t) = [\frac{-2k}{\ell_2} - \frac{6k}{\ell_2}\cot^2(\sqrt{k}\eta)]e^{(\sigma W(t)-\frac{1}{2}\sigma^2 t)}, \quad (37)$$

where $\eta = \frac{\Gamma(\beta+1)}{\alpha}(\eta_1 x^\alpha + \eta_2 y^\alpha + \eta_3 z^\alpha) - 4k(B\eta_3^3 + C\eta_3\eta_1^2 + C\eta_3\eta_2^2)t$.

Case2: If $k < 0$, then by using (25) we obtain

$$u(\eta) = \frac{-2k}{\ell_2} + \frac{6k}{\ell_2}\tanh^2(\sqrt{-k}\eta),$$

and

$$u(\eta) = \frac{-2k}{\ell_2} + \frac{6k}{\ell_2}\coth^2(\sqrt{-k}\eta).$$

Thus, the FSQZKE (2) has the solutions

$$\varphi_{2,3}(x,y,z,t) = [\frac{-2k}{\ell_2} + \frac{6k}{\ell_2}\tanh^2(\sqrt{-k}\eta)]e^{(\sigma W(t)-\frac{1}{2}\sigma^2 t)}, \quad (38)$$

and

$$\varphi_{2,4}(x,y,z,t) = [\frac{-2k}{\ell_2} + \frac{6k}{\ell_2}\coth^2(\sqrt{-k}\eta)]e^{(\sigma W(t)-\frac{1}{2}\sigma^2 t)}. \quad (39)$$

Case3: If $k = 0$, then by using (26) we obtain

$$u(\eta) = \frac{6}{\ell_2}\frac{1}{\eta^2}.$$

Thus the solution of FSQZKE (2) is

$$\varphi_{2,5}(x,y,z,t) = \frac{6}{\ell_2}\frac{1}{\eta^2}e^{(\sigma W(t)-\frac{1}{2}\sigma^2 t)}, \quad (40)$$

where $\eta = \frac{\Gamma(\beta+1)}{\alpha}(\eta_1 x^\alpha + \eta_2 y^\alpha + \eta_3 z^\alpha) - 4k(B\eta_3^3 + C\eta_3\eta_1^2 + C\eta_3\eta_2^2)t$.

Third set: The solution of Equation (9) in this case is

$$u(\eta) = \frac{-12k}{\ell_2} - \frac{6}{\ell_2}F^2(\eta) - \frac{6k^2}{\ell_2}F^{-2}(\eta).$$

For $F(\eta)$, there are three cases:

Case1: If $k > 0$, then by using (24) we obtain

$$\begin{aligned} u(\eta) &= \frac{-12k}{\ell_2} - \frac{6k}{\ell_2}\tan^2(\sqrt{k}\eta) - \frac{6k}{\ell_2}\cot^2(\sqrt{k}\eta) \\ &= -\frac{6k}{\ell_2}[\sec^2(\sqrt{k}\eta) + \csc^2(\sqrt{k}\eta)]. \end{aligned}$$

Thus, the FSQZKE (2) has the solution

$$\varphi_{3,1}(x,y,z,t) = -\frac{6k}{\ell_2}[\sec^2(\sqrt{k}\eta) + \csc^2(\sqrt{k}\eta)]e^{(\sigma W(t)-\frac{1}{2}\sigma^2 t)}, \quad (41)$$

where $\eta = \frac{\Gamma(\beta+1)}{\alpha}(\eta_1 x^\alpha + \eta_2 y^\alpha + \eta_3 z^\alpha) + 16k(B\eta_3^3 + C\eta_3\eta_1^2 + C\eta_3\eta_2^2)t$.

Case2: If $k < 0$, then by using (25) we obtain

$$\begin{aligned} u(\eta) &= \frac{-12k}{\ell_2} + \frac{6k}{\ell_2}\tanh^2(\sqrt{-k}\eta) + \frac{6k}{\ell_2}\coth^2(\sqrt{-k}\eta) \\ &= \frac{-6k}{\ell_2}[\operatorname{sech}^2(\sqrt{-k}\eta) - \operatorname{csch}^2(\sqrt{-k}\eta)]. \end{aligned}$$

Thus, the solution of FSQZKE (2) is

$$\varphi_{3,2}(x,y,z,t) = \frac{-6k}{\ell_2}[\operatorname{sech}^2(\sqrt{-k}\eta) - \operatorname{csch}^2(\sqrt{-k}\eta)]e^{(\sigma W(t) - \frac{1}{2}\sigma^2 t)}, \tag{42}$$

where $\eta = \frac{\Gamma(\beta+1)}{\alpha}(\eta_1 x^\alpha + \eta_2 y^\alpha + \eta_3 z^\alpha) + 16k(B\eta_3^3 + C\eta_3\eta_1^2 + C\eta_3\eta_2^2)t$.

Case3:If $k = 0$, then by using (26) we obtain

$$u(\eta) = \frac{6}{\ell_2}\frac{1}{\eta^2} + \frac{6}{\ell_2}\eta^2.$$

Thus, the solution of FSQZKE (2) is

$$\varphi_{3,3}(x,y,z,t) = \frac{6}{\ell_2}[\frac{1}{\eta^2} + \eta^2]e^{(\sigma W(t) - \frac{1}{2}\sigma^2 t)}. \tag{43}$$

Fourth set: The solution of Equation (9) in this case is

$$u(\eta) = \frac{8k}{\ell_2} - \frac{6}{\ell_2}F^2(\eta) - \frac{6k^2}{\ell_2}F^{-2}(\eta).$$

For $F(\eta)$, there are three cases:

Case1: If $k > 0$, then by using (24) we obtain

$$u(\eta) = \frac{8k}{\ell_2} - \frac{6k}{\ell_2}\tan^2(\sqrt{k}\eta) - \frac{6k}{\ell_2}\cot^2(\sqrt{k}\eta).$$

Thus, the FSQZKE (2) has the solutions

$$\varphi_{4,1}(x,y,z,t) = [\frac{8k}{\ell_2} - \frac{6k}{\ell_2}\tan^2(\sqrt{k}\eta) - \frac{6k}{\ell_2}\cot^2(\sqrt{k}\eta)]e^{(\sigma W(t) - \frac{1}{2}\sigma^2 t)}, \tag{44}$$

where $\eta = \frac{\Gamma(\beta+1)}{\alpha}(\eta_1 x^\alpha + \eta_2 y^\alpha + \eta_3 z^\alpha) - 14k(B\eta_3^3 + C\eta_3\eta_1^2 + C\eta_3\eta_2^2)t$.

Case2: If $k < 0$, then by using (25) we obtain

$$u(\eta) = \frac{8k}{\ell_2} + \frac{6k}{\ell_2}\tanh^2(\sqrt{-k}\eta) + \frac{6k}{\ell_2}\coth^2(\sqrt{-k}\eta).$$

Thus, the solution of FSQZKE (2) is

$$\varphi_{4,2}(x,y,z,t) = [\frac{8k}{\ell_2} + \frac{6k}{\ell_2}\tanh^2(\sqrt{-k}\eta) + \frac{6k}{\ell_2}\coth^2(\sqrt{-k}\eta)]e^{(\sigma W(t) - \frac{1}{2}\sigma^2 t)}. \tag{45}$$

Case3: If $k = 0$, then by using (26) we obtain

$$u(\eta) = \frac{6}{\ell_2}\frac{1}{\eta^2} + \frac{6}{\ell_2}\eta^2.$$

Thus, the FSQZKE (2) has the solution

$$\varphi_{4,3}(x,y,z,t) = \frac{6}{\ell_2}[\frac{1}{\eta^2} + \eta^2]e^{(\sigma W(t) - \frac{1}{2}\sigma^2 t)}, \tag{46}$$

where $\eta = \frac{\Gamma(\beta+1)}{\alpha}(\eta_1 x^\alpha + \eta_2 y^\alpha + \eta_3 z^\alpha) - 14k(B\eta_3^3 + C\eta_3\eta_1^2 + C\eta_3\eta_2^2)t$.

Remark 3. *Putting $\alpha = 1$, $\beta = \sigma = 0$ in Equations (33), (38) and (45), we obtain the similar solutions (50), (56) and (57) presented in [48] by extended F-expansion method.*

Remark 4. *Putting $\alpha = 1$, $\beta = \sigma = 0$ in Equations (31), (32), (34), (39), (44), and (45), we obtain the same solutions (3.26), (3.46), (3.22), (3.23), (3.54) and (3.61), respectively, reported in [43] by using extended generalized (G'/G)-expansion.*

4. Graphical Representation and Discussion

In this paper, the analytical solutions of FSQZKE were acquired. Many analytic solutions to FSQZKE, such as trigonometric, rational, elliptic, and hyperbolic solutions, were attained using modified F-expansion method and Jacobi elliptic function method. The first method, the Jacobi elliptic function, provided us with various solutions in the type of hyperbolic, trigonometric, and rational. While the second method, modified F-expansion, provided us with a variety of elliptic solutions. We introduced some graphical representations of the solutions using the Matlab program to better understand their behavior and features. The behaviors of the attained solutions can be controlled by switching the values of the free parameters. Consequently, switching the parameter values changes the nature of the graph. To show how the graph of the obtained solutions is impacted by the stochastic term and the fractional order, let us fix the next parameters $\eta_1 = 1$, $\eta_2 = -1$, $\eta_3 = 1$, $x \in [0,6]$, $t \in [0,2]$, $y = z = 1$, $A = 1$, $B = 0.5$, $C = 0.25$, $\ell_2 = \frac{1}{2}$.

First the impact of stochastic term:

We may conclude, from Figures 1 and 2, that there are several various sorts of solutions, including periodic solutions, dark solutions, and others, when the noise is disregarded (i.e., at $\sigma = 0$). When noise is taken into account, the surface after minor transit patterns considerably flattens down and gains strength by $\sigma = 1, 2$. This indicates that the stochastic term affects the FSQZKE solutions and causes them to stabilize around zero.

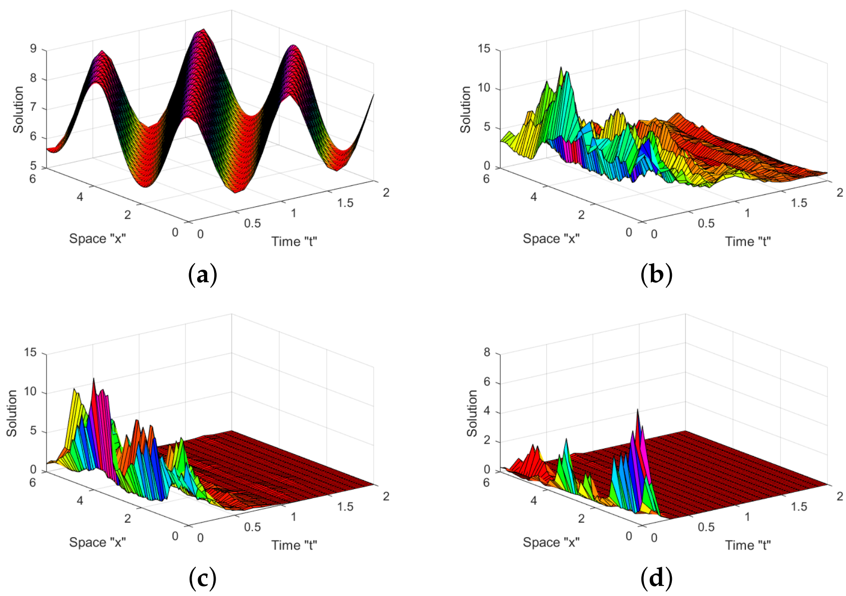

Figure 1. For Equation (33), (**a**–**d**) with $k = -1$, $\alpha = 1, \beta = 0, \lambda = -4$, indicate the 3D profiles (**a**) $\sigma = 0$ (**b**) $\sigma = 0.5$ (**c**) $\sigma = 1$ and (**d**) $\sigma = 2$.

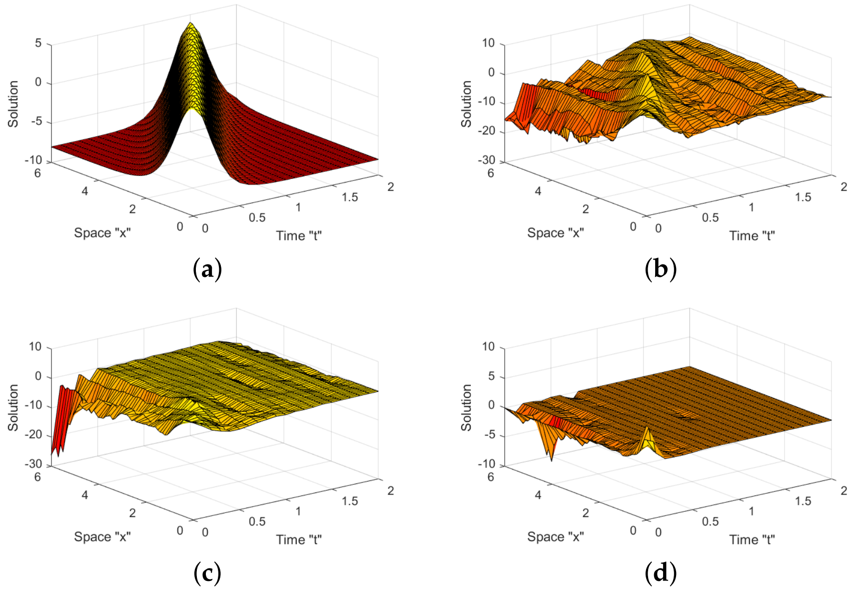

Figure 2. For Equation (16), (**a**–**d**) with $m = 0.5$, $\alpha = 1,\ \beta = 0,\ \lambda = -\sqrt{13}$, indicate the 3D profiles (**a**) $\sigma = 0$ (**b**) $\sigma = 0.5$ (**c**) $\sigma = 1$ and (**d**) $\sigma = 2$.

Second the impact of fractional order:

Finally, from Figures 3 and 4 we inferred that the curves do not overlap. Furthermore, the solutions shift to the left when the order of the M-truncated derivative increases.

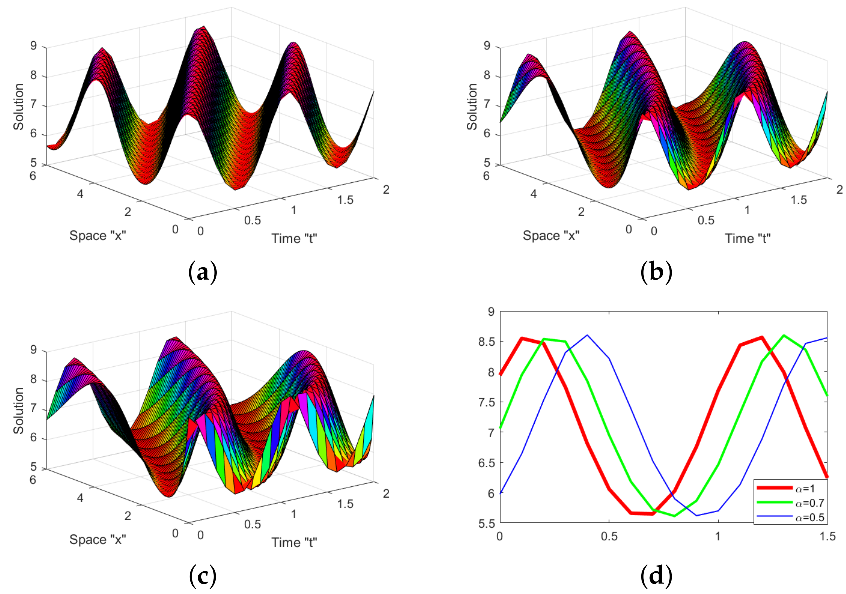

Figure 3. For Equation (33), (**a**–**c**) with parameters $k = -1,\ \sigma = 0,\ \lambda = -4$, indicate the 3D profiles and (**d**) denotes the 2D plot for different values of α at $t = 1.5$ and there is no overlap between the curves of the solution. (**a**) $\alpha = 1, \beta = 0$ (**b**) $\alpha = 0.7, \beta = 0.9$ and (**c**) $\alpha = 0.5,\ \beta = 0.9$.

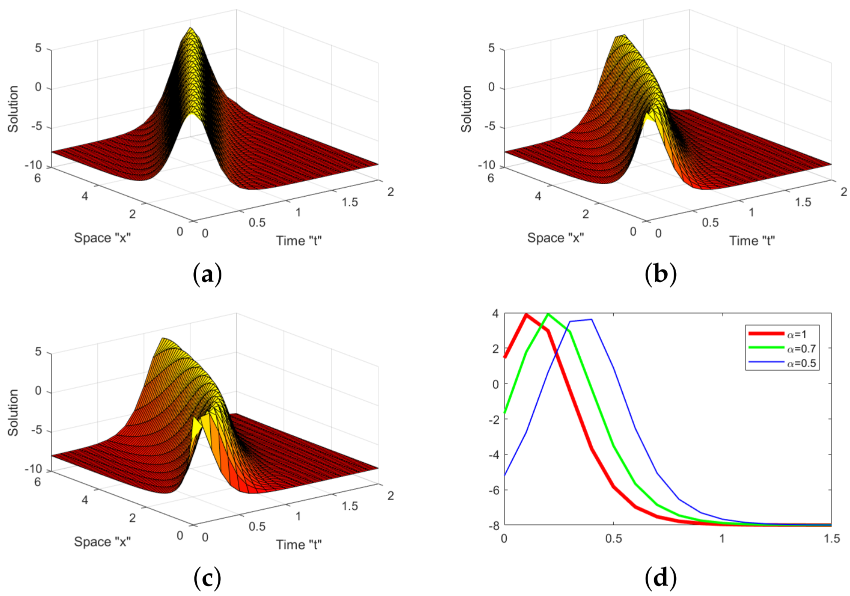

Figure 4. For Equation (16), (**a**–**c**) with parameters $m = 0.5, \sigma = 0,\ \lambda = -\sqrt{13}$ indicate the 3D profiles and (**d**) denotes the 2D plot for different values of α at $t = 1.5$ and there is no overlap between the curves of the solution. (**a**) $\alpha = 1, \beta = 0$ (**b**) $\alpha = 0.7, \beta = 0.9$ and (**c**) $\alpha = 0.5,\ \beta = 0.9$.

5. Conclusions

In this paper, we looked at the (3 + 1)-dimensional fractional-stochastic quantum Zakharov–Kuznetsov Equation (2) with M-truncated derivative. We obtained the trigonometric, hyperbolic, elliptic, and rational fractional-stochastic solutions of FSQZKE (2) by using the Jacobi elliptic function method and the modified F-expansion method. Additionally, we generalized a few previous results, such as those in [43,48]. The discovered solutions are crucial for deriving a wide range of exciting and complex phenomena. The impact of the stochastic term and fractional derivative on the analytical solution of the FSQZKE (2) was demonstrated using the Matlab program. Finally, we demonstrated how the FSQZKE solutions are impacted by the stochastic term and M-truncated derivative. In future work, we can study FSQZKE (2) with additive noise or with color multiplicative noise.

Author Contributions: Data curation, F.M.A.-A. and M.E.-M.; Formal analysis, W.W.M., F.M.A.-A., and C.C.; Funding acquisition, F.M.A.-A.; Methodology, C.C. and M.E.-M.; Project administration, W.W.M.; Software, W.W.M. and M.E.-M.; Supervision, C.C.; Visualization, F.M.A.-A.; Writing—original draft, M.E.-M.; Writing—review and editing, W.W.M. and C.C. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Acknowledgments: Princess Nourah bint Abdulrahman University Researcher Supporting Project number (PNURSP2023R 273), Princess Nourah bint Abdulrahman University, Riyadh, Saudi Arabia.

Conflicts of Interest: The authors declare no conflicts of interest.

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