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Study on Economic Data Forecasting Based on Hybrid Intelligent Model of Artificial Neural Network Optimized by Harris Hawks Optimization

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Abstract: To use different models for forecasting economic data suitably, three main basic models (the grey system model, time series analysis model, and artificial neural network (ANN) model) are analyzed and compared comprehensively. Based on the analysis results of forecasting models, one new hybrid intelligent model based on the ANN model and Harris hawks optimization (HHO) has been proposed. In this hybrid model, HHO is used to select the hyperparameters of the ANN and also to optimize the linking weights and thresholds of the ANN. At last, by using four economic data cases including two simple data sets and two complex ones, the analysis of the basic models and the proposed hybrid model have been verified comprehensively. The results show that the grey system model can suitably analyze exponential data sequences, the time series analysis model can analyze random sequences, and the ANN model can be applied to any kind of data sequence. Moreover, when compared with the basic models, the new hybrid model can be suitably applied for both simple data sets and complex ones, and its forecasting performance is always very suitable. In comparison with other hybrid models, not only for computing accuracy but also for computing efficiency, the performance of the new hybrid model is the best. For the least initial parameters used in the new hybrid model, which can be determined easily and simply, the application of the new hybrid model is the most convenient too.

Keywords: economic data forecasting; basic model; ANN model; hybrid intelligent model; HHO

MSC: 91B84

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1. Introduction

In economic activities, it is very crucial to grasp economic development. Therefore, the analysis of the economic data is key work. Generally, economic data are a nonlinear time series, such as the time series for the Gross Domestic Product (GDP), the time series of the yearly output, the time series of the stock index, etc. Therefore, forecasting the economic data time series is very important work for the government and researchers [1], by which future economic development can be mastered. Nowadays, there have been numerous studies in this field [2–4]. In those studies, three computing models (the grey system model, time series analysis model, and artificial neural network (ANN) model) are the main basic forecasting models for economic data time series. For those three basic models, the grey system model is the most typical forecasting method within economic system science [5]. Because economic activity is a typical complex behavior within social systems, it is very suitable to use the methods of system science for the analysis of economic activity [5]. Therefore, the grey system model has been widely used as one basic model for early economic data forecasting, and there have been many other methods proposed that use the grey system model [2–4]. The time series analysis model is derived from statistics science [6]. Because the analysis of economic data is actually a mathematical statistical problem, using a mathematical statistical model for economic data forecasting is very suitable. Therefore, the time series analysis model, which is a typical statistical model, has been widely applied for the early forecasting of economic data too [2,3]. Moreover, the time series analysis model can be used to construct other forecasting models easily, thus, it has become a basic model for economic data forecasting. Finally, the ANN model is one typical artificial intelligence model that can be used for the simple analysis of complex data series [7]. Therefore, it can be applied for the analysis of economic data very suitably. Currently, the ANN model has been applied for economic data forecasting widely and has become a typical basic model in this field too [4]. However, for the grey system model and time series analysis model, which are the traditional forecasting models, there have been some problems [8]. Although, as a modern forecasting model, the performance of the ANN model is superior, there are still some problems in the application of the ANN model [4]. Therefore, to improve the performance of the basic forecasting models, nowadays, many hybrid models based on the basic models are being proposed [4]. Although the application of hybrid forecasting models has become a hot topic, for numerous combinations of basic models and many newly proposed artificial intelligent methods, it is still urgently needed to develop a new hybrid intelligent model for economic data forecasting.

In this study, based on the analysis of previous researchers in this field, for the application of a suitable basic model, the three basic models have been analyzed, and their merits and demerits have been compared comprehensively. Then, a new hybrid intelligent model using the ANN model and a newly proposed intelligent method for economic data forecasting has been proposed. By applying these to four data cases, the performance of the three basic models and the new hybrid intelligent model have been analyzed. Finally, the comparison with other state-of-the-art hybrid intelligent models and applications for more complex data cases has been discussed.

The rest of this paper is as follows: Section 2 discusses the related work in this field. The comprehensive analysis of the basic forecasting models is conducted in Section 3. Section 4 gives the methodologies, including the HHO algorithm and new hybrid intelligent model for economic data forecasting. The case studies of four economic data series have been conducted in Section 5. Section 6 conducts the discussion. At last, the main conclusions of this study and future work are summarized in Section 7.

2. Related Works

Because the basic computing models can be used for forecasting economic data easily and simply, there have been numerous studies using three basic models. The main applications of the three basic models are summarized as follows.

Using the grey system model, Taiwan's unemployment rate in 2011 and the consumer price indexes (CPI) of the USA in 2011 have been predicted [9]. Using the grey system model, the yearly output of concave soil in the Xuyi county of China and the China Securities Index (CSI) 300 index data in 2010 all have been predicted [10]. The building material stock index in Taiwan has been analyzed using the grey forecasting model [11]. The energy consumption per unit GDP of China in 2015 has been predicted using the grey system model too [12]. In addition, the housing demand in Turkey in 2014 has also been predicted using the grey system model [13]. Moreover, using the time series analysis model, the electricity price in New South Wales from the Australian National Electricity Market in 2013 and close prices of the Larsen and Turbo (L&T) company stock in 2013 all have been predicted [14]. Taiwan's export in 2002 has been forecasted based on the time series analysis model too [15]. By using the time series analysis model, the returns for the Eurozone southern periphery stock markets were also predicted [16]. Furthermore, using the time series analysis model, based on the daily real-load electricity data for 709 individual households over an 18-month period in Ireland, the electricity demand was also predicted [17].

Different from the two basic models mentioned above, which belong to the traditional forecasting model as one artificial intelligence model, the artificial neural network (ANN) model has been widely used for economic data forecasting [18,19]. For example, based on the ANN model, the trend in the EUR/USD exchange rate in 2009 was predicted [20]. Using the ANN model, the stock price of Petrobras stock PETR4, traded in Petrobras, Brazil, was also predicted [21]. To predict the monthly average CPI data in American cities, the ANN model was used [22]. The USA real GDP growth rate for the period 1970–2021 was also predicted using the ANN model [23]. Moreover, this model was also applied to forecast energy-related data [24], such as solar radiation data, by a station in northwestern Sicily (Italy); wind speed data, by the U.S. National Renewable Energy Laboratory (NREL) and load demand data, by the Global Energy Forecasting Competition in 2012. Additionally, based on the ANN model, the pine sawtimber stumpage prices across 22 TimberMart-South regions in the USA were predicted [25]. The lithium mineral resource prices in China have also been forecasted using the ANN model [26]. In addition, based on the energy index of Tadawul Saudi Arabia, the Brent crude oil price was also predicted using the ANN model [27]. Finally, using the ANN model, based on the daily real-load electricity data for 709 individual households in Ireland, electricity demand was predicted [17].

We aim to improve the problems of the basic ANN model for its application [19]. Currently, a hybrid model employing the basic ANN model has become a hot topic for economic data forecasting. For example, using a hybrid model of ANN and a quantum genetic algorithm, in which the quantum genetic algorithm was used to tune the learning rates of the ANN model, the stock closing price prediction of the Chinese stock market was conducted [28]. Based on a hybrid model of ANN and improved elephant herding optimization, in which the improved elephant herding optimization was used to select the suitable weights and thresholds of the ANN model, building cooling and heating load forecasting was conducted [29]. Furthermore, using a hybrid model of ANN and the Bayesian optimization algorithm, in which the hyperparameters of the ANN model were determined using Bayesian optimization, the future international price of copper was predicted [30]. Moreover, based on a hybrid model of ANN and a genetic algorithm, in which the genetic algorithm was used to optimize the weights and thresholds of the ANN model, the maximum daily electricity price in Iran's electricity market was forecasted [31]. Using a hybrid model of ANN and particle swarm optimization, in which particle swarm optimization was used to set the initial weights of the ANN model, the electricity demand in Ghana was forecasted [32]. In addition, based on another hybrid model of ANN and particle swarm optimization, in which particle swarm optimization was used to set the optimal hyperparameters of the ANN model, the daily average electricity demand of the Ghana Grid Company was also forecasted [33]. Using the hybrid model of ANN and the gravitational search optimization algorithm, in which the hyperparameters of the ANN model were optimized using gravitational search optimization, electricity price and load in isolated power grids were forecasted [34]. Additionally, based on a hybrid model combining ANN and ant colony optimization, in which ant colony optimization was used to optimize the weights and biases of the ANN model, the short-term photovoltaic (PV) power for a 100 kW PV system in Beijing, China, was forecasted [35]. Moreover, using two hybrid models, one of ANN with gray wolf optimization and the other of ANN with the improved black hole algorithm, in which the hyperparameters of the ANN model were selected using gray wolf optimization and the improved black hole algorithm, respectively, stock market prediction (including price and index) was conducted [36,37]. To predict daily foreign exchange rates, a hybrid model of ANN and the modified water cycle algorithm was applied. For this hybrid model, the modified water cycle algorithm was used to select the hyperparameters of the ANN model [38]. Using a hybrid model of ANN and differential evolutionary particle swarm optimization, in which the differential evolutionary particle swarm optimization was used to optimize the weight and bias of the ANN model, daily peak-load forecasting for the Japan Meteorological Agency and TEPCO power grid was conducted [39]. To forecast the stock indices of some stock exchanges (daily indices of the S&P 500 and NASDAQ and the stock price of IBM), a hybrid model of ANN and a genetic algorithm was applied; in this hybrid model, the genetic algorithm was used to select the hyperparameters of the ANN model [40]. Additionally, using a hybrid model of ANN and improved simplified swarm optimization, in which the weight and bias of the ANN model were optimized using the improved simplified swarm optimization, the wind power in the Mai Liao Wind Farm (the most important wind farm in Taiwan) was forecasted [41]. Moreover, in another study [42], to predict the real GNP in USD billions in the USA, a hybrid model combining an ANN model and the biased random key genetic algorithm was proposed, in which the biased random key genetic algorithm was used both to optimize the weight and bias and to select the number of hidden neurons for the three-layer ANN model. Using a hybrid model of ANN and particle swarm optimization, in which the weights of the ANN model were optimized through the particle swarm optimization, SP500 stock daily prices were forecasted [43]. Moreover, to predict the closing prices of the Spanish, Italian, and German stock exchanges, a hybrid model combining an ANN model and the symbiotic organism search algorithm was proposed, in which the weights of the ANN model were optimized using the symbiotic organism search algorithm [44]. Finally, using a hybrid model of ANN and barnacle mating optimization, in which the barnacle mating optimization was used to determine the optimal weights of the ANN model, the stock price of Yahoo stock was forecasted [45].

From the analysis of the above studies, it can be found that hybrid models can solve this problem well, and their performances are satisfactory. Currently, for ANN models, there are two main factors that should be improved, namely, the selection of hyperparameters and the optimization of weight and bias. However, in most of the previous studies, only one of these aspects is considered; thus, the performance of this kind of hybrid model is somewhat restricted. Additionally, only in one study [42] were both aspects of an ANN model considered, but in that study, a three-layer ANN model was applied. Therefore, the two aforementioned aspects of ANN models were not completely considered in this study. In addition, the previously mentioned hybrid models are generally complicated as too many initial parameters need to be determined by users; thus, most of them are very difficult to use in real applications. Moreover, currently, there are no studies on the comprehensive analysis of the three basic models. Therefore, in this study, firstly, a comprehensive comparison study on the basic models is conducted. Secondly, one new hybrid intelligent model based on ANN and a new intelligent optimization, Harris hawks optimization (HHO), is proposed. In this hybrid model which considers the two aspects of ANN models completely, HHO is used both to select the hyperparameters of the ANN model and to optimize the linking weights and thresholds of the ANN. Finally, using some typical economic data cases, the application of the proposed new hybrid intelligent model is verified comprehensively.

3. Analysis of Basic Forecasting Models

A grey system model is a system science model dealing with a grey system whose information is partly known. It was proposed by a Chinese scholar in the 1980s [5]. It has developed quickly and has been applied extensively in the field of economic forecasting. The most commonly used and basic grey forecasting model is GM(1, 1), which indicates that one variable is employed in the model and adopts a first-order differential equation to match the data generated by the accumulated generating operation (AGO). The computing process of GM(1, 1) can be found in reference [5]. A time series analysis model is a statistical model used to study random sequences that diversify over time [6]. The most basic and most used model for economic data forecasting is the auto-regressive and moving average (ARMA) model, whose construction process can be found in reference [6]. Moreover, an ANN model is a good nonlinear artificial intelligence model that is able to approximate various nonlinearities in data [46] and can approximate a complicated time series very well [7]. The generally used ANN model is the backpropagation (BP) network, in which the learning algorithm is called an error backpropagation or BP algorithm. The construction process of the BP network can be found in reference [46].

3.1. Grey System Model

The construction process of a grey system model is essentially a curve approximation process. The merit of a grey system model is that the number of required samples is small and its construction process is simple. Moreover, a grey system model can be verified easily. However, a grey system model has demerits [47]. The first one is that the time series must be continuous and differentiable, and it must be expressed by a primary function. The second one is that a grey system model can only describe a process that has a monotone increase or decrease.

Through a detailed analysis of a grey system model, it can be found that it involves some problems. For example, the original time series is approximated by an exponential function arbitrarily when its variational rule is unknown and the approximative degree is not illuminated. Moreover, the method to confirm the approximative degree is also unknown.

When using a grey system model, the original time series must be operated on through accumulated generating operation (AGO). This operation can reduce the random error in a certain sequence. Additionally, for uncertain sequences, AGO may make the forecasting error larger. Therefore, for nonlinear economic data forecasting, grey system models must be used cautiously.

3.2. Time Series Analysis Model

The ARMA model is essentially a kind of linear autoregression model. It analyzes the statistical law of a dynamic data series that diversifies over time. The construction and the modality of this model are very simple, and this model has a very good statistical characteristics. However, the time series analysis model also has demerits [47]. The first one is that the time series must be a stationary and normal sequence, or the sequence must be a stationary random-time series. The second one is that the data of the time series must be expressed through the linear combination of the forepassed data.

In the real world, economic data time series are generally not stationary and normal random series; thus, there are very few instances of firsthand ARMA model application. There generally exists a problem when extracting the trend in the sequence and comparing it to the real economic time series. However, the variational rule of the original data series cannot be comprehensively grappled in advance, which makes the trend extraction very complicated. In fact, even though the real data series has no trend, it is also not stationary and normal. Moreover, generally, economic data series cannot be expressed through the linear combination of their forepassed data. Therefore, the above problems restrict the application of the time series analysis model.

3.3. ANN Model

ANN models are essentially a kind of nonlinear autoregression. There is no extra demand for real data series in an ANN model, and almost any data series can be analyzed using ANN. In the nonlinear time series forecasting field, the generally used ANN model is a BP network model, and the forecasting method is autoregression. Some problems also exist in ANN models, described below.

(1) Selection of the neuron number of the input layer

The neuron number of the input layer is actually the lingering time step. It is a key parameter for sample construction, and it is generally selected based on the user's experience. This method is very subjective. Another method to select the input neuron number is trial and error, which may consume a lot of time and may result in selecting the incorrect neuron number.

(2) Selection of the hidden-layer number and neuron number of the hidden layer

Based on the understanding of the Kolmogorov theorem, which is taken as the mathematic base of ANN [7], a three-layer network is generally used in the nonlinear time series forecasting field. In fact, a previous study [48] proved that only a four-layer network can approximate any continuous function with any precision. In addition, for three-layer networks, the neuron number of the hidden layer is the term number of the expanding formula for its corresponding function. To remember more samples and approximate a function, three-layer networks require many hidden neurons. However, the robustness of the three-layer network is very poor, and its computing efficiency decreases very rapidly as the number of hidden neurons increases.

(3) Selection of the neuron function

According to the Kolmogorov theorem, using the sigmoid activation function, an ANN model can approximate any continuous function. However, from the perspective of mathematical mapping, a previous study [49] proved that the hidden neuron activation function significantly affects the model character. When approximating a function, the different neuron activation function corresponds to a different radix function, which causes the generalization of the ANN model to differ significantly.

For clarity, the features, merits, and demerits of the three basic models are summarized in Table 1.

Table 1. Features, merits, and demits of the three basic models.

Models	Features	Merits	Demerits
Grey system model	Curve approximation	Small samples, simple construction, easy validity	Continuous and differentiable time series, monotonous time series, unknown approximative degree
Time series analysis model	Linear autoregression	Simple construction, good statistical characteristics	Stationary random time series, linear combination of forepassed data
ANN model	Nonlinear autoregression	No extra demand for real time series	Hard to select suitable construction and neuron function, hard to improve extrapolation

Moreover, the similarities and differences in the three basic models are summarized in Table 2.

Table 2. Similarities and differences in the three basic models.

Similarities	Differences
Requirement for time series (all equal-space time sequences)	Appropriate range (exponential trend sequence for grey system model, stationary normal random sequence for time series analysis model, no more special requirement for ANN model)
Construction process (all approximation process)	"Grey degree" (almost black for ANN model, greyer for time series analysis model, grey for grey system model)
Method to estimate parameters (all least mean squares (LMS) method)	Construction difficulty (most complicated for ANN model, complicated for time series analysis model, simple for grey system model)
Construction model (all grey system models)	

4. New Hybrid Intelligent Model for Economic Data Forecasting

From the above analysis, it can be seen that the ANN model is suitable for forecasting economic data series. However, the ANN model has many problems. To solve those problems, a hybrid intelligent model of ANN optimized using HHO (called HHO-ANN) is proposed here.

4.1. Harris Hawks Optimization (HHO)

HHO [50] is a new intelligence optimization algorithm inspired by the predation behavior of groups of Harris hawks. In a group of Harris hawks, during their predation process, some Harris hawks pounce on the prey from all directions, and the prey instinctively escapes through various methods. Correspondingly, the Harris hawks use various methods to chase the prey. Therefore, in HHO, based on the escape energy of the prey, different location update strategies are applied in three stages to simulate the searching and hunting characteristics of the Harris hawks. The location update strategies in the three stages are explained as follows.

4.1.1. Searching Stage

In this stage, the random location update strategy is applied. For this strategy, based on the random number (q) generated in the range of [0, 1], the location updates of the Harris hawk individuals are as follows.

If $q \geq 0.5$, the position of the Harris hawk individual is updated as

$$X(t+1) = X_{rand}(t) - r_1|X_{rand}(t) - 2r_2X(t)| \tag{1}$$

Otherwise, the position of the Harris hawk individual is updated as

$$X(t+1) = [X_{rabbit}(t) - X_m(t)] - r_3[lb + r_4(ub - lb)], \tag{2}$$

where $X(t)$ and $X(t+1)$ are the positions of the Harris hawk individual at the current and the next iteration. t is the number of iterations. $X_{rand}(t)$ is the position of a randomly selected Harris hawk individual. $X_{rabbit}(t)$ is the position of the prey (the individual with the best fitness value). r_1, r_2, r_3, r_4 are the random numbers in the range of [0, 1]. ub and lb are the upper and lower bounds of the searching scope. $X_m(t)$ is the average position of the Harris hawks group, which is

$$X_m(t) = \sum_{k=1}^{M} X_k(t)/M, \tag{3}$$

where $X_k(t)$ is the position of individual k and M is the population size of the Harris hawks.

4.1.2. Searching and Hunting Conversion Stage

In this stage, according to the escape energy of the prey (E), the HHO switches between the searching and hunting stages. The escape energy of the prey (E) is defined as

$$E = 2E_0(1 - \frac{t}{T}), \tag{4}$$

where E_0 is the initial escape energy of the prey, which is a random number in the range of $[-1, 1]$. T is the maximum number of iterations.

If the absolute value of the escape energy ($|E|$) is larger than or equal to 1, the HHO switches to the searching stage. Otherwise, the HHO switches to the hunting stage.

4.1.3. Hunting Stage

In this stage, according to the absolute value of the escape energy ($|E|$) and one random number (r) in the range of [0, 1], different location update strategies are used, which are the following.

(1) If $0.5 \leq |E| < 1$ and $r \geq 0.5$, the location update strategy is described as

$$X(t+1) = \Delta X(t) - E|J \cdot X_{rabbit}(t) - X(t)|, \tag{5}$$

where J is one random number in the range of [0, 2]. $\Delta X(t)$ is the difference between the prey's position and the current individual's position, which is

$$\Delta X(t) = X_{rabbit}(t) - X(t) \tag{6}$$

(2) If $|E| < 0.5$ and $r \geq 0.5$, the location update strategy is described as

$$X(t+1) = X_{rabbit}(t) - E|\Delta X(t)| \tag{7}$$

(3) If $0.5 \leq |E| < 1$ and $r < 0.5$, the location update strategy is described as

$$X(t+1) = \begin{cases} Y, f(Y) < f(X(t)) \\ Z, f(Z) < f(X(t)) \end{cases} \quad (8)$$

where $f(\cdot)$ is the fitness function. Y and Z can be described as

$$\begin{cases} Y = X_{rabbit}(t) - E|J \cdot X_{rabbit}(t) - X(t)| \\ Z = Y + S \times LF(DIM) \end{cases}, \quad (9)$$

where *DIM* is the dimension of the solved problem. *S* is a random vector whose dimension is *DIM*, in which the vector elements are the random numbers in the range of [0, 1]. *LF*(*DIM*) is the Levy flight vector whose dimension is *DIM*, which is described as

$$LF(DIM) = \frac{0.01 \times u \times \sigma}{|v|^{\frac{1}{c}}}, \quad (10)$$

where u and v are the standard normal distribution random numbers whose dimensions are *DIM*. c is one constant which is 1.5. σ can be described as

$$\sigma = \left\{ \frac{\Gamma(1+c) sin(\frac{\pi c}{2})}{c\Gamma(\frac{1+c}{2}) 2^{\frac{c-1}{2}}} \right\}^{\frac{1}{c}}, \quad (11)$$

where $\Gamma()$ is the Gamma function, which is $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt$.

(4) If $|E| < 0.5$ and $r < 0.5$, the location update strategy is described as

$$X(t+1) = \begin{cases} Y', f(Y) < f(X(t)) \\ Z, f(Z) < f(X(t)) \end{cases}, \quad (12)$$

where Y' can be described as

$$Y' = X_{rabbit}(t) - E|J \cdot X_{rabbit}(t) - X_m(t)| \quad (13)$$

Because HHO can search for the global optimum without the complex controlling parameters with a flexible algorithm structure and strong optimization performance, it was chosen for application. The detailed processes of HHO are as follows.

(1) Parameter initiation

The initial parameters of HHO, such as population size and maximum number of iterations, should be provided beforehand.

(2) Creation of initial population

In the HHO, the initial population is randomly generated in the solution space.

(3) Computation of fitness value

By using the fitness function from the solved problem, the fitness values for all the initial Harris hawk individuals can be obtained.

(4) Selection of prey

The Harris hawk individual whose fitness value is the best is selected as the prey.

(5) Location updating

According to the escape energy of the prey and the generated random numbers q and r, the different location update strategies are applied for all Harris hawk individuals, and the new Harris hawk population can be generated.

(6) Updating prey

If the fitness value of the best individual of the new Harris hawk population is better than the fitness value of the current prey, the current prey is replaced by the best individual.

(7) Termination criterion

In the original HHO, the termination criterion is the maximum number of iterations. However, using this criterion, the convergence of the algorithm relies on the maximum number of iterations and affects the computing efficiency. Therefore, here, new termination criteria were applied, which are as follows. The main termination condition was the amount of iterations performed without replacing the current prey. In this study, the number of iterations was five, determined via experience and test. Moreover, to avoid infinite iterations, a secondary condition, the maximum number of iterations, was also used.

(8) Output results

If the termination conditions are not satisfied, the process returns to step (5). Otherwise, the process is over, and at this time, the fitness value and the location of the prey are output.

The flow chart of the HHO is shown in Figure 1.

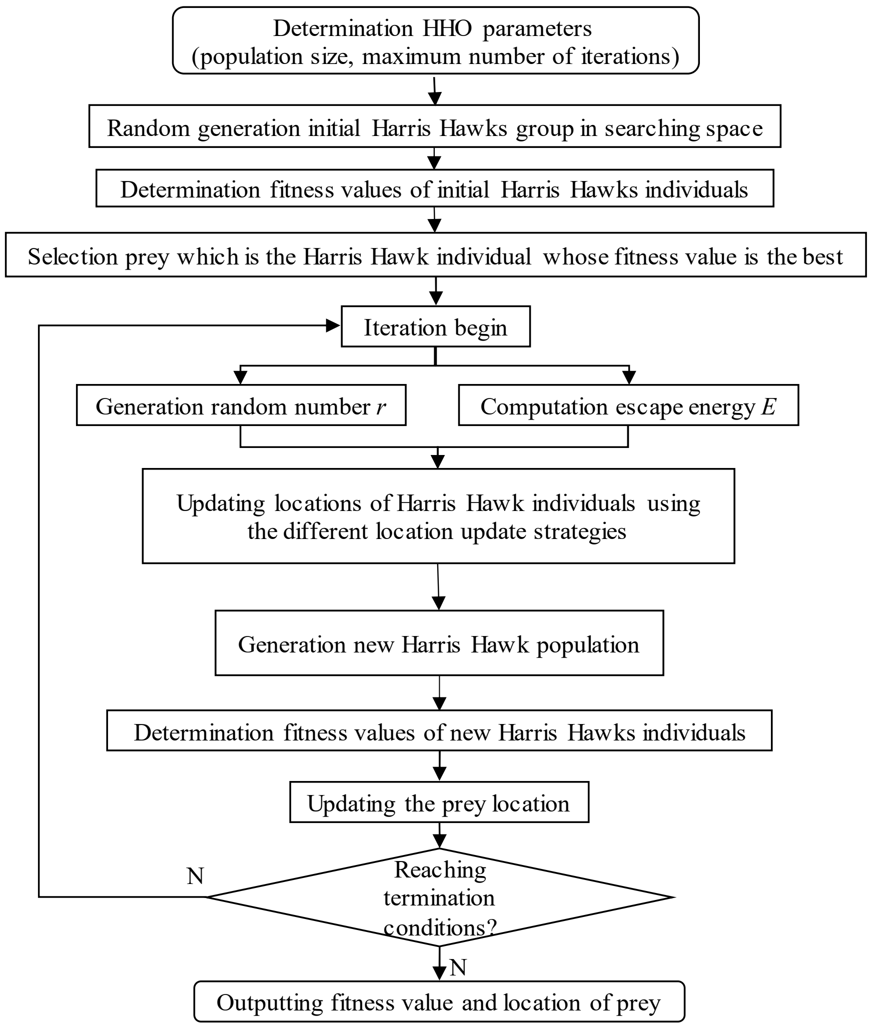


Figure 1. Flow chart of HHO.

4.2. New Hybrid Intelligent Model

To predict economic data using ANN, the first step is to construct the training samples. This process is described as follows.

Suppose the economic data series is $\{x(i), i = 1, 2, \ldots, n\}$. If the numbers of the input and output neurons are n_i and n_o, respectively, the number of the training samples is $n - n_i - n_o + 1$. The constructed training samples can be obtained as shown in Table 3.

Table 3. Training samples for economic data forecasting.

No. of Sample	Input Values	Output Values
1	$x(1), \ldots, x(n_i)$	$x(n_i + 1), \ldots, x(n_i + n_o)$
2	$x(2), \ldots, x(n_i + 1)$	$x(n_i + 2), \ldots, x(n_i + n_o + 1)$
J	$x(j), \ldots, x(n_i + j - 1)$	$x(n_i + j), \ldots, x(n_i + n_o + j - 1)$
…	…	…
$n - n_i - q_o + 1$	$x(n - n_i - n_o + 1), \ldots, x(n\text{-}n_o)$	$x(n - n_o + 1), \ldots, x(n)$

Generally, parameter n_o is the forecasting step, which can be easily determined based on experience. Therefore, to construct the samples, it is important to select parameter n_i. When determining the ANN structure, apart from the number of input neurons, the hidden-layer structure (the number of layers and neuron numbers in each layer) should be determined too. Because the computing results of ANN are significantly affected by its structure, it is critical and very difficult to determine the suitable ANN structure in advance. Therefore, the intelligent optimization method HHO was used to select the suitable ANN structure, which includes the number of input neurons, the number of layers, and the neuron numbers of each layer. Generally, the traditional BP algorithm is used to optimize the weights and thresholds of the ANN model. However, this kind of algorithm has some shortcomings, such as premature convergence, etc. To improve this method, here, the HHO was also used to optimize the weights and thresholds of the ANN model. Moreover, to improve the performance of the original ANN model, the new neuron activation function called the Softplus function was applied, which is expressed as follows:

$$f(z) = \log(1 + \exp(z)) \tag{14}$$

This function, which is the primitive function of the logistic sigmoid function and can be taken as the smoothed or "softened" version of Rectified Linear Units (max(1, z)), is closer to the activation model of real brain neurons.

Therefore, for the new hybrid intelligent model in this study, there are three structure parameters (the number of input neurons, the number of hidden layers, and the number of hidden-layer neurons), and the weights and thresholds of ANN are determined using the HHO. Here, the full-linking network [51] is used to make the hybrid intelligent model as simple as possible.

The detailed steps of the proposed hybrid intelligent model's process are as follows.

(1) The initial search ranges for the three structure parameters of ANN (the number of input neurons, the number of hidden layers, and the number of hidden-layer neurons) and the initial controlling parameters of the new hybrid intelligent model (maximum number of iterations, population size, and the number of output neurons of ANN) are all determined beforehand.

It should be noted that the initial conditions include the structure parameters of the ANN (number of input neurons, number of hidden layers, number of hidden-layer neurons, and number of output neurons) and the parameters of HHO (maximum number of iterations and population size). For the four structure parameters of ANN, only the search ranges of three of them (number of input neurons, number of hidden layers, and number of hidden-layer neurons) should be provided, as these only slightly affect the computing efficiency and do not affect the computing results. Additionally, the number of output neurons can be determined easily based on real economic data, and it only slightly affects the computing results and computing efficiency. Finally, the two parameters of HHO, which can be determined easily according to experience, also slightly affect the computing efficiency and computing results. Because the effect of those initial conditions on the computing results is not significant, in this study, they were determined according to experience and our tests.

(2) By using three randomly generated structure parameters in their search ranges and the given number of output neurons, one individual that represents one specific ANN structure can be obtained. Moreover, based on some randomly generated individuals whose number is the given population size, the initial population is created. Finally, it should be pointed out that for the generated individual, the fixed relationship between the number of hidden-layer neurons and the hidden layer is applied, which is determined beforehand.

(3) The fitness value of each created individual is computed using the following method.

a. Based on the economic data series, using the method described in Table 3, the learning samples can be generated. The samples are then divided into training samples (80% of the total samples) and testing samples (20% of the total samples).

b. The initial linking weights and thresholds of this specific ANN structure are randomly generated; thus, one ANN model is obtained. This process is repeated to generate a group of ANN models with the same structure whose number equals the population size of the HHO.

c. In the generated initial group of ANN models, each ANN model is trained using the training samples, and thus the square error (E) of the testing samples is taken as the fitness value.

d. For the initial generated group of ANN models, the individual with the best fitness value is selected as the prey.

e. According to the HHO, different location update strategies are used to adjust the linking weights and thresholds of each ANN model in the generated group of ANN models; then, a new group of ANN models can be created.

f. In the new group of ANN models, each ANN model is trained; thus, the square error (E) of the testing samples is taken as the fitness value.

g. For the newly generated group of ANN models, the prey is updated.

h. If the termination criteria of HHO are not reached, the computing process returns to step *e*. Otherwise, the fitness value of the final prey is the fitness value of the created individual.

(4) The individual whose fitness is the best is selected as the prey.

(5) According to the HHO, different location update strategies are used for each individual, and a new individual can be generated.

(6) The fitness values of each new individual are calculated using the method described in step (3). The new population can then be generated.

(7) The prey is updated.

(8) If the termination criteria of the HHO are reached, the algorithm stops. At this time, the prey which is the best individual in the current population is selected as the suitable HHO-ANN model for economic data forecasting.

The flow chart of the new hybrid intelligent model for economic data forecasting is shown in Figure 2.

It must be noted that, because the hybrid intelligence model is based on the ANN model, the proposed new hybrid intelligence model can be used to predict any kind of economic data. However, compared with the previous hybrid intelligent models, the new hybrid model considers both aspects of ANN models, which makes it easy to implement, and its initial parameters can be determined easily because their numbers are as small as possible.

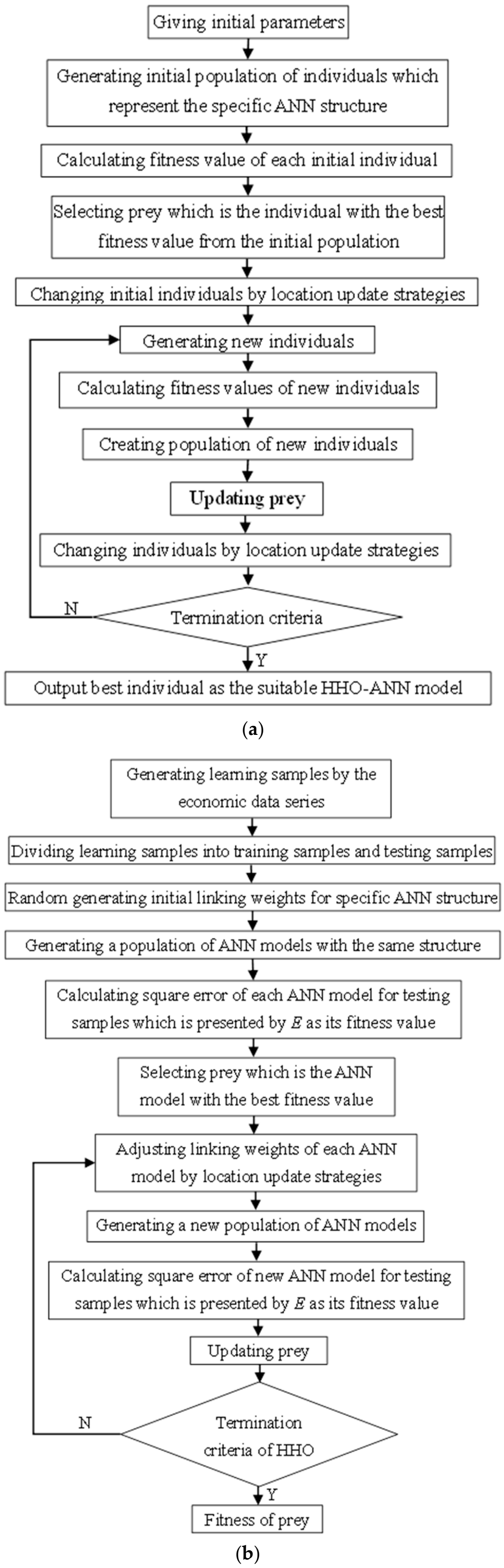


Figure 2. Flow chart of the new hybrid intelligent model. (**a**) Main flow chart of new hybrid model; (**b**) flow chart of fitness computation.

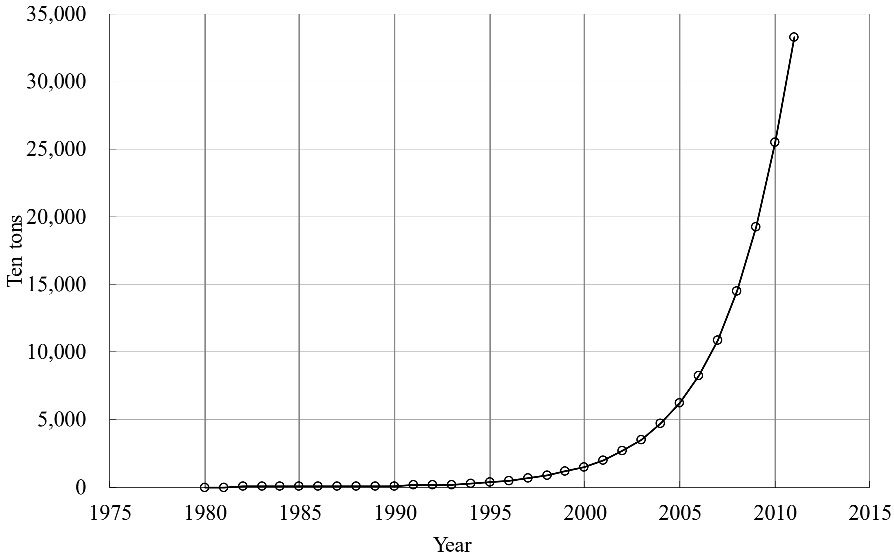
5. Case Studies

5.1. Studies on the Three Basic Models

To verify the analysis of the three basic models, two typical economic data series [10] were used. One of those data series is a monotone increasing data series and the other is a disordered data series. Using a typical and simple analysis, here, the widely used GM(1, 1) is used as the typical grey system model, the widely used ARMA(2, 1) model is used as the typical time series analysis model, and the widely used three-layer BP network is used as the typical ANN model.

5.1.1. First Case

The first economic data series is the yearly output of attapulgite clay in Xuyi County, Jiangsu, China, from 1980 to 2011. It is a monotone increasing time series (Figure 3a). For this economic data series, there are 32 data. In this study, the first 26 data were used to construct the forecasting model, and the last 6 were used to verify it.



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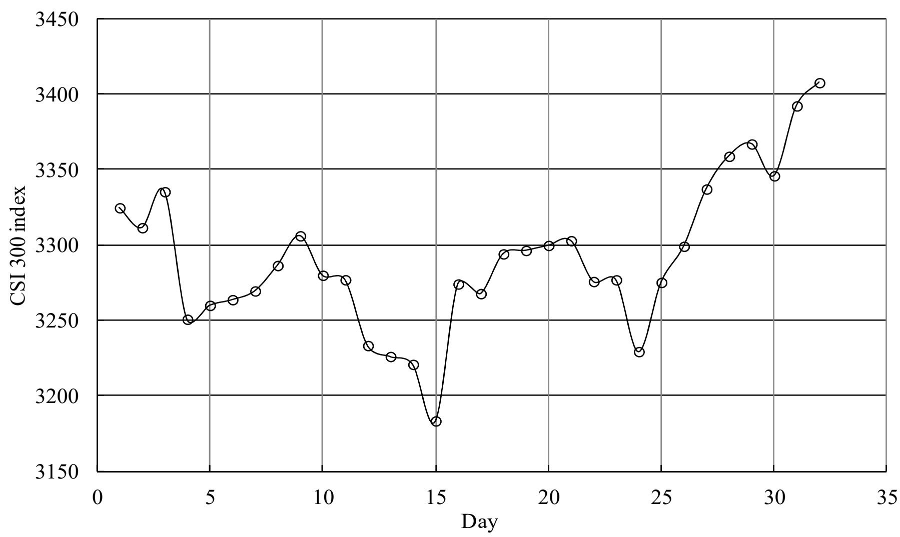
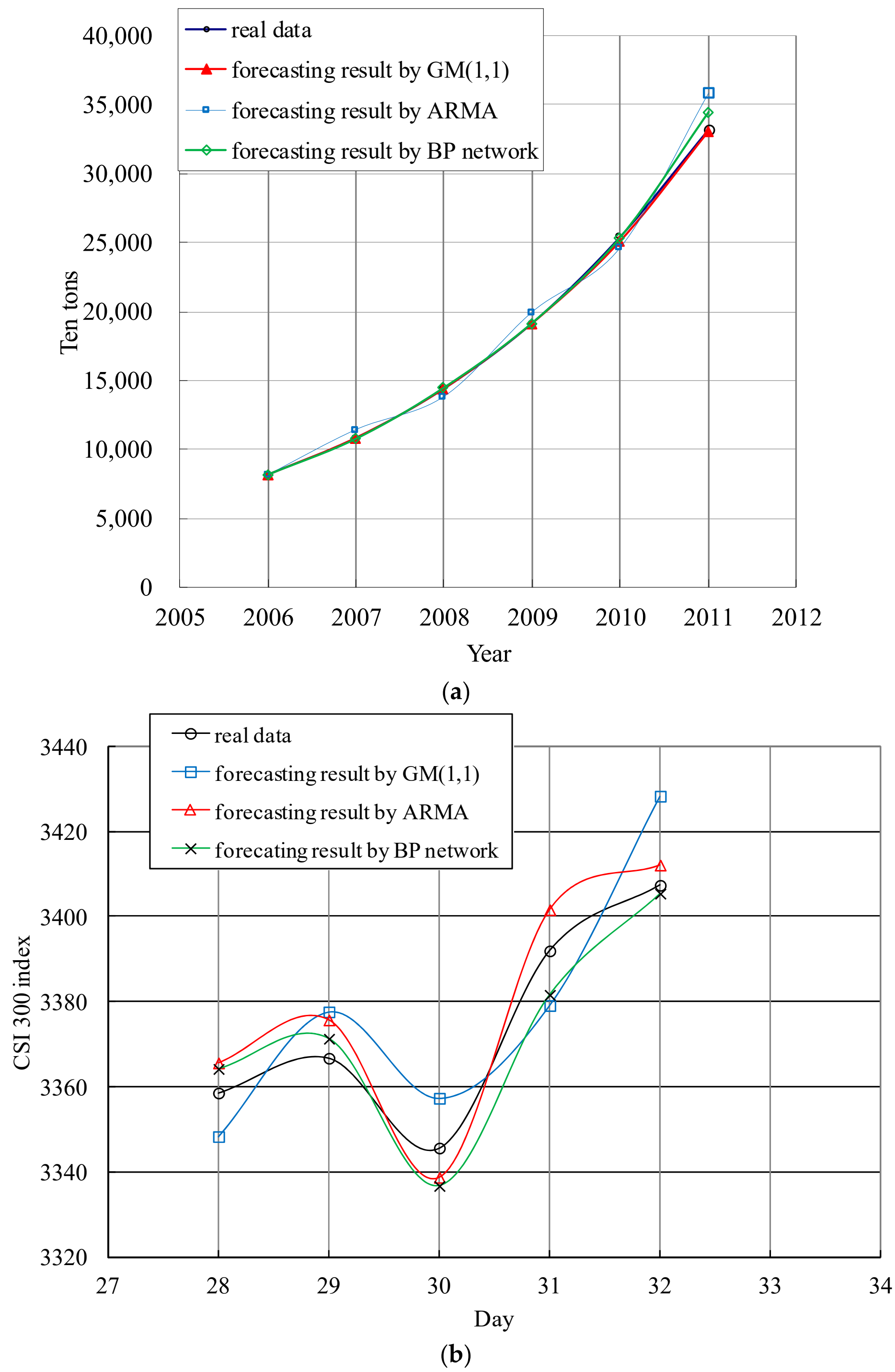


(**b**)

Figure 3. Real data for two typical economic data series. (**a**) First case; (**b**) Second case.

The forecasting results of the three basic models are shown in Figure 4a.



(**a**) (**b**)

Figure 4. Comparison of forecasting results using the three basic models for two typical economic data series. (**a**) First case; (**b**) Second case.

It must be noted that, for these economic data, based on the experiment and tests, the structure of the BP network model is 3-16-1.

As seen in Figure 4a, the forecasting results of the three basic models almost agree well with the real data. As time passes, the forecasting results of the ARMA model and the BP network model deviate further from the real data, especially for the last data point. The forecasting results of the ARMA model deviate more significantly. However, this phenomenon does not appear in the forecasting results of the GM(1, 1) model.

To compare these results more clearly, their computing accuracies described by the correlation coefficients are summarized in Table 4.

Table 4. Comparison of computing forecasting accuracy of the three basic models for two economic data series.

Model	First Economic Data Series	Second Economic Data Series
ANN model	0.9837	0.9845
Time series analysis model	0.8836	0.9784
Grey system model	0.9975	0.8215

As shown in Table 4, the correlation coefficient of GM(1, 1) is 0.9975, which is the largest. The correlation coefficient of the ARMA model is 0.8836, which is the smallest. Moreover, the correlation coefficient of the BP network model is 0.9877, which is acceptable.

From the results presented in Table 4 and Figure 4a, it can be concluded that, for this economic data case, which is a monotone increasing data series, the forecasting result obtained using the grey system model is the best, and that obtained using the time series analysis model is the worst. Moreover, the result obtained using the ANN model is acceptable. Therefore, the grey system model is the most suitable model for forecasting this monotone increasing data series, and the time series analysis model is not very suitable for forecasting this data series. Moreover, the ANN model is suitable for forecasting this data series. This conclusion is also in agreement with the results of a previous study [11]. Therefore, the analysis results in Section 3 can be verified by the forecasting results for this economic data series.

5.1.2. Second Case

The second economic data series is the CSI 300 index data series collected between 1 March 2010 and 2 April 2010; it is a disordered data series (Figure 3b). For this economic data series, there are also 32 data. In this study, the first 27 data were used to construct the forecasting model and the last 5 were used to verify it.

The forecasting results are shown in Figure 4b.

It must be noted that, for these economic data, based on the experiment and tests, the structure of the BP network model was determined to be 4-19-1.

As seen in Figure 4b, the forecasting results of the three basic models all reveal the basic changing law of the real data series. However, the computed results of the BP network model and the ARMA model are closer to the real ones than those of the GM(1, 1) model. Additionally, as time passes, the forecasting results of the GM(1, 1) model deviate from the real data, especially for the last data point. However, this phenomenon is not evident in the forecasting results of the BP network model or those of the ARMA model.

To compare these results more clearly, their computing accuracies described by the correlation coefficients are also summarized in Table 4. As shown in Table 4, the correlation coefficients of the BP network model and those of the ARMA model are similar, 0.9845 and 0.9784, respectively. The correlation coefficient of GM(1, 1) is the lowest, 0.8215. In other words, the forecasting results of the BP network model and those of the ARMA model are all suitable, although the forecasting result of the BP network is better. Moreover, the forecasting result of GM(1, 1) is not suitable.

From the results presented in Table 4 and Figure 4b, it can be concluded that, for this economic data series, which is a disordered data series, the forecasting result obtained using the ANN model is the best, and the result obtained using the time series analysis model is slightly worse than that of the ANN model. However, the forecasting result obtained using the grey system model is the worst. Therefore, the ANN model is the most suitable model for forecasting the disordered data series, and the grey system model is not very suitable for forecasting this data series. Moreover, the time series analysis model is also suitable. Therefore, the analysis results in Section 3 can be verified by the forecasting results for this economic data series as well.

From the analysis of the above two cases, it can be concluded that, for an exponential trend time sequence, the forecasting performance of the grey system model is the most suitable. For a disordered time sequence, the forecasting performance of the time series analysis model is suitable. Moreover, the ANN model can be used for all time series, and its forecasting performance is always acceptable. Those conclusions are in good agreement with the analysis results in Section 3, and they are also verified by the results in previous studies [17,52].

5.2. Studies on New Hybrid Intelligent Model

To verify the performance of the new hybrid intelligent model, four economic data series were used; these include the two abovementioned data series and two data groups of the Shanghai stock market, China, from September 2015–August 2018 [37]. For the first two time series, the amount of data is small and the changing law is relatively uncomplicated, meaning they can be called simple data cases, but for the last two, the amount of data is large and the changing law is complicated, meaning they can be called complex data cases.

5.2.1. Simple Data Cases

For the first two data cases, considering the amount of data, selected according to experience, n_o was determined to be 1. Based on the test and experience, the initial parameters for the new hybrid intelligent model are summarized in Table 5.

Table 5. Initial parameters for new hybrid intelligent model for the two simple data cases.

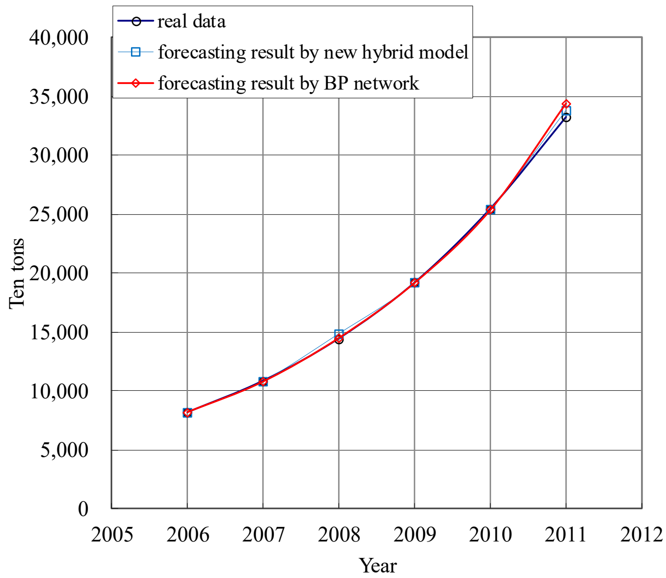
Parameter	Range	Parameter	Value
Number of input neurons	1–10	Maximum number of iterations	100
Number of hidden-layer neurons	1–50	Individual number in one population	20
Number of hidden layers	1–5		

Using the two simple datasets as shown in Figure 3 and the initial parameters in Table 5, the optimal structures of the new hybrid intelligent model were determined to be 4-14-1 and 4-16-11-1. The obtained optimal structures and datasets were applied to the suitable hybrid intelligent model. For the two simple data cases, the forecasting results were obtained, as shown in Figure 5. It must be noted that to compare the models clearly, the results of the basic ANN model (BP network) are also shown in Figure 5.

From the results depicted in Figure 5, it can be seen that the forecasting results obtained using the proposed new hybrid intelligent model are in good agreement with the real data series for both data cases. Additionally, for both cases, the forecasting results obtained using the new hybrid intelligent model are all better than those obtained using the basic ANN model. Moreover, because the changing law of the first data case is simpler, its forecasting result is better, not only for the new hybrid intelligent model but also for the BP network.

To show the performance of the new hybrid model more clearly, its computing accuracies for the two simple data cases also described by the correlation coefficients are summarized in Table 6. It must be noted that the results of the basic ANN model (BP network) are also shown in Table 6 to compare the results clearly.

As shown in Table 6, for all data cases, the correlation coefficients of the new hybrid intelligent model are all very high, 0.9962 and 0.9933, respectively. This shows that the forecasting accuracy of the new hybrid model is very high for the two data cases. In other words, the performance of the new hybrid model for simple data cases is very suitable. Moreover, compared with the new hybrid model, the forecasting accuracy of the basic ANN model is lower, and its correlation coefficients for the two data cases are only 0.9877 and 0.9845, respectively. Therefore, the proposed new hybrid intelligent model is a very suitable method for forecasting all kinds of simple economic data. Moreover, the initial parameters that should be determined beforehand for the new hybrid model can be determined easily and simply. Thus, the application of the new hybrid model is very convenient.



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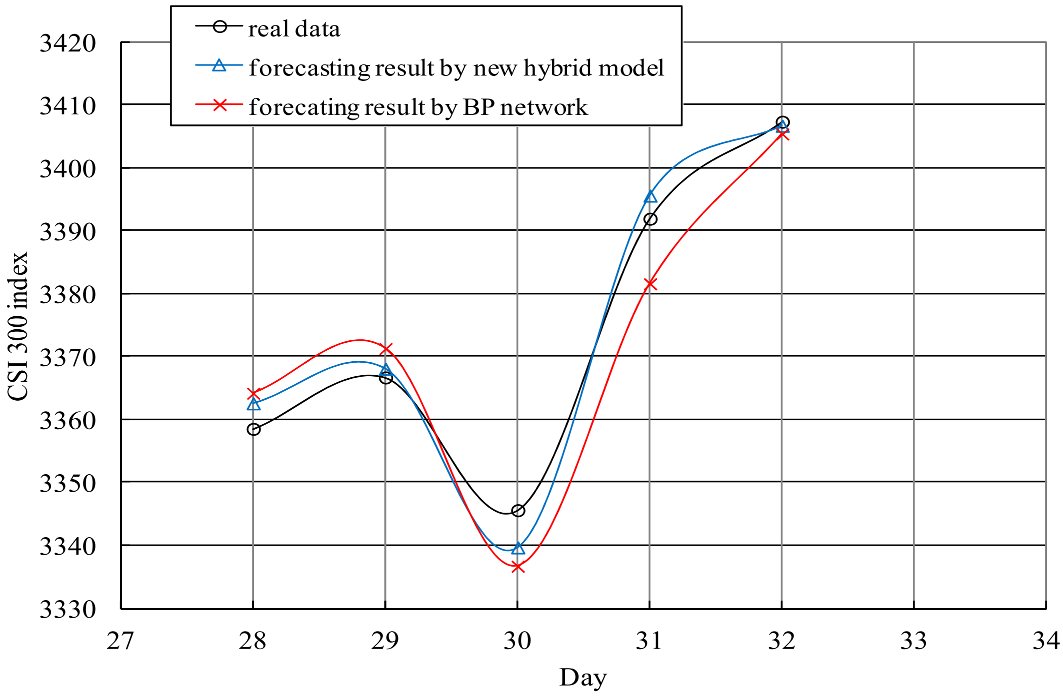


(**b**)

Figure 5. Forecasting results obtained using the new hybrid intelligent model for two simple data cases. (**a**) First case; (**b**) Second case.

Table 6. Forecasting accuracy of new hybrid intelligent model for the two simple economic data series.

Model	First Economic Data Series	Second Economic Data Series
ANN model	0.9877	0.9845
New hybrid intelligent model	0.9962	0.9933

5.2.2. Complex Data Cases

In the above section, the performance of the new hybrid model was verified through two simple data cases. Here, the performance of the new hybrid model is also analyzed using two complex data cases.

The datasets used in this section are the data series of the 733 stock index and that of daily turnover over three years in the period of 1 September 2015–31 August 2018 for the Shanghai stock market, China. These datasets are shown in Figure 6. It must be noted that the unit for the daily turnover dataset is ten billion, and there is no unit for the stock index.

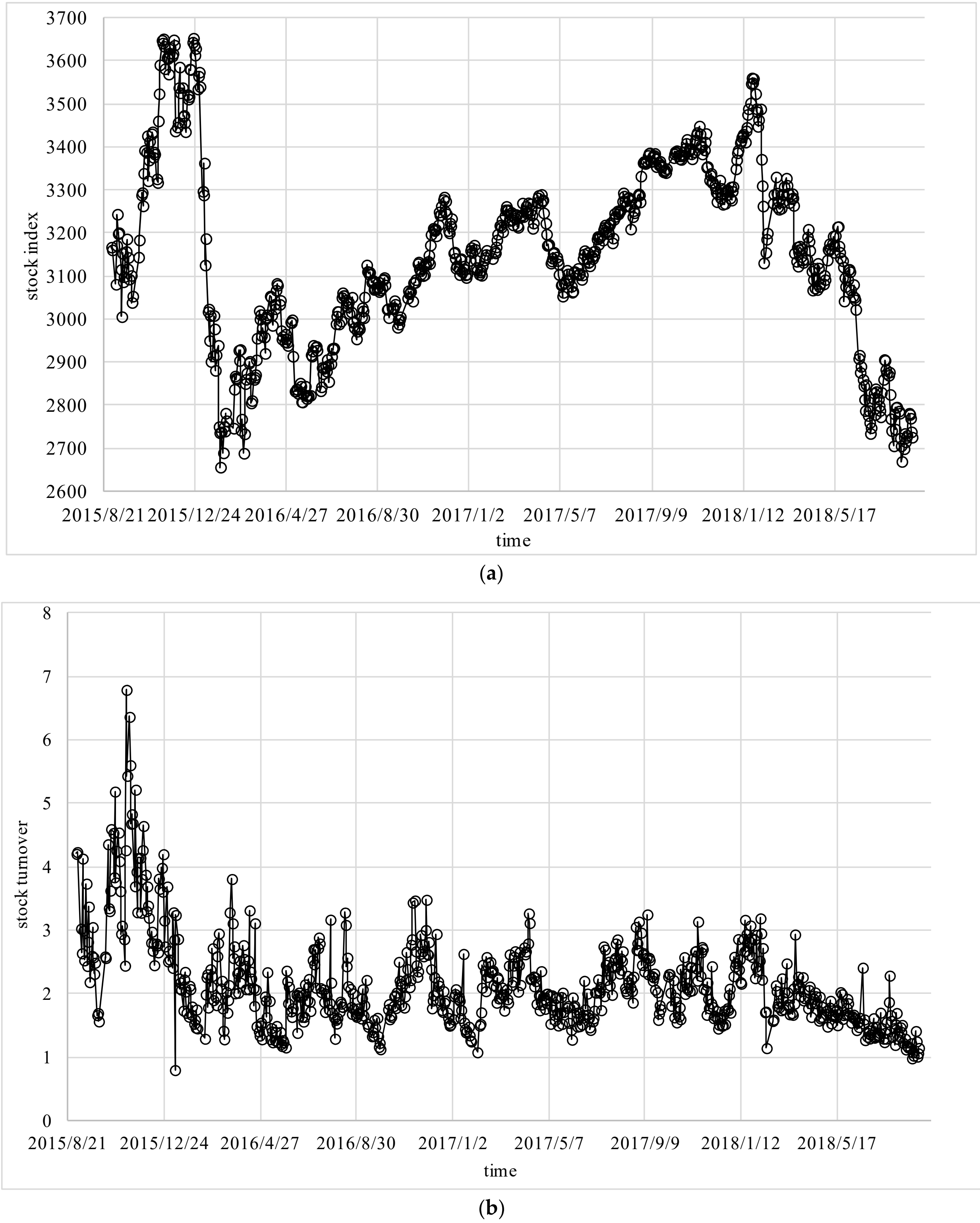


Figure 6. Stock market data for the Shanghai stock market, China, September 2015–August 2018. (**a**) Stock index; (**b**) daily turnover.

In this study, for the two data series, the first 30 months' data, comprising 606 data points, were used to construct the forecasting model, and the last 6 months' data, comprising 127 data points, were used to verify the performance of the constructed model.

For those two data cases, considering the amount of data, selected according to experience, n_0 was determined to be 10. Based on the test and experience, the initial parameters for the new hybrid intelligent model are summarized in Table 7.

Table 7. Initial parameters for new hybrid intelligent model for complex data cases.

Parameter	Range	Parameter	Value
Number of input neurons	1–300	Maximum number of iterations	300
Number of hidden-layer neurons	1–500	Individual number in one population	50
Number of hidden layers	1–5		

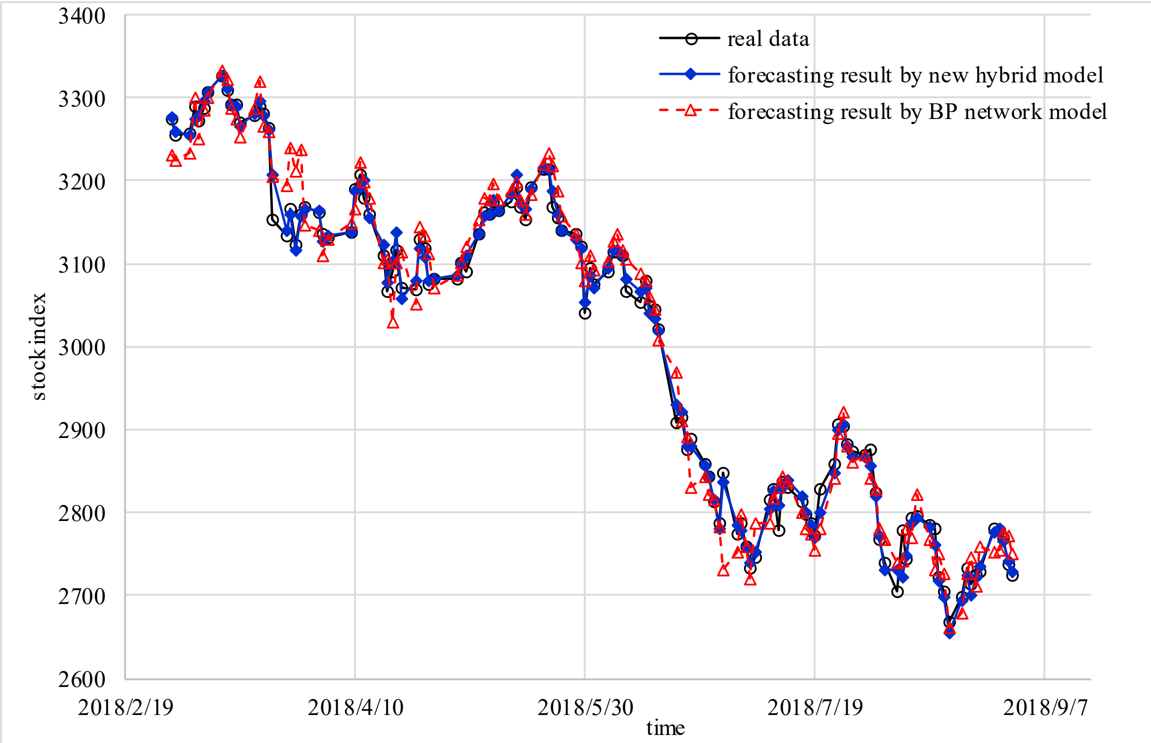
Using the two complex datasets as shown in Figure 6 and the initial parameters in Table 7, the optimal structures of the new hybrid intelligent model were determined to be 89-207-162-10 for the stock index and 85-210-148-10 for the daily turnover. The obtained optimal structures and datasets were applied to the suitable hybrid intelligent model. After computing, the forecasting results for two complex data cases were obtained, as shown in Figure 7. It must be noted that to compare the results clearly, the results of the basic ANN model (BP network) are also shown in Figure 7.

From the results presented in Figure 7, it can be seen that the forecasting results obtained using the proposed new hybrid intelligent model are also in good agreement with the real data series for both data cases. Additionally, for both data cases, the forecasting results obtained using the new hybrid intelligent model are all much better than those obtained using the basic ANN model.

To analyze the performance of the new hybrid model more clearly, for the two complex data cases, its computing accuracies described by the correlation coefficients are also summarized in Table 8. Moreover, to compare the results clearly, the results of the basic ANN model (BP network) are also shown in Table 8.

As shown in Table 8, for the two data cases (stock index and daily turnover), the correlation coefficients of the new hybrid intelligent model are all high, 0.9897 and 0.9873, respectively. Thus, the forecasting accuracy of the new hybrid model is also high for the two complex data cases. In other words, the performance of the new hybrid intelligent model on complex data cases is suitable. Moreover, compared with the new hybrid model, the forecasting accuracy of the basic ANN model is very low, and the correlation coefficients for the two data cases are only 0.8932 and 0.8862, respectively, which are much lower than those of the new hybrid intelligent model. Therefore, the proposed new hybrid intelligent model is also a very suitable method for forecasting complex economic data.

Moreover, from the results depicted in Tables 6 and 8, it can be seen that, for the complex data case, the forecasting accuracy is lower than that for the simple data case obtained using both models (the basic ANN model and the new hybrid intelligent model). That is to say, the performance of the two forecasting models decreases when they are used on complex data. However, the decrease in the performance of the basic ANN model is significant, while there is only a slight decrease in the performance of the new hybrid intelligent model. Therefore, the proposed new hybrid intelligent model can be applied very well for all kinds of economic data, and it is a very suitable model for economic data forecasting.



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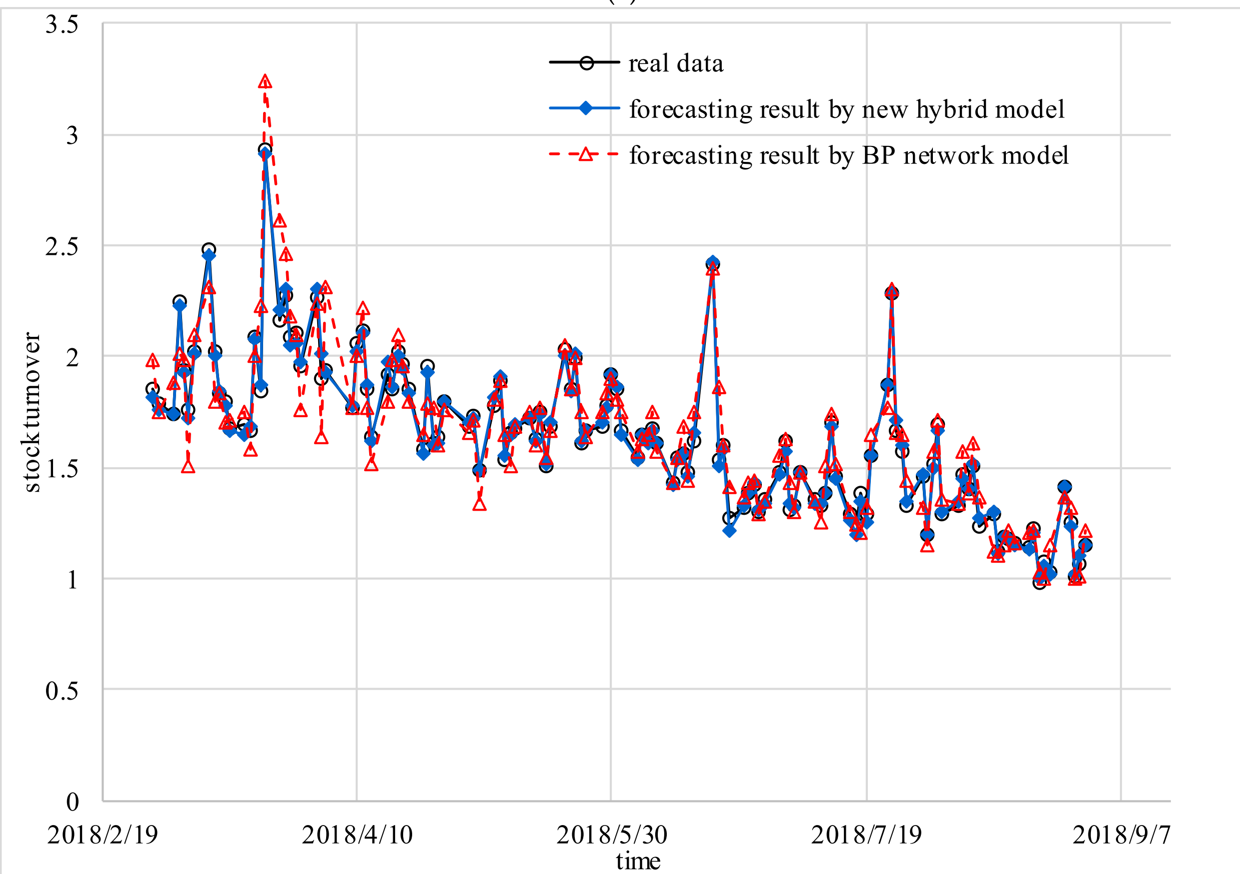

(**b**)

Figure 7. Forecasting results for stock market data of Shanghai stock market, China. (**a**) Stock index; (**b**) daily turnover.

Table 8. Forecasting accuracy of new hybrid intelligent model for the two complex economic data series.

Model	Stock Index	Daily Turnover
ANN model	0.8932	0.8862
New hybrid intelligent model	0.9897	0.9873

6. Discussion

6.1. Comparison with Other Hybrid Models

To analyze the performance of the new hybrid intelligent model proposed here in-depth, a study comparing its performance with that of some other hybrid intelligent models is conducted. The other hybrid models include the one proposed in reference [37] and two comparative hybrid models proposed here. For the first comparative hybrid model, the BP algorithm is used to optimize the linking weights of ANN, and the HHO is used to select the ANN's structure and the parameters of the BP algorithm, which is similar to that described in reference [37]. For the second comparative hybrid model, particle swarm optimization (PSO) is used both to select the hyperparameters of ANN and to optimize the linking weights and thresholds of ANN. Here, to compare the models clearly, the proposed new hybrid intelligent model is named HHO-ANN, the first comparative hybrid model is named HHO-BPNN, the second comparative hybrid model is named PSO-ANN, and the model proposed in reference [37] is named IBHA-MFNN. Moreover, in this comparison study, the two complex datasets of the stock market data of the Shanghai stock market, China, are used.

Because HHO-BPNN and IBHA-MFNN use similar methods, the same initial parameters are used for the two models; these are summarized in Table 9.

Table 9. Initial parameters for HHO-BPNN and IBHA-MFNN.

Parameter	Range	Parameter	Value
Number of input neurons	1–300	Maximum number of iterations	300
Number of hidden-layer neurons	1–500	Individual number in one population	50
Number of hidden layers	1–5	Iterating stop criterion for BP algorithm	500
Iterating step for BP algorithm	0.01–1	Iterating error criterion for BP algorithm	0.001
Inertia parameter for BP algorithm	0.0–0.99		

Moreover, because HHO-ANN and PSO-ANN use similar methods, some of the initial parameters for the two models are the same. To compare these models fairly, the same values are used for these identical initial parameters, as shown in Table 7. However, PSO-ANN has more initial parameters than HHO-ANN. Not including the initial parameters shown in Table 7, the initial parameters of PSO-ANN were determined according to the test and experience, and these are summarized in Table 10.

Table 10. Special initial parameters for PSO-ANN.

Minimum Inertia Weight	Maximum Inertia Weight	Learning Factors	Maximum Particle Velocity
0.2	0.9	2	6

To compare the models four hybrid models fairly, the other computing conditions are the same as those described in the above section.

The parameter optimization results obtained using the four hybrid models are summarized in Tables 11 and 12.

Using the parameter optimization results of the four hybrid models described in Tables 11 and 12, based on the two datasets of stock market data of the Shanghai stock market, China, the forecasting results were obtained; these are shown in Figure 8.

Table 11. Parameter optimization results of four hybrid models on stock index data.

Parameter	HHO-ANN	PSO-ANN	HHO-BPNN	IBHA-MFNN
Optimal structure	89-207-162-10	87-213-175-10	85-196-171-10	97-212-141-10
Iterating step for BP algorithm	-	-	0.297	0.324
Inertia parameter for BP algorithm	-	-	0.855	0.882

Table 12. Parameter optimization results of four hybrid models on daily turnover data.

Parameter	HHO-ANN	PSO-ANN	HHO-BPNN	IBHA-MFNN
Optimal structure	85-210-148-10	82-226-152-10	86-247-132-10	87-221-127-10
Iterating step for BP algorithm	-	-	0.305	0.283
Inertia parameter for BP algorithm	-	-	0.916	0.923

From the results depicted in Figure 8, it can be seen that the forecasting results obtained using the four hybrid models are all in agreement with the real data. However, the computed results obtained using the newly proposed HHO-ANN model are closer to the real ones than those obtained using the three other hybrid models; that is, the forecasting results of the HHO-ANN model are the best.

Moreover, to compare the four hybrid models in more detail, the computing accuracy described by the correlation coefficient and the computing efficiency described by the computing time are summarized in Table 13.

As seen in Table 13, for the two models in which the BP algorithm is used (HHO-BPNN and IBHA-FMNN), the computing results are similar. Additionally, by using HHO-BPNN, the correlation coefficients for the two datasets (stock index and daily turnover) are 0.9322 and 0.9316, respectively, which are slightly lower than those by IBHA-FMNN (0.9324 and 0.9318). Moreover, the computing times obtained using HHO-BPNN are 43 and 21, respectively, which are similar to those obtained using IBHA-FMNN (44 and 20). The reason for this may be that, for these two hybrid models, except for the different optimization methods used for the selection of the ANN structure, the main methods used are the same. Moreover, the optimization methods used for the selection of the ANN structure in the two models are all nature-inspired metaheuristic algorithms, whose difference is somewhat small. Therefore, the performance of HHO-BPNN is slightly better than that of IBHA-FMNN. However, the computing results of the two models (HHO-ANN and PSO-ANN) that use the same principal methods, not including the optimization methods they use (HHO and PSO), are very different. The correlation coefficients for the two datasets (stock index and daily turnover) obtained using the HHO-ANN are 0.9897 and 0.9873, respectively, which are larger than those obtained using PSO-ANN (0.9511 and 0.9502). Moreover, the computing times of HHO-ANN are 52 and 27, respectively, which are much shorter than those of PSO-ANN (124 and 62). Therefore, the computing results (both computing accuracy and computing efficiency) of HHO-ANN are all much better than those of PSO-ANN. In addition, for HHO-ANN, there are only five initial parameters (Table 7) that should be determined beforehand, which is much fewer than those for PSO-ANN (nine, as shown in Tables 7 and 10). Thus, the application of HHO-ANN is more convenient and simpler. Moreover, it also can be seen that the computed correlation coefficients of HHO-ANN and PSO-ANN are larger than those of HHO-BPNN and IBHA-FMNN; that is, the computing accuracies of HHO-ANN and PSO-ANN are higher than those of HHO-BPNN and IBHA-FMNN. However, the computing times of HHO-ANN and PSO-ANN are longer than those of the other two models (HHO-BPNN and IBHA-FMNN). In detail, the correlation coefficient of HHO-ANN is much larger than those of HHO-BPNN and IBHA-FMNN, but its computing time is, to some extent, shorter than those of the other two models. However, its number of initial parameters (five, as shown in Table 7) is much lower than those of HHO-BPNN and IBHA-FMNN (nine, as shown in Table 9); that is, HHO-ANN can be applied more conveniently. Thus, considering the computing results and application convenience comprehensively, the performance of HHO-ANN is much better than that of HHO-BPNN and IBHA-FMNN. Additionally, the correlation coefficient of PSO-ANN is, to some extent, larger than those of HHO-BPNN and IBHA-FMNN, but its computing time is much longer than those of the other two models—about three times longer. Moreover, PSO-ANN has nine initial parameters, the same as those for HHO-BPNN and IBHA-FMNN. Thus, considering the computing results and application convenience comprehensively, the performance of PSO-ANN is, to some extent, similar to that of HHO-BPNN and IBHA-FMNN. Therefore, the performance of the newly proposed HHO-ANN model is the best, and the application of HHO-ANN is also the most convenient.

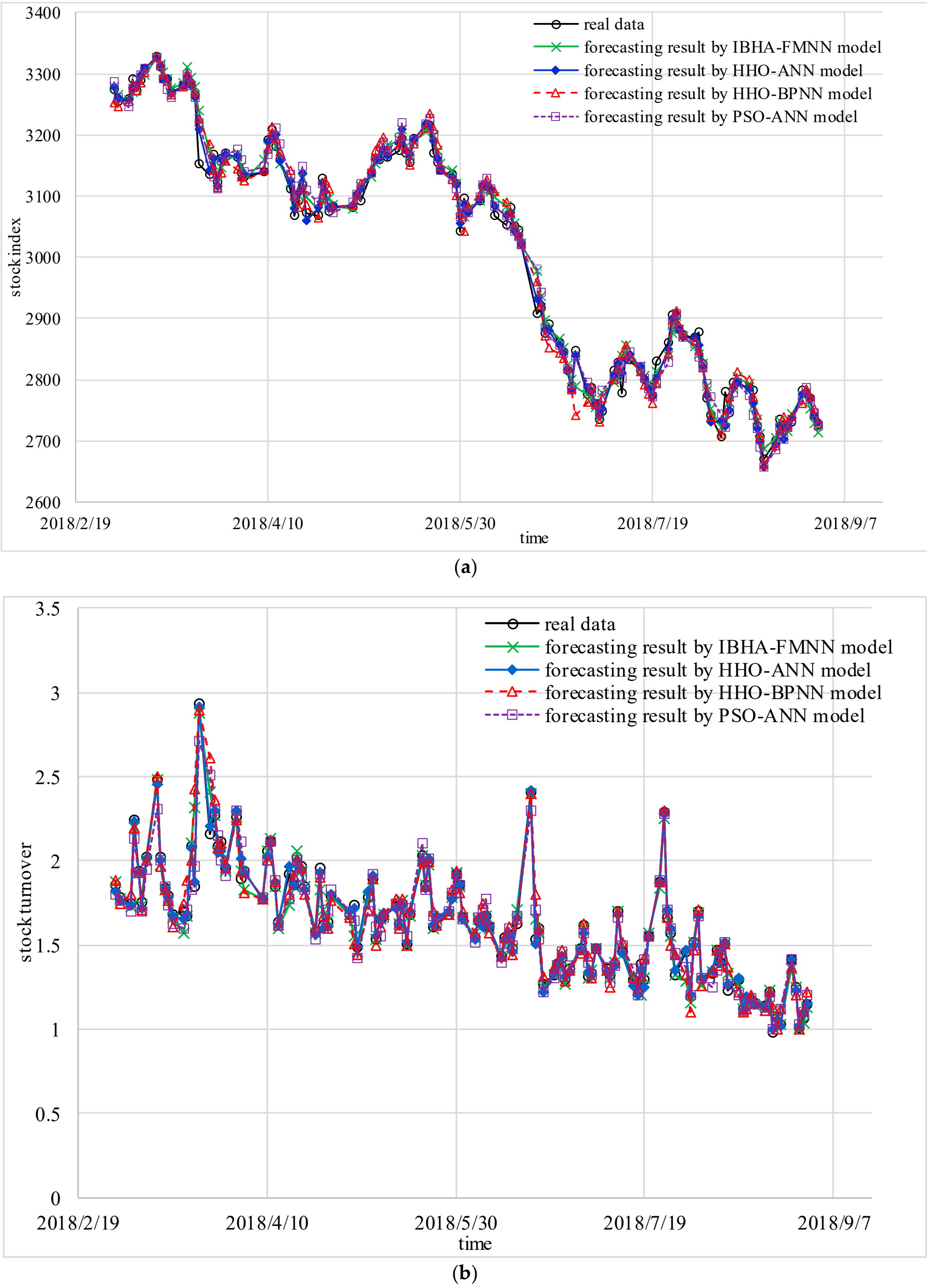

Figure 8. Comparison of forecasting results for stock market data obtained using four hybrid models. (**a**) Stock index; (**b**) Daily turnover.

Table 13. Computing results of four hybrid models on stock market data.

Model	Stock Index		Daily Turnover	
	Correlation Coefficient	**Computing Time (s)**	**Correlation Coefficient**	**Computing Time (s)**
HHO-ANN	0.9897	52	0.9873	27
PSO-ANN	0.9511	124	0.9502	62
HHO-BPNN	0.9322	43	0.9316	21
IBHA-FMNN	0.9324	44	0.9318	20

Finally, from the in-depth analysis of the comparison results, the academic contributions of this study can be summarized as follows. Analyzing the construction process of the new hybrid intelligent model, it can be seen that, for this model, the two aspects of ANN models have both been optimized, and the forecasting effect has been used to select the model; thus, the satisfactory computing accuracy of the constructed model can be guaranteed. In addition, because we used a suitable optimization algorithm, the reduction in the computing efficiency of the constructed model is slight, so it can be ignored. Therefore, in this study, based on the comprehensive consideration of both the computing accuracy and the computing efficiency, a hybrid intelligent model suitable for forecasting economic data was constructed.

6.2. Applications for More Complex Economic Data Cases

Generally, the greater the amount of economic data, the harder they are to be forecasted. Therefore, to analyze the application of the new hybrid intelligent model for more complex economic data on different conditions in-depth, six data cases were used: Cisco stock data, with 6329 records; Alcoa stock data, with 2528 records; American Express stock data, with 2528 records; Disney stock data, with 2528 records; IBM stock data, with 14,087 records and daily indices of NASDAQ, with 6286 records.

The original data for the first four cases were obtained from a previous study [53], and the last two were obtained from a public website (https://finance.yahoo.com, accessed on 10 March 2022). To compare the proposed model with the previously established hybrid models, for the first four cases, the last 800 data were used for forecasting, and for the last two cases, the last 20% of the total data (2817 and 1257) were used for forecasting. For those data cases, considering the amount of data, selected according to experience, n_o was determined to be 10, and the initial parameters for the new hybrid intelligent model were selected to be the same as those in Table 7.

The parameter optimization results obtained after the computation using the new hybrid intelligent model are summarized in Table 14.

Table 14. Parameter optimization results of new hybrid intelligent model for six data cases.

Data Case	Parameters
Cisco stock data	187-316-215-10
Alcoa stock data	117-236-182-10
American Express stock	105-252-180-10
Disney stock data	123-241-162-10
IBM stock data	258-406-268-10
Daily indices of NASDAQ	194-295-221-10

To compare the proposed model with other state-of-the-art hybrid models proposed in previous studies, the same evaluation indices for computing accuracy were applied. For the first four cases, the evaluation indices used were root mean square error (RMSE) and symmetric mean absolute percent error (SMAPE) [37]. For the last two cases, the mean absolute error (MAE) and relative root mean squared error (rRMSE) were used [37,40].

Using the parameter optimization results shown in Table 14, based on the six datasets, the forecasting results obtained using the new hybrid intelligent model were obtained. The computing accuracies for the first four data cases are summarized in Table 15.

Table 15. Forecasting results for the first four data cases obtained using the new hybrid intelligent model.

Data Case	Model	RMSE (%)	SMAPE (%)
Cisco stock data	New hybrid model	1.342	1.95
	IBHA-FMNN [37]	2.521	3.715
Alcoa stock data	New hybrid model	3.578	4.864
	IBHA-FMNN [37]	7.224	10.173
American Express stock	New hybrid model	3.85	5.124
	IBHA-FMNN [37]	7.736	11.021
Disney stock data	New hybrid model	3.485	4.931
	IBHA-FMNN [37]	7.042	9.52

It must be noted that, for an accurate comparison, the results obtained using the recently proposed IBHA-FMNN model [37] are also summarized in Table 15.

As seen in Table 15, for the four complex data cases, the computing errors made by the new hybrid intelligent model are very few; that is, the computing accuracy of the new hybrid model is very high. Moreover, the computing errors made by the new hybrid intelligent model number about one-half of those made by the IBHA-FMNN model; that is, the computing accuracy of the new hybrid model is much higher than that of the previous model.

The computing accuracies for the last two data cases are summarized in Table 16.

Table 16. Forecasting results for last two data cases obtained using the new hybrid intelligent model.

Data Case	Model	MAE (%)	rRMSE (%)
IBM stock data	New hybrid model	0.92	1.25
	IBHA-FMNN [37]	1.45	1.94
	GA-BPNN [40]	1.14	1.63
Daily indices of NASDAQ	New hybrid model	0.53	0.76
	IBHA-FMNN [37]	0.85	1.13
	GA-BPNN [40]	0.72	0.99

It must be noted that, for an accurate comparison, the results obtained using the recently proposed IBHA-FMNN model [37] and those obtained using the hybrid model named GA-BPNN, also recently proposed [40], are also summarized in Table 16.

As seen in Table 16, the computing errors made by the new hybrid intelligent model for the last two data cases, which are more complex, are also very few; that is, the computing accuracy of the new hybrid model is still very high. In comparison with the two previous models, the computing errors made by the new hybrid intelligent model number much fewer than those made by IBHA-FMNN, and also number fewer than those made by GA-BPNN; that is, the new hybrid model makes the fewest computing errors. Thus, the computing accuracy of the new hybrid model is the best.

Moreover, as seen in Tables 15 and 16, for all six data cases, the computing errors made by the new hybrid intelligent model are very few; that is, regardless of the complexity of the data case, the computing accuracy of the new hybrid model is always high. Though the computing accuracy of the new hybrid model is reduced as the complexity of the data cases increases, the degree of reduction is not significant. Therefore, for different kinds of complex data cases, the forecasting performance of the new hybrid model is always excellent. In other words, for any kind of economic data, the practical application of the new hybrid intelligent model is very satisfactory.

7. Conclusions and Future Work

Generally, economic data are complex nonlinear time series. Therefore, forecasting nonlinear time series is very important in the field of economy and finance. Nowadays, there are three basic computing models (grey system model, time series analysis model, and ANN model) for forecasting economic data. To use different models suitably and effectively, the three basic models were analyzed comprehensively. Moreover, based on the analysis results, one new hybrid intelligent model was proposed for economic data forecasting. In this new hybrid model, HHO is used to select the ANN structure and to optimize the linking weights and thresholds of ANN. Then, based on four economic datasets, the performances of the three basic models and that of the proposed new hybrid model were verified comprehensively. Finally, through comparison with other hybrid models, and the application of six more complex data cases, the academic contributions and practical applications of this study were verified. From these studies, the following conclusions can be drawn.

(1) The grey system model is suitable for the exponential trend time sequence. The time series analysis model is suitable for the disordered time sequence. Moreover, the ANN model can be used for all kinds of time series.

(2) For the two studied simple data cases, the forecasting accuracy of the new hybrid intelligent model is very high, and its correlation coefficients are 0.9962 and 0.9933, respectively. For the two complex data cases, the forecasting accuracy of the new hybrid model is also high, and its correlation coefficients are 0.9897 and 0.9873, respectively. Therefore, the proposed new hybrid model is a very suitable method for forecasting any kind of economic dataset.

(3) For two similar hybrid models (HHO-BPNN and IBHA-FMNN), the computing results are similar. However, for two other similar hybrid models (HHO-ANN and PSO-ANN), the computing results are very different. When using HHO-ANN, the computed correlation coefficient is much higher than those obtained using the three other models. Its computing time is also acceptable. Moreover, the number of initial parameters for HHO-ANN is the smallest. Therefore, the performance of HHO-ANN is the best, and its application is the most convenient as well.

(4) As the complexity of the data cases increases, the computing accuracy of the new hybrid model is reduced, but the degree of reduction is not significant. Therefore, regardless of the complexity of the data cases, the computing accuracy of the new hybrid model is always high. In other words, the practical application of the new hybrid intelligent model is very satisfactory.

Finally, it must be noted that economic development is very complex and may be affected by many factors, and some influencing factors are generally random and uncertain, which may not be described by economic data series. Therefore, the proposed hybrid model, which is a data-driven model, cannot consider some complex influence factors on economic development. In fact, economic development relies on this mechanism. Therefore, how to consider the mechanism of economic development in the new hybrid intelligence model will be addressed in our next study.

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