mathematics

Editorial

Analytical Methods in Wave Scattering and Diffraction Volume I

Nikolaos L. Tsitsas

School of Informatics, Aristotle University of Thessaloniki, 54124 Thessaloniki, Greece; ntsitsas@csd.auth.gr

Citation: Tsitsas, N.L. Analytical Methods in Wave Scattering and Diffraction Volume I. *Mathematics* **2023**, *11*, 954. https://doi.org/10.3390/math11040954

Received: 2 February 2023
Accepted: 7 February 2023
Published: 13 February 2023

Boundary value problems (BVPs) pertaining to scattering and radiation by devices that support novel wave phenomena are of primary importance in applied and computational mathematics, computational physics and engineering. Modeling BVPs with analytical or semi-analytical techniques is essential to obtain solutions with controllable accuracy and in a short execution time. These solutions can be considered as significant benchmarks and starting points for efficiently optimizing the device's parameters to achieve specific near- or far-field variations. Analytical and semi-analytical techniques with application domains including single- or multiple-particle scattering, metamaterials, direct and inverse scattering by inclusions in layered media, propagation in waveguides, resonators, and analysis of periodic or complex media, have received great attention. Techniques applied for the analytical modeling span from integral equation/differential equation-based methods to the generalized separation of variables and Fourier and eigenfunction series expansions, as well as to Galerkin-type methods.

The purpose of this Special Issue was to gather contributions from experts on analytical and semi-analytical techniques with application domains including, but not limited to, the domains mentioned above. Contributions with a main emphasis on numerical methods for wave phenomena were also welcome, provided that they exploited analytical means at certain stages of the procedures employed for the derivations of the solutions.

Following a thorough peer-review procedure by at least three (different) experts in the respective field, 13 papers have been accepted for publication in this Special Issue. All the papers were of high quality and representative of the areas they covered. Hereinafter, these published papers are briefly surveyed and classified according to the ascending order of their publication dates.

The paper authored by Lucido [1] investigates the surface plasmon resonances of a monolayer graphene disk, excited by an impinging plane wave. In the paper, an analytical technique is developed based on the Helmholtz decomposition and the Galerkin method. An integral equation is obtained by imposing the impedance boundary condition on the disk surface, assuming the graphene surface conductivity is given by the Kubo formalism. The problem is equivalently formulated as a set of one-dimensional integral equations for the harmonics of the surface current density. Then, a fast-converging Fredholm second-kind matrix operator equation is obtained by selecting the eigenfunctions of the most singular part of the integral operator, reconstructing the physical behavior of the unknowns, as expansion functions in a Galerkin scheme. Surface plasmon resonance frequencies are individuated by the peaks of the total scattering and the absorption cross-section, which are expressed in closed form. It is shown that the surface plasmon resonance frequencies can be tuned by the chemical potential of graphene.

The paper co-authored by Amirkulova, Gerges, and Norris [2] presents a gradient-based optimization (GBO) method for acoustic lens design and sound localization. The developed method combines semi-analytical optimization with acoustic reciprocity. An analytical formula is derived for the gradients of the pressure at the focal point, with respect to the positions of a set of cylindrical scatterers. The GBO algorithm maximizes the sound amplification at the focal point and enhances the localization by evaluating the pressure derivatives with respect to the cylinder positions and then perturbatively optimizing the position of each cylinder in the lens, while incorporating scattering between the cylindrical scatterers. The results of the GBO of the uni- and multi-directional broadband acoustic lens designs are presented, including several performance measures for the frequency dependence and the incidence angle. The method is illustrated for sound hard and sound soft cylinders, as well as for clusters of elastic thin shells in water.

The paper co-authored by Eremin and Wriedt [3] reports a generalization of the optical theorem for the case of a penetrable particle deposited near a transparent substrate and excited by a multipole of an arbitrary order and polarization. The derivation employs Maxwell's theory, Gauss's theorem, and a special representation for the multipole excitation. It is shown that the extinction cross-section can be evaluated by the calculation of some specific derivatives of the scattered field at the position of the multipole location, in addition to some finite integrals, which account for the multipole polarization and the presence of the substrate. In addition, the paper presents some specific examples for the excitation of a particle by an electric quadrupole.

The paper co-authored by Stefanidou, Vafeas and Kariotou [4] proposes an analytical method for electromagnetic wave scattering by a highly conducting impenetrable spherical object immerged in a lossless environment. The object is excited by an arbitrarily orientated low-frequency magnetic dipole. The scattering problem is formulated according to expansions of the implicated magnetic and electric fields in terms of the positive integer powers of the wave number of the medium. The static Rayleigh zeroth-order case and the initial three dynamic terms provide an excellent approximation of the obtained solution, while the terms of higher orders can be safely neglected. To this end, Maxwell's equations can be reduced to a finite set of interrelated elliptic partial differential equations, with each one accompanied by the perfectly electric conducting boundary conditions on the sphere and the necessary limiting behavior at infinity. The proposed method is based on the introduction of a suitable spherical coordinate system and yields compact solutions for the scattered components by means of the infinite series of spherical harmonic modes.

The paper co-authored by Golub and Doroshenko [5] develops a boundary integral equation method, by employing the Hankel transform of Green's matrices, for modelling wave scattering and analysis of the eigenfrequencies of circular, partially closed interface delaminations between dissimilar media. A more general case of partially closed circular delamination is introduced using the spring boundary conditions with non-uniform spring stiffness distribution. The unknown crack opening displacement is expanded as a Fourier series, with respect to the angular coordinate and in terms of the associated Legendre polynomials of the first kind via the radial coordinate. The problem is decomposed into a system of boundary integral equations and solved using the Bubnov–Galerkin method. The boundary integral equation method is compared with the meshless method and the published works for a homogeneous space with a circular open crack. Numerical results that showcase the efficiency and the convergence of the method are presented.

The paper co-authored by Qin, Fan, Li and Lei [6] presents an analytical time integrable time–space domain (ATI-TSD) method based on the identification of the minimum time–space domain for treatment on singularities in the three-dimensional time domain boundary element method (3D TD-BEM) formulation. A direct method to solve singular integrals in the 3D TD-BEM formulation for elastodynamic problems is proposed. The wavefront singularity is analytically eliminated in ATI-TSD, while the dual singularity can be treated by the direct method using Kutt's quadrature in the identified minimum time–space domain. Three benchmark examples are reported to verify the correctness and the applicability of the direct method for solving the singular integrals in 3D TD-BEM.

The paper authored by Sihvola [7] investigates a particular class of impedance boundary conditions, including a boundary condition that produces the so-called co-circular polarization reflector (CCPR). The analysis focuses on the possibilities of manipulating the polarization of the electromagnetic wave reflected from the CCPR surface, as well as the so-called matched waves associated with it. The characteristics of CCPR and its special cases (perfectly anisotropic boundary (PAB) and soft-and-hard surface (SHS)) are compared against more classical lossless boundaries, including perfect electric, perfect magnetic, and perfect electromagnetic conductors (PEC, PMC, and PEMC).

The paper co-authored by Ben Ali, Alshammari, Trabelsi, Alsaif, Kahouli, and Elleuch [8] presents a numerical investigation of photonic quasi-periodic generalized Fibonacci (GF) (m, n) sequences in the visible spectrum. The transfer matrix method is employed to study wave propagation through the photonic structures. First, its transmittance spectrum is compared to those of periodic and ordinary Fibonacci structures. It is shown that the GF structure permits one to obtain multi-photonic band gaps (PBGs) separated by several resonance modes. The variation in the parameter m of the GF (m, 1) structure allows for the tuning of the number, the position, and the width of these bands. By changing m, specific wavelengths of the plastic and glass optical fibers can be allowed or forbidden to transmit through the structure. In contrast, the variation in n for the GF (1, n) structure hides all the PBGs and only permits the appearance of several Kiessig fringes.

The paper co-authored by Charalambopoulos, Gergidis, and Vassilopoulou [9] presents a novel stochastic method for the time-reduced inverse scattering problem, governed by the Helmholtz equation, with external connected or disconnected obstacles supporting the Dirichlet-type boundary conditions. A series of novel stochastic representations of the scattered field are constructed, which constitute the probabilistic analogue of the deterministic integral representations that involve the famous Green's functions. Their advantage lies in their intrinsic probabilistic nature, enabling the solution of the direct and inverse scattering problem in the realm of local methods, which are strongly preferable in comparison with the traditional global methods. This locality reflects the ability to handle the scattered field only in small, bounded portions of the medium by monitoring suitable stochastic processes, which are confined in narrow sub-regions where data are available. Two different inverse scattering schemes are proposed facing reconstruction from the far field and near field data, respectively. The crucial characteristic of the inversion is that the reconstruction is fulfilled through stochastic experiments, taking place in the interior part of the conical regions whose base belong to the data region, while their vertices appropriately detect the supporting surfaces of the sought scatterers.

The paper co-authored by Tognolatti, Ponti, Santarsiero, and Schettini [10] presents an efficient Matlab computation of a 3D electromagnetic scattering problem, in which a plane wave impinges with a generic inclination onto a conducting ellipsoid of revolution. This solid is obtained by the rotation of an ellipse around one of its axes, which is also known as a spheroid. A fast code is developed to solve the electromagnetic scattering problem, using spheroidal vector wave functions, which are special functions used to describe physical problems in which a prolate or oblate spheroidal reference system is considered. Numerical results are presented both for TE and TM polarization of the incident wave and are validated by a comparison with the results obtained by a commercial electromagnetic simulator.

The paper co-authored by Gu and Cui [11] analyzes the electromagnetic scattering of a meta-surface, formed by a honeycomb substrate or periodically arranged metamaterials. A statistic estimation averaging method (SEAm) is developed through the transferred conditional structural probability density functions (PDFs) to realize the structural determinations by homogenization. The parameters estimated by SEAm are validated by comparing these with the measured results of a honeycomb structure. The method can be extended to estimate the parameters of an equivalent surface, including random scattering from information metamaterials. This study also gives a preliminary proposition on the state estimation method of information metasurfaces, which can interpret the modulation effect of wireless channels caused by the inhomogeneity of antenna wavefronts by statistical estimation average information entropy.

The paper co-authored by Vinogradova and Smith [12] develops a rigorous method for the accurate evaluation of the electromagnetic interaction between a thin metallic rod and a two-dimensional slotted cavity. Using the method of analytical regularization, the problem is transformed to well-conditioned coupled infinite systems of linear algebraic equations for the Fourier coefficients in the expansions of induced surface currents on the rod and slotted cavity. When truncated to finite size, their solutions exhibit fast convergence to the exact solution, as the order is increased. This feature makes it possible to investigate the spectral and scattering characteristics of the coupled cavity and rod for the desired level of accuracy. The complex eigenvalues for a slotted cavity in the presence of a thin rod and the dependence upon their relative location are investigated, particularly to find where there is significant or optimal enhancement of the Q factor. This can be useful in the design of advanced slot antennas and slotted waveguides.

Finally, the paper co-authored by Mastorakis, Papakanellos, Anastassiu, and Tsitsas [13] presents a hybrid of the method of auxiliary sources (MAS) with the fast multipole method (FMM), which provides a strategy for reducing the computational cost and for solving large-scale scattering problems without notable accuracy loss (and in a reasonable time). The hybrid MAS-FMM scheme is applied to the problem of electromagnetic scattering from an arbitrarily large array of lossless/lossy dielectric cylinders. Their numerical results are presented to verify the MAS and MAS-FMM schemes, as well as to illuminate the improvements that stemmed from the proposed hybridization (especially the improvements regarding the associated complexity and computational cost). A few concluding remarks are included, along with a list of possible future extensions.

As the Guest Editor of this Special Issue, I express my sincere gratitude to the 37 authors who contributed their research works. I am also grateful to a great number of dedicated reviewers, whose valuable comments and suggestions helped to improve the quality of the submitted papers, as well as to the MDPI editorial staff, whose help during the entire process of realizing this Special Issue was crucial.

Conflicts of Interest: The author declares no conflict of interest.

References

1. Lucido, M. Electromagnetic Scattering from a Graphene Disk: Helmholtz-Galerkin Technique and Surface Plasmon Resonances. *Mathematics* **2021**, *9*, 1429. [CrossRef]
2. Amirkulova, F.; Gerges, S.; Norris, A. Sound Localization through Multi-Scattering and Gradient-Based Optimization. *Mathematics* **2021**, *9*, 2862. [CrossRef]
3. Eremin, Y.A.; Wriedt, T. Generalization of the Optical Theorem to an Arbitrary Multipole Excitation of a Particle near a Transparent Substrate. *Mathematics* **2021**, *9*, 3244. [CrossRef]
4. Stefanidou, E.; Vafeas, P.; Kariotou, F. An Analytical Method of Electromagnetic Wave Scattering by a Highly Conductive Sphere in a Lossless Medium with Low-Frequency Dipolar Excitation. *Mathematics* **2021**, *9*, 3290. [CrossRef]
5. Golub, M.V.; Doroshenko, O.V. Analysis of Eigenfrequencies of a Circular Interface Delamination in Elastic Media Based on the Boundary Integral Equation Method. *Mathematics* **2022**, *10*, 38. [CrossRef]
6. Qin, X.; Fan, Y.; Li, H.; Lei, W. A Direct Method for Solving Singular Integrals in Three-Dimensional Time-Domain Boundary Element Method for Elastodynamics. *Mathematics* **2022**, *10*, 286. [CrossRef]
7. Sihvola, A. Co-Circular Polarization Reflector Revisited: Reflection Properties, Polarization Transformations, and Matched Waves. *Mathematics* **2022**, *10*, 641. [CrossRef]
8. Ben Ali, N.; Alshammari, S.; Trabelsi, Y.; Alsaif, H.; Kahouli, O.; Elleuch, Z. Tunable Multi-Band-Stop Filters Using Generalized Fibonacci Photonic Crystals for Optical Communication Applications. *Mathematics* **2022**, *10*, 1240. [CrossRef]
9. Charalambopoulos, A.; Gergidis, L.; Vassilopoulou, E. A Conditioned Probabilistic Method for the Solution of the Inverse Acoustic Scattering Problem. *Mathematics* **2022**, *10*, 1383. [CrossRef]
10. Tognolatti, L.; Ponti, C.; Santarsiero, M.; Schettini, G. An Efficient Computational Technique for the Electromagnetic Scattering by Prolate Spheroids. *Mathematics* **2022**, *10*, 1761. [CrossRef]
11. Gu, T.-T.; Cui, T.-J. The Statistical Estimation Averaging Method to Express the Effective Electromagnetic Parameters over a Planar Information Meta-Surface. *Mathematics* **2022**, *10*, 2589. [CrossRef]

12. Vinogradova, E.D.; Smith, P.D. *Q* Factor Enhancement of Open 2D Resonators by Optimal Placement of a Thin Metallic Rod in Front of the Longitudinal Slot. *Mathematics* **2022**, *10*, 2774. [CrossRef]
13. Mastorakis, E.; Papakanellos, P.J.; Anastassiu, H.T.; Tsitsas, N.L. Analysis of Electromagnetic Scattering from Large Arrays of Cylinders via a Hybrid of the Method of Auxiliary Sources (MAS) with the Fast Multipole Method (FMM). *Mathematics* **2022**, *10*, 3211. [CrossRef]