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Editorial

Advanced Numerical Methods in Computational Solid Mechanics

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Efficient numerical solving of nonlinear solid mechanics problems is still a challenging issue which concerns various fields: nonlinear behavior, micromechanics, contact mechanics, damage, crack propagation, rupture, etc. Numerical methods dedicated to such topics have been developed over many decades, but many fundamental and important challenges remain, such as multiscale methods which bridge different scales in time and space, efficient reduced-order models for variational inequalities or fully scalable nonlinear solvers. This Special Issue is dedicated to contributions which introduce or adapt advanced numerical methods for computational mechanics. It contains ten high-quality articles that were accepted after a careful reviewing process.

Concerning micromechanics, Belgrand et al. [1] investigated the effects of inclusion proximity in dilute matrix–inclusion composites. Using RVE simulations, they show that classical analytical estimators of the Mori–Tanaka type fail to predict the effective properties, particularly the first- and second-order moments per phase, when the inclusions become close. For spherical inclusions, deviations in the effective and phase properties, which can reach more than 10%, appear for proximity parameters lower than the radius of the spheres.

Papers [2–4] are devoted to contact problems. In [2], Wilayn and Heβ focus on functional gradient materials (cf. additive manufacturing). For these materials, which possess high stiffness variability, solving contact problems, including tangential fretting, becomes much more difficult. The authors propose a fictitious equivalent contact model in the mathematical space of the Abel transform. This model considerably simplifies the solution procedure without being an approximation. Furthermore, a closed-form analytical solution for the dissipated energy is derived which is applicable to a wide class of axisymmetric shapes and elastic inhomogeneities. Through this formulation, the well-known solution of Mindlin et al. is recovered as a special case.

The work of Le Berre et al. [3] is devoted to the reduced-order modeling of contact mechanics problems treated by Lagrange multipliers. The main issue concerns the compression of dual solutions which are highly nonlinear and must respect a non-negativity condition. To circumvent this issue, a hyper-reduction approach based on a reduced integration domain (RID) is considered. In order to obtain accurate dual forces and to respect the inf-sup condition, a greedy algorithm based on the condition number of the projected contact rigidity matrix is introduced for the primal basis enrichment. For large parametric variation of the contact zone, the authors proposed to cluster the parametric space first. The non-linear solution is then built through piece-wise low-rank (linear) approximations.

In the paper [4] by Gomez et al., the authors develop a discontinuous Galerkin (DG) numerical scheme for wave propagation in elastic solids with frictional contact interfaces. The DG method in space was chosen for its good parallelization properties. As the frictional interfaces (micro-cracks) implying non-linear and non-smooth variational problems concern only a limited number of degrees of freedom, the additional calculation cost remains low compared to the standard DG method. Emphasis is placed on contact conditions at the

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crack surface, including provisions for crack opening or closing and sliding with Coulomb friction. The proposed numerical algorithm is based on the leapfrog time scheme and an augmented Lagrangian algorithm to solve the frictional contact non-linear problems.

Also concerning parallel solving, but in terms of classical elliptic problems, Negrello et al. [5] propose a new nonlinear version of preconditioning for FETI-like (Finite Element Tearing and Interconnecting) nonlinear solvers. In a parallel framework, this preconditioning allows the communication between processors to be reduced. The extra cost of the implementation of this new strategy remains limited since classically preconditioned linear FETI problems are recovered at each Newton step.

Papers [6–8] are devoted to beam structures. The paper by Musa et al. [6] concerns the study of the bending of a completely free beam resting on a tensionless foundation. The authors have developed a simple iterative procedure based on the Ritz method for the analysis of a problem with distributed load. The relevance of the method is illustrated on symmetric and asymmetric problems. A parametric study then gives interesting mechanical results.

In [7], Saadatmorad et al. introduce an enhanced discrete wavelet transform for structural health monitoring of beams. This approach encompasses the challenging issue of finding a mother wavelet and vanishing moments in structural crack identification. It is based on a combination of statistical characteristics of vibrational mode shapes (thanks to a Pearson correlation) of the beam structure and their discrete wavelet transforms. The damage detection's accuracy is then greatly improved.

In [8], Ge et al. propose the use of the interpolation matrix method (IMM) to solve the critical buckling load of Timoshenko beams with axial functional gradient. After applying the Timoshenko beams theory, the deflection and rotation functions are decoupled and transformed into an eigenvalue problem, which is simplified thanks to the IMM. Finally, a matrix QR decomposition can be performed to obtain the critical buckling load corresponding deflection function.

The contribution of Sahla et al. [9] concerns the field of stress-based layerwise models for multilayered plates. The authors propose the use of a mixed finite-element formulation instead of the classical primal (displacement) approach. Tensile continuity between the elements is treated by introducing an additional Lagrange multiplier defined on the facets of the element. Thanks to a static condensation at the element level, the final system to be solved is small. The proposed mixed strategy is shown to be free from shear-locking in the thin plate limit and more accurate than a displacement approach for the same number of degrees of freedom.

Finally, Verelli et al. [10] explore the modeling of the butterfly swimming through a harmonically self-similar temporal partition. This partition is based on the generalized Fibonacci sequence and the golden ratio. The authors proposed quantitative indices to assess a (hidden) time-harmonised self-similar structure. This work also includes the validation of the model on experimental data from national and international swimmers.

The papers published in this Special Issue constitute a further advance in the field of numerical methods for computational solid mechanics. They push back some limits of the current methods and open the way to the methods of tomorrow.

Conflicts of Interest: The authors declare no conflict of interest.

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