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Distributed Traffic Signal Optimization at V2X Intersections

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Abstract: This paper presents our research on a traffic signal control system (TSCS) at V2X intersections. The overall objective of the study is to create an implementable TSCS. The specific objective of this paper is to investigate a distributed system towards implementation. The objective function of minimizing queue delay is formulated as the integral of queue lengths. The discrete queueing estimation is mixed with macro and micro traffic flow models. The novel proposed architecture alleviates the communication network bandwidth constraint by processing BSMs and computing queue lengths at the local intersection. In addition, a two-stage distributed system is designed to optimize offsets, splits, and cycle length simultaneously and in real time. The paper advances TSCS theories by contributing a novel analytic formulation of delay functions and their first degree of derivatives for a two-stage optimization model. The open-source traffic simulation engine Enhanced Transportation Flow Open-Source Microscopic Model (ETFOMM version 1.2) was selected as a simulation environment to develop, debug, and evaluate the models and the system. The control delay of the major direction, minor direction, and the total network were collected to assess the system performance. Compared with the optimized TSCS timing plan by the Virginia Department of Transportation, the system generated a 21% control delay reduction in the major direction and a 7% control delay reduction in the minor direction at just a 10% penetration rate of connected vehicles. Finally, the proposed distributed and centralized systems present similar performances in the case study.

Keywords: highway traffic operations; queue length model; vehicle detection; signal optimal control; basic safety messages

MSC: 90-10

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1. Introduction

Connected vehicle (CV) technology, or Vehicle-to-Everything (V2X) Communications, could provide mobility, safety, and environmental benefits to arterial traffic operations [1–3]. The National Highway Traffic Safety Administration (NHTSA) performed a detailed benefit study in 2016 [4]. A few review papers [5–7] have captured the dynamics of CV and its benefits after hundreds of papers were researched and summarized. Currently, connected vehicle technology is still in the simulation and/or experimental stages, and the full implementation of this technology (i.e., 100% penetration rates of connected vehicles) is still decades from completion.

The majority of traffic signals in the US are operated as actuated control systems and require a labor-intensive and costly traffic engineering study to periodically "re-time" traffic signal plans. The advanced generation of Adaptive Traffic Control Systems (ATCS, discussed in Section 2.1) has been developed to overcome the retiming process for a few decades. It has had limited success in market deployment. V2X brings a major advantage for CV-based ATCS over traditional ATCS since V2X alleviates the dedicated advanced sensor and sensor communication costs. V2X infrastructure needs to be deployed by state Departments of Transportation (DOT) and local agencies. However, budgetary, institutional, technological, and training constraints have cast uncertainty over deployment timelines. To help state DOTs and local agencies, the objective of the research was to create Traffic Signal Control System (TSCS) models and implementation algorithms for the early deployment of V2X (10% penetration rate, for example) and examine its mobility, safety, fuel, and emissions benefits [1–3].

The specific objective of the research presented in this paper was to investigate a distributed system towards implementation. This paper presents the core of the system and the novel models and algorithms. In the prior reports and Transportation Research Board (TRB) papers [1–3], it was assumed that Basic Safety Messages (BSMs) from each Roadside Unit (RSU) would be forwarded to a central computer for storing and processing without considering the communication network bandwidth and processing computing power. This might be acceptable for a rural community where the number of RSUs is of moderate quantity (under one hundred units, for example).

We are motivated to overcome the potential limitations in BSM communication and processing demand on centralized systems. In a future scenario with high percentages of CVs, like a metropolitan area, BSMs are broadcast to the RSUs at each intersection and then forwarded to the computer installed with the proposed systems. Let's assume congested intersections are processing 10,000 vehicles per hour, the RSU communication range is 1500 ft, the average spacing between vehicles is 50 ft, and vehicles communicate with RSUs 10 times per second at each intersection. This would result in having 833 BSMs per second. With 320 Bytes/BSM [8], 268,000 Bytes/s or about 262 MB per second per intersection will be expected. In an extreme case with 1000 such intersections, the demands for communication bandwidth and computing power at the central computer system could be as high as 250 GB/s. Such high communication and processing demands might overburden and crash the centralized system. To make the proposed algorithm work under the above scenario, a distributed system should be explored to see if there is still a significant benefit. [2]

IBM defines distributed computing as "A distributed computer system consisting of multiple software components on multiple computers but run as a single system. . . .Distributed computing aims to make such a network work as a single computer" [9]. IBM's definition of "Distributed" above is applied in this research. Most existing controllers in the US operate on (semi-)actuated and fixed time mode with time-based coordination between selected upstream and selected down-street intersections. The TSCS in operations and the market is a locally decentralized computing system, and it is not considered a distributed system since there is no single centralized computing system to run all controllers; each control system is run independently. Several advanced ATCS are distributed systems and will be discussed in Section 2.1 of the next section. One must notice that only one percentage of US TSCS are ATCS [10].

This paper bridges all gaps in TSCS at V2X intersections identified in a research review [11] published just in 2023. Our case studies confirmed that the implemented optimized control saves 40% of delays at a low penetration rate of 10%. (gap 1). We used a mesoscopic traffic model to advance vehicles within links and forward to the next links with the V2X information from BSMs and traditional infrastructure-based sensors for non-V2X vehicles. (gap 2 and 3). In addition, we simultaneously and continuously optimize cycle length, split, and offset in real-time in a rolling horizon time frame. On top of exceeding those challenges, we pioneered the distributed architecture necessary to alleviate BSM communication and processing challenges to TSCS. The novel analytic formula of the first degree of derivatives of queue delays lays the foundations for any researchers seeking a gradient-based heuristic optimization to use the queue delay model as objective functions. These implemented models, algorithms, and experiments prove the equivalent benefits of such systems to centralized systems. Finally, the TSCS proposed in this paper fits all features of ATCS; for simplicity, we use the term TSCS throughout the paper when referring to our proposed system.

We present a comprehensive literature review in Section 2 and introduce the objectives and approaches in Section 3. We discuss the methodologies and distributed models in detail in Section 4. After the present case studies in Section 5, we summarize our findings in the conclusion and the future directions in Section 6.

2. Literature Review

While research papers regarding TSCS at large are vast, a few review papers [12–15] have summarized a comprehensive spectrum of architectures, models, algorithms, and case studies. In this research, we have decided to narrow the literature research to those works related to the proposed system and those comparable to the proposed research. After briefing and discussing the existing state of practice in TSCS in Section 2.1, we present some unique research papers on TSCS with CV data in Section 2.2. In Section 2.3, we also discovered several papers that apply distributed architecture/computing in TSCS with/without CV data. We finally summarize our findings in the Section 2.4.

2.1. Traditional Distributed Traffic Signal Systems in Operations

In a report produced by the Florida Department of Transportation (FDOT) in 2016 [16], a research group summarized Adaptive Traffic Control Systems (ATCS). Five of the ATCSs were distributed systems, including Real-Time Hierarchical Optimized Distributed and Effective System (RHODES) [17], Optimization Policies for Adaptive Control (OPAC), Adaptive Control ACS Lite [18], Kadence [19], and InSync. RHODES estimated queue lengths and arrivals locally; the queue lengths and arrivals were then transferred from the local controllers to the central computer. RHODES used DSL communication via a modem and VME (Versa Module Europa, an American National Standards Institute (ANSI) and the IEEE standard), the speed of which could reach 320 MB/s to transport the signal messages (containing the signal event and signal time plan information) and the detector messages (containing detector event information). OPAC used local controllers to optimize timing plans without a fixed cycle length. Local controllers optimize green phases and the cycle length of each coordinated intersection, while central computers optimize offsets of coordinated intersections. OPAC used regular digital subscriber line (DSL) communication via a modem. Adaptive Control System (ACS) Lite used local programs to predict traffic status reports (i.e., volume, queue, speed, etc.). The National Transportation Communications for Intelligent Transportation Systems Protocol (NTCIP) protocol was used for communication. The traffic status message (i.e., traffic signal timing plan and traffic volume) would be transferred between local controllers, master controllers, and a centralized computer. ACS Lite and Kadence used the NTCIP protocol over dial-up channel service. The two types of NTCIP communications were center-to-field (C2F) and center-to-center (C2C). Kadence [18], based on ACS Lite, adjusted offset in local controllers. Cycle length and phase lengths were changed at the central level.

2.2. Traffic Signal Control Systems at V2X Intersections

We focus our reviews on TSCS optimizations using data from CV. In the real world, at V2X-equipped intersections, TSCSs can receive V2X data as BSM. It is noticed that in most research papers reviewed, converting BSMs to the data the system could accept is ignored. Summaries of a review research paper about adaptive signal control and coordination [13] have been added here for the reader's convenience. In the review, the authors listed five types of objective functions used in 22 ACSs: delay, queue length, waiting time, number of stops, and travel time (which is correlated to delay), among which there are thirteen delay functions and five queue length functions. Signal coordination was listed separately from ACS control parameters. There are nine papers considering coordinates, two overlapping with ACS. Authors detailed uses of online/offline CV data, rolling horizontal approaches, optimization algorithms, application functions, and the benefits in case of studies for 20 research papers. Finally, three challenges and the future directions for overcoming those challenges are identified. First, most of the existing CV-based ACS control methods do not address the CV penetration rate limitations in the foreseeable future. They identified nine studies with minimum penetration rates and found no methods that solved the low penetration rate of 10% and ultra-low rate of 5% in real-time (gap 1). Second, despite considerable improvements in traffic models in the last several decades, efficient and accurate models need to be improved (gap 2). Finally, predictive control strategies for CV and non-CV need further studies (gap 3). An earlier review [20] in 2017 and a recent review [21] in 2023 of these topics are also helpful, especially for those needing a starting point to understand the problem comprehensively. The 2017 review [20] found that only three out of twenty-six papers considered non-CV.

In FHWA reports and prior studies [1–3], several signal control strategies at V2X intersections were presented. Vehicle speeds and locations were extracted from BSMs to incorporate with upstream and stop-bar detectors as control system input. The control strategies adjusted or optimized offsets, splits, and cycle length. The simulation-based case study considered five different penetration rates of CVs (10–70%). Two case studies in various locations show that the strategies could generate more than a 40% reduction in control delay, a 25% reduction in fuel consumption and emissions on environment-related impacts, and about a 45% conflict reduction in safety, with a mere CV penetration rate of 10%. Feng et al. [22] presented an ATCS optimization model in a CV environment. Two-level optimization focused on queue length minimization (lower level) and total delay (upper level) minimization. In addition, estimating a non-CV queue is an integral part of the model. Li et al. noticed that Feng's models require a 25% minimum penetration rate to perform [11]. Zhang et al. [23] presented optimal control and coordination of connected and automated vehicles. The research focused on urban traffic intersections and minimized fuel consumption. In addition to those papers, several papers have attempted to optimize the TSCS timing plan and vehicle arrivals (referred to as trajectory planning). For example, Wu [24] formulated the control as problems of optimizing green time and sliding the green windows by adjusting the CVs' speed. Astarita et al. [25] presented an interesting paper using "connected vehicle" (any vehicle that reports its GPS position to its system) as input (no traditional loop detection) to Floating Car Data Adaptive Traffic Signals, which applied simple rules to adjust green time in fixed cycle intersections.

A few research studies are related to distributed TSCS with CV data. Islam [26,27] presented their traffic optimization research based on the cell transmission model (CTM) to distribute the computing powers at each intersection with the consideration of stop bar detector data and CV data. The models and algorithms were evaluated in VISSIM through the Component Object Model (COM) interface developed by Microsoft in the 1990s. Under 100% of the CV penetration rate, Agafonov et al. [28] proposed a cooperative control of vehicle trajectories and TSCS phases. The cooperative control combined a predictive control algorithm and a trajectory construction algorithm. They reported more significant savings in stop delays than other performance indexes like travel time and fuel consumption.

Artificial intelligence and deep learning have proven excellent approaches to the same problem in this research in recent decades, a few of which applied distributed intelligence as well. Li's review listed several research papers [11]. There is also a review of reinforcement-based TSCS [29]. For example, Mo et al.'s [30], Maadi et al. [31], and Chen's [32] research papers are well received. Due to the limitation of our scope of work and the word limitations, we did not conduct a full review of the approaches. Additionally, information about queue lengths, delays, and other outstanding traffic conditions can be obtained by combining data from a BSM and the sensory interpretation of an artificial backpropagation neural network (NN). This same ideology is proposed by Gao et al. [33], referred to as a queue length sensing model. Again, the BSM contributes information that would otherwise be unknown from the traditional loop and video detectors—things such as a vehicle's acceleration, position in the queue, speed, and others—and the NN is sophisticated enough to handle complex logic operations [33]. Through building a road network in VISSIM and verifying the results of the two sub-models in MATLAB, the researchers found that their queue length sensing model had an accuracy of 95% for a high penetration rate (70%) and an accuracy of 85% for a low penetration rate (10%). Additionally, the results were most favorable for mixed traffic conditions (i.e., saturated and under-saturated).

2.3. Distributed Traffic Signal Control Systems

Zhang et al. [34] developed models and algorithms in distributed cloud nodes that replicated intersection computing powers. The architecture enabled parallel processing of the data and distributed the Genetic Algorithms. Mehrabipour et al. [35] modeled the TSCS at the network level as Mixed Integer Linear Programming (MILP), distributing algorithms to the intersection level for solutions. Ahmed and Easa [36] addressed a real-time distributed signal control system. The system included "purely" distributed control logic, whereas a distributed system that is non-purely distributed requires neighboring controllers' information and needs a centralized computing system. In addition, a person-based hypothesis aimed to enhance area-wide control without centralized infrastructure (i.e., the system individually predicts queue congestion at local controllers). The local controller's function is to set up each phase's green time according to queue lengths.

In a distributed system such as the ones mentioned above, the TSCS must maintain real-time information for every approach to an intersection. The TSCS will be required to control the signal timings for some indefinite period after they are installed and must then adapt accordingly and efficiently as changes in traffic volume, as well as other unforeseeable events, occur. This ever-growing problem is often called the infinite horizon distributed control problem. To alleviate such a problem, Srinivasan et al. [37] produced research that may serve as a solution for the controller's need to make continuous inferences and improvements when faced with evolving traffic conditions. This research proposes a multiagent system based on computational intelligence and improves upon an earlier proposed simultaneous perturbation stochastic approximation neural network (SPSA-NN) system [37]. Simulation testing of the hybrid NN-based model was conducted within the PARAMICS framework. The simulated traffic network comprised 25 signalized intersections [37]. During the six-hour simulation, vehicle speeds recovered quickly at the conclusion of the second peak period, utilizing the hybrid NN-based multiagent system within its traffic network. The hybrid system reduced network-wide mean delay by 78% and stoppage time by 85% [37].

A plethora of research on distributed TSCSs in back-pressure modeling originated from research groups, mainly at the University of Minnesota. Several research papers also apply the Cell Transmit Model using distributed computing resources. Those models are generally macroscopic and are different from our approaches. The readers may refer to those papers for further information [34,38–44].

2.4. Summaries

Our literature review indicated research papers on TSCS at V2X intersections have been widely performed and have dramatically improved the performance of ACSs. We also identified significant gaps in research and implementation of research. It is noticed that in most research papers reviewed, converting BSMs to the data the system could accept is ignored. More importantly, BSM's processing power and communication bandwidth have not been discussed. Most of the existing CV-based ACS control methods do not address the low CV penetration rate limitations in the foreseeable future. In reality, a field study performed by Purdue University measured less than 5% connected vehicle penetration rate in 2021 [45]. Efficient and accurate models need to be improved. Finally, predictive control strategies to accommodate CV and non-CV need further studies.

3. Objectives and Approaches

3.1. Objectives

The objectives of this research paper were (1) to design a distributed TSCS, adding new models and algorithms and improve the models and algorithms in prior studies that fit the distributed system, (2) to implement the design, models, and algorithms, and (3) to perform a case study to compare the mobility benefits between distributed and centralized systems and (4) to improve the performance of distributed systems.

3.2. The Distributed System Approach

The proposed distributed system advances the state of the TSCS. NTCIP compliance and a central computer are the requirements for the system. The RSU and fiber network are a part of future V2X infrastructures. The hybrid Macro/Micro traffic model, or Intersection Queue Estimation Program (IQEP) [46], provides future real-time queue lengths of all approaches at each intersection.

For IQEP [46], a low-cost client computer might need to be added at the intersection if the computing resource in the TSCS is not powerful enough or IQEP is not allowed to be installed on the controller inside the cabinet. A computer at the data center performs TSCS optimization: the computer receives predicated queue lengths, traffic signal indications, phasing plans, etc., from each intersection in real time. The computer pushes the optimized TSCS timing plan back to the intersections. In a closed-loop system, updating the timing plan could be accomplished by the master controller. The feasibility of such architecture has been proven in a FHWA report [3].

In the reviewed traditional traffic signal system with V2X discussed in Section 2.2, vehicle trajectory is used as input to the objective functions in optimization algorithms at a centralized computer. BSMs, the only native data from V2X, are only available at the intersections. The computing resources and communications bandwidth are generally ignored. In our proposed distributed approaches, BSM is processed at the intersection level and objective function (delays), and the first degree of objective function (the queue length) is obtained through processing BSMs at the intersection level. The aggregated data, delay, and queue length are transmitted from the intersection to the central computer. There is no need to transmit the BSM to a central computer and process BSMs from all intersections at the central computer.

The literature review indicated a distributed system of TSCS usually distributed some sub-tasks to local computing powers (controllers). For example, the field implemented, tested, and deployed RHODES uses a local-level process to predict queues and arrivals [17]. We followed a similar distributed architecture as used in queue length prediction. With this architecture, the software program developed in this research optimizes offsets, splits, and cycle lengths of intersections sequentially within the central computer (Figure 1).

If IQEP [46] estimates queue lengths in major and minor directions once a cycle, the projection horizon is at least a cycle. To increase the accuracy of queue length estimation within a cycle, the prediction of queue lengths of major/minor direction is updated twice a cycle. When major or minor directions are shorter links, or RSU's communication range cannot cover the entire link length, the queue length prediction may not contain all vehicles that need to be adequately accounted for. In this case, the vehicles in an upstream link must be added.

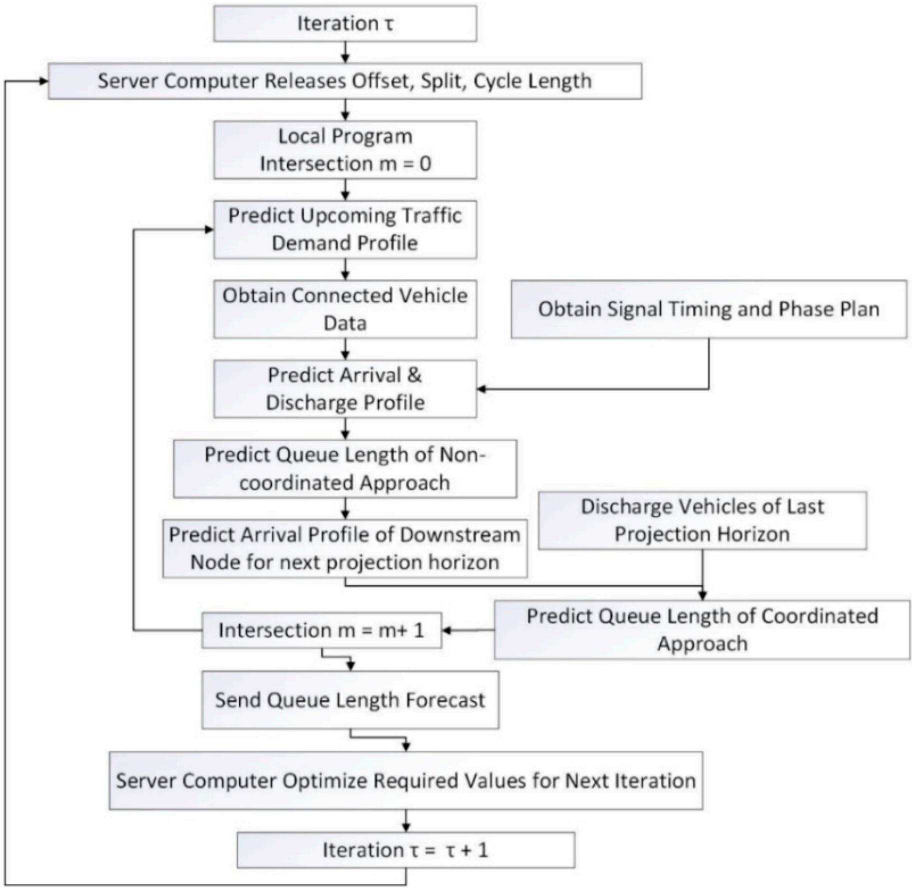

Figure 1. Algorithm logic of distributed TSCS.

4. Methodologies and Models

BSM and detector data are collected through simulation APIs with ETFOMM version 1.2 (ETFOMM is referred in this paper) [47] in our case studies. There are ETFOMM functions to convert BSM to vehicle trajectories [47]. Those data are fed into the IQEP model described in Section 4.1. In the world, BSMs are accessed through RSU, and detector information is obtained through traffic signal controllers via NTCIP. The queue length is then transmitted to a computer for optimization. The queue length from 4.1 feeds into the objective function and its first degree of derivatives is described in Section 4.2.

The objective function and the queue length are dynamically updated through the BSMs and detector information in real-time traffic. The optimization utilized a rolling horizon to optimize the signal timing plan consistently according to the traffic dynamics. Once the optimization is completed, the signal timing plan, including cycle length, offset, and phase lengths, is transmitted back to the computer within the traffic control cabinet, where the signal timing plan is changed through NTCIP. The performance of the proposed system is evaluated in terms of control delays generated by ETFOMM.

According to the Highway Capacity Manual, control delay is the measure of effectiveness at signalized intersections. Control delay best measures traffic flow efficiency and reduction in congestion at the intersection level. Queue delay, adopted as objective function, accounts for about 80% of control delays.

4.1. Forecast the Number of Vehicles in the Queue

The IQEP is based on traffic models discussed in this subsection. CV data and traditional loop detector data as part of actuated controllers are used for queue length forecasting. BSMs from CVs contain information such as speed and vehicle trajectories. The timestamps of vehicles arriving at an upstream detector and departing from a stop bar are collected from simulated detectors. The details of the queue length forecast are in the previous research [3,46].

4.2. The Objective Function and Its First Degree of Deviations

Table 1 lists the variables and parameters in the formula developed in the proposed models and algorithms. The derivative of queue delay with respect to decision variables was formulated to optimize control decision variables in Equation (2) (phase length, offsets, and splits). When traffic propagates from one intersection to the next intersection, the queue lengths during the projected time can be estimated. The total queue delay of all phases at all intersections is summarized as

$$\mathrm{F} = \mathrm{f}(X) = \min \sum_{m=1}^{M} \sum_{p=1}^{P} s_{m,p} = \sum_{m=1}^{M} \sum_{p=1}^{P} \int_{0}^{C_m} q_{m,p,t} d\lambda_t \tag{1}$$

where

$$X = \left(C_1, \ldots C_m\ ,\ O_1 \ldots O_m\ , g_{1,1} \ldots g_{1,P}, \ldots, g_{m,1} \ldots g_{m,P}\ \right)^T \tag{2}$$

Table 1. Set subscript parameters and variables used in the formulation.

m	The index of coordinated intersections (m = 1, 2, 3, . . ., and M)
M	Total number of coordinated intersections in a signalized arterial
d	Approach of each intersection (d = 1, 2, 3, . . ., and D)
D	Total number of approaches for an intersection
$p(, p_{(c)}, p_{(n)})$	The index of phases (on coordination directions and non-coordination directions) for an intersection (p = 1, 2, 3, . . ., and P)
$P(, P_{(c)}, P_{(n)})$	Total numbers of phases (on coordination directions and non-coordination directions) for an intersection
Δ	Time interval to calculate queue delays (1s is used)
t	The t'th time interval in the projection horizon (t = 1, 2, 3, . . . and T)
T	Total number of time intervals in the projection horizon
λ_t	Time since the beginning of the projection horizon $\lambda_t = \Delta \times t$
$F = f(X)$	The total queue delay function within the projection horizon (one cycle)
X	Traffic signal control variables, including cycle length, offset, and phase green time
$s_{m,p}$	The total queue delay of phase p at the m'th intersection within the projection horizon (one cycle)
$q_{m,d,t}$	The number of vehicles in the queue of approach d at intersection m at time interval t
O_m	Offset of intersection m
$R_{m,p}$	The duration of red indications before green phase p at intersection m (seconds)
$g_{m,p}$	The duration of green time of phase p at intersection m (seconds)
C_m	The cycle length of coordinated intersections
l_m	The total lost time of intersection m due to all red and startup loss time (seconds) for one cycle.
$\omega_{m,p}$	Vehicle movement of phase p at intersection m within projection horizon (%) (Turning Percentage)
$v_{m,d,t}$	Number of arrival vehicles joining the queue at intersection m at projection horizon t (vehs)
$\alpha_{m,d(n),t}$	Number of vehicles in the initial queue region at intersection m that remains in the queue at a time interval t (vehs, in a non-coordinated phase)
$\beta_{m,d(n),t}$	Number of vehicles in queue formulation region at intersection m at a time interval t (vehs, in a non-coordinated phase)
$\gamma_{1,m,d(n),t}$	Number of vehicles in progression formulation region one at intersection m at a time interval t (vehs, in a non-coordinated phase)

Table 1. *Cont.*

$\gamma_{2,m,d(n),t}$	Number of vehicles in progression formulation region two at intersection m at time interval t (vehs, in a non-coordinated phase)
$\eta_{m,p,t}$	Number of discharge vehicles in phase p at intersection m at projection horizon t (vehs)
$\eta_{m,d,t}$	Number of discharge vehicles in approach d at intersection m at projection horizon t (vehs)
$h_{m,p,t}$	Discharge headway of vehicle in phase p at intersection m at projection horizon t (vehs/s)
$h_{m,p,i}$	Any discharge headway of vehicle in phase p at intersection m at time i (vehs/s) where $i \leq t$
r	r'th iteration in seeking optimal cycle length, offset, and green time

The queue length is determined by arrival vehicles and discharge vehicles. CV and upstream detector data determine the number of arrival vehicles in each phase. Stop-bar detectors and CV data determine the discharge headway $h_{m,p,i}$. Then queue delay $s_{m,p}$ is the integral of queue lengths $q_{m,p,t}$ within one cycle, as shown in Equations (3) and (4). The terms at intersection m at turning movements corresponding to phase p are detailed. Equations (5)–(7) are the first-degree derivative of the delay objective function with respect to control variables of cycle length, offsets, and green splits. Equations (8)–(10) are the queue length formulation, the details of which can be found in Dr. Lei Zhang's dissertation [48].

Equations (12)–(19) are the constraints. Equation (11) ensures all cycle lengths in the corridor are the same (common cycle length). Equation (12) keeps the ring/barrier structure maintained. Equations (13) and (16) ensure the timing plan in each ring is enforced as cycle length. Equations (15)–(17) make sure the number of vehicles is positive. Equation (18) warrants the offset is less than the cycle length.

$$s_{m,p} = \int_0^{R_{m,p}} (\omega_{m,p} \times v_{m,d,\,t})d\lambda_t + \int_{R_{m,p}}^{R_{m,p}+g_{m,p}} (\omega_{m,p} \times v_{m,d,\,t} - \eta_{m,p,\,t})d\lambda_t + \int_{R_{m,p}+g_{m,pt}}^{C_m} (\omega_{m,p} \times v_{m,d,\,t})d\lambda_t \tag{3}$$

$$s_{m,p} = \int_0^{C_m} (\omega_{m,p} \times v_{m,d,\,t})d\lambda_t - \int_{R_{m,p}}^{R_{m,p}+g_{m,p}} \eta_{m,p,\,t}\, d\lambda_t \tag{4}$$

Therefore,

$$\frac{\partial(s_{m,p})}{\partial C_m} = (\omega_{m,p} \times v_{m,d,\,t}) \tag{5}$$

$$\frac{\partial(s_{m,p})}{\partial O_m} = \omega_{m,p} \times \int_0^{C_m} \frac{\partial(v_{m,d,\,t})}{\partial o_m} d\lambda_t \tag{6}$$

$$\frac{\partial(s_{m,p})}{\partial g_{m,p}} = -\eta_{m,p,t} \tag{7}$$

$$v_{m,d(n),t} - \eta_{m,d(n),\,t} = \alpha_{m,d(n),t} + \beta_{m,d(n),t} + \gamma_{2,m,d(n),t} + \gamma_{1,m,d(n),t} \tag{8}$$

$$v_{m,d(c),t} = \sum\nolimits_{p \in P} \eta_{m-1,d(c),0} - \theta_{m,d(c),t} \tag{9}$$

$$\eta_{m,p,t} = n \left\{ where \sum\nolimits_{i=R_{m,p}}^{t} h_{m,p,i} < (\lambda_t - R_{m,p}) \leq g_{m,p} \; and \sum\nolimits_{i=R_{m,p}}^{t+1} h_{m,p,i} < (\lambda_t - R_{m,p}) \leq g_{m,p} \right\} \tag{10}$$

S.T.

$$C_1 = C_2 = C_3 \ldots = C_m \tag{11}$$

$$\begin{cases} g_{m,1} + g_{\mathrm{m},2} + l_{m(1,2)} = g_{\mathrm{m},5} + g_{\mathrm{m},6} + l_{m(5,6)} \\ g_{\mathrm{m},3} + g_{\mathrm{m},4} + l_{m(3,4)} = g_{\mathrm{m},7} + g_{\mathrm{m},8} + l_{m(7,8)} \end{cases} \tag{12}$$

$$g_{m,1} + g_{\mathrm{m},2} + g_{\mathrm{m},3} + g_{\mathrm{m},4} + l_{m(1-4)} = C_m \tag{13}$$

$$g_{m,5} + g_{\mathrm{m},6} + g_{\mathrm{m},7} + g_{\mathrm{m},8} + l_{m(5-8)} = C_m \quad (14)$$

$$\alpha_{m,d(n),t} \geq 0 \quad (15)$$

$$\beta_{m,d(n),t} \geq 0 \quad (16)$$

$$\gamma_{1,m,d(n),t} \geq 0 \quad (17)$$

$$O_m < C_m \quad (18)$$

Given that the objective function is non-linear, the Newton–Raphson iteration method [48] was chosen to find the minimum point of the objective function $f(X)$. The selected method is advantageous when compared to other non-linear approaches due to having an efficient, quadratic order of convergence when it does come to converge to a solution [48]. The $(r + 1)$'th interactions can be found from r'th iterations in (19).

$$\left(X^T\right)^{r+1} = \left(X^T\right)^r - H^{-1}\left((X)^r\right)/f'\left((X)^r\right) \quad (19)$$

where H is the Hessen Matrix (the matrix of the second degree derivatives) of f at x.

4.3. Two-Stage Models in Distributed System

Two-stage optimization models in which control variables in major and minor directions are optimized in two different stages are explored. For the major direction, optimized variables contain offsets, green splits of the major direction, and the cycle length. For the minor direction, optimized variables include green splits of minor direction. The equations shown below are major and minor direction optimization objective functions.

$$\text{Major}: \ \min \sum\nolimits_{m=1}^{M} \sum\nolimits_{p_{(c)}=1}^{P_{(c)}} s_{m,p}\left(O_m, g_{m,p(c)}, C_m\right) \quad (20)$$

$$\text{Minor}: \ \min \sum\nolimits_{m\in M} \sum\nolimits_{p_{(n)}=1}^{P_{(n)}} s_{m,p}\left(g_{m,p(n)}\right) \quad (21)$$

As indicated in Figure 2, the boxes at the top show the optimization sequences. At the start of the green indication of the major direction, the minor direction signal timing plan optimization starts. The minor optimization ends before the major green indication ends. When the minor green indication starts, the major direction's optimization also starts.

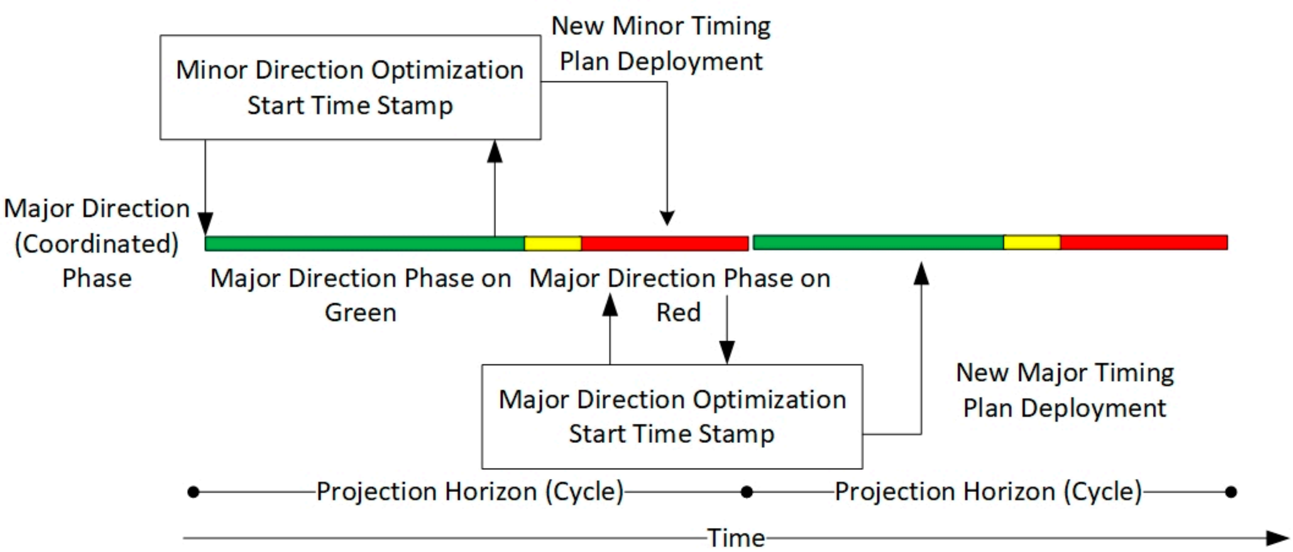

Figure 2. Optimal time step of the two-stage optimization distributed system.

For major direction optimizations, the objective function and its first degree of derivatives are the same as (1)–(7) except for the green times severing the major approaches. For an eight-phase dual ring signal controller configuration that effectively reduces the number of optimization variables by $4M$ (M is the total number of intersections),

$$\% = \frac{4M}{8M + M + 1} \approx \frac{4}{9} = 44.4\%$$

For minor direction optimization, the objective function is the same as (1). The optimized variables are minor splits. The first degree of derivatives is in (22). This effectively reduces the number of variables by 55.6% for eight-phase dual-ring National Electrical Manufacturers Association (NEMA) controllers.

$$\frac{\partial(s_{m,p})}{\partial g_{m,p(n)}} = -\eta_{m,p(n),T} \tag{22}$$

5. Case Studies

This research selected a microscopic traffic simulator, Enhanced Transportation Flow Open-source Microscopic Model (ETFOMM), for this case study. ETFOMM was developed based on CORridor SIMmulation (CORSIM) algorithms and concepts with updated traffic flow models, advanced computing technology, and advanced CV features [3,47]. ETFOMM's Application Programming Interface (API) or ETAPI is "built on the most recent Microsoft Windows Communication Foundation (WCF) technology". It is "the Most Advanced API for Mobile/Distributed Computing" [3,47], the feature of which perfectly fits this research.

5.1. Simulation Network Calibration and Case Studies

A simulation calibration is required before conducting a case study. The base case is an actuated signal plan optimized by Synchro and implemented in the field. For this case study, the network examined was Dolley Madison Boulevard in McLean, VA. The intersections are Georgetown Pike, Chain Bridge Road, Old Chain Bridge Road, and Old Dominion Drive @ Dolley Madison Blvd. The Virginia Department of Transportation (VDOT) provided the Synchro file with an optimized TSCS timing plan and a performance report. Calibration was performed to adjust simulation parameters. Traffic volumes, car-following sensitive factors, and startup loss times were the calibrated parameters. After the calibration, an average difference of 3.94% in control delays between ETFOMM simulation and VDOT performance report was achieved, which shows the calibration is acceptable. The details of the base case calibration are detailed in references [1–3,47].

The mobility benefits in this research are the percentages of control delay reductions; Figure 3 and Table 2 show the percentage saving of control delays. The savings are compared with the base case calibrated in the discussion above. The delays are divided into major and minor street ones. Figure 3 and Table 2 indicate the mobility benefits are the functions of control strategy (centralized and distributed with/without a real time limit) and penetration rate.

Table 2. Control delay reduction of the distributed system.

1. Two-Stage DS with Time Limit					
Summary	10%	25%	50%	60%	70%
Major Direction	−21.82%	−25.62%	−29.54%	−32.26%	−33.30%
Minor Direction	−7.43%	−9.54%	−10.81%	−11.63%	−12.27%

Table 2. *Cont.*

2. Two-Stage DS without Time Limit					
Summary	10%	25%	50%	60%	70%
Major Direction	−23.33%	−27.48%	−31.08%	−34.05%	−34.95%
Minor Direction	−8.37%	−10.61%	−11.83%	−12.68%	−13.32%
3. Two-Stage Centralized System without Time Limit and Full Optimization					
Summary	10%	25%	50%	60%	70%
Major Direction	−26.68%	−31.25%	−39.45%	−42.80%	−44.88%
Minor Direction	−10.35%	−11.48%	−14.92%	−15.83%	−16.75%

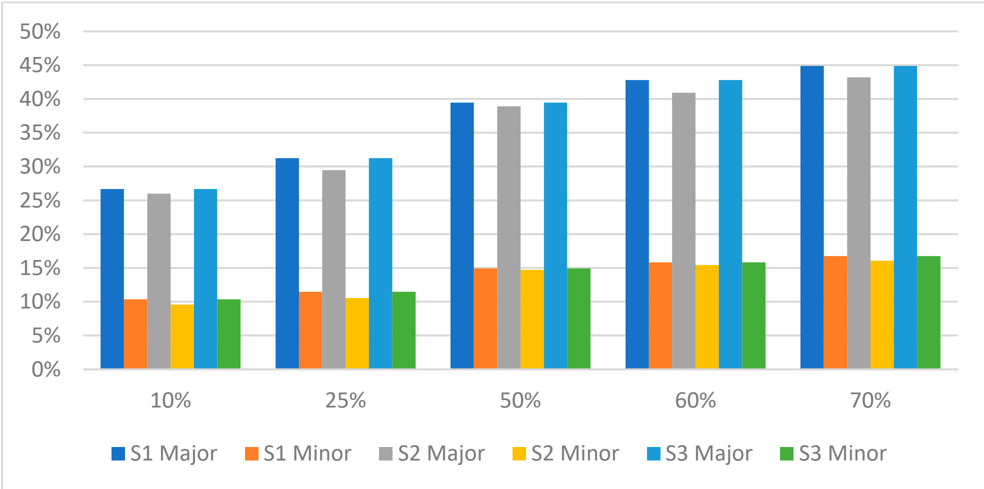

Figure 3. Control delay reduction of the distributed system and centralized system.

5.2. Mobility Benefit and Control Strategies

One critical consideration in a distributed real-time system is the time limitation on optimization. In a two-stage optimization strategy, the time limitation function is added to the optimization algorithm, which is a heuristic algorithm to guarantee a better solution in each iteration. The time limit function is a function that checks the CPU time for optimization within a cycle or within one stage. Suppose the CPU time consumed for the optimization program is longer than the actual cycle length; in that case, the optimization process is stopped, and the current feasible solution is the new signal timing plan.

Three scenarios are presented to showcase the impact of centralized and distributed systems, with/without considering the real time. S1, the proposed distributed system, described in Section 4.3 is in two-stage optimizations; all variables, including splits on major and minor directions, are optimized twice in a cycle. In the S2 (centralized system), all data are passed to the computer, and queue estimations are also performed on the central computer. S3 (centralized system) is a two-stage optimization; splits on major and minor are optimized sequentially in one of two stages. The two-stage centralized system (S3) could generate the same control delay reduction as a two-stage distributed system (S1) in the same configurations (no time limit, optimization model, or decision variables). The results of distributed and centralized systems in the same condition (without time limit and with all variables optimization) show that a distributed system (S1) could have almost the same performance as a centralized system (S3)

The first and second scenarios are two-stage optimizations, as shown in Table 2 and Figure 3, in which green times in major and minor directions are optimized sequentially just in one of two stages. The first scenario is limited by real time, while the second scenario does not have time limitations. In the third scenario, all variables are optimized simultaneously in each stage. All data are averaged from 10 simulation runs with different random number seeds. Two-stage optimization with a time limitation could generate a 33% control delay reduction in the major direction and a 12% reduction in the minor direction. The two-stage optimization without a time limit function and complete timing plan optimization (cycle length, split, and offset) could generate better benefits on control delay reduction to 44% in major and 16% in minor directions. The distributed system strategy demonstrated exemplary performance in the mobility of traffic improvement. Overall, the performance of the distributed systems (time limit and partial optimization) was degraded by 5–10% as compared with that of the centralized system shown above. However, distributed computing without time limits could recover about 2% of the performance. As the computer's computing power increases generation by generation, the optimized performance by time limits should be able to catch up with the optimization without time limitation.

5.3. Mobility Benefit and Penetration Rates

As discussed in the literature review, one research gap is the impact of the penetration rate on the mobility benefit, especially at the low penetration rate of 10%, and the mobility. With closing the gap in mind, we demonstrated the proposed systems are highly effective even when the penetration rate is low at 10%. In all three control strategies, the control delays are reduced by at least 21–27% (close to) on major streets while the control delays on major streets are improved as well (from 7.4% to 10.4%) even when real time limits the optimization outcomes. Another encouraging outcome from the case studies is with the increase of the penetration rate, the percentages of savings increased. That means the higher the penetration rate, the better the mobility benefit. When the penetration rate is more than 50%, the mobility benefit increase starts to slow down, although the increase is still visible. When the penetration rate reaches 50%, the control delays on major streets are reduced by close to at least 30% and could be as high as 33%. On the minor street, when the penetration rate is 25%, the mobility benefit is at least close to 10%. In the best-case scenario where there is a 70% penetration rate under centralized control strategies without a time limit, the control delay could be as high as almost 45% on the major street and 16% on the minor street.

Our case studies in this paper confirmed, once again, that with the proposed control strategies and models, mobility benefits are available to BOTH major and minor streets. We have the same findings as in all prior case studies [1–3]. This feature is much more desirable than the existing manual traffic signal re-timing plan in which the mobility benefits on the main streets increases at the expense of minor street mobilities.

6. Conclusions and Future Directions

This paper designed a distributed TSCS that processes BSM locally and passes queue length information to the central computer. Models and algorithms were detailed in this paper to provide solutions to optimize cycle length, offsets, and splits simultaneously in real time in a rolling horizon time frame. The models evolved into two-stage distributed models. The vehicle queue delays were established as an objective function. The first degree of derivatives of queue delays with respect to control variables (cycle length, offset, and green split) were explicitly formulated. The delay model took BSMs at a penetration rate as low as 10% and traditional loop detector data as input.

The results of the distributed system with a 10–70% penetration rate showed that the two-stage models in the distributed system could significantly reduce control delay. The two-stage models in the distributed system with two-set variables optimization performed similarly, with at least 30% in the major direction and 12% to 13% in the minor direction. At a low penetration rate of 10%, the proposed distributed TSCS can bring about 22% of mobility benefits on major streets and more than 7% mobility benefits in minor directions.

With proper design and implementation, the distributed system experiments demonstrated that TSCS control algorithms at V2X intersections could be altered and implemented in a distributed environment with minimum degrading of this optimization performance.

This will help researchers focus on models, algorithms, field tests, and implementation based on centralized systems in their initial development stages.

This paper is the first in R&D of distributed TSCS at V2X intersections to alleviate the communication bandwidth and computing power cost. The novel analytic formulation of delay functions and their first degree of derivatives for a two-stage optimization model are proposed in this research. They provide foundations for any non-linear optimization algorithms that are based on derivatives. The objective function and its derivatives were further obtained from the prior research's discrete queueing estimation mixed with macro and micro traffic flow models. This paper also filled gaps in TSCS at V2X intersections.

There still needs to be an assumption about the existing models. The infrastructure-based detector input and BSM are assumed to be synchronized. In the real world, BSM and detector input come from two sources of data. The existing models will need to be improved to account for this reality. Or a sensor fusion will need to be conducted to pre-process BSM and detector input to synchronize two sources of data.

We will pursue further development of the proposed system to seek funding to implement the proposed systems and field test the feasibility of the proposed systems. The proposed system will need to be further tested in more simulation case studies by enhancing the models and improving system reliabilities of algorithms. Once that is completed, a hardware/communication in the loop simulation will be conducted in library conditions to demonstrate the feasibility of the systems. After all those steps, a field study could be conducted.

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Conflicts of Interest: Li Zhang and Lei Zhang were employed by New Global Systems. The remaining author declares that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest. New Global Systems had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, or in the decision to publish the results.

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