mathematics

Article

Dynamic Event-Triggered Control for Delayed Nonlinear Markov Jump Systems under Randomly Occurring DoS Attack and Packet Loss

Haiyang Zhang [1,2,*], Huizhen Chen [2], Lianglin Xiong [3] and Yi Zhang [4]

[1] Faculty of Mechanical and Electrical Engineering, Kunming University of Science and Technology, Kunming 650500, China
[2] School of Mathematics and Computer Science, Yunnan Minzu University, Kunming 650500, China; 21213037570010@ymu.edu.cn
[3] School of Media and Information Engineering, Yunnan Open University, Kunming 650504, China; xionglianglin@ynou.edu.cn
[4] College of Computer Science, Sichuan University, Chengdu 610065, China; zhangyi@scu.edu.cn
* Correspondence: 20222002@kust.edu.cn

Abstract: This paper aims to address the exponential stability and stabilization problems for a class of delayed nonlinear Markov jump systems under randomly occurring Denial-of-Service (DoS) attacks and packet loss. Firstly, the stochastic characteristics of DoS attacks and packet loss are depicted by the attack success rate and packet loss rate. Secondly, a Period Observation Window (POW) method and a hybrid-input strategy are proposed to compensate for the impact of DoS attack and packet loss on the system. Thirdly, A Dynamic Event-triggered Mechanism (DETM) is introduced to save more network resources and ensure the security and reliability of the systems. Then, by constructing a general common Lyapunov functional and combining it with the DETM and other inequality analysis techniques, the less conservative stability and stabilization criteria for the underlying systems are derived. In the end, the effectiveness of our result is verified through two examples.

Keywords: delayed nonlinear Markov jump systems; attack success rate; packet loss rate; Dynamic Event-triggered Mechanism; exponential stability and stabilization

MSC: 93E15

check for updates

Citation: Zhang, H.; Chen, H.; Xiong, L.; Zhang, Y. Dynamic Event-Triggered Control for Delayed Nonlinear Markov Jump Systems under Randomly Occurring DoS Attack and Packet Loss. *Mathematics* **2024**, *12*, 1064. https://doi.org/10.3390/math12071064

Academic Editors: Tzong-Ru Tsai, Jianbao Chen and Yuhlong Lio

Received: 1 March 2024
Revised: 29 March 2024
Accepted: 31 March 2024
Published: 1 April 2024

1. Introduction

In recent decades, Markov Jump Systems (MJSs) have received widespread attention [1,2] due to their powerful ability to depict the mutation phenomenon that the parameter and structure of systems often encountered. It is worth mentioning that some undesirable dynamic behaviors, such as cyber attack [3,4], packet loss [5,6], time delay [7,8], non-linearity [9,10], often appear in the real systems, due to the openness of communication networks, limited bandwidth, external disturbance and signal propagation. As is well known, the stability is a prerequisite to ensure the normal operation of the system. However, such undesirable dynamic behaviors often lead to oscillation, chaos, and even instability. Therefore, it is very interesting to study the stability and stabilization problems of the Delayed Nonlinear MJSs (DNMJSs) under cyber attacks and packet loss.

Generally, there are two kinds of common cyber attack, named Denial-of-Service (DoS) attack and deception attack. Compared with deception attacks, a DoS attack is often launched by occupying communication resources to prevent the normal operation of the network and poses strong aggressiveness and ease of implementation, so it has become the most threatening form of cyber attack. Recently, many fruitful results on the stability and stabilization problems of DNMJSs under DoS attack or packet loss have been achieved [11,12]. From the perspective of characterization methods, Bernoulli processes [11–13] and Markov processes [14,15] are often used to model the stochastic properties of the DoS attack and packet loss. From the viewpoint of compensation strategies, the hold-input strategy and zero-input strategy are often adopted to deal with the impacts of DoS attacks and packet loss on the systems. The hold-input strategy is used to achieve faster and smoother stability of the systems in [16,17]. The stochastic behaviors of the packet loss are modeled by a Bernoulli process, and a zero-input strategy is adopted to compensate for the impacts of packet loss in [12]. However, DoS attacks and packet loss are rarely mentioned together, and few papers consider attack success rate and packet loss rate, which will limit its applicability in practical applications. Therefore, it is necessary to consider the impacts of attack success rate and packet loss rate on system performance at the same time, which prompted this paper.

On the other hand, with the increasing pressure of network communication, especially in the case of limited network resources, determining how to improve the resource utilization rate has become the focus of many scholars. To this end, the traditional time-triggered mechanisms, such as sampled-data control or impulsive control, have been widely used in recent years [18]. However, they cannot determine the triggered time on demand, which results in a waste of communication resources to a certain extent. For an event-triggered mechanism, the signals are transmitted only when the system state meets the preset triggered condition, which can effectively overcome the aforementioned obstacle. It is worth noting that according to the type of triggered parameters, event-triggered mechanisms are often divided into Static Event-Triggered Mechanisms (SETM) [19–22] and Dynamic Event-Triggered Mechanisms (DETM) [23,24]. Based on the Adaptive Event-Triggered Mechanism (AETM), ref. [19] considered the asynchronous stabilization problem of Markov jump interval type-2 fuzzy systems with cyber attacks, ref. [20] studied the reliable stabilization problem of Markovian jump complex dynamic networks with actuator faults. By the Event-Triggered Impulsive Mechanism (ETIM), ref. [21] discussed the security stabilization problem of stochastic networked control systems under cyber attacks, ref. [22] addressed the stabilization problem for stochastic switched systems with input constraints. Compared with the SETM, DETM contains a non-negative internal dynamic variable in the event-triggered condition which relies on the system's state and error state, which afford DETM greater advantages in reducing communication costs. Thus, determining how to use DETM to study the security control problem of delayed MJSs under a DoS attack and packet loss shall be an interesting topic.

Based on the points discussed above, this paper will further study the dynamic event-triggered security control problem for a class of DNMJSs under randomly occurring DoS attacks and packet loss. The main contributions of this paper are summarized as follows:

1. Two independent Bernoulli processes are introduced to describe the stochastic characteristics of attack success rate and packet loss rate during the action-period and sleeping-period, respectively.
2. Considering the physical properties of a randomly occurring DoS attack and packet loss, the POW method and hybrid-input strategy are proposed, which are very useful to depict the evolution law of DoS attack and packet loss.
3. By constructing a general common Lyapunov functional, combining with DETM and other inequality analysis techniques, the less conservative security stability criteria are obtained.

Notations: Throughout this paper, $\mathbb{R}$, $\mathbb{Z}_+$ represent the set of real numbers and the set of positive integer numbers, respectively. $\mathbb{R}^n$, $\mathbb{R}^{m\times n}$ and $\mathbb{S}_+^{n\times n}$ stand for a n-dimensional Euclidean space, the set of $m \times n$ real matrices and the set of symmetric positive definite matrices, respectively. The symbol $*$ denotes the symmetric entry in the symmetric matrix. $sym\{G\} = G + G^T$. $\langle x, y\rangle$ represents the inner product of vectors $x, y \in \mathbb{R}^n$. $(\Omega, \mathcal{F}_t, \mathcal{P})$ denotes a complete probability space, where Ω is a sample space of events, $\mathcal{F}_t$ is a sigma-algebra of events, $\mathcal{P}$ is a probability measure on $\mathcal{F}_t$.

2. Problem Formulation and Preliminary

2.1. System Description

Consider the following DNMJSs:

$$\dot{x}(t) = A(\bar{\lambda}_t)x(t) + A_q(\bar{\lambda}_t)x(t-q(t)) + f(t,x(t)) + B(\bar{\lambda}_t)u(t), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $q(t)$ denotes the time-varying delay that satisfies $0 \le q(t) \le q, 0 \le \dot{q}(t) \le \mu < 1, x(s) = \varphi(s), \forall s \in [-q,0]$ is the initial condition, $u(t) \in \mathbb{R}^m$ is the control input, $A(\bar{\lambda}_t) \in \mathbb{R}^{n\times n}, A_q(\bar{\lambda}_t) \in \mathbb{R}^{n\times n}, B(\bar{\lambda}_t) \in \mathbb{R}^{n\times m}$ are known constant matrices, $f(t,x(t))$ is the external disturbance that satisfies Assumption 1, $\{\bar{\lambda}_t, t \ge 0\}$ denotes the right continuous homogeneous Markov jump process on the probability space $(\Omega, \mathcal{F}_t, \mathcal{P})$ and taken values in a finite set $\Gamma = \{1,2,\cdots,S\}$ with the transition rate matrix $\Pi_\lambda = \{\lambda_{ij}\}$ given by

$$\Pr\{\bar{\lambda}_{t+\Delta} = j | \bar{\lambda}_t = i\} = \begin{cases} \lambda_{ij}\Delta + o(\Delta), & i \ne j \\ 1 + \lambda_{ij}\Delta + o(\Delta), & i = j \end{cases} \tag{2}$$

where $\Delta > 0$, $\lim\limits_{\Delta\to 0} o(\Delta)/\Delta = 0$. $\lambda_{ij} \ge 0 (i,j \in \Gamma, i \ne j)$ represents the transition rate from the mode i at time t to the mode j at time $t+\Delta$, and $\lambda_{ii} = -\sum_{j=1,j\ne i}^{S} \lambda_{ij}$. For convenience, denote $A(\bar{\lambda}_t) = A_i, A_q(\bar{\lambda}_t) = A_{qi}, B(\bar{\lambda}_t) = B_i$, when $\bar{\lambda}_t = i$.

Assumption 1 ([25]). *For $\hat{x}_1, \hat{x}_2 \in \mathbb{R}^n$, $f(t,x)$ satisfies one-sided Lipschitz:*

$$\langle f(t,\hat{x}_1) - f(t,\hat{x}_2), \hat{x}_1 - \hat{x}_2 \rangle \le \rho_0 \|\hat{x}_1 - \hat{x}_2\|^2,$$

where $\rho_0 \in \mathbb{R}$ is one-sided Lipschitz constant.

Assumption 2 ([25]). *For $\hat{x}_1, \hat{x}_2 \in \mathbb{R}^n$, $f(t,x)$ is quadratic inter-bounded, if*

$$\|f(t,\hat{x}_1) - f(t,\hat{x}_2)\|^2 \le \beta_0 \|f(t,\hat{x}_1) - f(t,\hat{x}_2)\|^2 + \alpha_0 \langle \hat{x}_1 - \hat{x}_2, f(t,\hat{x}_1) - f(t,\hat{x}_2) \rangle$$

holds, where $\beta_0, \alpha_0 \in \mathbb{R}$ are known constants.

Definition 1 ([9]). *The system (1) is exponentially mean-square stable, if there are constants $a > 0$ and $c > 0$, such that*

$$E\left\{\|x(t)\|^2\right\} \le a e^{-ct} \sup_{-q\le s\le 0} E\left\{\|\varphi(s)\|^2\right\}.$$

Definition 2 ([9]). *For the Lyapunov functional $V(x(t),\bar{\lambda}_t)$, its infinitesimal operator is defined as follows:*

$$LV(x(t),\bar{\lambda}_t) = \lim_{\Delta\to 0^+} \frac{1}{\Delta}\left[E\left\{ V(x(t+\Delta),\bar{\lambda}_{t+\Delta}) \middle| \; x(t),\bar{\lambda}_t\right\} - V(x(t),\bar{\lambda}_t)\right]. \tag{3}$$

Lemma 1 ([10]). *For any vectors $\zeta_1(t), \zeta_2(t)$, $\sigma_1(t), \sigma_2(t) \in \mathbb{R}$ satisfying $\sigma_1(t) + \sigma_2(t) = 1$, and matrices $Z \in \mathbb{R}^{n\times n}$, $\aleph_1, \aleph_2 \in \mathbb{S}_+^{n\times n}$, the following inequality holds*

$$\frac{1}{\sigma_1(t)}\zeta_1^T(t)\aleph_1\zeta_1(t) + \frac{1}{\sigma_2(t)}\zeta_2^T(t)\aleph_2\zeta_2(t) \ge \begin{bmatrix} \zeta_1(t) \\ \zeta_2(t) \end{bmatrix}^T \begin{bmatrix} \aleph_1 & Z \\ * & \aleph_2 \end{bmatrix} \begin{bmatrix} \zeta_1(t) \\ \zeta_2(t) \end{bmatrix} \tag{4}$$

subject to $\begin{bmatrix} \aleph_1 & Z \\ * & \aleph_2 \end{bmatrix} > 0.$

2.2. DoS Attack and Packet Loss

The openness and complexity of the communication network often leads to DoS attacks and packet loss, which can reduce or even destroy the performance of the system. Thus, this paper shall consider the DoS attack and packet loss in the communication network between the zero-order holder (ZOH) and the actuator (as shown in Figure 1), and the DoS attack and packet loss have the following characteristics: (1) the DoS attack and packet loss will not occur in a same time interval; (2) the DoS attack and packet loss will occur randomly.

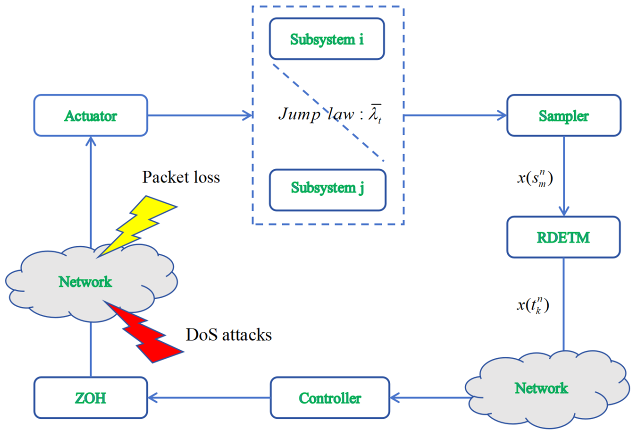

Figure 1. The framework of system (1) under DoS attack and packet loss.

To depict such kind of DoS attack and packet loss more intuitively, a Periodic Observation Window (POW) method is proposed to model the first characteristic of DoS attack and packet loss. Specifically, the nth POW is designed as $[n\ell, (n+1)\ell)$, which can be divided into $\left[n\ell, n\ell + \ell_{off}\right)$ and $\left[n\ell + \ell_{off}, (n+1)\ell\right)$, where $n \in \{0, \mathbb{Z}_+\}$, ℓ is an observation period. $\left[n\ell, n\ell + \ell_{off}\right)$ is the sleeping-period of DoS attack, $\left[n\ell + \ell_{off}, (n+1)\ell\right)$ is the action-period of DoS attack, and DoS attack and packet loss will occur in the action-period and sleeping-period, respectively. It is worth noting that during the sleeping-period, the control signal cannot be transmitted to the actuator if the packet loss occurs, and during the action-period, the control signal cannot be transmitted to the actuator if the attack succeeds.

Furthermore, two random variables $\varsigma_s(t)$ and $\varsigma_a(t)$ are introduced to model the second characteristic of DoS attack and packet loss, which are independent of each other and obey the Bernoulli distribution, i.e.,

$$\varsigma_s(t) = \begin{cases} 0, & \text{Packet losses} \\ 1, & \text{Packet does not loss} \end{cases} \quad \forall t \in \left[n\ell, n\ell + \ell_{off}\right),$$

$$\varsigma_a(t) = \begin{cases} 0, & \text{Attack succeeds} \\ 1, & \text{Attack does not succeed} \end{cases} \quad \forall t \in \left[n\ell + \ell_{off}, (n+1)\ell\right).$$

From the property of Bernoulli distribution, it is easy to see that $\Pr\{\varsigma_s(t) = 1\} = \varsigma_s$, $\Pr\{\varsigma_s(t) = 0\} = 1 - \varsigma_s$, $\Pr\{\varsigma_a(t) = 1\} = \varsigma_a$, $\Pr\{\varsigma_a(t) = 0\} = 1 - \varsigma_a$, where $\varsigma_s, \varsigma_a \in (0, 1)$ represent the expectation of random variables.

Remark 1. *From the view of defense, the POW method can provide an effective way for defenders to monitor the cyber attack, and also provide a feasible strategy for defenders to compensate for the adverse impacts of Dos attack and packet loss. Furthermore, compared with the existing literature, the characteristics of DoS attack and packet loss considered in this paper is more in line with the actual situation.*

2.3. Dynamic Event-Triggered Mechanism

In order to further reduce the burden of network transmission, a DETM shall be introduced in this section. To this end, it is assumed that the system state is sampled with a fixed sampling period h, the mth sampling instant is denoted as s_m^n and satisfies $s_{m+1}^n - s_m^n = h$. Furthermore, the kth triggering instant is denoted as t_k^n and satisfies the following DETM:

$$t_{k+1}^n = t_k^n + \min_{j^n \in \mathbb{Z}_+} \{j^n h \mid F_{etc}(t_k^n + j^n h) \geq \delta\phi(t_k^n + j^n h)\}, \tag{5}$$

where $F_{etc}(t_k^n + j^n h) = e^T(t_k^n + j^n h) W e(t_k^n + j^n h) - \sigma x^T(t_k^n) W x(t_k^n)$, $\sigma \in (0,1)$ is a triggering threshold, W is a weighting matrix to be determined, $e(t_k^n + j^n h) = x(t_k^n) - x(t_k^n + j^n h)$ stands for the error state between the current sampling state and the latest triggered state. $\phi(t)$ is a dynamic variable that satisfies the following dynamic rule:

$$\dot{\phi}(t) = -2v_1\phi(t) - \delta\phi(s_m^n) + x^T(s_m^n)\Xi x(s_m^n), \quad t \in [s_m^n, s_{m+1}^n) \tag{6}$$

where $v_1 > 0$ and $\delta > 0$ are the given constants, $\Xi > 0$ is a weighting matrix to be determined. The initial condition is $\phi(0) = \phi_0 \geq 0$.

Remark 2. *It is easy to see that $\{t_k^n\} \subseteq \{s_m^n\}$, which implies the triggering interval $t_{k+1}^n - t_k^n \geq h > 0$, thus the DETM can avoid the Zeno behavior naturally. Furthermore, as reported in [25], for the given constants $\phi_0 \geq 0$, $v_1 > 0$, $h > 0$ and a weighting matrix $\Xi > 0$, there is always a constant δ satisfying*

$$0 < \delta \leq -2v_1 + \frac{2v_1}{1 - e^{-2v_1 h}}, \tag{7}$$

such that the dynamic variable $\phi(t)$ satisfies $\phi(t) \geq 0$ for $t \in [0, \infty)$. In addition, the DETM designed in this paper relies on the current sampling states of system, and the dynamic variable can be adjusted dynamically with the sampling instants, which results in the data transmission rate being reduced to a large extent. The detailed algorithm of DETM is given in the Algorithm 1.

Algorithm 1 The algorithm of DETM

Step 1: Initialize the parameters $\ell, \ell_{off}, \gamma, q, \sigma, \delta, v_1, v_2, \mu, \alpha_0, \beta_0, \rho_0, \phi_0$;
Step 2: Compute matrices $W, \mathcal{K}_i$ by solving LMIs (40)–(43) in Theorem 2;
Step 3: Input the initial time t_0 and the initial state $x(t_0)$ of systems;
  For $t \in [t_0, t_{end}) \cap \left\{ \left[D_n^{off}, D_n^{on}\right) \cup \left[D_n^{on}, D_{n+1}^{off}\right) \right\}$;
Step 4: Update the system state $x(t_k^n + j^n h)$ and the dynamic variable $\phi(t_k^n + j^n h)$;
Step 5: Compute the error state $e(t_k^n + j^n h)$ and check the DETM;
  If the triggering condition holds, go to **Step 6:**
Step 6: Update and save the triggering instant $t_{k+1}^n \leftarrow t_k^n + j^n h$, the triggering state $x(t_{k+1}) \leftarrow x(t_k^n + j^n h)$;
  Else Update the sampling instant $t_k^n + (j_k^n + 1)h$, and return to **Step 4:**
  End
 End

2.4. Control Input Strategy

In order to compensate for the adverse influence on the systems from the DoS attack and packet loss, this paper shall adopt the hybrid-input strategy, i.e., the zero-input strategy is adopted when the DoS attack and packet loss occur, otherwise the hold-input strategy is adopted. Then, combining with the DETM and the physical characteristics of DoS attack and packet loss, the control input can be designed as

$$u(t) = \begin{cases} \varsigma_s(t)K_i x(t_k^n), & t \in \left[t_k^n, t_{k+1}^n\right) \cap \left[n\ell, n\ell + \ell_{off}\right), \\ \varsigma_a(t)K_i x(t_k^n), & t \in \left[t_k^n, t_{k+1}^n\right) \cap \left[n\ell + \ell_{off}, (n+1)\ell\right), \end{cases} \tag{8}$$

where $K_i \in \mathbb{R}^{m\times n}$ is the controller gain matrix to be determined, $k \in \{1, \cdots, k_s^n, k_s^n + 1, \cdots, k_a^n\}$, where $k_s^n \triangleq \max\{k \in \mathbb{Z}_+ \mid t_k^n < n\ell + \ell_{off}\}$, $k_a^n \triangleq \max\{k \in \mathbb{Z}_+ \mid t_k^n < (n+1)\ell\}$.

Remark 3. *References [16,17] only adopt the hold-input strategy when the system is subjected to DoS attack and packet loss. Compared to this case, the hybrid-input strategy adopted in this paper can greatly combat the influence on system from randomly occurring DoS attack and packet loss.*

2.5. Model Transformation

In this section, the input delay method and interval decomposition approach shall be used to describe the control input under the randomly occurring DoS attack and packet loss. Firstly, the relationship between the triggering instants t_k^n and the sleeping-period and action-period of POW should be discussed as follows:

(A) During the sleep-period $\left[n\ell, n\ell + \ell_{off}\right)$:

$$\begin{cases} \Pi^n_{\iota^n_{1,1}} = \left[n\ell + \iota^n_{1,1}h, n\ell + (\iota^n_{1,1}+1)h\right), \iota^n_{1,1} = 0,1,2,\cdots,(\bar{\iota}^n_{1,1}-1), \\ \Pi^n_{\iota^n_{1,k}} = \left[t^n_k + \iota^n_{1,k}h, t^n_k + \left(\iota^n_{1,k}+1\right)h\right), \iota^n_{1,k} = 0,1,2,\cdots,(\bar{\iota}^n_{1,k}-1), \\ \Pi^n_{\iota^n_1} = \left[t^n_{k^n_s} + \iota^n_1 h, t^n_{k^n_s} + (\iota^n_1+1)h\right), \iota^n_1 = 0,1,2,\cdots,(\bar{\iota}^n_1-1), \end{cases} \tag{9}$$

where $\bar{\iota}^n_{1,1} \triangleq \max\{\iota^n_{1,1} \in \{0,\mathbb{Z}_+\} \mid n\ell + \iota^n_{1,1}h \le t^n_1\}$, $\bar{\iota}^n_{1,k} \triangleq \max\{\iota^n_{1,k} \in \{0,\mathbb{Z}_+\} \mid t^n_k + \iota^n_{1,k}h \le t^n_{k+1}\}$ and $\bar{\iota}^n_1 \triangleq \max\{\iota^n_1 \in \{0,\mathbb{Z}_+\} \mid t^n_{k^n_s} + \iota^n_1 h \le n\ell + \ell_{off}\}$. Note that if $t^n_1 = n\ell$ then $\Pi^n_{\iota^n_{1,1}} = \{t^n_1\}$, otherwise $t^n_1 = n\ell + \bar{\iota}^n_{1,1}h$. Similarly, $t^n_{k+1} = t^n_k + \bar{\iota}^n_{1,k}h$, $n\ell + \ell_{off} = t^n_{k^n_s} + \bar{\iota}^n_1 h$, and $k = 1,2,\cdots,(k^n_s-1)$.

(B) During the action-period $\left[n\ell + \ell_{off}, (n+1)\ell\right)$:

$$\begin{cases} \Pi^n_{\iota^n_{2,1}} = \left[n\ell + \ell_{off} + \iota^n_{2,1}h, n\ell + \ell_{off} + (\iota^n_{2,1}+1)h\right), \iota^n_{2,1} = 0,1,2,\cdots,(\bar{\iota}^n_{2,1}-1), \\ \Pi^n_{\iota^n_{2,k}} = \left[t^n_k + \iota^n_{2,k}h, t^n_k + \left(\iota^n_{2,k}+1\right)h\right), \iota^n_{2,k} = 0,1,2,\cdots,(\bar{\iota}^n_{2,k}-1), \\ \Pi^n_{\iota^n_2} = \left[t^n_{k^n_a} + \iota^n_2 h, t^n_{k^n_a} + (\iota^n_2+1)h\right), \iota^n_2 = 0,1,2,\cdots,(\bar{\iota}^n_2-1), \end{cases} \tag{10}$$

where $\bar{\iota}^n_{2,1} \triangleq \max\{\iota^n_{2,1} \in \{0,\mathbb{Z}_+\} \mid n\ell + \iota^n_{2,1}h \le t^n_{k^n_s+1}\}$, $\bar{\iota}^n_{2,k} \triangleq \max\{\iota^n_{2,k} \in \{0,\mathbb{Z}_+\} \mid t^n_k + \iota^n_{2,k}h \le t^n_{k+1}\}$ and $\bar{\iota}^n_2 \triangleq \max\{\iota^n_2 \in \{0,\mathbb{Z}_+\} \mid t^n_{k^n_a} + \iota^n_2 h \le (n+1)\ell\}$. Note that if $t^n_{k^n_s+1} = n\ell + \ell_{off}$, then $\Pi^n_{\iota^n_{2,1}} = \{t^n_{k^n_s+1}\}$, otherwise $t^n_{k^n_s+1} = n\ell + \ell_{off} + \bar{\iota}^n_{2,1}h$. Similarly, $t^n_{k+1} = t^n_k + \bar{\iota}^n_{2,k}h$, $(n+1)\ell = t^n_{k^n_a} + \bar{\iota}^n_2 h$, and $k = k^n_s+1, k^n_s+2, \cdots, (k^n_a-1)$.

Based on the interval decomposition in (9) and (10), for $\forall t \in [n\ell, (n+1)\ell)$, it follows from the input delay method that

$$\gamma^n_k(t) = \begin{cases} t - t^{n-1}_{k^{n-1}_a} - \hat{\iota}^{n-1}_2 h, t \in \hat{\Pi}^n_1, \hat{\iota}^{n-1}_2 = \bar{\iota}^{n-1}_2 + 1, \cdots, \bar{\iota}^{n-1}_2 + \bar{\iota}^n_{1,1} - 1, \\ t - t^n_1 - \hat{\iota}^n_{1,k}h, t \in \hat{\Pi}^n_2, \hat{\iota}^n_{1,k} = 0,1,\cdots,\bar{\iota}^n_{1,k}-1, \\ \qquad k = 1,2,\cdots,(k^n_s-1), \\ t - t^n_{k^n_s} - \hat{\iota}^n_1 h, t \in \hat{\Pi}^n_3, \hat{\iota}^n_1 = 0,1,2,\cdots,\bar{\iota}^n_1, \bar{\iota}^n_1+1, \cdots, \bar{\iota}^n_1 + \bar{\iota}^n_{2,1}, \\ t - t^n_{k^n_s+1} - \hat{\iota}^n_{2,k}h, t \in \hat{\Pi}^n_4, \hat{\iota}^n_{2,k} = 0,1,2,\cdots,\bar{\iota}^n_{2,k}-1, \\ \qquad k = k^n_s+1, k^n_s+2, \cdots, (k^n_a-1), \\ t - t^n_{k^n_a} - \hat{\iota}^n_2 h, t \in \hat{\Pi}^n_5, \hat{\iota}^n_2 = 0,1,2,\cdots,\bar{\iota}^n_2 - 1, \end{cases} \tag{11}$$

where $\hat{\Pi}_1^n = \Pi^{n-1}_{\check{\iota}_2^{n-1}-1} \cup \left\{ \cup_{\iota_{1,1}^n=0}^{\check{\iota}_{1,1}^n-1} \Pi^n_{\iota_{1,1}^n} \right\}, \hat{\Pi}_2^n = \cup_{\iota_{1,k}^n=0}^{\check{\iota}_{1,k}^n-1} \Pi^n_{\iota_{1,k}^n}, \hat{\Pi}_3^n = \left\{ \cup_{\iota_1^n=0}^{\check{\iota}_1^n-1} \Pi^n_{\iota_1^n} \right\} \cup \left\{ \cup_{\iota_{2,1}^n=0}^{\check{\iota}_{2,1}^n-1} \Pi^n_{\iota_{2,1}^n} \right\}, \hat{\Pi}_4^n = \cup_{\iota_{2,k}^n=0}^{\check{\iota}_{2,k}^n-1} \Pi^n_{\iota_{2,k}^n}, \hat{\Pi}_5^n = \cup_{\iota_2^n=0}^{\check{\iota}_2^n-1} \Pi^n_{\iota_2^n}$. It is easy to find that $\gamma_k^n(t)$ is a piece-wise continuous function, which satisfies $0 \leq \gamma_k^n(t) < \gamma = h$ and

$$e_k^n(t) = x(t_k^n) - x(t - \gamma_k^n(t)), \ \ \forall t \in \hat{\Pi}^n = \cup_{\iota^n=1}^{5} \hat{\Pi}^n_{\iota^n}. \tag{12}$$

Combining with (8) and (12), the DNMJSs (1) can be rewritten as the following switched systems:

$$\begin{cases} \dot{x}(t) = A_i x(t) + A_{qi} x(t - q(t)) + f(t,x) + u_s(t), \\ u_s(t) = \varsigma_s(t) B_i K_i (x(t - \gamma_k^n(t)) + e_k^n(t)), t \in \left[D_n^{on}, D_n^{off}\right), \\ \dot{x}(t) = A_i x(t) + A_{qi} x(t - q(t)) + f(t,x) + u_a(t), \\ u_a(t) = \varsigma_a(t) B_i K_i (x(t - \gamma_k^n(t)) + e_k^n(t)), t \in \left[D_n^{off}, D_{n+1}^{on}\right), \end{cases} \tag{13}$$

where $\left[D_n^{off}, D_n^{on}\right) = \hat{\Pi}^n \cap \left[n\ell, n\ell + \ell_{off}\right), \left[D_n^{on}, D_{n+1}^{off}\right) = \hat{\Pi}^n \cap \left[n\ell + \ell_{off}, (n+1)\ell\right)$.

3. Main Results

Before presenting the main results, the following vectors need to be given.

$$\begin{aligned} \varpi(t) =& col\{x(t), x(t - q(t)), x(t - q), x(t - \gamma_k^n(t)), x(t - \gamma), f(t,x), e_k^n(t)\}, \\ \Gamma_{1i} =& A_i \bar{e}_1 + A_{qi} \bar{e}_2 + \varsigma_s B_i K_i (\bar{e}_4 + \bar{e}_7) + \bar{e}_6, \\ \Gamma_{2i} =& A_i \bar{e}_1 + A_{qi} \bar{e}_2 + \varsigma_a B_i K_i (\bar{e}_4 + \bar{e}_7) + \bar{e}_6, \\ \bar{\Gamma}_{1i} =& A_i \Upsilon_i \bar{e}_1 + A_{qi} \Upsilon_i \bar{e}_2 + \varsigma_s B_i Y_i (\bar{e}_4 + \bar{e}_7) + \bar{e}_6, \\ \bar{\Gamma}_{2i} =& A_i \Upsilon_i \bar{e}_1 + A_{qi} \Upsilon_i \bar{e}_2 + \varsigma_a B_i Y_i (\bar{e}_4 + \bar{e}_7) + \bar{e}_6, \\ \hat{\Gamma}_{1i} =& A_i \Upsilon_i \bar{e}_1 + B_i Y_i (\bar{e}_4 + \bar{e}_7) + \bar{e}_6, \hat{\Gamma}_{2i} = A_i \Upsilon_i \bar{e}_1 + \bar{e}_6, \\ \Pi_1 =& \left[\bar{e}_1^T - \bar{e}_4^T, \bar{e}_4^T - \bar{e}_5^T\right]^T, \Pi_2 = \left[\bar{e}_1^T - \bar{e}_2^T, \bar{e}_2^T - \bar{e}_3^T\right]^T, \\ \bar{e}_b =& \left[0_{n \times (b-1)n} \ I_n \ 0_{n \times (7-b)n}\right], \ \ b = 1, 2, \cdots, 7. \end{aligned}$$

In this section, the exponential stability and stabilization criteria for the system (13) under the randomly occurring DoS attack and packet loss are established in terms of LMIs.

Theorem 1. *For given positive scalars $\ell, \ell_{off}, q, \sigma, h, \delta, v_1, v_2, \varepsilon_1, \varepsilon_2, \mu$, scalars $\alpha_0, \beta_0, \rho_0$, $v_1 + v_2 > 0$ satisfying (7) and*

$$-v_2 \ell + (v_1 + v_2)\ell_{off} > 0, \tag{14}$$

if there exist matrices $W_1, W_2, W_3, J_1, J_2, W, \Xi, P_i \in \mathbb{S}_+^{n \times n}, K_i \in \mathbb{R}^{m \times n}$, and matrices $M_1, M_2 \in \mathbb{R}^{n \times n}$, such that the following LMIs hold:

$$\begin{bmatrix} J_1 & M_1 \\ * & J_1 \end{bmatrix} \geq 0, \tag{15}$$

$$\begin{bmatrix} J_2 & M_2 \\ * & J_2 \end{bmatrix} \geq 0, \tag{16}$$

$$\Phi_{1i} < 0, \tag{17}$$

$$\Phi_{2i} < 0, \tag{18}$$

where

$$
\begin{aligned}
\Phi_{1i} =& sym\Big\{\bar{e}_1^T P_i \Gamma_{1i}\Big\} + \bar{e}_1^T\Big(2v_2 P_i + \textstyle\sum_{j=1}^{S} \lambda_{ij} P_j\Big)\bar{e}_1 + \Phi_0, \\
\Phi_{2i} =& sym\Big\{\bar{e}_1^T P_i \Gamma_{2i}\Big\} + \bar{e}_1^T\Big(-2v_2 P_i + \textstyle\sum_{j=1}^{S} \lambda_{ij} P_j\Big)\bar{e}_1 + \Phi_0, \\
\Phi_0 =& sym\Big\{(\varepsilon_1\rho_0 + \varepsilon_2\beta_0)\bar{e}_1^T\bar{e}_1 + (\varepsilon_2\alpha_0 - \varepsilon_1)\bar{e}_1^T\bar{e}_6 - \varepsilon_2\bar{e}_6^T\bar{e}_6\Big\} \\
&+ \bar{e}_1^T(W_1 + W_2 + W_3)\bar{e}_1 - e^{-2v_1\gamma}\bar{e}_5^T W_1\bar{e}_5 - e^{-2v_1 q}(1-\mu)\bar{e}_2^T W_2\bar{e}_2 \\
&- e^{-2v_1 q}\bar{e}_3^T W_3\bar{e}_3 + \Gamma_{2i}^T\Big(\gamma^2 J_1 + q^2 J_2\Big)\Gamma_{2i} + \bar{e}_4^T\Xi\bar{e}_4 + \sigma(\bar{e}_4 + \bar{e}_7)^T W(\bar{e}_4 + \bar{e}_7) \\
&- \bar{e}_7^T W\bar{e}_7 - e^{-2v_1\gamma}\Pi_1^T\begin{bmatrix} J_1 & M_1 \\ * & J_1 \end{bmatrix}\Pi_1 - e^{-2v_1 q}\Pi_2^T\begin{bmatrix} J_2 & M_2 \\ * & J_2 \end{bmatrix}\Pi_2.
\end{aligned}
$$

then the system (13) *is exponentially mean-square stable.*

Proof of Theorem 1. Construct the following Lyapunov functional:

$$
U\big(x(t), \bar{\lambda}_t, t\big) = V\big(x(t), \bar{\lambda}_t\big) + \phi(t), \tag{19}
$$

where

$$
\begin{aligned}
V\big(x(t), \bar{\lambda}_t\big) =& x^T(t)P\big(\bar{\lambda}_t\big)x(t) + \int_{t-\gamma}^{t} e^{-2v_1(t-\xi)}x^T(\xi)W_1 x(\xi)d\xi \\
&+ \int_{t-q(t)}^{t} e^{-2v_1(t-\xi)}x^T(\xi)W_2 x(\xi)d\xi \\
&+ \int_{t-q}^{t} e^{-2v_1(t-\xi)}x^T(\xi)W_3 x(\xi)d\xi \\
&+ \gamma\int_{t-\gamma}^{t}\int_{s}^{t} e^{-2v_1(t-\xi)}\dot{x}^T(\xi)J_1\dot{x}(\xi)d\xi ds \\
&+ q\int_{-q}^{0}\int_{t+s}^{t} e^{-2v_1(t-\xi)}\dot{x}^T(\xi)J_2\dot{x}(\xi)d\xi ds.
\end{aligned}
$$

According to Definition 2, it follows:

$$
\begin{aligned}
LV\big(x(t), \bar{\lambda}_t\big) \le& -2v_1 V\big(x(t), \bar{\lambda}_t\big) + 2x^T(t)P_i\dot{x}(t) + 2v_1 x^T(t)P_i x(t) \\
&+ x^T(t)\Big(\textstyle\sum_{j=1}^{S} \lambda_{ij}P_j\Big)x(t) + x^T(t)(W_1 + W_2 + W_3)x(t) \\
&- e^{-2v_1\gamma}x^T(t-\gamma)W_1 x(t-\gamma) - e^{-2v_1 q}x^T(t-q)W_3 x(t-q) \\
&- e^{-2v_1 q}(1-\mu)x^T(t-q(t))W_2 x(t-q(t)) \\
&+ \dot{x}^T(t)\Big(\gamma^2 J_1\Big)\dot{x}(t) - \gamma\int_{t-\gamma}^{t} e^{-2v_1(t-\xi)}\dot{x}^T(\xi)J_1\dot{x}(\xi)d\xi \\
&+ \dot{x}^T(t)\Big(q^2 J_2\Big)\dot{x}(t) - q\int_{t-q}^{t} e^{-2v_1(t-\xi)}\dot{x}^T(\xi)J_2\dot{x}(\xi)d\xi.
\end{aligned} \tag{20}
$$

By using the Jensen integral inequality in [10] and Lemma 1, with the help of (15) and (16), the last two integral quadratic terms of (20) can be rewritten as:

$$
\begin{aligned}
&-\gamma\int_{t-\gamma}^{t} e^{-2v_1(t-\xi)}\dot{x}^T(\xi)J_1\dot{x}(\xi)d\xi - q\int_{t-q}^{t} e^{-2v_1(t-\xi)}\dot{x}^T(\xi)J_2\dot{x}(\xi)d\xi \\
\le& -e^{-2v_1\gamma}\varpi^T(t)\Pi_1^T\begin{bmatrix} J_1 & M_1 \\ * & J_1 \end{bmatrix}\Pi_1\varpi(t) - e^{-2v_1 q}\,\varpi^T(t)\Pi_2^T\begin{bmatrix} J_2 & M_2 \\ * & J_2 \end{bmatrix}\Pi_2\varpi(t).
\end{aligned} \tag{21}
$$

From Assumptions 1 and 2, we have

$$2\varepsilon_2\beta_0 x^T(t)x(t)+2\varepsilon_2\alpha_0 x^T(t)f(t,x)-2\varepsilon_2 f^T(t,x)f(t,x),\\ +2\varepsilon_1\rho_0 x^T(t)x(t)-\varepsilon_1 x^T(t)f(t,x)-\varepsilon_1 f^T(t,x)x(t)\geq 0, \tag{22}$$

where the scalars $\varepsilon_1>0$ and $\varepsilon_2>0$.

Case A: During the sleeping-period $[n\ell, n\ell+\ell_{off})$, one can obtain from (5) and (6) that,

$$\begin{aligned}\dot{\phi}(t)\leq&-2v_1\phi(t)+x^T(t-\gamma_k^n(t))\Xi x(t-\gamma_k^n(t))-(e_k^n(t))^T We_k^n(t)\\&+\sigma[x(t-\gamma_k^n(t))+e_k^n(t)]^T W[x(t-\gamma_k^n(t))+e_k^n(t)].\end{aligned}\tag{23}$$

Combining with (20)–(23), we have

$$LU(t)\leq -2v_1U(t)+\varpi^T(t)\{\Phi_{1i}\}\varpi(t). \tag{24}$$

Thus, it follows from (17) and (24) that,

$$LU(t)\leq -2v_1U(t). \tag{25}$$

Then, for $t\in\left[D_n^{off}, D_n^{on}\right)$, one can obtain from (25) and Dynkin formula [2] that

$$\begin{aligned}EU(t)\leq& e^{-2v_1\left(t-D_n^{off}\right)}EU\left(D_n^{off}\right)\\ \leq& e^{-2v_1\left(t-\ell_{n-1}^{on}\right)+2(v_1+v_2)\left(\ell-\ell_{off}\right)}EU\left(D_{n-1}^{on}\right)\\ \leq& e^{-2v_1\left(t-\ell_{n-1}^{off}\right)+2(v_1+v_2)\left(\ell-\ell_{off}\right)}EU\left(D_{n-1}^{off}\right)\\ &\vdots\\ \leq& e^{-2v_1\left(t-\ell_0^{off}\right)+2n(v_1+v_2)\left(\ell-\ell_{off}\right)}EU\left(D_0^{off}\right),\end{aligned}\tag{26}$$

It follows from $D_0^{off}=0$ that

$$EU(t)\leq e^{-2v_1t+2n(v_1+v_2)\left(\ell-\ell_{off}\right)}U(0), \tag{27}$$

For $t\geq D_n^{off}\geq n\ell$, from (14), we have

$$EU(t)\leq e^{-2\lambda n}U(0), \tag{28}$$

where $\lambda=-v_1\ell+(v_1+v_2)\ell_{off}$. And because of $t\leq D_n^{on}=n\ell+\ell_{off}$, then $n\geq\frac{t-\ell_{off}}{\ell}$. Thus,

$$EU(t)\leq e^{2\lambda\frac{\ell_{off}}{\ell}}e^{-\frac{2\lambda}{\ell}t}U(0). \tag{29}$$

Case B: During the action-period $[n\ell+\ell_{off},(n+1)\ell)$, one can obtain from (5), (6) and $v_1+v_2>0$ that

$$\begin{aligned}\dot{\phi}(t)=&-2v_1\phi(t)-\delta\phi(t-\gamma_k^n(t))+x^T(t-\gamma_k^n(t))\Xi x(t-\gamma_k^n(t))\\ \leq&\, 2v_2\phi(t)+x^T(t-\gamma_k^n(t))\Xi x(t-\gamma_k^n(t))-(e_k^n(t))^T We_k^n(t)\\ &+\sigma[x(t-\gamma_k^n(t))+e_k^n(t)]^T W[x(t-\gamma_k^n(t))+e_k^n(t)],\end{aligned}\tag{30}$$

Combining with (20), (21) and (30),

$$LU(t) \leq 2v_2 U(t) + \varpi^T(t)\Phi_{2i}\varpi(t). \tag{31}$$

According to (18) and (31), it follows that

$$LU(t) \leq 2v_2 U(t). \tag{32}$$

Then, for $t \in \left[D_n^{on}, D_{n+1}^{off}\right)$, one can obtain from (32) and Dynkin formula [2] that

$$\begin{aligned} EU(t) \leq & e^{2v_2(t-D_n^{on})} EU(D_n^{on}) \\ \leq & e^{2v_2\left(t-D_n^{off}\right)-2(v_1+v_2)\ell_{off}} EU\left(D_n^{off}\right) \\ \leq & e^{2v_2\left(t-\ell_{n-1}^{on}\right)-2(v_1+v_2)\ell_{off}} EU\left(D_{n-1}^{on}\right) \\ & \vdots \\ \leq & e^{2v_2\left(t-\ell_0^{on}\right)-2n(v_1+v_2)\ell_{off}} EU\left(D_0^{on}\right) \\ \leq & e^{2v_2\left(t-\ell_0^{off}\right)-2(n+1)(v_1+v_2)\ell_{off}} EU\left(D_0^{off}\right), \end{aligned} \tag{33}$$

From $D_0^{off} = 0$, we have

$$EU(t) \leq e^{2v_2 t - 2(n+1)(v_1+v_2)\ell_{off}} U(0), \tag{34}$$

For $t \leq D_{n+1}^{off} = (n+1)\ell$, it can be obtained from (14)

$$EU(t) \leq e^{-2\lambda(n+1)} U(0), \tag{35}$$

And because of $t \leq (n+1)\ell$,

$$EU(t) \leq e^{-\frac{2\lambda}{\ell}t} U(0). \tag{36}$$

Denote $\eta_1 = \min\limits_{i\in\{1,\cdots,S\}}\{\lambda_{\min}(P_i)\}$, $\eta_2 = \min\limits_{i\in\{1,\cdots,S\}}\{\lambda_{\max}(P_i)\}$, $\eta_3 = \eta_2 + \gamma\lambda_{\max}(W_1) + q\lambda_{\max}(W_2) + q\lambda_{\max}(W_3) + \gamma^2\lambda_{\max}(J_1) + q^2\lambda_{\max}(J_2)$. From (29) and (36), there is a scalar $d > 1$ satisfies

$$EU(t) \geq \eta_1 E\left\{\|x(t)\|^2\right\}, \tag{37}$$

$$U(0) \leq d\eta_3 E\left\{\sup_{-\gamma\leq s\leq 0}\|\varphi(s)\|_\gamma^2\right\} + \|\phi(0)\|. \tag{38}$$

Furthermore, for given φ and $\phi(0)$, there always exists a scalar $\eta_4 > 0$ such that $\|\phi(0)\| \leq \eta_4 E\{\sup\limits_{-\gamma\leq s\leq 0}\|\varphi(s)\|_\gamma^2\}$. Thus, combining with (29) and (36)–(38), we have

$$E\left\{\|x(t)\|^2\right\} \leq \frac{\vartheta\eta_5}{\eta_1} e^{-\frac{2\lambda}{\ell}t} E\left\{\sup_{-\gamma\leq s\leq 0}\|\varphi(s)\|_\gamma^2\right\}, \tag{39}$$

where $\vartheta = \max\left\{e^{2\lambda\frac{\ell_{off}}{\ell}}, 1\right\}$ and $\eta_5 = d\eta_3 + \eta_4$. Therefore, system (13) is exponentially mean-square stable. The proof is finished. □

Next, based on Theorem 1, we shall to solve the controller gain matrix K_i and the weighting matrices Ξ_i and W_i in the DETM.

Theorem 2. *For given positive scalars $\ell, \ell_{off}, \gamma, q, \sigma, \delta, v_1, v_2, \varepsilon_1, \varepsilon_2, \mu, \kappa_1, \kappa_2$, scalars $\alpha_0, \beta_0, \rho_0$, and $v_1 + v_2 > 0$, satisfying (7) and (14), if there exist matrices $\bar{W}_{1i}, \bar{W}_{2i}, \bar{W}_{3i}, \bar{J}_{1i}, \bar{J}_{2i}, \bar{\Xi}_i, \bar{W}_i, \Upsilon_i \in \mathbb{S}_+^n$ and matrices $\bar{M}_{1i}, \bar{M}_{2i} \in \mathbb{R}^{n\times n}$, such that the following LMIs hold:*

$$\begin{bmatrix} \bar{J}_{1i} & \bar{M}_{1i} \\ * & \bar{J}_{1i} \end{bmatrix} \geq 0, \tag{40}$$

$$\begin{bmatrix} \bar{J}_{2i} & \bar{M}_{2i} \\ * & \bar{J}_{2i} \end{bmatrix} \geq 0, \tag{41}$$

$$\begin{bmatrix} \bar{\Phi}_{1i} & * & * & * & * \\ \gamma\bar{\Gamma}_{1i} & -(2\kappa_1\Upsilon_i - \kappa_1^2\bar{J}_{1i}) & * & * & * \\ q\bar{\Gamma}_{2i} & 0 & -(2\kappa_2\Upsilon_i - \kappa_2^2\bar{J}_{2i}) & * & * \\ \bar{\Phi}_{14i} & 0 & 0 & \bar{\Phi}_{44} & * \\ \Phi_{15} & 0 & 0 & 0 & \Phi_{55} \end{bmatrix} < 0, \tag{42}$$

$$\begin{bmatrix} \bar{\Phi}_{2i} & * & * & * & * \\ \gamma\bar{\Gamma}_{1i} & -(2\kappa_1\Upsilon_i - \kappa_1^2\bar{J}_{1i}) & * & * & * \\ q\bar{\Gamma}_{2i} & 0 & -(2\kappa_2\Upsilon_i - \kappa_2^2\bar{J}_{2i}) & * & * \\ \bar{\Phi}_{14i} & 0 & 0 & \bar{\Phi}_{44} & * \\ \Phi_{15} & 0 & 0 & 0 & \Phi_{55} \end{bmatrix} < 0, \tag{43}$$

where

$$\begin{aligned}
\bar{\Phi}_{1i} =& sym\left\{\bar{e}_1^T\bar{\Gamma}_{1i}\right\} + \bar{e}_1^T(2v_1\Upsilon_i + \lambda_{ii}\Upsilon_i)\bar{e}_1 + \bar{\Phi}_{0i}, \\
\bar{\Phi}_{2i} =& sym\left\{\bar{e}_1^T\bar{\Gamma}_{2i}\right\} + \bar{e}_1^T(-2v_2\Upsilon_i + \lambda_{ii}\Upsilon_i)\bar{e}_1 + \bar{\Phi}_{0i}, \\
\bar{\Phi}_{0i} =& sym\left\{(\varepsilon_2\alpha_0 - \varepsilon_1)\bar{e}_1^T\Upsilon_i\bar{e}_6 - \varepsilon_2\bar{e}_6^T\bar{e}_6\right\} - e^{-2v_1\gamma}\bar{e}_5^T\bar{W}_{1i}\bar{e}_5 \\
&+ \bar{e}_1^T(\bar{W}_{1i} + \bar{W}_{2i} + \bar{W}_{3i})\bar{e}_1 - e^{-2v_1 q}(1-\mu)\bar{e}_2^T\bar{W}_{2i}\bar{e}_2 \\
&- e^{-2v_1 q}\bar{e}_3^T\bar{W}_{3i}\bar{e}_3 + \bar{e}_4^T\bar{\Xi}_i\bar{e}_4 - \bar{e}_7^T\bar{W}_i\bar{e}_7 + \sigma(\bar{e}_4 + \bar{e}_7)^T\bar{W}_i(\bar{e}_4 + \bar{e}_7) \\
&- e^{-2v_1\gamma}\,\Pi_1^T\begin{bmatrix} \bar{J}_{1i} & \bar{M}_{1i} \\ * & \bar{J}_{1i} \end{bmatrix}\Pi_1 - e^{-2v_1 q}\Pi_2^T\begin{bmatrix} \bar{J}_{2i} & \bar{M}_{2i} \\ * & \bar{J}_{2i} \end{bmatrix}\Pi_2, \\
\bar{\Phi}_{14i} =& \begin{bmatrix} \sqrt{\lambda_{i1}}\Upsilon_i & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \sqrt{\lambda_{i(i-1)}}\Upsilon_i & 0 & 0 & 0 & 0 & 0 & 0 \\ \sqrt{\lambda_{i(i+1)}}\Upsilon_i & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \sqrt{\lambda_{iS}}\Upsilon_i & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
\bar{\Phi}_{44} =& diag\{-\Upsilon_1, \cdots, -\Upsilon_{i-1}, -\Upsilon_{i+1}, \cdots, -\Upsilon_S\}, \\
\Phi_{15} =& \begin{bmatrix} (\varepsilon_1\rho_0 + \varepsilon_2\beta_0) & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \Phi_{55} = -\frac{1}{2}(\varepsilon_1\rho_0 + \varepsilon_2\beta_0).
\end{aligned}$$

then, the system (13) is exponentially mean-square stable. Moreover, we obtain $K_i = \Upsilon_i\Upsilon_i^{-1}$, $\Xi_i = \Upsilon_i^T\Xi\Upsilon_i$ and $W_i = \Upsilon_i^TW\Upsilon_i$.

Proof of Theorem 2. Denote $\Upsilon_i = P_i^{-1}$, $G_i = \Upsilon_i G \Upsilon_i$, $G \in \{W_1, W_2, W_3, J_1, J_2, \Xi, W, N_1, N_2, M_1, M_2\}$, $Y_i = K_i\Upsilon_i$, $G_u = \{\Upsilon_i, \Upsilon_i, \Upsilon_i, \Upsilon_i, \Upsilon_i, I_n, \Upsilon_i, I_n, I_n, \hat{\Upsilon}_i, I_n\}$ and $\hat{\Upsilon}_i = diag\underbrace{\{I_n, \cdots, I_n\}}_{S-1}$. Pre- and post-multiplying (15) and (16) by G_u, we have (40) and (41). Pre- and post-multiplying (17) and (18) by G_u, respectively, then by using $-J_1^{-1} = -\Upsilon_i J_{1i}{}^{-1}\Upsilon_i \leq -2\kappa_1\Upsilon_i + \kappa_1^2 J_{1i}$, $-J_2^{-1} = -\Upsilon_i J_{2i}{}^{-1}\Upsilon_i \leq -2\kappa_2\Upsilon_i + \kappa_2^2 J_{2i}$ and Schur complement, we can obtain (42) and (43), respectively. □

Next, when the time-varying delay, packet loss and DoS occurring rate are excluded, the system (13) can be rewritten as follows:

$$\begin{cases} \dot{x}(t) = A_i x(t) + f(t,x) + u_s(t), \\ u_s(t) = B_i K_i (x(t - \gamma_k^n(t)) + e_k^n(t)), t \in \left[D_n^{on}, D_n^{off}\right), \\ \dot{x}(t) = A_i x(t) + f(t,x), t \in \left[D_n^{off}, D_{n+1}^{on}\right). \end{cases} \tag{44}$$

Then, by set the matrices $\bar{W}_{2i} = \bar{W}_{3i} = \bar{J}_{2i} = \bar{M}_{2i} = 0$ in Theorem 1, one can obtain the following Corollary.

Corollary 1. *For given positive scalars $\ell, \ell_{off}, \gamma, q, \sigma, \delta, v_1, v_2, \varepsilon_1, \varepsilon_2, \mu, \kappa_1, \kappa_2$, scalars $\alpha_0, \beta_0, \rho_0$, and $v_1 + v_2 > 0$, satisfying (7) and (14), if there exist matrices $\bar{W}_{1i}, \bar{J}_{1i}, \bar{\Xi}_i, \bar{W}_i, Y_i \in \mathbb{S}_+^n$ and matrices $\bar{M}_{1i} \in \mathbb{R}^{n\times n}$, such that the following LMIs hold:*

$$\begin{bmatrix} \bar{J}_{1i} & \bar{M}_{1i} \\ * & \bar{J}_{1i} \end{bmatrix} \geq 0, \tag{45}$$

$$\begin{bmatrix} \bar{\Phi}_{1i} & * & * & * & * \\ \gamma\hat{\Gamma}_{1i} & -(2\kappa_1 Y_i - \kappa_1^2 \bar{J}_{1i}) & * & * & * \\ q\hat{\Gamma}_{2i} & 0 & -(2\kappa_2 Y_i - \kappa_2^2 \bar{J}_{2i}) & * & * \\ \bar{\Phi}_{14i} & 0 & 0 & \bar{\Phi}_{44} & * \\ \Phi_{15} & 0 & 0 & 0 & \Phi_{55} \end{bmatrix} < 0, \tag{46}$$

$$\begin{bmatrix} \bar{\Phi}_{2i} & * & * & * & * \\ \gamma\hat{\Gamma}_{1i} & -(2\kappa_1 Y_i - \kappa_1^2 \bar{J}_{1i}) & * & * & * \\ q\hat{\Gamma}_{2i} & 0 & -(2\kappa_2 Y_i - \kappa_2^2 \bar{J}_{2i}) & * & * \\ \bar{\Phi}_{14i} & 0 & 0 & \bar{\Phi}_{44} & * \\ \Phi_{15} & 0 & 0 & 0 & \Phi_{55} \end{bmatrix} < 0, \tag{47}$$

where

$$\begin{aligned} \bar{\Phi}_{1i} =& sym\left\{\bar{e}_1^T \hat{\Gamma}_{1i}\right\} + \bar{e}_1^T (2v_1 Y_i + \lambda_{ii} Y_i)\bar{e}_1 + \bar{\Phi}_{0i} + \hat{\Phi}_i, \\ \bar{\Phi}_{2i} =& sym\left\{\bar{e}_1^T \hat{\Gamma}_{2i}\right\} + \bar{e}_1^T (-2v_2 Y_i + \lambda_{ii} Y_i)\bar{e}_1 + \bar{\Phi}_{0i}, \\ \bar{\Phi}_{0i} =& sym\left\{(\varepsilon_2\alpha_0 - \varepsilon_1)\bar{e}_1^T Y_i \bar{e}_6 - \varepsilon_2 \bar{e}_6^T \bar{e}_6\right\} \\ & - e^{-2v_1\gamma}\bar{e}_5^T \bar{W}_{1i}\bar{e}_5 + \bar{e}_1^T(\bar{W}_{1i})\bar{e}_1 + \bar{e}_4^T \Xi_i \bar{e}_4, \\ \hat{\Phi}_i =& -\bar{e}_7^T \bar{W}_i \bar{e}_7 + \sigma(\bar{e}_4 + \bar{e}_7)^T \bar{W}_i(\bar{e}_4 + \bar{e}_7), \\ \bar{\Phi}_{14i} =& \begin{bmatrix} \sqrt{\lambda_{i1}} Y_i & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \sqrt{\lambda_{i(i-1)}} Y_i & 0 & 0 & 0 & 0 & 0 & 0 \\ \sqrt{\lambda_{i(i+1)}} Y_i & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \sqrt{\lambda_{iS}} Y_i & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\ \bar{\Phi}_{44} =& diag\{-Y_1, \cdots, -Y_{i-1}, -Y_{i+1}, \cdots, -Y_S\}, \\ \Phi_{15} =& \begin{bmatrix} (\varepsilon_1\rho_0 + \varepsilon_2\beta_0) & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \Phi_{55} = -\frac{1}{2}(\varepsilon_1\rho_0 + \varepsilon_2\beta_0). \end{aligned}$$

then, the system (13) is exponentially mean-square stable. Moreover, we obtain $K_i = Y_i Y_i^{-1}$, $\Xi_i = Y_i^T \Xi Y_i$ and $W_i = Y_i^T W Y_i$.

4. Numerical Example

Example 1. *Consider the system* (13) *with the following parameters:*

$$A_1 = \begin{bmatrix} -1 & 0 \\ 0.8 & -1.4 \end{bmatrix}, A_{q1} = \begin{bmatrix} 0.85 & 1.75 \\ 0 & -1.6 \end{bmatrix}, B_1 = \begin{bmatrix} 0.7 \\ 0.4 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 1 & 0.8 \\ 0 & -1.2 \end{bmatrix}, A_{q2} = \begin{bmatrix} -0.49 & 0 \\ -1.45 & -0.19 \end{bmatrix}, B_2 = \begin{bmatrix} 0.4 \\ 0.7 \end{bmatrix},$$

and other parameters are given as $\ell = 2$, $h = 0.05$, $v_1 = 0.2$, $v_2 = 0.5$, $\sigma = 0.1$, $\delta = 0.5$, $\varepsilon_1 = 0.1$, $\varepsilon_2 = 0.6$, $\alpha_0 = -0.2$, $\beta_0 = 0.1$, $\rho_0 = -0.4$, $\mu = 0.68$, $\kappa_1 = 0.17$, $\kappa_2 = 0.15$, $q = 0.02$, *the attack success rate and packet loss rate are set as* $\varsigma_s = 0.7$, $\varsigma_a = 0.3$, *respectively. The mode transition rate is given as* $\Pi_\lambda = \begin{bmatrix} -3 & 3 \\ 5 & -5 \end{bmatrix}$.

Based on the above parameters, by using the LMI toolbox of Matlab 2018a to solve the LMIs in Theorem 2, we can obtain: $\ell_{off} = 1.7s$ and

$$K_1 = [-5.8501 \quad -3.0855], K_2 = [-4.5665 \quad -2.7979],$$
$$W_1 = \begin{bmatrix} 75.0127 & 16.2939 \\ 16.2939 & 37.2556 \end{bmatrix}, W_2 = \begin{bmatrix} 38.0562 & 14.8402 \\ 14.8402 & 41.0452 \end{bmatrix},$$
$$\Xi_1 = \begin{bmatrix} 5.4950 & -0.0791 \\ -0.0791 & 7.4208 \end{bmatrix}, \Xi_2 = \begin{bmatrix} 3.4657 & 1.5007 \\ 1.5007 & 7.6517 \end{bmatrix}.$$

Furthermore, let the nonlinear function $f(t,x) = -0.2\sin(-1.2x(t))$, the initial condition $x(0) = col\{0.5, -0.5\}$, $\phi_0 = 2$, combining with the above gain matrices, the simulation results of systems (13) are given in the following figures. As shown in Figure 2, system (13) cannot achieve stability without control. As shown in Figure 3, the system state gradually reaches the stable state under DETM. Figure 4 is the control input of system (13) under DETM. Figure 5 shows the relationship between triggered instants and intervals. In summary, this example demonstrates that DETM (5) can not only stabilize the system under the influence of DoS attack and packet loss, but also alleviate network communication pressure to a certain extent.

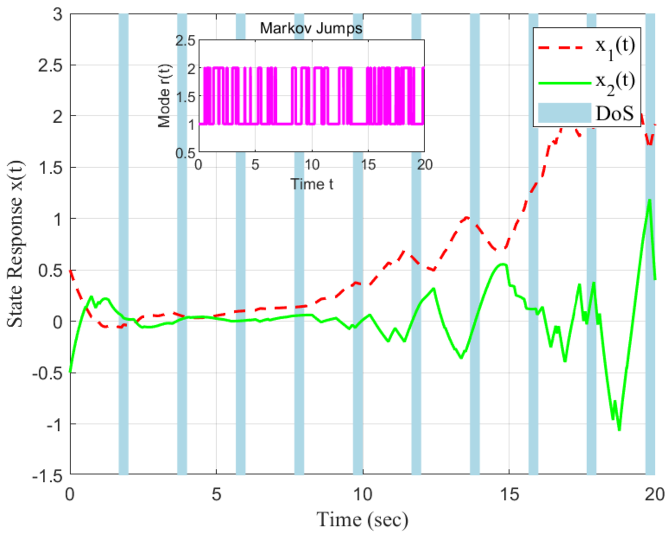

Figure 2. The state response of system (13) without control.

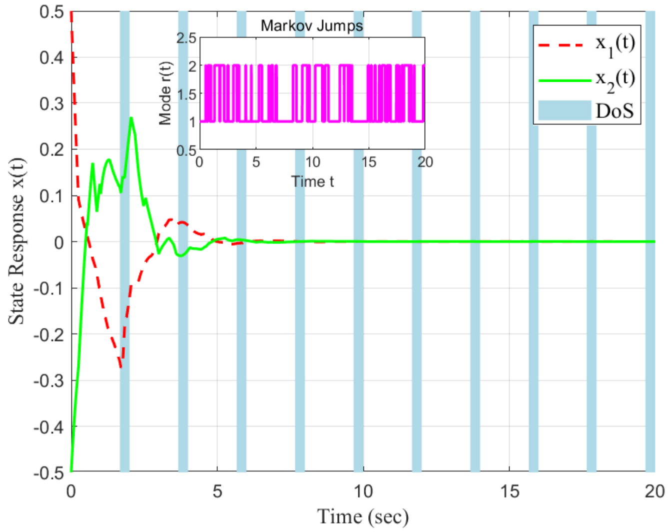

Figure 3. The state response of system (13) under DETM.

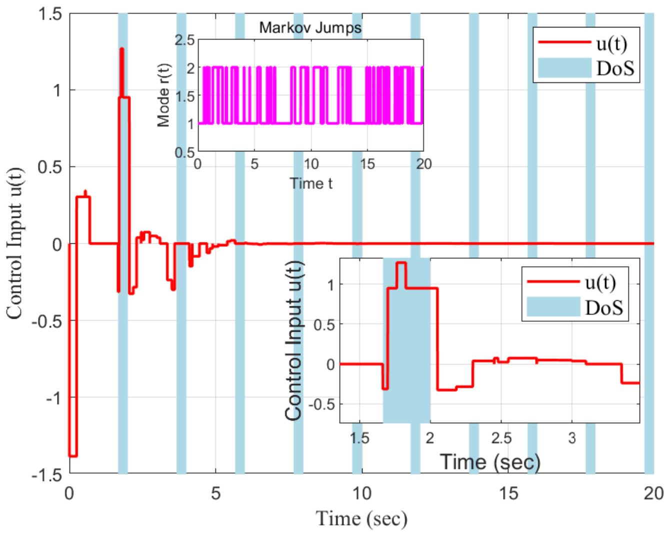

Figure 4. The control input of system (13) under DETM.

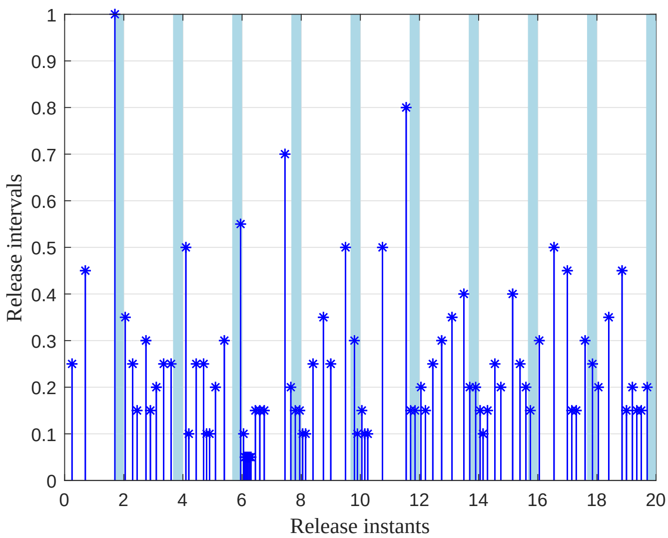

Figure 5. The release instants and intervals of DETM.

Example 2. *Consider the system* (44) *with the following parameters [25]:*

$$A_1 = \begin{bmatrix} -1.4 & 0 \\ 1 & -1 \end{bmatrix}, B_1 = \begin{bmatrix} 0.8 \\ 0.5 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 0.5 \\ 0 & -1.5 \end{bmatrix}, B_2 = \begin{bmatrix} 0.5 \\ 0.8 \end{bmatrix},$$

and other parameters are given as $\ell = 2, h = 0.1, v_1 = 0.1, v_2 = 0.7, \sigma = 0.1, \delta = 0.5, \varepsilon_1 = 0.1,$ $\varepsilon_2 = 0.6, \alpha_0 = -0.3, \beta_0 = 0.1, \rho_0 = -0.3, \mu = 0.1,$*, respectively. The mode transition rate is given as* $\Pi_\lambda = \begin{bmatrix} -3 & 3 \\ 5 & -5 \end{bmatrix}$*. It is worth noting that the above parameters come from a practical system, named robot arm system of single-link rigid robot. The simplified diagram is shown in Figure 6 and the detailed model transformation can be found in [25].*

Based on the above parameters, by using the Matlab LMI toolbox to solve the LMIs in Theorem 1, we can obtain: $\ell_{off} = 1.5s$ and

$$\begin{aligned} K_1 =& [-1.2103 \quad -0.5742], K_2 = [-2.3486 \quad -0.7193], \\ W_1 =& \begin{bmatrix} 13.2459 & 2.3973 \\ 2.3973 & 9.9921 \end{bmatrix}, W_2 = \begin{bmatrix} 10.8423 & 1.8592 \\ 1.8592 & 10.0822 \end{bmatrix}, \\ \Xi_1 =& \begin{bmatrix} 1.2945 & -0.5230 \\ -0.5230 & 2.2985 \end{bmatrix}, \Xi_2 = \begin{bmatrix} 1.1017 & -0.3870 \\ -0.3870 & 2.8863 \end{bmatrix}. \end{aligned}$$

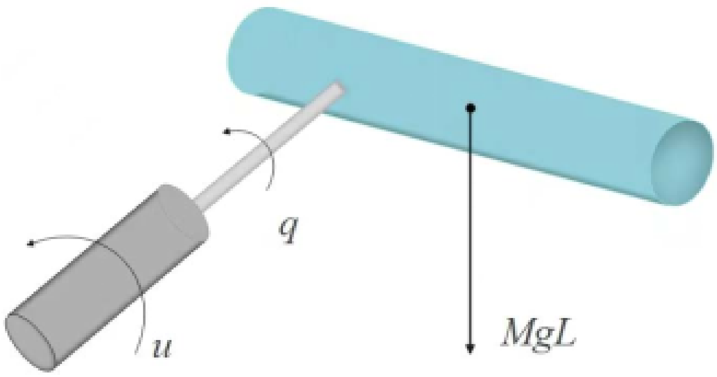

Figure 6. Robot arm system of single-link rigid robot.

Furthermore, let the nonlinear function $f(t, x) = 0.6 \sin(x(t))$, the initial condition $x(0) = col\{0.3, -0.5\}$, $\phi_0 = 2$, combining with the above gain matrices, the simulation results of systems (13) are given in the following figures. As shown in Figure 7, system (44) cannot achieve stability without control. As shown in Figures 8 and 9, the system state both gradually reaches the stable state under DETM and SETM, respectively. Figures 10 and 11 are the control input of system (44) under DETM and SETM, respectively. Figures 12 and 13 show the relationship between triggered instants and intervals of DETM and SETM, respectively. In summary, this example demonstrates that DETM (5) can not only stabilize the system under the influence of DoS attack, but also alleviate network communication pressure to a larger extent.

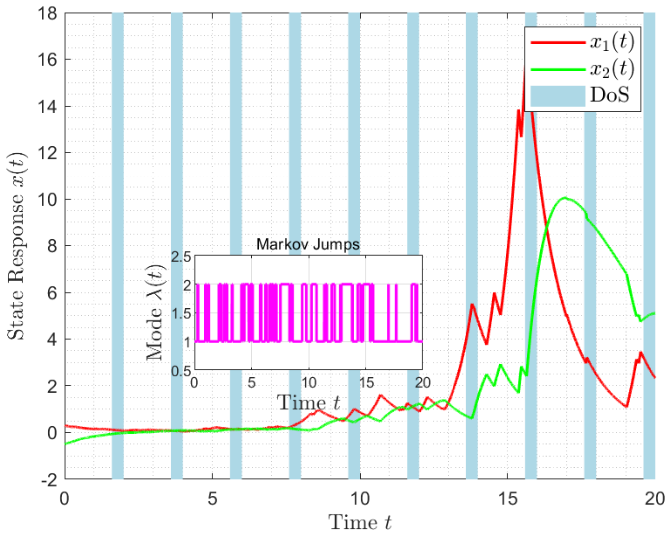

Figure 7. The state response of system (44) without control.

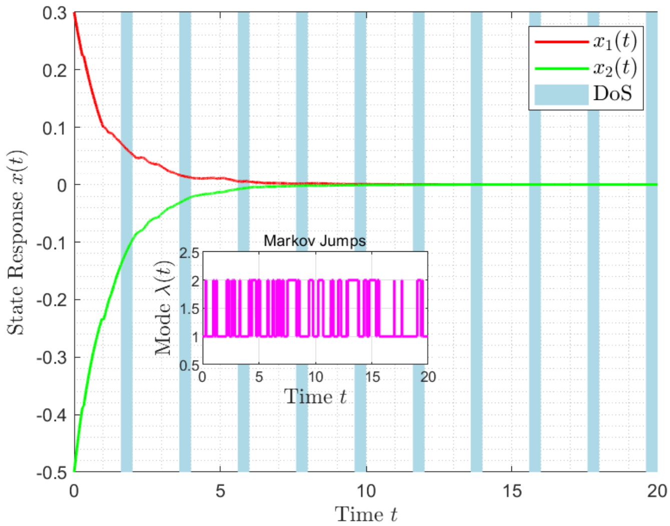

Figure 8. The state response of system (44) under DETM.

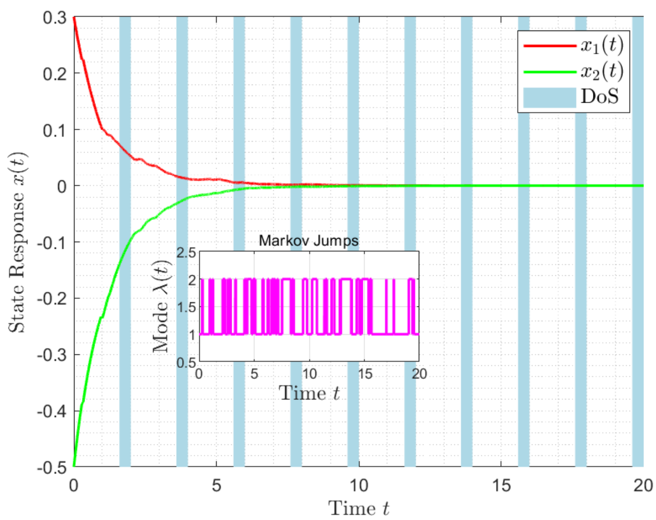

Figure 9. The state response of system (44) under SETM.

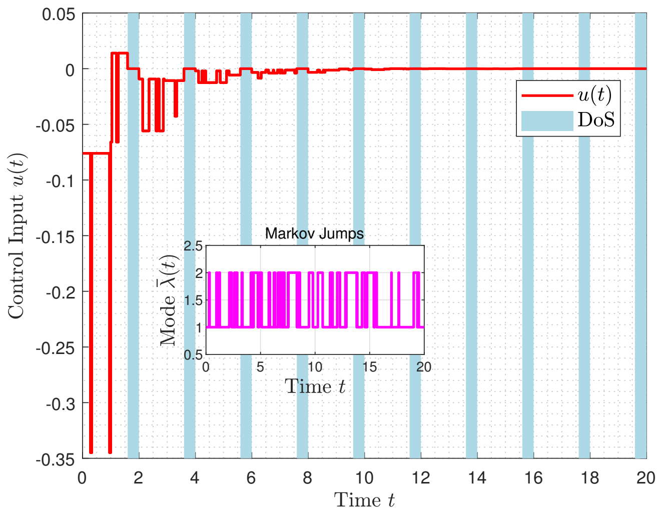

Figure 10. The control input of system (44) under DETM.

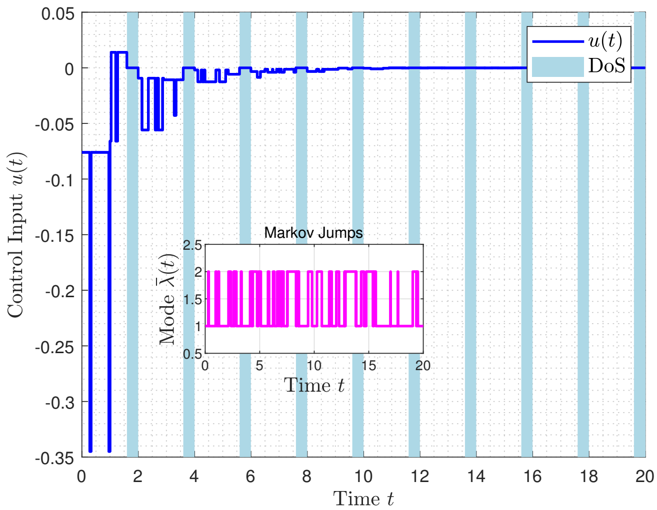

Figure 11. The control input of system (44) under SETM.

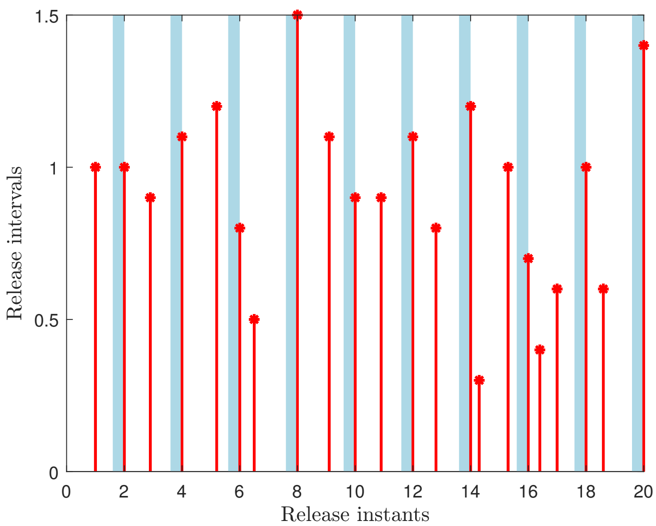

Figure 12. The release instants and intervals of DETM.

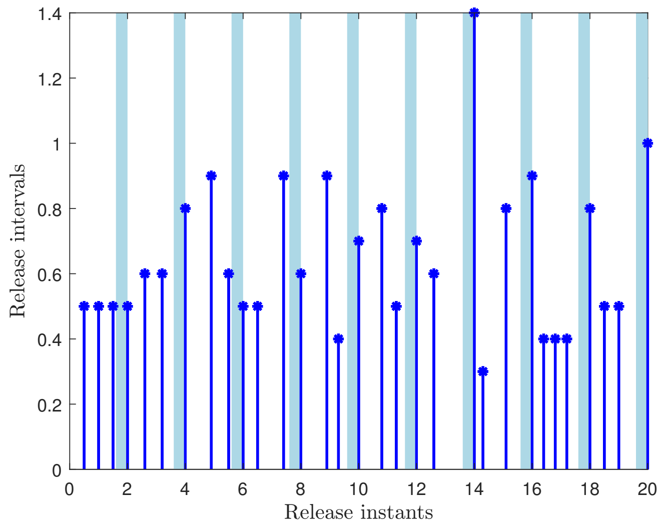

Figure 13. The release instants and intervals of SETM.

5. Conclusions

This paper has studied the exponential stability and stabilization problems of a class of DNMJSs under randomly occurring DoS attacks and packet loss. The attack success rate and packet loss rate have been introduced to describe the stochastic characteristics of DoS attacks and packet loss. The POW method has been proposed to depict the switching property of DoS attacks and packet loss. Furthermore, the hybrid-input strategy have been adopted to compensate for the impacts of DoS attacks and packet loss on the systems. By constructing a general common Lyapunov functional, and combining with the DETM, and other analysis approaches, the less conservative stability criteria are derived in the form of linear matrix inequality. Finally, a numerical example and a practical example were used to verify the validity of our results. In the future, the results obtained in this paper shall be applied to investigate other analyses and syntheses problems of Markov jump systems under cyber attacks and packet loss.

Author Contributions: Conceptualization, H.Z.; methodology, L.X.; software, Y.Z.; validation, H.C.; investigation, writing—original draft preparation, H.C.; writing—review and editing, H.Z.; visualization, Y.Z.; supervision, L.X. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the National Nature Science Foundation of China under Grants 12061088, 12261104, and 61833005, the Basic Research Youth Fund Project of Yunnan Science and Technology Department under Grant 202201AU070046, the Scientific Research Fund Project of Yunnan Provincial Department of Education under Grant 2022J0447.

Data Availability Statement: Data are contained within the article.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Zhang, H.; Qiu, Z.; Xiong, L.; Jiang, G. Stochastic stability analysis for neutral-type Markov jump neural networks with additive time-varying delays via a new reciprocally convex combination inequality. *Int. J. Syst. Sci.* **2019**, *50*, 970–988. [CrossRef]
2. Liu, X.; Zhang, H.; Yang, J.; Chen, H. Stochastically exponential synchronization for Markov jump neural networks with time-varying delays via event-triggered control scheme. *Adv. Differ. Equ.* **2021**, *2021*, 74. [CrossRef]
3. Sun, K.; Wang, Y.; Zhuang, G.; Wang, J. Asynchronous secure controller design for singularly perturbation stochastic semi-Markov jump CPSs with the memory-based dynamic event-triggered scheme against complex cyber-attacks. *Commun. Nonlinear Sci. Numer. Simul.* **2023**, *125*, 107408. [CrossRef]
4. Xie, W.; Zeng, Y.; Shi, K.; Wang, X.; Fu, Q. Hybrid event-triggered filtering for nonlinear Markov jump systems with stochastic cyber-attacks. *IEEE Access* **2020**, *9*, 248–258. [CrossRef]
5. Liu, X.; Zhang, H.; Wu, T.; Shu, J. Stochastic exponential stabilization for Markov jump neural networks with time-varying delays via adaptive event-triggered impulsive control *Complexity* **2020**, *2020*, 3956549. [CrossRef]

6. Yang, C.; Yao, W.; Wang, Y.; Ai, X. Resilient event-triggered load frequency control for multi-area power system with wind power integrated considering packet losses. *IEEE Access* **2021**, *9*, 78784–78798. [CrossRef]
7. Zhao, S.; Zhang, H.; Xiong, L.; Wen, S.; Cao, J.; Zhang, Y. Resilient adaptive event-triggered synchronization control of piecewise-homogeneous Markov jump delayed neural networks under aperiodic DoS attacks *Int. J. Robust Nonlinear Control.* **2022**, *34*, 1493–1521. [CrossRef]
8. Zhou, Y.; Ji, X. Static Output Feedback Control for Nonlinear Time-Delay Semi-Markov Jump Systems Based on Incremental Quadratic Constraints. *Math. Comput. Appl.* **2023**, *28*, 30. [CrossRef]
9. Li, H.; Shi, P.; Yao, D. Adaptive sliding-mode control of Markov jump nonlinear systems with actuator faults. *IEEE Trans. Autom. Control* **2016**, *62*, 1933–1939. [CrossRef]
10. Park, P.; Lee, W.I.; Lee, S.Y. Auxiliary function-based integral inequalities for quadratic functions and their applications to time-delay systems. *J. Frankl. Inst.* **2015**, *352*, 1378–1396. [CrossRef]
11. Su, L.; Ye, D. Observer-based output feedback H_∞ control for cyber-physical systems under randomly occurring packet dropout and periodic DoS attacks. *ISA Trans.* **2019**, *95*, 58–67. [CrossRef]
12. Lu, R.; Shi, P.; Su, H.; Wu, Z.G.; Lu, J. Synchronization of general chaotic neural networks with nonuniform sampling and packet missing: A switched system approach. *IEEE Trans. Neural Netw. Learn. Syst.* **2016**, *29*, 523–533. [CrossRef]
13. Li, L.; Zhang, G.; Ou, M. A New Method to Non-Fragile Piecewise H_∞ Control for Networked Nonlinear Systems with Packet Dropouts. *IEEE Access* **2020**, *8*, 196102–196111. [CrossRef]
14. Qiu, L.; Yao, F.; Zhong, X. Stability analysis of networked control systems with random time delays and packet dropouts modeled by Markov chains. *J. Appl. Math.* **2013**, *2013*, 715072. [CrossRef]
15. Zhang, Y.; Xie, S.; Ren, L.; Zhang, L. A new predictive sliding mode control approach for networked control systems with time delay and packet dropout. *IEEE Access* **2019**, *7*, 134280–134292. [CrossRef]
16. Huang, L.; Guo, J.; Li, B. Observer-based dynamic event-triggered robust H_∞ control of networked control systems under DoS attacks. *IEEE Access* **2021**, *9*, 145626–145637. [CrossRef]
17. Li, H.; Li, X.; Zhang, H. Optimal control for discrete-time NCSs with input delay and Markovian packet losses: Hold-input case. *Automatica* **2021**, *132*, 109806. [CrossRef]
18. Qu, H.; Zhao, J. Stabilisation of switched linear systems under denial of service. *IET Control Theory Appl.* **2020**, *14*, 1438–1444. [CrossRef]
19. Ran, G.; Li, C.; Sakthivel, R.; Han, C.; Wang, B.; Liu, J. Adaptive event-triggered asynchronous control for interval type-2 fuzzy Markov jump systems with cyber attacks. *IEEE Trans. Control Netw. Syst.* **2022**, *9*, 88–99. [CrossRef]
20. Hou, M.; Liu, D.; Ma, Y. Adaptive event-triggered control of Markovian jump complex dynamic networks with actuator faults. *Neurocomputing* **2022**, *491*, 273–287. [CrossRef]
21. Hu, Z.; Mu, X. Event-triggered impulsive control for stochastic networked control systems under cyber attacks. *IEEE Trans. Syst. Man, Cybern. Syst.* **2021**, *52*, 5636–5645. [CrossRef]
22. Liu, B.; Sun, Z.; Li, M.; Liu, D.N. Stabilization via event-triggered impulsive control with constraints for switched stochastic systems. *IEEE Trans. Cybern.* **2021**, *52*, 11834–11846. [CrossRef] [PubMed]
23. Zhang, H.; Chen, Z.; Ao, W.; Shi, P. Improved Dynamic Event-Triggered Robust Control for Flexible Robotic Arm Systems with Semi-Markov Jump Process. *Sensors* **2023**, *23*, 5523. [CrossRef] [PubMed]
24. Lin, Y.; Shi, T. Static Output Feedback l_2-l_∞ Asynchronous Control of Markov Jump Systems under Dynamic Event-Triggered Scheme. *IEEE Access* **2022**, *10*, 97748–97757. [CrossRef]
25. Zhao, N.; Shi, P.; Xing, W. Dynamic event-triggered approach for networked control systems under denial of service attacks. *Int. J. Robust Nonlinear Control* **2021**, *31*, 1774–1795. [CrossRef]