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Barrier Option Under Lévy Model : A PIDE and Mellin Transform Approach

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Abstract: We propose a stochastic model to develop a partial integro-differential equation (PIDE) for pricing and pricing expression for fixed type single Barrier options based on the Itô-Lévy calculus with the help of Mellin transform. The stock price is driven by a class of infinite activity Lévy processes leading to the market inherently incomplete, and dynamic hedging is no longer risk free. We first develop a PIDE for fixed type Barrier options, and apply the Mellin transform to derive a pricing expression. Our main contribution is to develop a PIDE with its closed form pricing expression for the contract. The procedure is easy to implement for all class of Lévy processes numerically. Finally, the algorithm for computing numerically is presented with results for a set of Lévy processes.

Keywords: Barrier option pricing; Lévy process; numerical inverse Mellin transform; simulation

1. Introduction

Barrier options are derivatives with a pay-off that depends on whether a reference entity has crossed a certain boundary. Common examples are the knock-in and knock-out call and put options that are activated or deactivated when the underlying crosses a specified Barrier-level. Barrier and Barrier-type options belong to the most widely traded exotic options in the financial markets.

A class of models that has been shown to be capable of generating a good fit of observed call and put option price data is formed by the infinite activity Lévy models, such as normal inverse Gaussian, CGMY and Meixner. This class of models has been extensively studied and we refer for background and further references to the book by [1]. In this paper, we consider Barrier options driven by Lévy processes with infinite activity. This class contains many of the Lévy models used in financial modelling as the fore-mentioned ones.

Several approaches have been proposed during the last few years. The calculation of first-passage distributions and Barrier option prices in (specific) Lévy models has been investigated in a number of papers. In [2], the authors proposed a Laplace transformed based approach to compute the prices and greeks of Barrier options for a class of Lévy process with Wiener-Hopf factorisation. The authors of [3] calculated prices and deltas of double Barrier options under the Black-Scholes model. For spectrally one-sided Lévy processes with a Gaussian component [4] derived a method to evaluate first-passage distributions. The authors of [5–7] followed a transform approach to obtain Barrier prices for a jump-diffusion with exponential jumps. In the setting of infinite activity Lévy processes with jumps in two directions Cont and [8] investigated discretisation of the associated integro-differential equations. In [9], the author employed Fourier methods to investigate Barrier option prices for Lévy processes of regular exponential type. These approaches are based on exponential Lévy process with a risk neutral measure considering a complete market, involving extremely complex techniques and applicable for a specific class of Lévy process.

Summarizing all the issues in the previous work, we find a few challenges in pricing the Barrier option under Lévy processes. First of all, the Lévy market is incomplete and more than one measure exists leading to multiple prices for a single contract and hedging is not possible. Therefore, the pricing model requires the selection of the correct measure from the market and finding market price of risk with the help of market price available by calibration method with better goodness of fit. Secondly, as the distribution of the underlying stock prices is unknown, in general no explicit analytical expression is available. Finally, it is also difficult to derive a closed form expression of the contract. Our model is proposed to take care of all the challenges. The approach first developed a PIDE for pricing and solved it using Mellin transform and its inverse. In [10], the author proposed a similar method for Asian options of arithmetic type but used Fourier transform instead of Mellin transform. The advantage of our model is that it has a closed form expression of the Mellin transform applicable for any class of Lévy processes and the standard inverse Mellin transform can be applied to construct prices. The Mellin transform based method for option pricing was proposed earlier by [11–13] for pricing American options.

The organization of different sections in this paper is as follows. Section 2 recalls some basic facts about exponential Lévy processes and provides a model used in this paper. Section 3 derives the partial integro-differential equation (PIDE) for the option pricing of Barrier options. It also provides a pricing formula in terms of the inverse Mellin transform. Numerical results are provided in Section 4 and a brief conclusion is provided in Section 5.

2. Model with Lévy Processes

We denote the stock-price of the underlying asset at a given time t by $S(t)$. It is well known that contrary to the Brownian process the log-return of stock-price (that is, $\log(S(t))$) is neither Gaussian, nor homogeneous and it does *not* have independent increments (see, e.g., [14]). Thus, we study the return considering the stock price as the exponential Lévy process described by the following equations:

$$\begin{aligned} S(t) &= S(0)e^{Z(t)}, \\ dZ(t) &= \mu dt + \sigma dW(t) + \int_{\mathbb{R}} x\widetilde{N}(dt, dx) \end{aligned} \tag{1}$$

with $\widetilde{N}(dt, dx) = N(dt, dx) - \nu(dx)dt$, where N is the jump measure of Z and $W(t)$ is the Brownian motion. The Lévy triplet for Z is (μ, σ^2, ν) with respect to some measure $\mathbb{P}$.

For convenience, we assume $S(0) = 1$ for the rest of the paper. The parameters σ, and μ are called the *volatility* and *drift* of stock price respectively. We assume that $Z(t)$ has finite moments $\int_{|x|\geq 1} |x|^p \nu(dx) < \infty$, for all positive integer p (see [15]). The examples of such a class of Lévy processes are the infinite activity processes like VG, NIG, CGMY, Meixner processes. Some of these processes are described in Appendix B. Details of these processes are also described in [1].

We briefly describe the procedure of finding the equivalent martingale measure. All the details are provided in the Appendix A. To find an equivalent martingale measure $\mathbb{Q}$ for the stock-price process $S(t)$, let Y be a Lévy type stochastic integral of the form

$$dY(t) = G(t)dt + F(t)W(t) + \int_{\mathbb{R}-\{0\}} H(t, x)\widetilde{N}(ds, dx)$$

where $\sqrt{G(t)}, F(t) \in \mathcal{P}_2(t)$ and $H \in \mathcal{P}_2(t, \mathbb{R} - \{0\})$ for each $t \geq 0$ (where $\mathcal{P}_2$ is defined in the Appendix A). The equivalent martingale measure $\mathbb{Q}$, on a fixed time interval $[0, T]$, satisfies $\frac{d\mathbb{Q}}{d\mathbb{P}} = e^{Y(T)}$, for $0 \leq t \leq T$.

Clearly, the Lévy triplet of Z with respect to $\mathbb{Q}$ in terms of the Lévy triplet with respect to $\mathbb{P}$ is given by

$$\left(\mu_Q, \sigma^2, e^{H(t,x)}\nu(dx)\right), \quad \mu_Q = \mu + \sigma F(t) + \int_{\mathbb{R}} x(e^{H(t,x)} - 1)\nu(dx) \tag{2}$$

We make the following assumption related to the nature of the function $H(t, x)$.

Assumption 1. $\int_{|x|\geq 1} e^x \nu_Q(dx) = \int_{|x|\geq 1} e^{x+H(t,x)}\nu(dx) < \infty$.

Therefore, with respect to the equivalent martingale measure $\mathbb{Q}$, the dynamics of $S(t)$ is given by

$$\frac{dS(t)}{S(t-)} = r\,dt + \sigma dW_Q(t) + \int_{\mathbb{R}} (e^x - 1)\widetilde{N}_Q(dt, dx) \tag{3}$$

It is clear from Equations (2) and (A3) that there are non-unique ways (depending on various choices of $F(t)$ and $H(t, x)$) of selecting density function Y. The choice for the equivalent martingale measure $\mathbb{Q}$ in this paper will be the *Föllmer-Schweizer minimal measure* which minimizes the quadratic risk of the associated cost function. In this procedure there is an unique measure $\mathbb{Q}$ for which $\frac{d\mathbb{Q}_t}{d\mathbb{P}_t} = e^{Y(t)}$, so that

$$d(e^{Y(t)}) = e^{Y(t)}\;P(t)\left(\sigma dW(t) + \int_{\mathbb{R}} x\widetilde{N}(dt, dx)\right)$$

for some adapted process $P(t)$ which satisfies

$$\sigma P(t) = F(t), \quad xP(t) = e^{H(t,x)} - 1$$

for $t \geq 0$ and $x \in \mathbb{R}$. We define

$$\rho_1 = \int_{\mathbb{R}} x^2\nu(dx), \quad \rho_2 = \int_{\mathbb{R}} x(e^x - 1 - x1_{|x|<1})\nu(dx), \quad \rho_3 = \int_{\mathbb{R}} (e^x - 1 - x1_{|x|<1})\nu(dx)$$

Then we obtain the following expression from Equation (A3).

$$P(t) = \frac{r - \mu - \frac{\sigma^2}{2}}{1 + \rho_1 + \rho_2 + \rho_3} = \rho \tag{4}$$

We note that given r and the Lévy triplet of Z with respect to measure $\mathbb{P}$, *i.e.*, (μ, σ^2, ν), Equation (4) gives a constant function , for $P(t) = \rho$. Thus, $F(t) = \sigma\rho$ is also constant. On the other hand, $H(t, x)$ is a function of x alone and it is given by $H(t, x) = \log(1 + \rho x)$. Consequently, the Lévy density $\nu_Q(dx) = (1 + \rho x)\nu(dx)$. The derived parameter ρ is also known as the *market price of risk* for the Lévy market.

In [16] it is shown that this method coincides with the general procedure described by Föllmer and Schweitzer (see [17]) which works by constructing a replicating portfolio of value $V(t) = \alpha(t)\;S(t) + \beta(t)\;\mathcal{W}(t)$ and discounting it to obtain $\widetilde{V}(t) = \alpha(t)\widetilde{S}(t) + \beta(t)\mathcal{W}(0)$. If we now define the cumulative cost $C(t) = \widetilde{V}(t) - \int_0^t \alpha(t)d\widetilde{S}(s)$, then $\mathbb{Q}$ minimizes the risk $E\left[\left(C(T) - C(t)\right)^2|\mathcal{F}_t\right]$.

3. Pricing Barrier Options

In this section, we present two main theorems related to single Barrier options. Let S be the stock price and B is a fixed single Barrier. In general, there are four different types of Barrier options according to the payoff functions. Let T be the time of expiry of the option. For *fixed strike* (K) call and put Up-And-Out Barrier options payoffs are given by $(S - K)^+, 0 \leq S \leq B$ and $(K - S)^+, 0 \leq S \leq B$ respectively. For *fixed strike* call and put Down-And-Out Barrier options the payoffs are given by $(S - K)^+, B \leq S$ and $(K - S)^+, B \leq S$ respectively. In this section, we develop a technique for pricing fixed strike call for both Up-And-Out and Down-And-Out options. Option pricing for other type options can be done by a very similar procedure. We first show that the price of the both Up-And-Out and Down-And-Out Barrier option is given by a PIDE.

For the convenience of notation, in this section, we write simply W and $\widetilde{N}$ in lieu of W_Q and $\widetilde{N}_Q$ respectively. Since in this section we mostly work with the equivalent martingale measure $\mathbb{Q}$ this abuse of notation will not create any confusion. However, we will keep the notation for the Lévy density with respect to $\mathbb{P}$ and $\mathbb{Q}$ as the same as in the previous section, viz. ν and ν_Q respectively. For the Föllmer Schweizer minimal equivalent martingale measure $\mathbb{Q}$,

$$\nu_Q(dx) = (1+\rho x)\nu(dx)$$

where ρ is given by Equation (4). Also, assume the Lévy density corresponding to Lévy measures ν_Q and ν are denoted as $w_Q(x)$ and $w(x)$ respectively. Thus for the Föllmer Schweizer case

$$w_Q(x) = (1+\rho x)w(x) \tag{5}$$

Theorem 1. *The price of Up-And-Out and Down-And-Out Barrier call option* $C(t, S(t))$, *where the stock-price dynamics is described by Equation* (1), *is given by*

$$\begin{aligned} \frac{\partial C(t,S)}{\partial t} &+ rS\frac{\partial C}{\partial S}(t,S) + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2}(t,S) - rC(t,S) \\ &+ \int_{\mathbb{R}} \nu_Q(dx)\left[C(t, Se^x) - C(t,S) - S(e^x-1)\frac{\partial C}{\partial S}(t,S)\right] = 0 \end{aligned} \tag{6}$$

with final condition

$$C(T,S) = (S-K)^+, 0 \le S \le B \text{ for Up-And-Out option} \tag{7}$$

$$= (S-K)^+, B \le S < \infty \text{ for Down-And-Out option} \tag{8}$$

Proof. Under an equivalent martingale measure $\mathbb{Q}$, the Up-And-Out and Down-And-Out Barrier call option can be written as

$$C(t, S(t)) = e^{-r(T-t)} E_{\mathcal{Q}}\left[H(S_T)|\mathcal{F}_t\right]$$

where

$$\begin{aligned} H(S_t) &= \Big(S(t)-K\Big)^+ \mathbb{1}_{S(t)\le B} \text{ for Up-And-Out option} \\ &= \Big(S(t)-K\Big)^+ \mathbb{1}_{S(t)\ge B} \text{ for Down-And-Out option} \end{aligned}$$

From the dynamics of the stock price under $\mathbf{Q}$ is given by Equation (3). We define the continuous part and jump of $S(t)$ by

$$dS^c(t) = S(t-)rdt + \sigma S(t-)dW(t)$$

and

$$\Delta S = S(t) - S(t-)$$

respectively.
The continuous part of S(t) is defined to be

$$dS^c(t) \quad = \quad rS(t)dt + \sigma S(t)dW(t)$$

Now $S(t)$ has a smooth C^2 density with derivative vanishing at infinity and so $C(t, S(t))$ is a smooth function of S and we can apply Itô formula.Let us consider $S(t) = S$ and $\widetilde{C}(t, S(t)) = e^{r(T-t)}C(t, S(t))$ and if we can apply Itô's formula to this function,

$$\begin{aligned} d\widetilde{C}(t, S(t)) &= e^{r(T-t)}\left[\frac{\partial C}{\partial t} + rS\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 C}{\partial S^2} - rC \right. \\ &\left. + \int_{\mathbb{R}}\left(C(t, Se^x) - C(t, S) - (e^x - 1)S\frac{\partial C}{\partial S}\right)\nu_Q(dx)\right] dt \\ &+ e^{r(T-t)}\frac{\partial C}{\partial S}\sigma S dW(t) \\ &+ e^{r(T-t)}\int_{\mathbb{R}}\left\{C(t, Se^x) - C(t, S)\right\}\widetilde{N}(dt, dx) \\ &= a(t)dt + dM(t) \end{aligned}$$

where

$$\begin{aligned} a(t) &= e^{r(T-t)}\left[\frac{\partial C}{\partial t} + rS\frac{\partial C}{\partial S} + \frac{1}{2}S^2\sigma^2\frac{\partial^2 C}{\partial S^2} - rC \right. \\ &+ \left. \int_{\mathbb{R}}\left(C(t, Se^x) - C(t, S) - S(e^x - 1)\frac{\partial C}{\partial S}\right)\nu_Q(dx)\right] \end{aligned}$$

and

$$dM(t) = e^{r(T-t)}\frac{\partial C}{\partial S}\sigma S dW(t) + e^{r(T-t)}\int_{\mathbb{R}}\left\{C(t, Se^x) - C(t, S)\right\}\widetilde{N}(dt, dx)$$

Clearly, $M(t)$ is a Martingale. By construction $\widetilde{C}(t, S(t)) = E[H(S(t))|\mathcal{F}_t]$ and $M(t)$ both are martingales, then $\widetilde{C}(t, S(t)) - M(t)$ is also a martingale. But $\widetilde{C}(t, S(t)) - M(t) = \int_0^t a(s)ds$ is a continuous process with finite variation. Thgerefore, we must have $a(t) = 0$ almost surely. Thus, we obtain the partial integro-differential equation (PIDE),

$$\begin{aligned} \frac{\partial C(t, S)}{\partial t} &+ rS\frac{\partial C}{\partial S}(t, S) + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 C}{\partial S^2}(t, S) - rC(t, S) \\ &+ \int_{\mathbb{R}}\nu_Q(dx)\left[C(t, Se^x) - C(t, S) - S(e^x - 1)\frac{\partial C}{\partial S}(t, S)\right] = 0 \end{aligned} \tag{9}$$

for $0 \le t \le T$ and $0 < S < \infty$ and $C(t, S) \to \infty$ as $S \to \infty$ with the boundary conditions are

Up and Out Barrier Option

$$\begin{aligned} &C(t, 0) = 0, 0 \le t \le T, \\ &C(t, B) = 0, 0 \le t < T \\ &C(T, S) = (S - K)^+, 0 \le S \le B \end{aligned}$$

Down and Out Barrier Option

$$\begin{aligned} &C(t, 0) = 0, 0 \le t \le T \\ &C(t, B) = 0, 0 \le t < T \\ &C(T, S) = (S - K)^+, B \le S < \infty \end{aligned}$$

□

Theorem 2. *The Mellin transform of the price of Barrier option* $C(t, S(t))$ *is given by*

$$C(t, S(t)) = S\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\left(\frac{K}{S}\right)^{-\eta}\left[H(\eta)e^{\psi(\eta)(T-t)}\right]d\eta \tag{10}$$

with

$$H(\eta) = \begin{cases} \frac{1}{\eta(\eta+1)} - \left[\frac{(K/B)^{\eta}}{\eta} - \frac{(K/B)^{\eta+1}}{\eta+1}\right] & \textit{for Up-And-Out option} \\ \frac{(\frac{K}{B})^{\eta}}{\eta} - \frac{(\frac{K}{B})^{\eta+1}}{\eta+1}, \text{ if } \frac{K}{B} \leq 1 & \textit{for Down-And-Out option} \\ \frac{1}{\eta(\eta+1)} \text{ if } \frac{K}{B} \geq 1 & \textit{for Down-And-Out option} \end{cases}$$

and

$$\psi(\eta) = -\frac{1}{2}\sigma^2\eta(\eta+1) + r\eta + I(\eta) \tag{11}$$

with

$$I(\eta) = \int_{\mathbb{R}} \nu_Q(dx) \left[e^{(\eta+1)x} - (1+\eta)e^x + \eta\right] \tag{12}$$

Proof. Let us assume that $y = \frac{K}{S(t)}$, then

$$\begin{aligned} C(t,S) &= e^{-r(T-t)}E_{\mathcal{Q}}\left[H(S_t)|\mathcal{F}_t\right] \\ &= S(t)f(t,y) \end{aligned}$$

where

$$\begin{aligned} f(t,y) &= E_{\mathcal{Q}}\left[(1-y)^+|\mathcal{F}_t\right].\mathbb{1}_{y\geq\frac{K}{B}}, \text{ for Up-And-Out} \\ &= E_{\mathcal{Q}}\left[(1-y)^+|\mathcal{F}_t\right].\mathbb{1}_{y\leq\frac{K}{B}}, \text{ for Down-And-Out} \end{aligned}$$

Using above we have as follows,

$$\begin{aligned} \frac{\partial f}{\partial t} &- ryf_y - \frac{1}{2}\sigma^2y^2f_{yy} \\ &+ \int_{\mathbb{R}} \nu_Q(dx)\left[e^x\left\{f\left(t, ye^{-x}\right) - f(t,y)\right\} + (e^x-1)yf_y\right] = 0 \end{aligned} \tag{13}$$

with the following boundary conditions

(1) Up and Out Barrier Option

$$\begin{aligned} f(T,y) &= (1-y)^+, \text{when } \infty > y \geq \frac{K}{B} \\ &= 0 \text{ else} \end{aligned}$$

(2) Down and Out Barrier Option

$$\begin{aligned} f(T,y) &= (1-y)^+, \text{when } 0 \leq y \leq \frac{K}{B} \leq 1 \\ &= 0 \text{ else} \end{aligned}$$

Now, the Mellin transform of the PIDE, gives us,

$$\begin{aligned} \frac{d\hat{f}(t,\eta)}{dt} &+ r\eta\hat{f}(t,\eta) - \frac{1}{2}\sigma^2\eta(\eta+1)\hat{f}(t,\eta) \\ &+ \int_{\mathbb{R}} \nu_Q(dx)\left[e^{(\eta+1)x} - (\eta+1)e^x + \eta\right]\hat{f}(t,\eta) = 0 \end{aligned}$$

At boundary condition $t = T$, $\hat{f}(T,\eta) = \hat{H}(\eta)$, and we can write

$$\hat{f}(t,\eta) = \hat{H}(\eta)e^{\psi(\eta)(T-t)} \tag{14}$$

where

$$\psi(\eta) = -\frac{1}{2}\sigma^2\eta(\eta+1) + r\eta + I(\eta)$$

and

$$I(\eta) = \int_{\mathbb{R}} \nu_Q(dx)\left[e^{(\eta+1)x} - (1+\eta)e^x + \eta\right]$$

Mellin Tranform of the boundary condition $\hat{H}(\eta)$ Up-and-Out Barrier option

$$\begin{aligned} \hat{H}(\eta) = \hat{f}(T,\eta) &= \int_{K/B}^{1} (1-y)y^{\eta-1}\,dy \\ &= \frac{1}{\eta(\eta+1)} - \left[\frac{(K/B)^{\eta}}{\eta} - \frac{(K/B)^{\eta+1}}{\eta+1}\right] \end{aligned} \tag{15}$$

and for Down-and-Out Barrier option is

$$\begin{aligned} \hat{H}(\eta) = \hat{f}(T,\eta) &= \int_{0}^{\frac{K}{B}} (1-y)y^{\eta-1}\,dy \\ &= \frac{\left(\frac{K}{B}\right)^{\eta}}{\eta} - \frac{\left(\frac{K}{B}\right)^{\eta+1}}{\eta+1},\ \text{if } \frac{K}{B} \le 1 \\ &= \int_{0}^{1} (1-y)y^{\eta-1}\,dy = \frac{1}{\eta(\eta+1)} \text{ if } \frac{K}{B} \ge 1 \end{aligned} \tag{16}$$

Hence, we can derive the expression for Call price for the both type of options described in Equation (10) .

□

Theorem 3. *The Mellin transform of the sensitivities of Barrier option is given by*

$$\Delta(t,S(t)) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} (\eta+1)\left(\frac{K}{S}\right)^{-\eta}\left[H(\eta)e^{\psi(\eta)(T-t)}\right]d\eta \tag{17}$$

$$\Gamma(t,S(t)) = \frac{1}{S}\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \eta(\eta+1)\left(\frac{K}{S}\right)^{-\eta}\left[H(\eta)e^{\psi(\eta)(T-t)}\right]d\eta \tag{18}$$

$$\Theta(t,S(t)) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \psi(\eta)\left(\frac{K}{S}\right)^{-\eta}\left[H(\eta)e^{\psi(\eta)(T-t)}\right]d\eta \tag{19}$$

with

$$H(\eta) = \begin{cases} \frac{1}{\eta(\eta+1)} - \left[\frac{(K/B)^{\eta}}{\eta} - \frac{(K/B)^{\eta+1}}{\eta+1}\right] & \textit{for Up-And-Out option} \\ \frac{\left(\frac{K}{B}\right)^{\eta}}{\eta} - \frac{\left(\frac{K}{B}\right)^{\eta+1}}{\eta+1},\ \text{if } \frac{K}{B} \le 1 & \textit{for Down-And-Out option} \\ \frac{1}{\eta(\eta+1)} \text{ if } \frac{K}{B} \ge 1 & \textit{for Down-And-Out option} \end{cases}$$

and

$$\psi(\eta) = -\frac{1}{2}\sigma^2\eta(\eta+1) + r\eta + I(\eta) \tag{20}$$

with

$$I(\eta) = \int_{\mathbb{R}} \nu_Q(dx)\left[e^{(\eta+1)x} - (1+\eta)e^{x} + \eta\right] \tag{21}$$

Proof. Since

$$C(t,S(t)) = S\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\left(\frac{K}{S}\right)^{-\eta}\left[H(\eta)e^{\psi(\eta)(T-t)}\right]d\eta$$

and

$$\Delta(t,S(t)) = \frac{\partial C}{\partial S};\Gamma(t,S(t)) = \frac{\partial^2 C}{\partial S^2};\Theta(t,S(t)) = \frac{\partial C}{\partial t}$$

By differentiating, we will have the desired result. □

4. Numerical Results

As the Lévy market is incomplete, there exists more than one or mathematically infinite number of equivalent martingale measures. We describe a method to determine an unique Lévy measure ν from the market data by using *non-parametric calibration*. Given observed market prices of options, we follow the non-parametric approach for identification of the Lévy measure.

Let us consider the (observed) market prices $C^*(T_i, S_i, B)$, $i = 1, \ldots, n$, for a set of liquid put options. The objective is to find constants ν such that

$$C^{\nu}(T_i, S_i, B) = C^*(T_i, S_i, B), \tag{22}$$

where C^{ν} is the option price computed for parameters ν. The popular approach to non-linear least squares is

$$(\nu^*) = \arg \inf_{\nu} \sum_{i=1}^{N} \{C^{\nu}(T_i, S_i, B) - C^*(T_i, S_i, B)\}^2$$

The usual formulations of the inverse problems via nonlinear least squares are ill-posed and in [18] a regularization method is proposed on relative entropy. In [18] the calibration problem is reformulated into problem of finding a risk-neutral jump-diffusion model that reproduces the observed option prices and has the smallest possible relative entropy with respect to a chosen prior model. In the calibration for the present paper we use this technique. The following parameters estimated by calibration of S&P 500 options (1970 to 2001) in [1], has been considered for computing the prices

Algorithm 1 Algorithms for computing the Barrier Call option

Require: Initial time t and stock price $S(t)$, Maturity time T, Stock growth r and Volatility σ , Lévy triplet (m, k, ν) and put price available from Market.
Ensure: $C(t, S(t))$

{**Step 1**}
Estimate the Lévy triplet (m, k, ν)

$$H(\eta) = \begin{cases} \frac{1}{\eta(\eta+1)} - \left[\frac{(K/B)^{\eta}}{\eta} - \frac{(K/B)^{\eta+1}}{\eta+1}\right] & \text{for Up-And-Out option} \\ \frac{\left(\frac{K}{B}\right)^{\eta}}{\eta} - \frac{\left(\frac{K}{B}\right)^{\eta+1}}{\eta+1}. \text{ if } \frac{K}{B} \leq 1 & \text{for Down-And-Out option} \\ \frac{1}{\eta(\eta+1)} \text{ if } \frac{K}{B} \geq 1 & \text{for Down-And-Out option} \end{cases}$$

{**Step 2**}
for $n \leftarrow 1, L$ **do**
  Evaluate $I(n) = \int_{\mathbb{R}} \nu_Q(dx) \left[e^{(n+1)x} - (1+n)e^x + n\right]$ using Clenshaw Curtis quadrature rule in the Appendix C taking examples of Levy Process from Appendix B
  $\psi(n) = -\frac{1}{2}\sigma^2 n(n+1) + rn + I(n)$
  $C(t, n) = H(n) e^{\psi(n)(T-t)}$
  $fVal(n) = \frac{P(t,n)}{2^n \Gamma(n)}$
end for
for $k \leftarrow 1, L$ **do**
  temp=0
  for $n \leftarrow 1, k$ **do**
    $temp = temp + (-1)^{n-1} \binom{k-1}{n-1} fVal(n)$
  end for $C(k) = temp$;
end for
for $k \leftarrow 1, L$ **do**
  $C(t, S(t)) = C(t, S(t)) + C(k) * e^{-\frac{S}{2}} L_{k-1}\left(\frac{S}{2}\right)$;
end for
$C(t, S(t))$

Algorithm 1 describes the procedure for computing the call price of the both Down-And-Out and Up-And-Out Barrier options. We have used above calibrated parameters to plot the call price plot against the Time-to-Maturity and Initial stock price for NIG, CGMY and Meixner processes in Figures 1–6. This help us to understand how the call price changes with the change in stock price and maturity. The change of call price and sensitivities are also computed with the change of parameters such as volatility σ, Interest rate r, initial stock price S_0 and Barrier B.

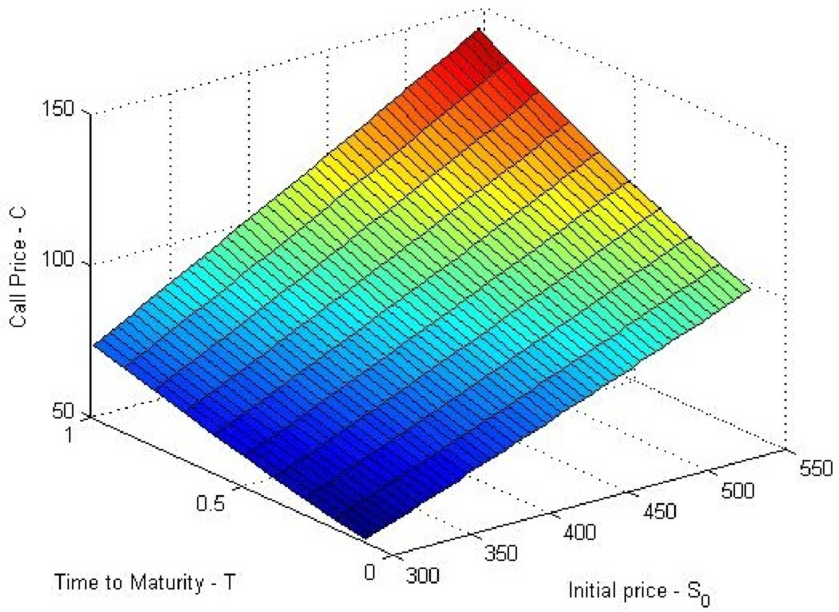

Figure 1. Down-And-Out call with NIG process with Stock Price S_0 = 450, Strike price K = 150, Barrier B = 350, σ = 0.1812, r = 0.167 and Time to maturity T = 1.1.

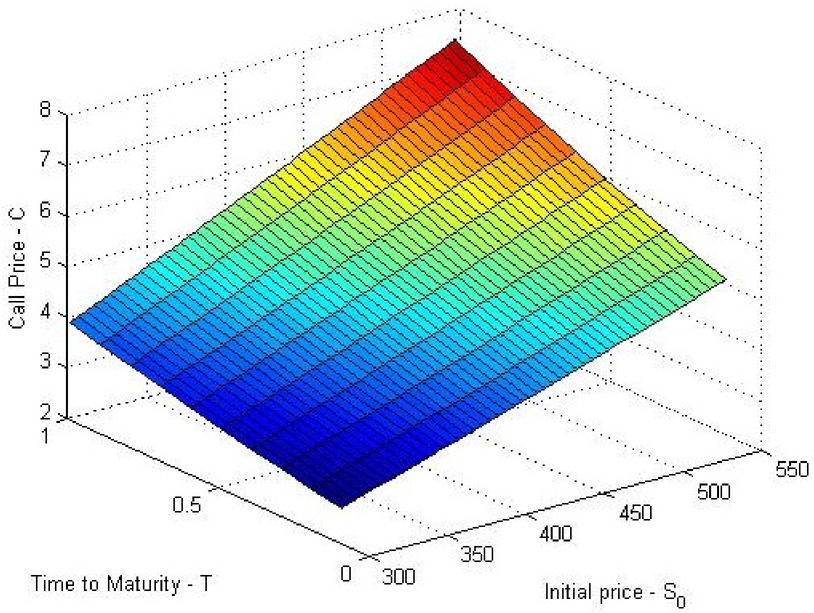

Figure 2. Up-And-Out call with NIG ($\alpha = 6.1882$, $\beta = -3.8941$, $\delta = 0.1622$) with Stock Price S_0 = 450, Strike price K = 150, Barrier B = 350, σ = 0.1812, r = 0.167 and Time to maturity T = 1.1.

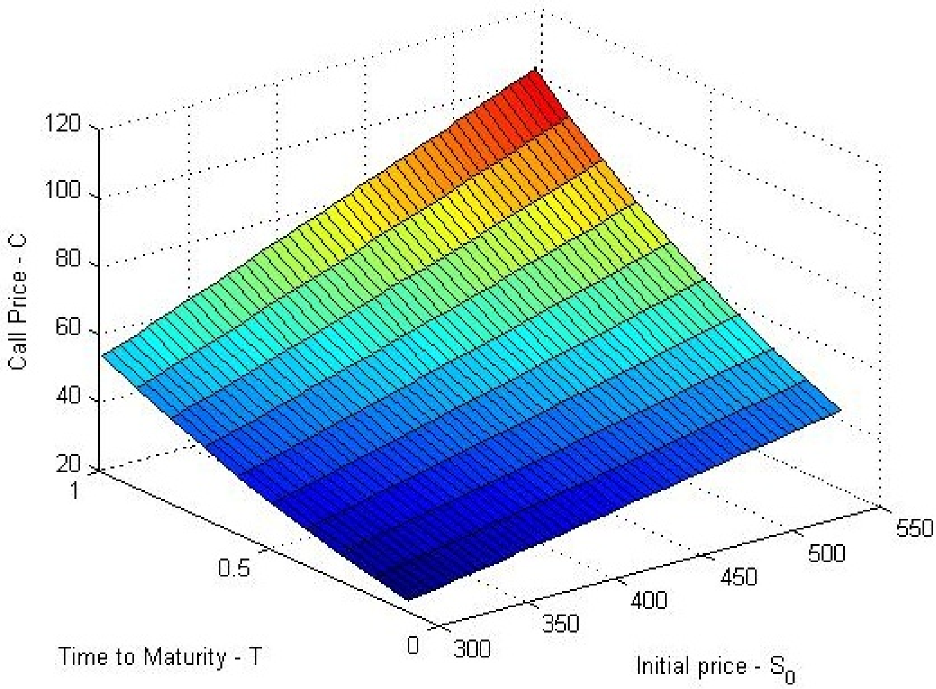

Figure 3. Down-And-Out call with CGMY(C = 0.0244, G = 0.0765, M = 7.5515, Y = 1.2945) with Stock Price S_0 = 450, Strike price K = 150, Barrier B = 350, σ = 0.1812, r = 0.167 and Time to maturity T = 1.1.

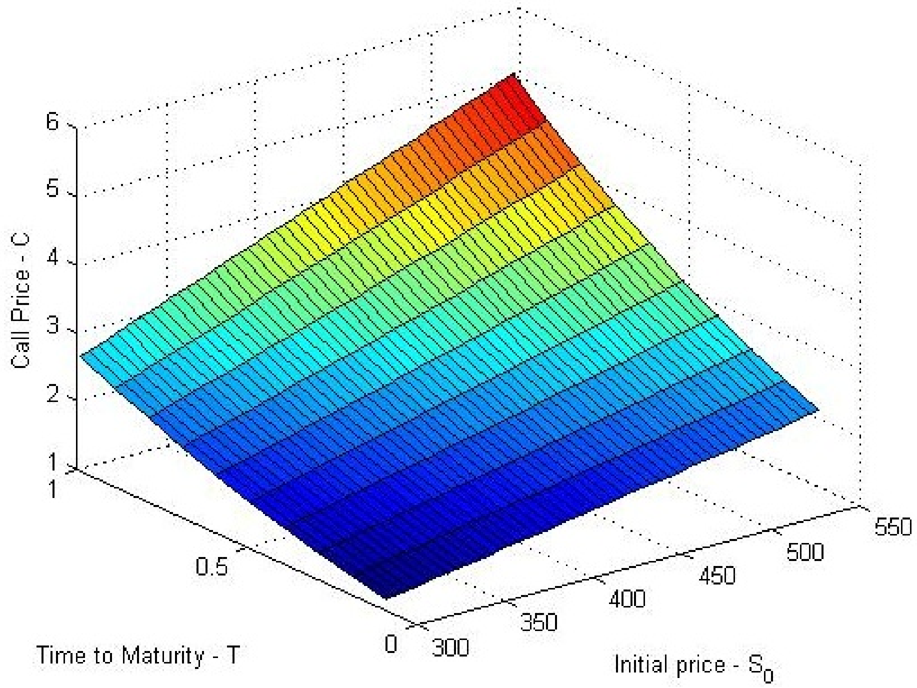

Figure 4. Up-And-Out call with CGMY(C = 0.0244, G = 0.0765, M = 7.5515, Y = 1.2945) with Stock Price S_0 = 450, Strike price K = 150, Barrier B = 350, σ = 0.1812, r = 0.167 and Time to maturity T = 1.1.

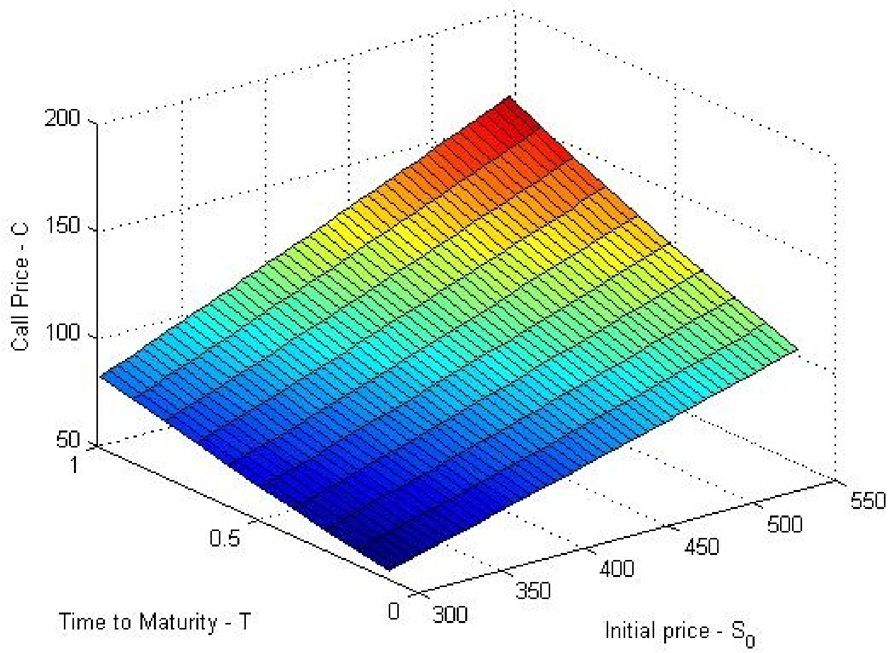

Figure 5. Down-And-Out call with Meixner($\alpha = 0.3977$, $\beta = -1.494$, $\delta = 0.3462$) with Stock Price $S_0 = 450$, Strike price K = 150, Barrier B = 350, $\sigma = 0.1812$, r = 0.167 and Time to maturity T = 1.1.

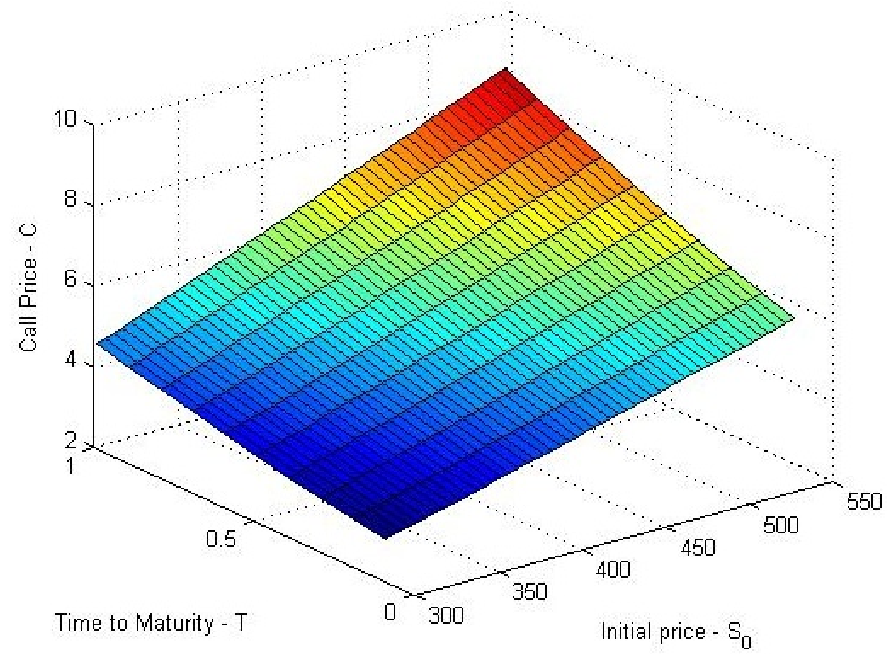

Figure 6. Up-And-Out call with Meixner($\alpha = 0.3977$, $\beta = -1.494$, $\delta = 0.3462$) with Stock Price $S_0 = 450$, Strike price K = 150, Barrier B = 350, $\sigma = 0.1812$, r = 0.167 and Time to maturity T = 1.1.

In Table 1 we provide the calibration results for the given data set with three different processes (as Lévy density)- NIG, CGMY and Meixner. The Algorithm 1 used to compute the call price and sensitivities and result listed in Tables 2–5. This result is also generated with the change of time-to-maturity, growth and volatility of the stock for different types of Lévy process.

Table 1. Estimated parameters for Levy processes.

Model	Parameters			
NIG	$\alpha = 6.1882$	$\beta = -3.8941$	$\delta = 0.1622$	
CGMY	$C = 0.0244$	$G = 0.0765$	$M = 7.5515$	$Y = 1.2945$
Meixner	$\alpha = 0.3977$	$\beta = -1.494$	$\delta = 0.3462$	

Table 2. Change in Call Price with different types of Lévy Process.

t	r	σ	NIG (α, β, δ) Call		CGMY (C, G, M, Y) Call		Meixner (α, β, δ) Call	
			Down-Out	Up-Out	Down-Out	Up-Out	Down-Out	Up-Out
1	0.167	0.5	8.8249	9.5132	8.4641	9.1207	8.8331	9.5222
	0.167	0.2	8.9626	9.6631	8.8112	9.4983	8.9689	9.6699
0.8	0.167	0.5	8.6620	9.3362	7.6423	8.2272	8.6863	9.3626
	0.167	0.2	9.0740	9.7843	8.6218	9.2924	9.0932	9.8052
0.5	0.167	0.5	8.4232	9.0767	6.5559	7.0477	8.4706	9.1282
	0.167	0.2	9.2436	9.9691	8.3454	8.9918	9.2827	10.0117

Call option with stock Initial value S = 300, Strike price K = 150, Barrier B = 450 and Time to maturity T = 1.1.

Table 3. Call Price & Sensitivities change with Barrier.

Barrier (B)	Call		Delta		Gamma		Theta	
	Down-Out	Up-Out	Down-Out	Up-Out	Down-Out	Up-Out	Down-Out	Up-Out
250	15.8960	3.4417	0.0410	0.0058	8.7909	1.9033	−0.6604	−0.1417
300	14.0080	5.3296	0.0374	0.0094	7.7468	2.9474	−0.5825	−0.2196
350	12.4266	6.9111	0.0342	0.0126	3.8220	6.8722	−0.5172	−0.2850

Option with Stock price S = 350, K = 150, $\sigma = 0.1812$, r = 0.167, Time to maturity T = 1.1 and NIG $(\alpha = 6.1882, \beta = -3.8941, \delta = 0.1622)$ as Lévy Process.

Table 4. Change of Delta and Gamma over Stock price change.

S_0	NIG (α, β, δ)				CGMY (C, G, M, Y)				Meixner (α, β, δ)			
	Delta		Gamma		Delta		Gamma		Delta		Gamma	
	Down-Out	Up-Out	Down-Out	Up-Out	Down-Out	Up-Out	Down-Out	Up-Out	Down-Out	Up-Out	Down-Out	Up-Out
350	0.03	0.01	6.87	3.82	0.02	0.01	4.97	4.61	0.02	0.01	5.53	5.13
400	0.03	0.01	9.09	5.05	0.02	0.01	6.58	6.09	0.03	0.01	7.32	6.78
450	0.04	0.01	11.64	6.45	0.03	0.01	8.42	7.80	0.03	0.02	9.38	8.67

Barrier Call option with Strike price K = 150, Barrier B = 350, $\sigma = 0.1812$, r = 0.167 and Time to maturity T = 1.1.

Table 5. Change of Theta over Time to expire.

t	NIG (α, β, γ) Theta		CGMY (C, G, M, Y) Theta		Meixner (α, β, δ) Theta	
	Down-Out	Up-Out	Down-Out	Up-Out	Down-Out	Up-Out
0.4	−0.6851	−0.3763	0.7374	0.6763	−0.5246	−0.4801
0.6	−0.6753	−0.3710	0.7536	0.6910	−0.5175	−0.4736
0.8	−0.6655	−0.3657	0.7702	0.7061	−0.5104	−0.4672
1.0	−0.6560	−0.3605	0.7871	0.7215	−0.5034	−0.4608

Barrier Call option with Stock Price S_0 = 450, Strike price K = 150, Barrier B = 350, $\sigma = 0.1812$, r = 0.167 and Time to maturity T = 1.1.

The Sensitivities like Delta, Gamma and Theta of the option with respect to initial stock price S_t and t will be denoted by

$$\Delta = \frac{\partial}{\partial S}C(S,B,t);\Gamma = \frac{\partial^2}{\partial S^2}C(S,B,t);\Theta = \frac{\partial}{\partial t}C(S,B,t)$$

Using the above equations for sensitivities, we will check how the Call, Delta, Gamma & Theta changes with the change of Barrier for a specific type of Lévy process (in this case NIG) in the Table 3.

The Call Price and Sensitivities (Delta, Gamma and Theta) computed (Tables 4 and 5) for different types of Lévy process with its parameters.

5. Conclusions

In this paper, we have focused on three types of Lévy process with infinite activity but finite moments to option pricing and compared the results. We developed alternative techniques to compute prices and sensitives of the Barrier options. Here, we first determined the modified Lévy process under measure for incomplete market followed by development of a Partially Integro-Differential Equation and subsequently used the Mellin transform technique to get an expression for options. The expression computed numerically with a class of Lévy process with infinite activity where the distribution of the process is unknown.

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Appendix

A. Derivation of the Stock Dynamics under the Equivalent Martingale Measure

Define $\mathcal{P}_2(t,E)$ to be the set of all equivalence class of mappings $f : [0,t] \times \mathcal{B}(\mathbb{R}) \times \Omega \to \mathbb{R}$ which coincide everywhere with respect to $\rho_\Sigma \times \mathbb{P}$, and satisfy the following conditions

1. f is predictable.
2. $\mathbb{P}\left(\int_0^t \int_E |f(s,x)|^2 \rho_\Sigma(ds,dx) < \infty\right) = 1,$

where ρ_Σ is a σ-finite measure on $\mathbb{R}^+ \times E$. Analogously it is possible to define $\mathcal{P}_2(t)$. Let Y be a Lévy type stochastic integral of the form

$$dY(t) = G(t)dt + F(t)W(t) + \int_{\mathbb{R}-\{0\}} H(t,x)\widetilde{N}(ds,dx)$$

where $\sqrt{G(t)}, F(t) \in \mathcal{P}_2(t)$ and $H \in \mathcal{P}_2(t, \mathbb{R}-\{0\})$ for each $t \geq 0$.

The goal is to find the equivalent martingale measure $\mathbb{Q}$, on a fixed time interval $[0,T]$, for which $\frac{d\mathbb{Q}}{d\mathbb{P}} = e^{Y(T)}$, for $0 \leq t \leq T$. We consider the associated process e^Y is a martingale and hence $G(t)$ is determined by $F(t)$ and $H(t)$. With respect to the new measure $\mathbb{Q}$, $W_{\mathbb{Q}}(t) = W(t) - \int_0^t F(s)\,ds$ is a Brownian motion and

$$\widetilde{N}_{\mathbb{Q}}(t,E) = \widetilde{N}(t,E) - \int_0^t \int_E (e^{H(s,x)} - 1)\nu(dx)\,ds$$

is a martingale (see [19], Section 5.6.3). Thus with respect to the new measure $\mathbb{Q}$ the dynamics of Z is given by

$$dZ(t) = \left(\mu + \sigma F(t) + \int_{\mathbb{R}} x(e^{H(s,x)} - 1)\nu(dx)\right)\,dt + \sigma dW_{\mathbb{Q}}(t) + \int_{\mathbb{R}} x\widetilde{N}_{\mathbb{Q}}(dt,dx) \tag{A1}$$

Also,

$$\int_{\mathbb{R}} x\widetilde{N}_Q(dt, dx) = \int_{\mathbb{R}} x(N(dt, dx) - \nu_Q(dx)dt)$$

where

$$\nu_Q(dx) = e^{H(t,x)}\nu(dx) \tag{A2}$$

is the Lévy measure with respect to $\mathbb{Q}$. Thus from Equations (A1) and (A2) it is clear that the Lévy triplet of Z with respect to $\mathbb{Q}$ in terms of Lévy triplet with respect to $\mathbb{P}$ is given by

$$\left(\mu_Q, \sigma^2, e^{H(t,x)}\nu(dx)\right), \quad \mu_Q = \mu + \sigma F(t) + \int_{\mathbb{R}} x(e^{H(t,x)} - 1)\nu(dx)$$

Remark A1. *Using Girsanov's theorem (see [20]), there exist a deterministic process $\beta(t)$ and a measurable non-negative deterministic process $Y(t,x)$ such that*

$$\mu_Q = \mu + \sigma^2\beta(t) + \int_{\mathbb{R}} x(Y-1)\nu(dx), \quad {\sigma_Q}^2 = \sigma^2, \quad \nu_Q(dx) = Y\nu(dx)$$

Comparing with Equation (2) we obtain $\beta(t) = \frac{F(t)}{\sigma}$ and $Y(t,x) = e^{H(t,x)}$.

With respect to $\mathbb{Q}$, $e^{-rt}S(t) = e^{-rt+Z(t)}$ is a martingale. By Proposition 3.18(2), [21] and Equation (2), we thus obtain (since we have Assumption 1)

$$\frac{\sigma^2}{2} + \mu_Q + \int_{\mathbb{R}} (e^x - 1 - x1_{|x|\le 1})\nu_Q(dx) = r \tag{A3}$$

Therefore the dynamics of stock-price is given by the following theorem.

Theorem A1. *With respect to the equivalent martingale measure $\mathbb{Q}$, the dynamics of $S(t)$ is given by*

$$\frac{dS(t)}{S(t-)} = r\,dt + \sigma dW_Q(t) + \int_{\mathbb{R}} (e^x - 1)\widetilde{N}_Q(dt, dx)$$

Proof. Using results for exponential of a Lévy process (see Proposition 8.20, [21]) we obtain,

$$\frac{dS(t)}{S(t-)} = \left(\mu_Q + \frac{\sigma^2}{2} + \int_{\mathbb{R}} (e^x - 1 - x1_{|x|\le 1})\nu_Q(dx)\right)\,dt + \sigma dW_Q(t) + \int_{\mathbb{R}} (e^x - 1)\widetilde{N}_Q(dt, dx)$$

Thus the proof follows from Equation (A3). □

B. Examples of Lévy Process

B.1. Lévy Process with Infinite Activity

We have considered the following Lévy processes with infinite activity but $\int_{\mathbb{R}} x^2\nu(dx)\mathbb{1}_{\{x<1\}} < \infty$.

1. The Normal Inverse Gaussian

   The NIG distribution with parameters $\alpha > 0, \alpha < \beta < \alpha$ and $\delta > 0, NIG(\alpha, \beta, \delta)$, has a characteristic function

   $$E\left[e^{iuX}\right] = \exp\Big(-\delta\big(\sqrt{\alpha^2 - (\beta + iu)^2} - \sqrt{\alpha^2 - \beta^2}\big)\Big)$$

   The Lévy measure is given by

   $$\nu_{NIG}(dx) = \frac{\delta\alpha}{\pi}\,\frac{\exp\,(\beta x)\,K_1(\alpha|x|)}{|x|}dx \tag{B1}$$

where $K_\lambda(x)$ is the modified Bessel function of third kind with index λ.

An NIG process has no Brownian component and its Lévy triplet is

$$[\gamma, 0, \nu_{NIG}(dx)], \text{ where}$$

$$\gamma = \frac{2\delta\alpha}{\pi}\int_0^1 \sinh(\beta x)\ K_1(\alpha x)dx$$

2. The CGMY Process

The CGMY(C, G, M, Y) distribution is four parameter distribution with characteristic function

$$E\left[e^{iuX}\right] = \exp\Big(C\Gamma(-Y)\big((M-iu)^Y - M^Y + (G+iu)^Y - G^Y\big)\Big)$$

The Lévy measure of this process admits the representation

$$\nu_{CGMY}(dx) \quad = \quad C\Big(\frac{e^{-Mx}}{x^{1+y}}\mathbb{1}_{x>0} + C\frac{e^{Gx}}{|x|^{1+y}}.\mathbb{1}_{x<0}\Big)dx \text{ when } \ C, G, M > 0 \text{ and } y < 2 \quad \text{(B2)}$$

The CGMY process is a pure jump Lévy process with Lévy triplet

$$[\gamma, 0, \nu_{CGMY}(dx)]$$

where

$$\gamma \quad = \quad C\Big(\int_0^1 x^{-Y}e^{-Mx}dx - \int_{-1}^0 |x|^{-Y}e^{Gx}dx\Big)$$

The characteristic function of the pure jump KoBol process of order $\nu \in (0,2), \nu \neq 1$ is given by

$$\phi_{KoBol}(u) \quad = \quad \exp\Big(-i\mu u + c\Gamma(-\nu)\big[\lambda_+^\nu - (\lambda_+ + iu)^\nu - (-\lambda_-)^\nu - (-\lambda_- - iu)^\nu\big]\Big) \quad \text{(B3)}$$
$$\text{where } c > 0, \mu \in \mathbb{R}, \lambda_- < -1 < 0 < \lambda_+$$

An ordinary KoBoL process is obtained from this definition by specializing to the case where $\nu_+ = \nu_- = \nu$ and $c_+ = c_- = c$ The relation between these parameters and the parameters C, G, M, Y is as follows: $C = c, G = \lambda_+, M = -\lambda_-, Y = \nu$

3. The Meixner Process

The Meixner process is defined by $Meixner(\alpha, \beta, \delta), \alpha > 0, -\pi < \beta < \pi, \delta > 0$ then Lévy measure is defined by

$$\nu_{Meixner}(dx) = \delta\frac{\exp(\beta x/\alpha)}{x\ \sinh(\pi x/\alpha)}dx \quad \text{(B4)}$$

Since $\int_{-1}^{+1}|x|\nu(dx) = \infty$, the process is of infinite variation but moments of all order exists. The first parameter of Lévy triplet

$$\gamma = \alpha\delta\ \tan\,(\beta/2) - 2\delta\int_1^\infty \frac{\sinh(\beta x/\alpha)}{\sinh(\pi x/\alpha)}dx$$

It has no Brownian part and a pure jump part governed by the Lévy measure.

The Lévy triplet is given by

$$[\gamma, 0, \nu_{Meixner}(dx)]$$

C. Numerical Techniques

C.1. Computing $I(\eta)$ *by Clenshaw Curtis Quadrature Rule*

In this section, we will use Clenshaw-Curtis rule for integration [22] to calculate the integral $I(\eta)$ because of its high accuracy level and low computational time. According to Clenshaw-Curtis rule for integration, any integral in $[-1,1]$ can be written with the help of interpolation polynomial $L_n(x)$ as

$$I_n(f) = \int_{-1}^{1} f(x)dx = \int_{-1}^{1} L_n(x)dx = \int_{-1}^{1} \sum_{k=0}^{M} c_k T_k(x)dx = \sum_{k=0}^{M} c_k \mu_k \tag{C1}$$

where $\mu_k = \int_{-1}^{1} T_k(x)dx$ are the moments of the Chaebyshev polynomials, $c_k = \frac{2}{M}\sum_{j=0}^{M} f(x_j)\cos\left(\frac{kj\pi}{M}\right)$ which is the real part of an FFT, and $x_j = \cos(j\pi/M)$.
The μ_k can be written,

$$\mu_k = \int_{-1}^{1} T_k(x)dx = \begin{cases} 0 & \text{if k odd} \\ 2/(1-k^2) & \text{if k even} \end{cases}$$

A fast and accurate algorithm for computing the weights in the Fejér and Clenshaw-Curtis rules in $O(M\log M)$ computation has been given by [22]. The weights are obtained as the inverse FFT of certain vectors given by explicit rational expression.
Converting the any integration from interval $[a,b]$ to $[-1,1]$, we have

$$\int_a^b f(x)dx = \frac{b-a}{2}\int_{-1}^{1} f\left(\frac{b-a}{2}x + \frac{a+b}{2}\right)dx$$

C.2. Properties of Mellin Transform

The Mellin transform of real valued function $\phi(z)$ defined on $(0,\infty)$ where Mellin transform with respect to s which is a real number, is definded as

$$\mathcal{M}\{\phi(z)\} = \Phi(s) = \int_0^{\infty} z^{s-1}\phi(z)dz, \quad s \in \mathbb{R}$$

where its inverse is

$$\mathcal{M}^{-1}\{\Phi(s)\} = \phi(z) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} z^{-s}\Phi(s)ds, \quad c > 0$$

There are some interesting properties of Mellin Transform on scaling and derivaties of first and second order available as follows (See [23],

$$\mathcal{M}\{\phi(az)\} = a^{-s}\Phi(s)$$

$$\mathcal{M}\{z\frac{\partial\phi(z)}{\partial z}\} = -s\Phi(s)$$

$$\mathcal{M}\{z^2\frac{\partial^2\phi(z)}{\partial z^2}\} = (-1)^2 s(s+1)\Phi(s)$$

C.3. Numerical Mellin Inversion

The Mellin transform is defined by the formulae [19]:

$$\Phi(s) = \int_0^\infty z^{s-1}\phi(z)dz, \quad \mathrm{Re}(s) > 0 \tag{C2}$$

and its inverse is

$$\phi(z) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} z^{-s}\Phi(s)ds, \quad c > 0$$

where one-to-one correspondence is denoted as follows, if the inverse $\Phi(s)$ function exists:

$$\phi(z) \leftrightarrow \Phi(s) \text{ or } \Phi(s) = \mathcal{M}\{\phi(z)\}$$

The numerical Mellin inverse is first presented by [24] and later by [25].We have followed the approach proposed by [24] and can write the numerical inverse of Mellin as,

$$\phi(t) \simeq \sum_{s=1}^{N} c_s e^{-\frac{t}{2}} L_{s-1}\left(\frac{t}{2}\right) \tag{C3}$$

where

$$c_s = \sum_{n=1}^{s} (-1)^{n-1}\binom{s-1}{n-1} H_n, \quad s = 1(1)N \tag{C4}$$

and

$$H_s \equiv H(s) \equiv \frac{\Phi(s)}{2^s\Gamma(s)} \tag{C5}$$

Now, we have observed that H_s is defined in integer domain and so $\Phi(s)$. But, in real case it is quite likely that the Mellin transform $\Phi(s^*) = \mathcal{M}\{f(t)\}$ will have a strip of existence for $s^* \in (a^*, b^*)$ where s^* is not an integer rather real.In such case, we will apply a linear transform as to keep H_s defined in integer domain as follows,

$$s^* = As + B, \quad s \in [1, N] \tag{C6}$$

with

$$A = \frac{b^* - a^*}{N-1}, \quad B = \frac{a^*N - b^*}{N-1} \text{which maps the interval } [1, N] \text{ onto } [a^*, b^*]$$

Since the function exists in interval $[a^*, b^*]$ we can invert $\Phi(As + B)$ to recover the function $g(t)$ with the following

$$\mathcal{M}\{g(t)\} = G(s) \equiv \Phi(s^*) = \Phi(As + B), \quad s \in [1, N] \tag{C7}$$

and thereafter original function $f(t) = \mathcal{M}^{-1}\Phi(s)$ can be derived by the following transformation:

$$f(t) = A\frac{g(t^A)}{t^B}$$

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