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Power Aggregation Operators and VIKOR Methods for Complex q-Rung Orthopair Fuzzy Sets and Their Applications

Harish Garg ^{1,*}, Jeonghwan Gwak ^{2,*}, Tahir Mahmood ³ and Zeeshan Ali ³

¹ School of Mathematics, Thapar Institute of Engineering and Technology, Deemed University, Patiala 147004, India

² Department of Software, Korea National University of Transportation, Chungju 27469, Korea

³ Department of Mathematics and Statistics, International Islamic University, Islamabad 44000, Pakistan; tahirbakhhat@iiu.edu.pk (T.M.); zeeshan.phdma102@iiu.edu.pk (Z.A.)

* Correspondence: harishg58iitr@gmail.com (H.G.); james.han.gwak@gmail.com (J.G.)

Received: 2 March 2020; Accepted: 30 March 2020; Published: 5 April 2020



Abstract: The aim of this paper is to present the novel concept of Complex q-rung orthopair fuzzy set (Cq-ROFS) which is a useful tool to cope with unresolved and complicated information. It is characterized by a complex-valued membership grade and a complex-valued non-membership grade, the distinction of which is that the sum of q-powers of the real parts (imaginary parts) of the membership and non-membership grades is less than or equal to one. To explore the study, we present some basic operational laws, score and accuracy functions and investigate their properties. Further, to aggregate the given information of Cq-ROFS, we present several weighted averaging and geometric power aggregation operators named as complex q-rung orthopair fuzzy (Cq-ROF) power averaging operator, Cq-ROF power geometric operator, Cq-ROF power weighted averaging operator, Cq-ROF power weighted geometric operator, Cq-ROF hybrid averaging operator and Cq-ROF power hybrid geometric operator. Properties and special cases of the proposed approaches are discussed in detail. Moreover, the VIKOR (“VIseKriterijumska Optimizacija I Kompromisno Resenje”) method for Cq-ROFSs is introduced and its aspects discussed. Furthermore, the above mentioned approaches apply to multi-attribute decision-making problems and VIKOR methods, in which experts state their preferences in the Cq-ROF environment to demonstrate the feasibility, reliability and effectiveness of the proposed approaches. Finally, the proposed approach is compared with existing methods through numerical examples.

Keywords: fuzzy sets; complex fuzzy sets; Pythagorean fuzzy sets; q-rung orthopair fuzzy sets; complex q-rung orthopair fuzzy sets; power aggregation operators

1. Introduction

Multiple attribute decision making (MADM) is one of the important branches of decision science for which the principle goal is to find the best alternatives under the set of the different attributes. In it, each alternative is accessed by an expert or group of experts under the different attributes and their ratings are expressed either as a crisp number or as a linguistic number. However, in today's challenging world, it is very common that the uncertainty plays a vital role in almost every decision-making process. Thus there is a need to handle the uncertainties in the analysis. To address this, Zadeh [1] stated the theory of fuzzy set (FS) by defining the membership degree to each element. After their appearance, several researchers have extended their theory to its more generalized ones namely, intuitionistic fuzzy set (IFS) [2], interval-valued IFS (IVIFS) [3], linguistic IVIFS [4], cubic IFS [5], Pythagorean fuzzy set

(PFS) [6,7], q-rung orthopair fuzzy set [8,9]. In the existing IFS, each element is rated by an expert in terms of membership degree μ and non-membership degree η such that $\mu, \eta \in [0,1]; \mu + \eta \leq 1$, while in PFS, these conditions are extended over the unit circle as $\mu, \eta \in [0,1]; \mu^2 + \eta^2 \leq 1$. Since their existence, several researchers have presented the different kinds of algorithms for solving the MADM problems by using aggregation operators (AOs), similarity or distance measures, hybrid operators and so forth. The several existing approaches under the IFS or PFS environments are classified into three aspects as below:

- (1) **Approaches based on operators:** Under it, several AOs are defined by the researchers to aggregate the intuitionistic fuzzy numbers (IFNs) and Pythagorean fuzzy numbers (PFNs). For instance, Xu and Yager [10] defined the geometric operators while Garg [11,12] defined the interactive AOs based on the hesitance degree for the pairs of intuitionistic fuzzy numbers (IFNs). He et al. [13] presented some interactive geometric AOs. In terms of PFSs, Yager [7] defined the weighted averaging AOs for Pythagorean fuzzy numbers (PFNs). Garg [14,15] extended their AOs with Einstein t-norm operations and hence presented some generalized AOs to solve the MADM problems. Zhang [16] presented ordered weighted while Zeng et al. [17] defined the hybrid AOs for PFSs. Garg [18] presented neutrality aggregation operators for PFNs to solve the MADM problems.
- (2) **Approaches based on measures:** The information measure such as similarity or distance measures, correlation coefficients and so forth also play a vital role in the process of MADM problems. In the IFS environment, Li and Cheng [19] defined the similarity measures for IFNs. Garg and Kumar [20] defined the similarity measures for IFNs by using connection numbers of IFNs. Ye [21] defined the cosine similarity measures for IFNs. However, under the PFS environment, Peng and Yang [22] defined the subtraction and division operations between the two or more PFNs. Yager and Abbasov [23] studied the relationship between the Pythagorean and complex numbers. Peng [24] defined some new operations between the PFSs. Wei and Wei [25] defined the similarity measure for PFS to solve the decision-making problems. Garg [26] defined the correlation coefficient between the pairs of PFNs to solve the MADM problems.
- (3) **Approaches based on hybrid operators:** Some researchers have utilized the extended operators to solve the MADM or MADM problems such as applications of the problems by using TOPSIS (“Technique for Order Preference with respect to the Similarity to the Ideal Solution”) [27], Bonferroni mean [28], Maclaurin symmetric mean [29] and so forth. However, for PFSs, Nie et al. [30] presented partitioned Bonferroni mean AOs to solve the DMPs with Shapley fuzzy measure. Wei and Lu [31] presented a power AOs for PFS to solve MADM problems. Gao [32] presented Hamacher Prioritized weighted AOs for PFNs. Apart from them, some interactive AOs for PFNs have been proposed by researchers in References [33–35].

From the above extensive literature, it has been noted that IFS and PFS are widely applied in various decision making problems. However, it is observed that their approaches are limited in nature. For example, when an expert provides their information in terms of membership degree as 0.8 and non-membership degree as 0.7 then it is clearly seen that neither $\mu + \eta \leq 1$ nor $\mu^2 + \eta^2 \leq 1$ holds. Hence, IFSs and PFSs cannot describe the information effectively. To manage such a situation, a concept of q-ROFS was introduced by Yager [9] in which each element is characterized as a membership degree μ, η such that $\mu, \eta \in [0,1]; \mu^q + \eta^q \leq 1$, with $q \geq 1$ integer. The q-ROFS is more effective and more general than the existing methods for handling uncertain and unpredictable information in real-decision problems with an independent degree parameter q . Clearly, when $q = 1$ and $q = 2$, the q-ROFS reduces to IFS and PFS respectively. Thus, the IFS and PFS are the special cases of the q-ROFS. The geometrical interpretation of q-ROFS is shown in Figure 1. Under the q-ROFS environment, scholars have utilized it to solve the MADM problems. Liu and Wang [36] explored the q-rung fuzzy weighted average and geometric operators. Wei et al. [37] proposed the Heronian mean operator based on the q-ROFS to solve the MADM problems. Peng et al. [38] proposed an exponential operation and aggregation operators for the q-ROFS. Liu and Wang [39] presented the q-rung orthopair fuzzy Archimedean Bonferroni

mean operators for decision-making problems. Liu and Liu [40] proposed q-rung orthopair fuzzy Bonferroni mean operators and Wei et al. [41] proposed q-rung orthopair fuzzy Maclaurin symmetric mean operators. Xing et al. [42] impersonated the point operators for q-ROFSs. Garg and Chen [43] developed the neutrality aggregation operators for q-ROFS to solve the MADM problems. Peng and Dai [44] acted the multi-parametric similarity measures and its based method to solve the MADM problem under q-ROFS environment.

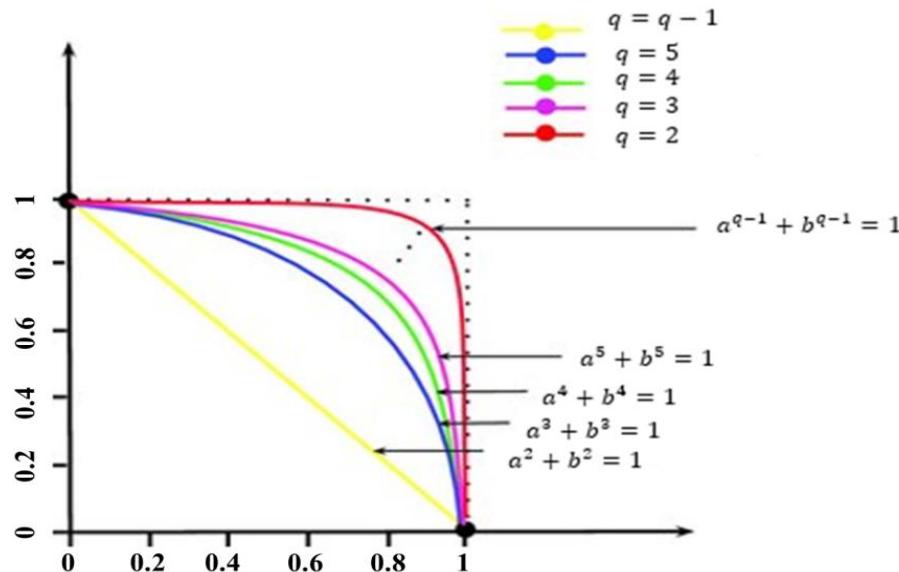


Figure 1. Geometrical interpretation of q-rung orthopair fuzzy sets. Here $a = \mu$, $b = \eta$.

With the growing technologies these days, it is perceived that the above stated and similar approaches are limited in nature and are unable to handle the information which are vary with the phase of time such as “medical diagnosis”, “biometric recognition” and so forth. To deal with this, Alkouri and Salleh [45] presented the concept of the complex IFS (CIFS) in which each element is characterized by complex-valued membership and complex-valued non-membership grades represented in polar coordinates over the unit disc. Later on, Kumar and Bajaj [46] initiated the concept of the distance measures and entropy for complex intuitionistic soft sets. Rani and Garg [47] presented the distance measures to compute the degree of separation between two CIFSs. In terms of AOs, Garg and Rani [48] and Rani and Garg [49] presented weighted and power AOs for solving the MADM problems. Alkouri and Salleh [50] defined the complex intuitionistic fuzzy relations. Recently, Garg and Rani [51] presented the generalized Bonferonni mean AOs for decision-making problems. Garg and Rani [52] presented various measures for CIFS. Ullah et al. [53] extended the concept of CIFS to a complex Pythagorean fuzzy set (CPFS). However, to study the several approaches to solve the MADM under the CIFS environment, we refer to References [54–56].

Considering the flexibility of the complex fuzzy sets and q-ROFS to describe the uncertain information in the data, this paper presents a new concept named the complex q-rung orthopair fuzzy set (Cq-ROFS). To address it completely, in this paper, we extend the concept of CIFS to Cq-ROFS by taking the advantages of q-ROFS. The properties of the stated sets are also investigated. The proposed Cq-ROFS is defined under the constraint that the sum of q-powers of the real parts (imaginary parts) of the membership and non-membership grades is less than or equal to one. Further, in contrast to the standard VIKOR strategy, we have defined the VIKOR method for solving decision-making problems. Holding all the above points in mind, the primary objectives of the work are:

- (1) To establish the concept of Cq-ROFS and its fundamental properties to handle uncertain information in the data.

- (2) To define several powers weighted AOs to aggregate the different Cq-ROFS. Several properties and their special cases are also discussed in detail.
- (3) In view of the merits of the VIKOR model in considering the compromise between group utility maximization and individual regret minimization, this paper introduced the novel VIKOR method for Cq-ROFS.
- (4) Two new MADM methods, based on power operators and VIKOR methods are presented to solve the decision making problems.
- (5) The presented approaches have been demonstrated with several numerical examples to show their superiority and advantages.

The rest of the manuscript is organized as follows: in Section 2, some basic concepts of the existing sets are reviewed. In Section 3, we present a new concept of Cq-ROFS and its fundamental properties. In Section 4, we define the various power aggregation operators for Cq-ROFS and discuss their properties. In Section 5, two new MADM methods are presented to solve the problems. In Section 6, we illuminate the stated methods with several examples and compare their methods with the existing ones. Finally, Section 7 concludes the paper.

2. Preliminaries

In this section, we review the basic concept of q-ROFSs, CIFSs and their operational laws over the universal set X .

2.1. Basic Concepts

Definition 1. An IFS R on finite universal set X is given by [2]:

$$R = \{(x, \mu'_R(x), \eta'_R(x)) : x \in X\} \quad (1)$$

where $\mu'_R(x)$ and $\eta'_R(x)$ represent the “membership degree” and “non-membership degree”, respectively, with a condition: $0 \leq \mu'_R(x) + \eta'_R(x) \leq 1$.

Definition 2. A CIFS R on finite universal set X is given by [45]:

$$R = \{(x, \mu'_R(x), \eta'_R(x)) : x \in X\} \quad (2)$$

where $\mu'_R(x) = \mu_R(x)e^{i2\pi(W_{\mu_R})}$ and $\eta'_R(x) = \eta_R(x)e^{i2\pi(W_{\eta_R})}$, represents the complex-valued truth and complex-valued falsity degrees, with a conditions: $0 \leq \mu_R(x) + \eta_R(x) \leq 1, 0 \leq W_{\mu_R}(x) + W_{\eta_R}(x) \leq 1$.

Definition 3. For a two CIFNs $R_1 = (\mu_1(x)e^{i2\pi(W_{\mu_1})}, \eta_1(x)e^{i2\pi(W_{\eta_1})})$ and $R_2 = (\mu_2(x)e^{i2\pi(W_{\mu_2})}, \eta_2(x)e^{i2\pi(W_{\eta_2})})$ and $\gamma > 0$ be any real number [45]. Then

- (1) $R_1 \oplus R_2 = ((\mu_1 + \mu_2 - \mu_1\mu_2)e^{i2\pi(W_{\mu_1} + W_{\mu_2} - W_{\mu_1}W_{\mu_2})}, (\eta_1\eta_2)e^{i2\pi(W_{\eta_1}W_{\eta_2})});$
- (2) $R_1 \otimes R_2 = ((\mu_1\mu_2)e^{i2\pi(W_{\mu_1}W_{\mu_2})}, (\eta_1 + \eta_2 - \eta_1\eta_2)e^{i2\pi(W_{\eta_1} + W_{\eta_2} - W_{\eta_1}W_{\eta_2})});$
- (3) $\gamma R_1 = ((1 - (1 - \mu_1)^\gamma)e^{i2\pi(1 - (1 - \mu_1)^\gamma)}, \eta_1^\gamma e^{i2\pi W_{\eta_1}^\gamma});$
- (4) $\gamma R_1 = (\mu_1^\gamma e^{i2\pi W_{\mu_1}^\gamma}, (1 - (1 - \eta_1)^\gamma)e^{i2\pi(1 - (1 - \eta_1)^\gamma)}).$

Definition 4. A PFS R on finite universal set X is given by [7]:

$$R = \{(x, \mu'_R(x), \eta'_R(x)) : x \in X\} \quad (3)$$

with a condition: $0 \leq \mu'_R(x) + \eta'_R(x) \leq 1$, where $\mu'_R, \eta'_R : X \rightarrow [0, 1]$.

Definition 5. A CPFS R on finite universal set X is given by [53]:

$$R = \{(x, \mu'_R(x), \eta'_R(x)) : x \in X\} \quad (4)$$

where $\mu'_R(x) = \mu_R(x)e^{i2\pi(W_{\mu_R})}$ and $\eta'_R(x) = \eta_R(x)e^{i2\pi(W_{\eta_R})}$, represents the complex-valued truth and complex-valued falsity degrees, with a conditions: $0 \leq \mu_R^2(x) + \eta_R^2(x) \leq 1, 0 \leq W_{\mu_R}^2(x) + W_{\eta_R}^2(x) \leq 1$, where $\mu_R, W_{\mu_R}, \eta_R, W_{\eta_R} \in [0, 1]$.

Definition 6. For a two CPFNs $R_1 = (\mu_1(x)e^{i2\pi(W_{\mu_1})}, \eta_1(x)e^{i2\pi(W_{\eta_1})})$ and $R_2 = (\mu_2(x)e^{i2\pi(W_{\mu_2})}, \eta_2(x)e^{i2\pi(W_{\eta_2})})$ and $\gamma > 0$ be any real number [53]. Then

- (1) $R_1 \oplus R_2 = \left((\mu_1^2 + \mu_2^2 - \mu_1^2 \mu_2^2)^{\frac{1}{2}} e^{i2\pi(W_{\mu_1}^2 + W_{\mu_2}^2 - W_{\mu_1}^2 W_{\mu_2}^2)^{\frac{1}{2}}}, (\eta_1 \eta_2) e^{i2\pi(W_{\eta_1} W_{\eta_2})} \right);$
- (2) $R_1 \otimes R_2 = \left((\mu_1 \mu_2) e^{i2\pi(W_{\mu_1} W_{\mu_2})}, (\eta_1^2 + \eta_2^2 - \eta_1^2 \eta_2^2)^{\frac{1}{2}} e^{i2\pi(W_{\eta_1}^2 + W_{\eta_2}^2 - W_{\eta_1}^2 W_{\eta_2}^2)^{\frac{1}{2}}} \right);$
- (3) $\gamma R_1 = \left((1 - (1 - \mu_1^2)^\gamma)^{\frac{1}{2}} e^{i2\pi(1 - (1 - W_{\mu_1}^\gamma))^{\frac{1}{2}}}, \eta_1^\gamma e^{i2\pi W_{\eta_1}^\gamma} \right);$
- (4) $\gamma R_1 = \left(\mu_1^\gamma e^{i2\pi W_{\mu_1}^\gamma}, (1 - (1 - \eta_1^2)^\gamma)^{\frac{1}{2}} e^{i2\pi(1 - (1 - W_{\eta_1}^\gamma))^{\frac{1}{2}}} \right).$

Definition 7. A q -ROFS R on finite universal set X is given by [9]:

$$R = \{(x, \mu'_R(x), \eta'_R(x)) : x \in X\} \quad (5)$$

with a condition: $0 \leq \mu'^q_R(x) + \eta'^q_R(x) \leq 1$. The hesitancy degree is denoted and defined by $\theta_R = (1 - \mu'^q_R(x) - \eta'^q_R(x))^{\frac{1}{q}}$. The q -rung orthopair fuzzy number is given by $R = (\mu'_R(x), \eta'_R(x))$.

Definition 8. A family of data R_i ($i = 1, 2, \dots, n$), then the power averaging operator is given by [57]:

$$PA(R_1, R_2, \dots, R_n) = \frac{\sum_{j=1}^n ((1 + {}^T(R_j)) R_j)}{\sum_{j=1}^n (1 + {}^T(R_j))}$$

where $\sum_{i=1}^n \sup_{i \neq j} (R_i, R_j) = {}^T(R_i)$ and $\sup_{i \neq j} (R_i, R_j)$ is called support degree, which holds the axioms: $\sup_{i \neq j} (R_i, R_j) \in [0, 1]; \sup_{i \neq j} (R_i, R_j) = \sup_{j \neq i} (R_j, R_i); \text{ and } \sup_{i \neq j} (R_i, R_j) \geq \sup_{k \neq l} (R_k, R_l) \Rightarrow |R_i - R_j| \leq |R_k - R_l|$.

2.2. Standard VIKOR Method

VIKOR (“VIseKriterijumska Optimizacija I Kompromisno Resenje”) [58] is a standard and important method to solve the decision making problem. In the VIKOR method, alternatives are assessed by considering the compromise between group utility maximization and individual regret minimization. The standard procedure for solving the decision making problems with the VIKOR method is summarized below:

Consider m alternatives X_1, X_2, \dots, X_m and n attributes $\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n$ with respect to weight vectors such that $w = (w_1, w_2, \dots, w_n)^T$, $\sum_{j=1}^n w_j = 1$ and the compromise ranking by VIKOR methods starts with the form of L_p -metric [59].

$$L_{pi} = \left\{ \sum_{j=1}^n \left[\left(\frac{R_j^* - R_{ij}}{R_j^* - R_j^-} \right) \right]^p \right\}^{\frac{1}{p}}, \quad 1 \leq p \leq \infty, \quad i = 1, 2, \dots, m$$

In the VIKOR method, the maximum group utility can be gotten by $\min S_i$ and minimum individual regret can be gotten by $\min S'_i$, where $S_i = L_{1,i}$ and $S'_i = L_{\infty,i}$. The various steps involved in the VIKOR method are summarized as follows:

Step 1: Computing the virtual positive ideal x_j^* and the virtual negative ideal x_j^- values under the attributes \tilde{A}_j , we have

$$x_j^* = \max_i(x_{ij}), \quad x_j^- = \min_i(x_{ij})$$

Step 2: Computing the values of group utility S_i and S'_i , we have

$$S_i = \frac{\sum_{j=1}^n w_j \| R_j^* - R_{ij} \|}{\| R_j^* - R_j^- \|} \text{ and } S'_i = \frac{\max_j w_j \| R_j^* - R_{ij} \|}{\| R_j^* - R_j^- \|}$$

Step 3: Computing the values of $Q_i, i = 1, 2, \dots, m$, we have

$$Q_i = \frac{v(S_i - S^*)}{(S^- - S^*)} + \frac{(1-v)(S'_i - S'^*)}{(S'^- - S'^*)},$$

where $S^* = \min_i(S_i)$, $S^- = \max_i(S_i)$, $S'^* = \min_i(S'_i)$ and $S'^- = \max_i(S'_i)$, the symbol v is the balance parameter which can balance the group of utility and individual regret.

Step 4: Using the values of S, S' and Q and ranking the alternatives, then we will obtain the compromise solution.

Step 5: When we get the compromise solution $X^{(1)}$ in steps 4, then it satisfied the following two conditions.

- **Condition 1: Acceptable advantages:** $Q(X^{(2)}) - Q(X^{(1)}) \geq \frac{1}{m-1}$, where $Q(X^{(2)})$ is the Q value in the second position of all ranking alternatives produced by the value of Q and m number of alternatives.
- **Condition 2: Acceptable stability:** Alternative $X^{(1)}$ must also in the first position of all ranking alternatives produced by the values of S or S' .

If one of the above conditions is not met, then we collected the compromise alternatives and not one compromise solution. If condition 2 does not hold, then we will examine the alternatives $X^{(1)}$ and $X^{(2)}$ should be the compromise solution. If condition 2 does not hold, then the maximum M eximane by the formula $Q(X^{(M)}) - Q(X^{(1)}) < MQ = \frac{1}{m-1}$, we examined the alternatives $X^{(1)}, X^{(2)}, \dots, X^{(m)}$ are the compromise solution.

3. Proposed Cq-ROFS

In this section, we present the new concept of Complex q-rung orthopair fuzzy set (Cq-ROFS) and state their properties.

Definition 9. A Cq-ROFS R on finite universal set X is given by:

$$R = \{(x, \mu'_R(x), \eta'_R(x)) : x \in X\}, \quad (6)$$

where $\mu'_R(x) = \mu_R(x)e^{i2\pi(W_{\mu_R})}$ and $\eta'_R(x) = \eta_R(x)e^{i2\pi(W_{\eta_R})}$, represents the complex-valued truth and complex-valued falsity degrees, with a conditions: $0 \leq \mu_R^q(x) + \eta_R^q(x) \leq 1, 0 \leq W_{\mu_R}^q(x) + W_{\eta_R}^q(x) \leq 1$. The complex q -rung orthopair fuzzy number (Cq -ROFN) is denoted by: $R = (\mu_R(x)e^{i2\pi(W_{\mu_R})}, \eta_R(x)e^{i2\pi(W_{\eta_R})})$.

Remark 1. The hesitancy degree is denoted and defined by $\theta_R = (1 - \mu_R^q(x) - \eta_R^q(x))^{\frac{1}{q}} e^{i2\pi(1 - W_{\mu_R}^q(x) - W_{\eta_R}^q(x))^{\frac{1}{q}}}$.

Definition 10. For a two Cq -ROFNs $R_1 = (\mu_1(x)e^{i2\pi(W_{\mu_1})}, \eta_1(x)e^{i2\pi(W_{\eta_1})})$ and $R_2 = (\mu_2(x)e^{i2\pi(W_{\mu_2})}, \eta_2(x)e^{i2\pi(W_{\eta_2})})$, then

- (1) $R_1 \subseteq R_2$ iff $\mu_1(x) \lesssim \mu_2(x), \eta_1(x) \geq \eta_2(x)$ and $W_{\mu_1} \lesssim W_{\mu_2}, W_{\eta_1} \geq W_{\eta_2}$.
- (2) $R_1 = R_2$ if $\mu_1(x) = \mu_2(x), \eta_1(x) = \eta_2(x)$ and $W_{\mu_1} = W_{\mu_2}, W_{\eta_1} = W_{\eta_2}$.
- (3) $R_1^c = (\eta_1(x)e^{i2\pi(W_{\eta_1})}, \mu_1(x)e^{i2\pi(W_{\mu_1})})$.

To compare two or more Cq -ROFNs, we define the score and accuracy functions for them as

Definition 11. For Cq -ROFN $R_1 = (\mu_1(x)e^{i2\pi(W_{\mu_1})}, \eta_1(x)e^{i2\pi(W_{\eta_1})})$, a score function for it is defined as

$$S(R_1) = \mu_1^q - \eta_1^q + W_{\mu_1}^q - W_{\eta_1}^q \quad (7)$$

while an accuracy function H is stated as

$$H(R_1) = \mu_1^q + \eta_1^q + W_{\mu_1}^q + W_{\eta_1}^q \quad (8)$$

Definition 12. For two Cq -ROFNs R_1 and R_2 and by using Definition 11, we define an order relation between them as follows

- (1) If $S(R_1) > S(R_2)$ then $R_1 > R_2$, similarly if $H(R_1) > H(R_2)$ then $R_1 > R_2$.
- (2) If $S(R_1) = S(R_2)$ and $H(R_1) > H(R_2)$ then $R_1 > R_2$.

Definition 13. For a two Cq -ROFNs $R_1 = (\mu_1(x)e^{i2\pi(W_{\mu_1})}, \eta_1(x)e^{i2\pi(W_{\eta_1})})$ and $R_2 = (\mu_2(x)e^{i2\pi(W_{\mu_2})}, \eta_2(x)e^{i2\pi(W_{\eta_2})})$ and $\gamma > 0$ be any real number. Then

- (1) $R_1 \oplus R_2 = \left((\mu_1^q + \mu_2^q - \mu_1^q \mu_2^q)^{\frac{1}{q}} e^{i2\pi(W_{\mu_1}^q + W_{\mu_2}^q - W_{\mu_1}^q W_{\mu_2}^q)^{\frac{1}{q}}}, (\eta_1 \eta_2)e^{i2\pi(W_{\eta_1} W_{\eta_2})} \right);$
- (2) $R_1 \otimes R_2 = \left((\mu_1 \mu_2)e^{i2\pi(W_{\mu_1} W_{\mu_2})}, (\eta_1^q + \eta_2^q - \eta_1^q \eta_2^q)^{\frac{1}{q}} e^{i2\pi(W_{\eta_1}^q + W_{\eta_2}^q - W_{\eta_1}^q W_{\eta_2}^q)^{\frac{1}{q}}} \right);$
- (3) $\gamma R_1 = \left((1 - (1 - \mu_1^q)^\gamma)^{\frac{1}{q}} e^{i2\pi(1 - (1 - W_{\mu_1}^q)^\gamma)^{\frac{1}{q}}}, \eta_1^\gamma e^{i2\pi W_{\eta_1}^\gamma} \right);$
- (4) $\gamma R_1 = \left(\mu_1^\gamma e^{i2\pi W_{\mu_1}^\gamma}, (1 - (1 - \eta_1^q)^\gamma)^{\frac{1}{q}} e^{i2\pi(1 - (1 - W_{\eta_1}^q)^\gamma)^{\frac{1}{q}}} \right).$

Theorem 1. For two Cq-ROFNs R_1 and R_2 with real number $\gamma > 0$, then $R_1 \oplus R_2$, γR_1 , $R_1 \otimes R_2$ and R_1^γ as defined in Definition 13, are Cq-ROFNs.

Proof. For two Cq-ROFNs $R_1 = (\mu_1(x)e^{i2\pi(W_{\mu_1})}, \eta_1(x)e^{i2\pi(W_{\eta_1})})$ and $R_2 = (\mu_2(x)e^{i2\pi(W_{\mu_2})}, \eta_2(x)e^{i2\pi(W_{\eta_2})})$ and by definition of Cq-ROFN, we have

$$0 \leq \mu_i, W_{\mu_i}, \eta_i, W_{\eta_i} \leq 1, \text{ then } 0 \leq \mu_i^q + \eta_i^q \leq 1 \text{ and } 0 \leq W_{\mu_i}^q + W_{\eta_i}^q \leq 1$$

Consider $R_3 = R_1 \oplus R_2$ and by Definition 13, we consider $R_3 = (\mu_3(x)e^{i2\pi(W_{\mu_3})}, \eta_3(x)e^{i2\pi(W_{\eta_3})})$, where $\mu_3 = (1 - \prod_{i=1}^2 (1 - \mu_i^q))^{\frac{1}{q}}$, $W_{\mu_3} = (1 - \prod_{i=1}^2 (1 - W_{\mu_i}^q))^{\frac{1}{q}}$, $\eta_3 = \prod_{i=1}^2 \eta_i$ and $W_{\eta_3} = \prod_{i=1}^2 W_{\eta_i}$. Since, $0 \leq \mu_1, \mu_2 \leq 1$, then $0 \leq 1 - \mu_i \leq 1$ and hence for $q \geq 1$ we have $0 \leq (1 - \prod_{i=1}^2 (1 - \mu_i^q))^{\frac{1}{q}} \leq 1$. On the other hand, $0 \leq \eta_1, \eta_2 \leq 1$, than $0 \leq \prod_{i=1}^2 \eta_i \leq 1$. Moreover, $0 \leq \mu_i^q + \eta_i^q \leq 1$, $i = 1, 2$, then

$$\mu_3^q + \eta_3^q = 1 - \prod_{i=1}^2 (1 - \mu_i^q) + \prod_{i=1}^2 \eta_i^q \leq 1 - \prod_{i=1}^2 (1 - \mu_i^q) + \prod_{i=1}^2 (1 - \mu_i^q) = 1.$$

And we have $\mu_3^q + \eta_3^q \geq 0$, so $0 \leq \mu_3^q + \eta_3^q \leq 1$ similaily, $0 \leq W_{\mu_3}^q + W_{\eta_3}^q \leq 1$. Hence $R_1 \oplus R_2$ proves. Similarly it can be proving that γR_1 , $R_1 \otimes R_2$ and R_1^γ are also Cq-ROFNs. \square

Theorem 2. For any three Cq-ROFNs R_1, R_2 and R_3 , the following axioms are hold true

- (1) $R_1 \oplus R_2 = R_2 \oplus R_1$;
- (2) $R_1 \otimes R_2 = R_2 \otimes R_1$;
- (3) $(R_1 \oplus R_2) \oplus R_3 = R_1 \oplus (R_2 \oplus R_3)$;
- (4) $(R_1 \otimes R_2) \otimes R_3 = R_1 \otimes (R_2 \otimes R_3)$.

Proof. Straightforward. \square

Theorem 3. For any two Cq-ROFNs R_1 and R_2 with γ_1 and γ_2 are a positive real numbers, the following axioms are hold true

- (1) $\gamma_1(R_1 \oplus R_2) = \gamma_1 R_1 \oplus \gamma_1 R_2$;
- (2) $(R_1 \otimes R_2)^{\gamma_1} = R_1^{\gamma_1} \otimes R_2^{\gamma_1}$;
- (3) $(\gamma_1 + \gamma_2)R_1 = \gamma_1 R_1 \oplus \gamma_2 R_1$;
- (4) $R_1^{\gamma_1 + \gamma_2} = R_1^{\gamma_1} \otimes R_1^{\gamma_2}$

Proof. We examine part (1) and part (3), the others are straightforward.

(1) Let $R_1 = (\mu_1(x)e^{i2\pi(W_{\mu_1})}, \eta_1(x)e^{i2\pi(W_{\eta_1})})$ and $R_2 = (\mu_2(x)e^{i2\pi(W_{\mu_2})}, \eta_2(x)e^{i2\pi(W_{\eta_2})})$ be two Cq-ROFNs. Then

$$\begin{aligned}\gamma_1(R_1 \oplus R_2) &= \gamma_1 \left(\begin{array}{l} \left(1 - \prod_{j=1}^2 \left(1 - (\mu_j)^q \right) \right)^{\frac{1}{q}} \cdot e^{i2\pi(1 - \prod_{j=1}^2 (1 - (W_{\mu_j})^q))^{\frac{1}{q}}}, \\ \left(\prod_{j=1}^2 \eta_j \right) \cdot e^{i2\pi(\prod_{j=1}^2 W_{\eta_j})} \end{array} \right) \\ &= \left(\begin{array}{l} \left(1 - \prod_{j=1}^2 \left(1 - (\mu_j)^q \right)^{\gamma_1} \right)^{\frac{1}{q}} \cdot e^{i2\pi(1 - \prod_{j=1}^2 (1 - (W_{\mu_j})^q)^{\gamma_1})^{\frac{1}{q}}}, \\ \left(\prod_{j=1}^2 \eta_j \right)^{\gamma_1} \cdot e^{i2\pi(\prod_{j=1}^2 W_{\eta_j})^{\gamma_1}} \end{array} \right) \\ &= \left(\begin{array}{l} \left(1 - (1 - \mu_1^q)^{\gamma_1} \right)^{\frac{1}{q}} e^{i2\pi(1 - (1 - W_{\mu_1}^q)^{\gamma_1})^{\frac{1}{q}}}, \\ \eta_1^{\gamma_1} e^{i2\pi W_{\eta_1}^{\gamma_1}} \end{array} \right) \\ &\oplus \left(\begin{array}{l} \left(1 - (1 - \mu_2^q)^{\gamma_1} \right)^{\frac{1}{q}} e^{i2\pi(1 - (1 - W_{\mu_2}^q)^{\gamma_1})^{\frac{1}{q}}}, \\ \eta_2^{\gamma_1} e^{i2\pi W_{\eta_2}^{\gamma_1}} \end{array} \right) \\ &= \gamma_1 R_1 \oplus \gamma_1 R_2\end{aligned}$$

(3) Let $R_1 = (\mu_1(x)e^{i2\pi(W_{\mu_1})}, \eta_1(x)e^{i2\pi(W_{\eta_1})})$ and $R_2 = (\mu_2(x)e^{i2\pi(W_{\mu_2})}, \eta_2(x)e^{i2\pi(W_{\eta_2})})$ be two Cq-ROFNs. Then

$$\begin{aligned}(\gamma_1 + \gamma_2)R_1 &= (\gamma_1 + \gamma_2)(\mu_1(x)e^{i2\pi(W_{\mu_1})}, \eta_1(x)e^{i2\pi(W_{\eta_1})}) \\ &= \left(\left(1 - (1 - \mu_1^q)^{\gamma_1 + \gamma_2} \right)^{\frac{1}{q}} e^{i2\pi(1 - (1 - W_{\mu_1}^q)^{\gamma_1 + \gamma_2})^{\frac{1}{q}}}, \eta_1^{\gamma_1 + \gamma_2} e^{i2\pi W_{\eta_1}^{\gamma_1 + \gamma_2}} \right) \\ &= \left(\begin{array}{l} \left(1 - (1 - \mu_1^q)^{\gamma_1} \right)^{\frac{1}{q}} e^{i2\pi(1 - (1 - W_{\mu_1}^q)^{\gamma_1})^{\frac{1}{q}}}, \\ \eta_1^{\gamma_1} e^{i2\pi W_{\eta_1}^{\gamma_1}} \end{array} \right) \\ &\oplus \left(\begin{array}{l} \left(1 - (1 - \mu_1^q)^{\gamma_2} \right)^{\frac{1}{q}} e^{i2\pi(1 - (1 - W_{\mu_1}^q)^{\gamma_2})^{\frac{1}{q}}}, \\ \eta_1^{\gamma_2} e^{i2\pi W_{\eta_1}^{\gamma_2}} \end{array} \right) \\ &= \gamma_1 R_1 \oplus \gamma_2 R_1\end{aligned}$$

Hence $(\gamma_1 + \gamma_2)R_1 = \gamma_1 R_1 \oplus \gamma_2 R_1$. \square

4. Proposed Power Aggregation Operators for Cq-ROFS

In this section, we define the power weighted averaging and geometric AOs for a collection of Cq-ROFNs. Several of their properties are also investigated.

4.1. The Cq-ROFPA Operator

Let Γ be the collections of Cq-ROFNs. Here, we introduce the concept of Complex q-rung orthopair power averaging (Cq-ROFPA) and Complex q-rung orthopair power weighted averaging (Cq-ROFPWA) operators as follows:

Definition 14. The Cq-ROFPA operator is a mapping $Cq-ROFPA : \Gamma^n \rightarrow \Gamma$ given by:

$$Cq-ROFPA(R_1, R_2, \dots, R_n) = \bigoplus_{j=1}^n \frac{1+T(R_j)}{\sum_{j=1}^n (1+T(R_j))} R_j \quad (9)$$

where

$$\text{Sup}(R_j) = \sum_{\substack{j=1 \\ i \neq j}}^n \text{Sup}(R_j, R_i) \quad (10)$$

and $\text{Sup}(R_j, R_i)$ represents is the support for R_j from R_i , with the conditions: $\text{Sup}(R_j, R_i) \in [0, 1]$; $\text{Sup}(R_j, R_i) = \text{Sup}(R_i, R_j)$; and $\text{Sup}(R_j, R_i) \geq \text{Sup}(R_s, R_t)$, if $d(R_j, R_i) < d(R_s, R_t)$, where d is distance measure.

By using the operational laws of Cq-ROFNs and by Definition 14, we examined the following result.

Theorem 4. For the collections of “ n ” Cq-ROFNs $R_j = (\mu_j e^{i2\pi(W_{\mu_j})}, \eta_j e^{i2\pi(W_{\eta_j})})$, the aggregated value obtained through Definition 14 is again a Cq-ROFN and given by

$$\begin{aligned} \text{Cq-ROFPW}\tilde{\Delta}(R_1, R_2, \dots, R_n) &= \oplus_{j=1}^n \frac{1 + \text{T}(R_j)}{\sum_{j=1}^n (1 + \text{T}(R_j))} R_j \\ &= \left(\begin{array}{l} \left(1 - \prod_{j=1}^n (1 - \mu_j^q)^{\frac{(1 + \text{T}(R_j))}{\sum_{j=1}^n (1 + \text{T}(R_j))}} \right)^{\frac{1}{q}} \cdot e^{i2\pi(1 - \prod_{j=1}^n (1 - w_{\mu_j}^q))^{\frac{(1 + \text{T}(R_j))}{\sum_{j=1}^n (1 + \text{T}(R_j))}}} \\ \left(\prod_{j=1}^n (\eta_j)^{\frac{(1 + \text{T}(R_j))}{\sum_{j=1}^n (1 + \text{T}(R_j))}} \right) \cdot e^{i2\pi(\prod_{j=1}^n (w_{\eta_j})^{\frac{(1 + \text{T}(R_j))}{\sum_{j=1}^n (1 + \text{T}(R_j))}})} \end{array} \right), \end{aligned} \quad (11)$$

where $\text{T}(R_j) = \sum_{\substack{j=1 \\ i \neq j}}^n \text{Sup}(R_j, R_i)$ is defined in Equation (10).

Proof. For two Cq-ROFNs $R_1 = (\mu_1(x) e^{i2\pi(W_{\mu_1})}, \eta_1(x) e^{i2\pi(W_{\eta_1})})$ and $R_2 = (\mu_2(x) e^{i2\pi(W_{\mu_2})}, \eta_2(x) e^{i2\pi(W_{\eta_2})})$. For simplicity, we consider $z_j = \frac{(1 + \text{T}(R_j))}{\sum_{j=1}^n (1 + \text{T}(R_j))} \in [0, 1]$ and hence by applying the operational laws of Cq-ROFNs, we get

$$\begin{aligned} z_1 R_1 &= \left(\left(1 - (1 - \mu_1^q)^{z_1} \right)^{\frac{1}{q}} e^{i2\pi(1 - (1 - W_{\mu_1}^q)^{z_1})^{\frac{1}{q}}}, \eta_1^{z_1} e^{i2\pi W_{\eta_1}^{z_1}} \right) \\ z_2 R_2 &= \left(\left(1 - (1 - \mu_2^q)^{z_2} \right)^{\frac{1}{q}} e^{i2\pi(1 - (1 - W_{\mu_2}^q)^{z_2})^{\frac{1}{q}}}, \eta_2^{z_2} e^{i2\pi W_{\eta_2}^{z_2}} \right) \end{aligned}$$

Using the additive law in Definition 13, we have

$$\begin{aligned} \text{Cq-ROFPW}\tilde{\Delta}(R_1, R_2) &= \left(\begin{array}{l} \left(1 - (1 - \mu_1^q)^{z_1} \right)^{\frac{1}{q}} e^{i2\pi(1 - (1 - W_{\mu_1}^q)^{z_1})^{\frac{1}{q}}}, \\ \eta_1^{z_1} e^{i2\pi W_{\eta_1}^{z_1}} \end{array} \right) \\ &\oplus \left(\begin{array}{l} \left(1 - (1 - \mu_2^q)^{z_2} \right)^{\frac{1}{q}} e^{i2\pi(1 - (1 - W_{\mu_2}^q)^{z_2})^{\frac{1}{q}}}, \\ \eta_2^{z_2} e^{i2\pi W_{\eta_2}^{z_2}} \end{array} \right) \\ &= \left(\begin{array}{l} \left(1 - \prod_{j=1}^2 \left(1 - \mu_j^q \right)^{z_j} \right)^{\frac{1}{q}} e^{i2\pi(1 - \prod_{j=1}^2 (1 - W_{\mu_j}^q)^{z_j})^{\frac{1}{q}}}, \\ \left(\prod_{j=1}^2 (\eta_j)^{z_j} \right) e^{i2\pi(\prod_{j=1}^2 (W_{\eta_j})^{z_j})} \end{array} \right) \end{aligned}$$

so the result is true for $n = 2$. Next, we assume that the result is holds for $n = m$.

$$Cq - ROFP\tilde{A}(R_1, R_2, \dots, R_m) = \left(\begin{array}{l} \left(1 - \prod_{j=1}^m (1 - \mu_j^q)^{z_j} \right)^{\frac{1}{q}} \cdot e^{i2\pi(1 - \prod_{j=1}^m (1 - W_{\mu_j}^q)^{z_j})^{\frac{1}{q}}}, \\ \left(\prod_{j=1}^m (\eta_j)^{z_j} \right) \cdot e^{i2\pi(\prod_{j=1}^m (W_{\eta_j})^{z_j})} \end{array} \right)$$

Then for $n = m + 1$, we have

$$\begin{aligned} Cq - ROFP\tilde{A}(R_1, R_2, \dots, R_{m+1}) &= \left(\begin{array}{l} \left(1 - \prod_{j=1}^m (1 - \mu_j^q)^{z_j} \right)^{\frac{1}{q}} \cdot e^{i2\pi(1 - \prod_{j=1}^m (1 - W_{\mu_j}^q)^{z_j})^{\frac{1}{q}}}, \\ \left(\prod_{j=1}^m (\eta_j)^{z_j} \right) \cdot e^{i2\pi(\prod_{j=1}^m (W_{\eta_j})^{z_j})} \end{array} \right) \\ &\oplus \left(\begin{array}{l} \left(1 - (1 - \mu_{m+1}^q)^{z_{m+1}} \right)^{\frac{1}{q}} e^{i2\pi(1 - (1 - W_{\mu_{m+1}}^q)^{z_{m+1}})^{\frac{1}{q}}}, \\ \eta_{m+1}^{z_{m+1}} e^{i2\pi W_{\eta_{m+1}}^{z_{m+1}}} \end{array} \right) \\ &= \left(\begin{array}{l} \left(1 - \prod_{j=1}^{m+1} (1 - \mu_j^q)^{z_j} \right)^{\frac{1}{q}} \cdot e^{i2\pi(1 - \prod_{j=1}^{m+1} (1 - W_{\mu_j}^q)^{z_j})^{\frac{1}{q}}}, \\ \left(\prod_{j=1}^{m+1} (\eta_j)^{z_j} \right) \cdot e^{i2\pi(\prod_{j=1}^{m+1} (W_{\eta_j})^{z_j})} \end{array} \right) \end{aligned}$$

Hence, the result is true for $n = m + 1$, so the Equation (11) holds true for all natural numbers. \square

The established operators based on Cq-ROFSs is more powerful due to its constraint, that is, the sum of q-powers of the real parts (imaginary parts) of the membership and non-membership grades is less than or equal to one.

To further explain its working for different numbers, we verified with the help of an example discussed below.

Example 1. Let $R_1 = (0.9e^{i2\pi(0.8)}, 0.4e^{i2\pi(0.3)})$, $R_2 = (0.53e^{i2\pi(0.63)}, 0.47e^{i2\pi(0.57)})$, $R_3 = (0.96e^{i2\pi(0.66)}, 0.33e^{i2\pi(0.23)})$ and $R_4 = (0.89e^{i2\pi(0.50)}, 0.41e^{i2\pi(0.31)})$ are four Cq-ROFNs for $q = 4$, then we examine $d(R_s, R_t)$ for all $s, t \in \{1, 2, 3, 4\}, s \neq t$. Using the distance measure as defined in below equation

$$d(R_s, R_t) = \frac{(|\mu_s^q - \mu_t^q| + |\eta_s^q - \eta_t^q| + |W_{\mu_s}^q - W_{\mu_t}^q| + |W_{\eta_s}^q - W_{\eta_t}^q|)}{4}, \quad s, t = 1, 2, 3, 4; \quad (12)$$

and hence we compute the measurement values of each numbers and get

$$d(R_1, R_2) = \frac{|0.9^4 - 0.53^4| + |0.4^4 - 0.47^4| + |0.8^4 - 0.63^4| + |0.3^4 - 0.57^4|}{4} = 0.22$$

Similarly, $d(R_1, R_3) = 0.085$, $d(R_1, R_4) = 0.082$, $d(R_2, R_3) = 0.24$, $d(R_2, R_4) = 0.20$ and $d(R_3, R_4) = 0.098$. Subsequently, we can obtain the support degree, such that

$$Sup(R_1, R_2) = 1 - d(R_1, R_2) = 1 - 0.22 = 0.78$$

Similarly, $\text{Sup}(R_1, R_3) = 0.915$, $\text{Sup}(R_1, R_4) = 0.92$, $\text{Sup}(R_2, R_3) = 0.77$, $\text{Sup}(R_2, R_4) = 0.80$ and $\text{Sup}(R_3, R_4) = 0.9$. Furthermore, we use Equation (10) and get

$$\begin{aligned} \mathbb{T}(R_1) &= \sum_{\substack{k=1 \\ k \neq j}}^4 \text{Sup}(R_k, R_j) = \text{Sup}(R_1, R_2) + \text{Sup}(R_1, R_3) + \text{Sup}(R_1, R_4) \\ &= 0.78 + 0.915 + 0.92 = 2.62 \end{aligned}$$

Similarly, $\mathbb{T}(R_2) = 2.34$, $\mathbb{T}(R_3) = 2.58$ and $\mathbb{T}(R_4) = 2.62$. Then,

$$\sum_{i=1}^4 (1 + \mathbb{T}(R_i)) = (1 + 2.62) + (1 + 2.34) + (1 + 2.58) + (1 + 2.62) = 14.16$$

$$z_1 = \frac{(1 + \mathbb{T}(R_1))}{\sum_{i=1}^4 (1 + \mathbb{T}(R_i))} = \frac{1 + 2.62}{14.16} = 0.26$$

Similarly $z_2 = 0.24$, $z_3 = 0.26$ and $z_4 = 0.26$. Moreover,

$$\prod_{j=1}^4 (1 - \mu_j^q)^{z_j} = (1 - (0.9)^4)^{0.26} \times (1 - (0.53)^4)^{0.24} \times (1 - (0.96)^4)^{0.26} \times (1 - (0.89)^4)^{0.26} = 0.35$$

$$\prod_{j=1}^4 (1 - W_{\mu_j}^q)^{z_j} = (1 - 0.8^4)^{0.26} \times (1 - 0.63^4)^{0.24} \times (1 - 0.66^4)^{0.26} \times (1 - 0.5^4)^{0.26} = 0.78$$

$$\prod_{j=1}^4 (\eta_j)^{z_j} = (0.4)^{0.26} (0.47)^{0.24} (0.33)^{0.26} (0.41)^{0.26} = 0.39$$

and

$$\prod_{j=1}^4 (W_{\eta_j})^{z_j} = (0.3)^{0.26} (0.57)^{0.24} (0.23)^{0.26} (0.31)^{0.26} = 0.32$$

Then, by using $Cq - ROFP\tilde{\mathbf{A}}$ operator, we get

$$Cq - ROFP\tilde{\mathbf{A}}(R_1, R_2, R_3, R_4) = \left(\begin{array}{c} (1 - 0.35)^{\frac{1}{4}} e^{i2\pi(1-0.78)^{\frac{1}{4}}} \\ (0.39)e^{i2\pi(0.32)} \end{array} \right) = \left(\begin{array}{c} (0.898)e^{i2\pi(0.68)} \\ (0.39)e^{i2\pi(0.32)} \end{array} \right)$$

It is clear that the aggregated value is again q -ROFNs.

The proposed $Cq - ROFP\tilde{\mathbf{A}}$ operator satisfies certain properties, which are stated as below. Here, we have taken $z_j = \frac{(1 + \mathbb{T}(R_j))}{\sum_{j=1}^n (1 + \mathbb{T}(R_j))}$.

Property 1. When all R_j ($j = 1, 2, \dots, n$) are equal, that is $R_j = R$ for all j , then

$$Cq - ROFP\tilde{\mathbf{A}}(R_1, R_2, \dots, R_n) = R \quad (13)$$

Proof. When $R_j = R$ for all j , then, we have $\mu_j = \mu$, $\eta_j = \eta$, $W_{\mu_j} = W_\mu$, $W_{\eta_j} = W_\eta$ and hence by definition of Cq-ROFPA operator, we get

$$\begin{aligned} Cq - ROFP\tilde{\Delta}(R_1, R_2, \dots, R_n) &= \left(\begin{array}{l} \left(1 - \prod_{j=1}^n (1 - \mu^q)^{z_j} \right)^{\frac{1}{q}} \cdot e^{i2\pi(1 - \prod_{j=1}^n (1 - W_\mu^q)^{z_j})^{\frac{1}{q}}}, \\ \left(\prod_{j=1}^n (\eta)^{z_j} \right) \cdot e^{i2\pi(\prod_{j=1}^n (W_\eta)^{z_j})} \end{array} \right) \\ &= \left(\begin{array}{l} \left(1 - (1 - (\mu)^q)^{\sum_{j=1}^n z_j} \right)^{\frac{1}{q}} \cdot e^{i2\pi(1 - (1 - (w_\mu)^q)^{\sum_{j=1}^n z_j})^{\frac{1}{q}}}, \\ ((\eta)^{\sum_{j=1}^n z_j}) \cdot e^{i2\pi((w_\eta)^{\sum_{j=1}^n z_j})} \end{array} \right) \\ &= (\mu \cdot e^{i2\pi(w_\mu)}, \eta \cdot e^{i2\pi(w_\eta)}) = R \end{aligned}$$

□

Property 2. Let $R_j (j = 1, 2, \dots, n)$ be the family of Cq-ROFNs and let $R^- = \min_j(R_j)$, $R^+ = \max_j(R_j)$. Then

$$R^- \leq Cq - ROFP\tilde{\Delta}(R_1, R_2, \dots, R_n) \leq R^+ \quad (14)$$

Proof. By Theorem 4, we considered $Cq - ROFP\tilde{\Delta}(R_1, R_2, \dots, R_n) = R$, then

$$R = (\mu_R e^{i2\pi(W_{\mu_R})}, \eta_R e^{i2\pi(W_{\eta_R})})$$

For Cq-ROFN R_j , we first verify the real part of the complex-valued membership grade:

$$\begin{aligned} \min_j(\mu_j) &\leq \mu_j \leq \max_j(\mu_j) \\ \Rightarrow \min_j(\mu_j^q) &\leq \mu_j^q \leq \max_j(\mu_j^q) \\ \Rightarrow 1 - \max_j(\mu_j^q) &\leq 1 - \mu_j^q \leq 1 - \min_j(\mu_j^q) \end{aligned}$$

which implies that

$$\begin{aligned} (1 - \max_j(\mu_j^q))^{z_j} &\leq (1 - \mu_j^q)^{z_j} \leq (1 - \min_j(\mu_j^q))^{z_j} \\ \Rightarrow \left(1 - \prod_{j=1}^n (1 - \max_j(\mu_j^q))^{z_j} \right)^{\frac{1}{q}} &\geq \left(1 - \prod_{j=1}^n (1 - \mu_j^q)^{z_j} \right)^{\frac{1}{q}} \geq \left(1 - \prod_{j=1}^n (1 - \min_j(\mu_j^q))^{z_j} \right)^{\frac{1}{q}} \\ \Rightarrow \left(1 - (1 - \max_j(\mu_j^q))^{\sum_{j=1}^n z_j} \right)^{\frac{1}{q}} &\geq \left(1 - \prod_{j=1}^n (1 - \mu_j^q)^{z_j} \right)^{\frac{1}{q}} \geq \left(1 - (1 - \min_j(\mu_j^q))^{\sum_{j=1}^n z_j} \right)^{\frac{1}{q}} \\ \Rightarrow \max_j(\mu_j) &\geq \left(1 - \prod_{j=1}^n (1 - \mu_j^q)^{z_j} \right)^{\frac{1}{q}} \geq \min_j(\mu_j) \end{aligned}$$

Similarly, we can obtain

$$\max_j(W_{\mu_j}) \geq \left(1 - \prod_{j=1}^n (1 - W_{\mu_j}^q)^{z_j} \right)^{\frac{1}{q}} \geq \min_j(W_{\mu_j})$$

$$\max_j(\eta_j) \geq \left(1 - \prod_{j=1}^n (1 - \eta_j^q)^{z_j} \right)^{\frac{1}{q}} \geq \min_j(\eta_j)$$

and

$$\max_j(W_{\eta_j}) \geq \left(1 - \prod_{j=1}^n (1 - W_{\eta_j}^q)^{z_j}\right)^{\frac{1}{q}} \geq \min_j(W_{\eta_j})$$

Then, by using the Equation (7), we have

$$\begin{aligned} S(R) &= \mu_R^q - \eta_R^q + W_{\mu_R}^q - W_{\eta_R}^q \\ &\leq \max_j(\mu_j^q) - \min_j(\eta_j^q) + \max_j(W_{\mu_j}^q) - \min_j(W_{\eta_j}^q) = S(R^+) \end{aligned}$$

and

$$\begin{aligned} S(R) &= \mu_R^q - \eta_R^q + W_{\mu_R}^q - W_{\eta_R}^q \\ &\geq \min_j(\mu_j^q) - \max_j(\eta_j^q) + \min_j(W_{\mu_j}^q) - \max_j(W_{\eta_j}^q) = S(R^-) \end{aligned}$$

Hence, $R^- \leq Cq - ROFP\tilde{\bar{A}}(R_1, R_2, \dots, R_n) \leq R^+$ \square

Property 3. Let $R_j (j = 1, 2, \dots, n)$ be the permutation of $R'_j (j = 1, 2, \dots, n)$. Then

$$Cq - ROFP\tilde{\bar{A}}(R_1, R_2, \dots, R_n) = Cq - ROFP\tilde{\bar{A}}(R'_1, R'_2, \dots, R'_n) \quad (15)$$

Proof. Let $R_j (j = 1, 2, \dots, n)$ be the permutation of $R'_j (j = 1, 2, \dots, n)$. Then

$$\begin{aligned} Cq - ROFP\tilde{\bar{A}}(R'_1, R'_2, \dots, R'_n) &= \frac{\oplus_{j=1}^n ((1 + T(R'_j)) R'_j)}{\sum_{j=1}^n (1 + T(R'_j))} = \frac{\oplus_{j=1}^n ((1 + T(R_j)) R_j)}{\sum_{j=1}^n (1 + T(R_j))} \\ &= Cq - ROFP\tilde{\bar{A}}(R'_1, R'_2, \dots, R'_n) \end{aligned}$$

Hence, $Cq - ROFP\tilde{\bar{A}}(R_1, R_2, \dots, R_n) = Cq - ROFP\tilde{\bar{A}}(R'_1, R'_2, \dots, R'_n)$ holds. \square

Remark 2. The monotonicity properties cannot be held for the Cq -ROFPA operator.

In the above definition of the Cq -ROFPA operator, it is considered that all the values are equally important; however, in reality, different attributes/numbers play different roles during the aggregation. Hence, we have defined the complex q -rung orthopair power weighted average (Cq -ROFPWA) operator for a collection of Cq -ROFNs.

Definition 15. For a collection of “ n ” Cq -ROFNs R_j , the Cq -ROFPWA operator is defined as

$$\begin{aligned} Cq - ROFPW\tilde{\bar{A}}(R_1, R_2, \dots, R_n) &= \oplus_{j=1}^n \frac{w_j(1+T(R_j))}{\sum_{j=1}^n w_j(1+T(R_j))} R_j \\ &= \left\{ \left(1 - \prod_{j=1}^n (1 - \mu_j^q)^{\left(\frac{w_j(1+T(R_j))}{\sum_{j=1}^n w_j(1+T(R_j))} \right)} \right)^{\frac{1}{q}} \cdot e^{i2\pi(1 - \prod_{j=1}^n (1 - w_j(1+T(R_j)))^{\left(\frac{w_j(1+T(R_j))}{\sum_{j=1}^n w_j(1+T(R_j))} \right)}} \right\} \\ &= \left\{ \left(\prod_{j=1}^n (\eta_j)^{\left(\frac{w_j(1+T(R_j))}{\sum_{j=1}^n w_j(1+T(R_j))} \right)} \right) \cdot e^{i2\pi(\prod_{j=1}^n (w_{\eta_j})^{\left(\frac{w_j(1+T(R_j))}{\sum_{j=1}^n w_j(1+T(R_j))} \right)}} \right\} \end{aligned} \quad (16)$$

where

$$\tau(R_j) = \sum_{\substack{j=1 \\ i \neq j}}^n w_i \text{Sup}(R_j, R_i) \quad (17)$$

represent the support of the numbers.

Example 2. Consider the information given in Example 1 and by taking their weights are $W = (0.4, 0.2, 0.1, 0.3)^T$. By utilizing the information, we get $\sum_{i=1}^4 w_i(1 + \tau(R_i)) = 3.42$ and $z_1 = \frac{w_1(1 + \tau(R_1))}{\sum_{i=1}^4 w_i(1 + \tau(R_i))} = 0.43$. Similarly, $z_2 = 0.20, z_3 = 0.11$ and $z_4 = 0.32$, we get $\prod_{j=1}^4 (1 - \mu_j^q)^{z_j} = 0.37$, $\prod_{j=1}^4 (1 - W_{\mu_j}^q)^{z_j} = 0.74$, $\prod_{j=1}^4 (\eta_j)^{z_j} = 0.39$ and $\prod_{j=1}^4 (W_{\eta_j})^{z_j} = 0.31$. Thus by using Equation (16), we get $Cq - ROFPWA(R_1, R_2, R_3, R_4) = ((0.90)e^{i2\pi(0.71)}, (0.39)e^{i2\pi(0.31)})$.

Similar to Cq-ROFPA operator, the Cq-ROFPWA operator also satisfies certain properties which are stated (without proof) as follows.

(1) (Idempotency) When $R_j = R$ for all j , then

$$Cq - ROFPWA(R_1, R_2, \dots, R_n) = R \quad (18)$$

(2) (Boundedness) Let $R^- = \min_j(R_j), R^+ = \max_j(R_j)$ Then,

$$R^- \leq Cq - ROFPWA(R_1, R_2, \dots, R_n) \leq R^+ \quad (19)$$

(3) (Commutativity) Let $R_j (j = 1, 2, \dots, n)$ be the permutation of $R'_j (j = 1, 2, \dots, n)$. Then,

$$Cq - ROFPWA(R_1, R_2, \dots, R_n) = Cq - ROFPWA(R'_1, R'_2, \dots, R'_n) \quad (20)$$

Next, we provide a complex q-rung orthopair fuzzy power ordered weighted averaging (Cq-ROFPOWA) operator as:

Definition 16. The Cq-ROFPOWA operator is given by

$$\begin{aligned} Cq - ROFPWA(R_1, R_2, \dots, R_n) &= \oplus_{j=1}^n \frac{w_j(1 + \tau(R_{O(j)}))}{\sum_{j=1}^n w_j(1 + \tau(R_{O(j)}))} R_{O(j)} \\ &= \left\{ \left(1 - \prod_{j=1}^n (1 - \mu_{O(j)}^q)^{\left(\frac{w_j(1 + \tau(R_{O(j)}))}{\sum_{j=1}^n w_j(1 + \tau(R_{O(j)}))} \right)^{\frac{1}{q}}} \cdot e^{i2\pi(1 - \prod_{j=1}^n (1 - \mu_{O(j)}^q)^{\left(\frac{w_j(1 + \tau(R_{O(j)}))}{\sum_{j=1}^n w_j(1 + \tau(R_{O(j)}))} \right)^{\frac{1}{q}}}} \right) \right. \\ &\quad \left. \left(\prod_{j=1}^n (\eta_{O(j)})^{\left(\frac{w_j(1 + \tau(R_{O(j)}))}{\sum_{j=1}^n w_j(1 + \tau(R_{O(j)}))} \right)} \cdot e^{i2\pi(\prod_{j=1}^n (\eta_{O(j)})^{\left(\frac{w_j(1 + \tau(R_{O(j)}))}{\sum_{j=1}^n w_j(1 + \tau(R_{O(j)}))} \right)}} \right) \right\}, \end{aligned} \quad (21)$$

where $(O(1), O(2), \dots, O(n))$ is a permutation of $(1, 2, \dots, n)$ such that $R_{O(j-1)} \geq R_{O(j)}$ for all $j = 1, 2, \dots, n$. Furthermore, $w_j (j = 1, 2, \dots, n)$ is the collection of weights such that

$$w_j = g\left(\frac{P_j}{\mathbb{T}V}\right) - g\left(\frac{P_{j-1}}{\mathbb{T}V}\right), P_j = \sum_{i=1}^j V_{O(i)}, \mathbb{T}V = \sum_{i=1}^n V_{O(i)}, V_{O(i)} = 1 + \mathbb{T}(R_{O(i)}), \quad (22)$$

where $\mathbb{T}(R_{O(i)})$ represents the support of the j th largest Cq-ROFNs, that is

$$\mathbb{T}(R_{O(i)}) = \sum_{\substack{j=1 \\ i \neq j}}^n Sup(R_{O(j)}, R_{O(i)}) \quad (23)$$

$Sup(R_{O(j)}, R_{O(i)})$ denotes the support of the j th largest Cq-ROFNs and $g : [0, 1] \rightarrow [0, 1]$ is a basic unit-interval monotonic (BUM) function exhibiting the following properties: $g(0) = 0, g(1) = 1$ and $g(x) \geq g(y)$ if $x > y$.

The Cq-ROFPOWA operator satisfies certain properties as follows.

- (1) (Idempotency) When $R_j = R$ for all j , then

$$Cq-ROFPOW\tilde{A}(R_1, R_2, \dots, R_n) = R \quad (24)$$

- (2) (Boundedness). Let $R^- = \min_j(R_j), R^+ = \max_j(R_j)$ Then,

$$R^- \leq Cq-ROFPOW\tilde{A}(R_1, R_2, \dots, R_n) \leq R^+ \quad (25)$$

- (3) (Commutativity). Let $R_j (j = 1, 2, \dots, n)$ be the permutation of $R'_j (j = 1, 2, \dots, n)$. Then,

$$Cq-ROFPOW\tilde{A}(R_1, R_2, \dots, R_n) = Cq-ROFPOW\tilde{A}(R'_1, R'_2, \dots, R'_n) \quad (26)$$

Next, we provide a complex q-rung orthopair fuzzy power hybrid averaging (Cq-ROFPHA) operator below:

Definition 17. The Cq-ROFPHA operator is given by

$$Cq-ROFPW\tilde{A}(R_1, R_2, \dots, R_n) = \oplus_{j=1}^n \frac{w_j(1+\mathbb{T}(\dot{R}_{O(j)}))}{\sum_{j=1}^n w_j(1+\mathbb{T}(\dot{R}_{O(j)}))} \dot{R}_{O(j)} \\ = \left\{ \begin{array}{l} \left(1 - \prod_{j=1}^n (1 - (\dot{\mu}_{O(j)})^q)^{\frac{w_j(1+\mathbb{T}(\dot{R}_{O(j)}))}{\sum_{j=1}^n w_j(1+\mathbb{T}(\dot{R}_{O(j)}))}} \right)^{\frac{1}{q}} \cdot e^{i2\pi(1-\prod_{j=1}^n (1-(w_{\dot{\mu}_{O(j)}})^q)^{\frac{w_j(1+\mathbb{T}(\dot{R}_{O(j)}))}{\sum_{j=1}^n w_j(1+\mathbb{T}(\dot{R}_{O(j)}))}})}, \\ \left(\prod_{j=1}^n (\dot{\eta}_{O(j)})^{\frac{w_j(1+\mathbb{T}(\dot{R}_{O(j)}))}{\sum_{j=1}^n w_j(1+\mathbb{T}(\dot{R}_{O(j)}))}} \right) \cdot e^{i2\pi(\prod_{j=1}^n (w_{\dot{\eta}_{O(j)}})^{\frac{w_j(1+\mathbb{T}(\dot{R}_{O(j)}))}{\sum_{j=1}^n w_j(1+\mathbb{T}(\dot{R}_{O(j)}))}})} \end{array} \right\}, \quad (27)$$

where $\dot{R}_j (\dot{R}_j = n\Omega_j R_j), (O(1), O(2), \dots, O(n))$ is a permutation of $(1, 2, \dots, n)$ such that $\dot{R}_{O(j-1)} \geq \dot{R}_{O(j)}$ for all $j = 1, 2, \dots, n$. Furthermore, $\Omega_j (j = 1, 2, \dots, n), \sum_{j=1}^n \Omega_j = 1$ and $w_j (j = 1, 2, \dots, n)$ is the collection of weights such that

$$w_j = g\left(\frac{P_j}{\mathbb{T}V}\right) - g\left(\frac{P_{j-1}}{\mathbb{T}V}\right), P_j = \sum_{i=1}^j V_{O(i)}, \mathbb{T}V = \sum_{i=1}^n V_{O(i)}, V_{O(i)} = 1 + \mathbb{T}(\dot{R}_{O(i)}), \quad (28)$$

where $\text{Sup}(\dot{R}_{O(i)})$ represents the support of the j th largest Cq-ROFNs, that is

$$\text{Sup}(\dot{R}_{O(i)}) = \sum_{\substack{j=1 \\ i \neq j}}^n \text{Sup}(\dot{R}_{O(j)}, \dot{R}_{O(i)}) \quad (29)$$

$\text{Sup}(\dot{R}_{O(j)}, \dot{R}_{O(i)})$ denotes the support of the j -th largest Cq-ROFNs and $g : [0, 1] \rightarrow [0, 1]$ is a BUM function exhibiting the following properties: $g(0) = 0, g(1) = 1$ and $g(x) \geq g(y)$ if $x > y$.

Remark 3. If $w = \left(\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n}\right)^T$, then the Cq-ROFHA operator is converted to the Cq-ROFWA operator; if $\Omega = \left(\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n}\right)^T$, then the Cq-ROFHA operator is converted to the Cq-ROFOWA operator.

4.2. The Cq-ROFPG Operators

In this section, we define the weighted power aggregation operators for the collections of Cq-ROFNs named as Complex q-rung orthopair fuzzy Power geometric (Cq-ROFPG) and Complex q-rung orthopair fuzzy Power weighted geometric (Cq-ROFPWG) operators. The presented operators are also discussed with help of some remarks.

Definition 18. For a collection of Cq-ROFNs R_j ($j = 1, 2, \dots, n$), the Cq-ROFPG operator is given by:

$$Cq\text{-ROFPG}(R_1, R_2, \dots, R_n) = \otimes_{j=1}^n \left(R_j\right)^{\frac{(1+\text{T}(R_j))}{\sum_{j=1}^n (1+\text{T}(R_j))}} \quad (30)$$

where

$$\text{T}(R_j) = \sum_{\substack{j=1 \\ i \neq j}}^n \text{Sup}(R_j, R_i) \quad (31)$$

Theorem 5. The aggregated value by considering Cq-ROFPG operators is also a Cq-ROFN, where

$$\begin{aligned} Cq\text{-ROFPG}(R_1, R_2, \dots, R_n) &= \otimes_{j=1}^n \left(R_j\right)^{\frac{(1+\text{T}(R_j))}{\sum_{j=1}^n (1+\text{T}(R_j))}} \\ &= \left(\prod_{j=1}^n \left(\mu_j \right)^{\frac{(1+\text{T}(R_j))}{\sum_{j=1}^n (1+\text{T}(R_j))}} \right) e^{i2\pi \left(\prod_{j=1}^n (w_{\eta_j})^{\frac{(1+\text{T}(R_j))}{\sum_{j=1}^n (1+\text{T}(R_j))}} \right)}, \\ &= \left(\left(1 - \prod_{j=1}^n \left(1 - \left(\eta_j \right)^q \right)^{\frac{(1+\text{T}(R_j))}{\sum_{j=1}^n (1+\text{T}(R_j))}} \right)^{\frac{1}{q}} \cdot e^{i2\pi \left(1 - \prod_{j=1}^n \left(1 - (w_{\eta_j})^q \right)^{\frac{(1+\text{T}(R_j))}{\sum_{j=1}^n (1+\text{T}(R_j))}} \right)} \right)^{\frac{1}{q}}, \end{aligned} \quad (32)$$

where

$$\text{T}(R_j) = \sum_{\substack{j=1 \\ i \neq j}}^n \text{Sup}(R_j, R_i) \quad (33)$$

Proof. As similar to the above. \square

Remark 4. The Cq -ROFPG operator satisfies the similar properties as those of Cq -ROFPA operator.

Definition 19. The Cq -ROFPWG operator is given by:

$$\begin{aligned} Cq - ROFPWG(R_1, R_2, \dots, R_n) &= \otimes_{j=1}^n (R_j)^{\frac{w_j(1+\mathbb{T}(R_j))}{\sum_{j=1}^n w_j(1+\mathbb{T}(R_j))}} \\ &= \left(\prod_{j=1}^n (\mu_j)^{\left(\frac{w_j(1+\mathbb{T}(R_j))}{\sum_{j=1}^n w_j(1+\mathbb{T}(R_j))} \right)} \cdot e^{i2\pi(\prod_{j=1}^n (w_{\mu_j})^{\left(\frac{w_j(1+\mathbb{T}(R_j))}{\sum_{j=1}^n w_j(1+\mathbb{T}(R_j))} \right)})}, \right. \\ &\quad \left. \left(1 - \prod_{j=1}^n \left(1 - (\eta_j)^q \right)^{\left(\frac{w_j(1+\mathbb{T}(R_j))}{\sum_{j=1}^n w_j(1+\mathbb{T}(R_j))} \right)} \right)^{\frac{1}{q}} \cdot e^{i2\pi(1 - \prod_{j=1}^n (1 - (w_{\eta_j})^q)^{\left(\frac{w_j(1+\mathbb{T}(R_j))}{\sum_{j=1}^n w_j(1+\mathbb{T}(R_j))} \right)})^{\frac{1}{q}}} \right) \end{aligned} \quad (34)$$

where

$$\mathbb{T}(R_j) = \sum_{\substack{j=1 \\ i \neq j}}^n w_i \text{Sup}(R_j, R_i) \quad (35)$$

Next, we examined the properties of the Cq -ROFPWG operator.

(1) (Idempotency) When $R_j = R$ for all j , then

$$Cq - ROFPWG(R_1, R_2, \dots, R_n) = R \quad (36)$$

(2) (Boundedness) Let $R^- = \min_j(R_j)$, $R^+ = \max_j(R_j)$ Then

$$R^- \leq Cq - ROFPWG(R_1, R_2, \dots, R_n) \leq R^+ \quad (37)$$

(3) (Commutativity). Let $R_j (j = 1, 2, \dots, n)$ be the permutation of $R'_j (j = 1, 2, \dots, n)$. Then

$$Cq - ROFPWG(R_1, R_2, \dots, R_n) = Cq - ROFPWG(R'_1, R'_2, \dots, R'_n) \quad (38)$$

Moreover, we provide a complex q-rung orthopair fuzzy power ordered weighted geometric (Cq -ROFPOWG) operator below:

Definition 20. The Cq-ROFPOWG operator is given by:

$$Cq - ROFPWG(R_1, R_2, \dots, R_n) = \otimes_{j=1}^n (R_{O(j)})^{\frac{w_j(1+T(R_{O(j)}))}{\sum_{j=1}^n w_j(1+T(R_{O(j)}))}} \\ = \left(\prod_{j=1}^n (\mu_{O(j)})^{\left(\frac{w_j(1+T(R_{O(j)}))}{\sum_{j=1}^n w_j(1+T(R_{O(j)}))} \right)} \cdot e^{i2\pi(\prod_{j=1}^n (w_{\mu_{O(j)}})^{\frac{w_j(1+T(R_{O(j)}))}{\sum_{j=1}^n w_j(1+T(R_{O(j)}))}})}, \right. \\ \left. \left(1 - \prod_{j=1}^n (1 - (\eta_{O(j)})^q)^{\frac{w_j(1+T(R_{O(j)}))}{\sum_{j=1}^n w_j(1+T(R_{O(j)}))}} \right)^{\frac{1}{q}} \cdot e^{i2\pi(1 - \prod_{j=1}^n (1 - (w_{\eta_{O(j)}})^q)^{\frac{w_j(1+T(R_{O(j)}))}{\sum_{j=1}^n w_j(1+T(R_{O(j)}))}})} \right)^{\frac{1}{q}} \quad (39)$$

where $(O(1), O(2), \dots, O(n))$ is a permutation of $(1, 2, \dots, n)$ such that $R_{O(j-1)} \geq R_{O(j)}$ for all $j = 1, 2, \dots, n$. Further, $w_j (j = 1, 2, \dots, n)$ is the collection of weights such that

$$w_j = g\left(\frac{P_j}{TV}\right) - g\left(\frac{P_{j-1}}{TV}\right), \quad P_j = \sum_{i=1}^j V_{O(i)}, \quad TV = \sum_{i=1}^n V_{O(i)}, \quad V_{O(i)} = 1 + T(R_{O(i)}) \quad (40)$$

where $T(R_{O(i)})$ represents the support of the j th largest Cq-ROFNs, that is

$$T(R_{O(i)}) = \sum_{\substack{j=1 \\ i \neq j}}^n Sup(R_{O(j)}, R_{O(i)}) \quad (41)$$

$Sup(R_{O(j)}, R_{O(i)})$ denotes the support of j th largest Cq-ROFNs and $g : [0, 1] \rightarrow [0, 1]$ is a basic unit-interval monotonic (BUM) function having properties: $g(0) = 0, g(1) = 1$ and $g(x) \geq g(y)$ if $x > y$.

Next, we examined the properties of Cq-ROFPWG operator.

(1) (Idempotency) When $R_j = R$ for all j , then

$$Cq - ROFPWG(R_1, R_2, \dots, R_n) = R \quad (42)$$

(2) (Boundedness) Let $R^- = \min_j(R_j), R^+ = \max_j(R_j)$ Then

$$R^- \leq Cq - ROFPWG(R_1, R_2, \dots, R_n) \leq R^+ \quad (43)$$

(3) (Commutativity). Let $R'_j (j = 1, 2, \dots, n)$ be the permutation of $R_j (j = 1, 2, \dots, n)$. Then

$$Cq - ROFPWG(R_1, R_2, \dots, R_n) = Cq - ROFPWG(R'_1, R'_2, \dots, R'_n) \quad (44)$$

Moreover, we provide a complex q-rung orthopair fuzzy power hybrid geometric (Cq-ROFPHG) operator below:

Definition 21. The Cq-ROFPHG operator is given by:

$$\begin{aligned} \text{Cq-ROFPG}(R_1, R_2, \dots, R_n) &= \otimes_{j=1}^n (\dot{R}_{O(j)})^{\frac{w_j(1+\mathbb{T}(\dot{R}_{O(j)}))}{\sum_{j=1}^n w_j(1+\mathbb{T}(\dot{R}_{O(j)}))}} \\ &= \left(\prod_{j=1}^n (\dot{\mu}_{O(j)})^{\frac{w_j(1+\mathbb{T}(\dot{R}_{O(j)}))}{\sum_{j=1}^n w_j(1+\mathbb{T}(\dot{R}_{O(j)}))}} \right) \cdot e^{i2\pi(\prod_{j=1}^n (w_{\dot{\mu}_{O(j)}})^{\frac{w_j(1+\mathbb{T}(\dot{R}_{O(j)}))}{\sum_{j=1}^n w_j(1+\mathbb{T}(\dot{R}_{O(j)}))}})}, \end{aligned} \quad (45)$$

$$\left(\left(1 - \prod_{j=1}^n (1 - (\dot{\eta}_{O(j)})^q)^{\frac{w_j(1+\mathbb{T}(\dot{R}_{O(j)}))}{\sum_{j=1}^n w_j(1+\mathbb{T}(\dot{R}_{O(j)}))}} \right)^{\frac{1}{q}} \cdot e^{i2\pi(1 - \prod_{j=1}^n (1 - (w_{\dot{\eta}_{O(j)}})^q)^{\frac{w_j(1+\mathbb{T}(\dot{R}_{O(j)}))}{\sum_{j=1}^n w_j(1+\mathbb{T}(\dot{R}_{O(j)}))}})} \right)^{\frac{1}{q}}$$

where $\dot{R}_j = n\Omega_j R_j$, $(O(1), O(2), \dots, O(n))$ is a permutation of $(1, 2, \dots, n)$ such that $\dot{R}_{O(j-1)} \geq \dot{R}_{O(j)}$ for all $j = 1, 2, \dots, n$. Further, $\Omega_j (j = 1, 2, \dots, n)$, $\sum_{j=1}^n \Omega_j = 1$ and $w_j (j = 1, 2, \dots, n)$ is the collection of weights such that

$$w_j = g\left(\frac{P_j}{\mathbb{T}V}\right) - g\left(\frac{P_{j-1}}{\mathbb{T}V}\right), \quad P_j = \sum_{i=1}^j V_{O(i)}, \quad \mathbb{T}V = \sum_{i=1}^n V_{O(i)}, \quad V_{O(i)} = 1 + \mathbb{T}(\dot{R}_{O(i)}) \quad (46)$$

where $\mathbb{T}(\dot{R}_{O(i)})$ represents the support of the j th largest Cq-ROFNs, that is

$$\mathbb{T}(\dot{R}_{O(i)}) = \sum_{\substack{j=1 \\ i \neq j}}^n \text{Sup}(\dot{R}_{O(j)}, \dot{R}_{O(i)}) \quad (47)$$

$\text{Sup}(\dot{R}_{O(j)}, \dot{R}_{O(i)})$ denotes the support of j th largest Cq-ROFNs and $g : [0, 1] \rightarrow [0, 1]$ is a basic unit-interval monotonic (BUM) function having properties: $g(0) = 0$, $g(1) = 1$ and $g(x) \geq g(y)$ if $x > y$.

Remark 5. If $w = \left(\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n}\right)^\top$, then the Cq-ROFHG operator is converted to the Cq-ROFWG operator and if $\Omega = \left(\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n}\right)^\top$, then the Cq-ROFHG operator is converted to the Cq-ROFOWG operator.

5. Proposed MADM Methods

In this section, we offer a novel MADM method based on the proposed AOs and VIKOR method under the Cq-ROFS environment.

Consider m alternatives and n attributes X_1, X_2, \dots, X_m and $\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n$ with respect to weight vectors such that $w = (w_1, w_2, \dots, w_n)^\top$, $\sum_{j=1}^n w_i = 1$. Suppose that $R = (r_{ij})_{m \times n} = \left(\mu_{ij}e^{i2\pi W_{\mu_{ij}}}, \eta_{ij}e^{i2\pi W_{\eta_{ij}}}\right)_{m \times n}$ is the complex q-rung orthopair fuzzy matrix, which consists of Cq-ROFNs and each pair contains complex-valued membership and complex-valued non-membership grades, where $\mu_{ij}e^{i2\pi W_{\mu_{ij}}}$ represents the degree that the alternative \tilde{A}_i satisfies the attributes X_i given by the decision maker and $\eta_{ij}e^{i2\pi W_{\eta_{ij}}}$ represents the degree that the alternative \tilde{A}_i does not satisfy the attributes X_i given by the decision makers.

5.1. Algorithm 1: Method Based on AOs

The steps of the algorithm based on proposed Cq-ROFPWA and Cq-ROFPWG operators are summarized as follows:

Step 1: Computing the supports:

$$Sup(R_{ij}, R_{ik}) = 1 - d(R_{ij}, R_{ik}), \quad j, k = 1, 2, \dots, n,$$

which holds the support conditions; without loss of generality, we calculated $d(R_{ij}, R_{ik})$ using the new proposed approach:

$$d(R_{ij}, R_{ik}) = \frac{(|\mu_{ij}^q - \mu_{ik}^q| + |\eta_{ij}^q - \eta_{ik}^q| + |W_{\mu_{ij}}^q - W_{\mu_{ik}}^q| + |W_{\eta_{ij}}^q - W_{\eta_{ik}}^q|)}{4}, \quad j, k = 1, 2, \dots, n$$

Step 2: Computing the weighted support

$$\tau(R_{ij}) = \sum_{\substack{k=1 \\ k \neq j}}^n w_j Sup(R_{ij}, R_{ik})$$

Furthermore, we compute the weighted ζ_{ij} ($j = 1, 2, \dots, n$) associated with the Cq-ROFNs R_{ij} ($i = 1, 2, \dots, n, j = 1, 2, \dots, m$):

$$\zeta_{ij} = \frac{w_j(1 + \tau(R_{ij}))}{\sum_{j=1}^n w_j(1 + \tau(R_{ij}))}$$

where $\zeta_{ij} \geq 0$ and $\sum_{j=1}^n \zeta_{ij} = 1$, ($i = 1, 2, \dots, m$).

Step 3: Aggregate the Cq-ROFNs by using either Cq-ROFPWA or Cq-ROFPWG operator into the collective value $R_i = (\mu_i(x)e^{j2\pi(W_{\mu_i})}, \eta_i(x)e^{j2\pi(W_{\eta_i})})$ as

$$R_i = Cq - ROFPW\tilde{A}(R_{i1}, R_{i2}, \dots, R_{in}) = \oplus_{j=1}^n \frac{w_j(1 + \tau(R_{ij}))R_{ij}}{\sum_{j=1}^n w_j(1 + \tau(R_{ij}))}$$

or

$$R_i = Cq - ROFPWG(R_{i1}, R_{i2}, \dots, R_{in}) = \otimes_{j=1}^n (R_{ij})^{\frac{w_j(1 + \tau(R_{ij}))}{\sum_{j=1}^n w_j(1 + \tau(R_{ij}))}}$$

Step 4: Compute the score values of aggregated Cq-ROFNs R_i by using the Equation

$$S(R_i) = \mu_i^q - \eta_i^q + W_{\mu_i}^q - W_{\eta_i}^q$$

If for any two indices i_1, i_2 , the value of $S(R_{i_1})$ and $S(R_{i_2})$ are equal then compute the degree of accuracy values for such indices as

$$H(R_i) = \mu_i^q + \eta_i^q + W_{\mu_i}^q + W_{\eta_i}^q$$

Step 5: Ranking the alternatives \tilde{A}_i ($i = 1, 2, \dots, m$) based on the computed score values and choose the best one.

5.2. Algorithm 2: Method Based on VIKOR

The following are the steps of the proposed method summarized to find the best alternative by using the proposed VIKOR method.

Step 1: Normalize the decision matrix, there are two types of attribute such as benefits B and cost C types attributes, the normalized can be done by the following formula;

$$r_{ij} = \begin{cases} R_{ij}^c & j \in B \\ R_{ij}^- & j \in C \end{cases}$$

Step 2: Computing the virtual positive ideal x_j^* and the virtual negative ideal x_j^- values under the attributes \tilde{A}_j , we have

$$x_j^* = \max_i(x_{ij}), \quad x_j^- = \min_i(x_{ij})$$

Step 3: Computing the values of group utility S_i and S'_i , we have

$$S_i = \frac{\sum_{j=1}^n w_j (\|R_j^* - R_{ij}\|)}{(\|R_j^* - R_j^-\|)}$$

$$S'_i = \frac{\max_j w_j (\|R_j^* - R_{ij}\|)}{(\|R_j^* - R_j^-\|)},$$

where $\|R_i, R_j\|$ represents the distance between two Cq-ROFNs defined as:

$$d(R_i, R_j) = \|R_i, R_j\| = \frac{1}{2} \sum_{i=1}^n \left(|\mu_i^q - \mu_j^q| + |\eta_i^q - \eta_j^q| + |W_{\mu_i}^q - W_{\mu_j}^q| + |W_{\eta_i}^q - W_{\eta_j}^q| \right)$$

Step 4: Computing the values of $Q_i, i = 1, 2, \dots, m$, we have

$$Q_i = \frac{v(S_i - S^*)}{(S^- - S^*)} + \frac{(1-v)(S'_i - S'^*)}{(S'^- - S'^*)},$$

where $S^* = \min_i(S_i)$, $S^- = \max_i(S_i)$, $S'^* = \min_i(S'_i)$ and $S'^- = \max_i(S'_i)$, the symbol v is the balance parameter which can balance the group of utility and individual regret. There are three possibility:

- (a) If $v > 0.5$ represents the maximum group utility being more than minimum individual regret.
- (b) If $v < 0.5$ represents the minimum individual regret being more than maximum group utility.
- (c) If $v = 0.5$ represents the maximum group utility and minimum individual regret are of the same importance.

Step 4: Using the values of S, S' and Q and ranking the alternatives, then we will obtain the compromise solution.

Step 5: When we get the compromise solution $X^{(1)}$ in steps 4, then it satisfied the following two conditions.

- **Condition 1: Acceptable advantages:** $Q(X^{(2)}) - Q(X^{(1)}) \geq \frac{1}{m-1}$, where $Q(X^{(2)})$ is the Q value in the second position of all ranking alternatives produced by the value of Q and m number of alternatives.
- **Condition 2: Acceptable stability:** Alternative $X^{(1)}$ must also be in the first position of all ranking alternatives produced by the values of S or S' .

6. Illustrative Examples

In this section, the above defined algorithms have been demonstrated through several numerical examples.

Example 3. In this numerical example, we consider the potential evaluation of emerging technology commercialization with complex intuitionistic fuzzy information. For it, consider a five possible emerging technology enterprises $\tilde{A}_i (i = 1, 2, 3, 4, 5)$ and their attributes which are represented by $G_i (i = 1, 2, 3, 4)$, defined as G_1 (Technical Advancement), G_2 (Potential and Risk Market), G_3 (Financial Conditions) and G_4 (Employment Creation). The weight vector for the attributes is taken as $w = (0.3, 0.4, 0.2, 0.1)$. The normalized rating values of the alternatives under each attributes are summarized in Table 1.

Table 1. Decision matrix for Example 3.

	G_1	G_2	G_3	G_4
\tilde{A}_1	$\left(\begin{array}{l} 0.5e^{i2\pi(0.6)} \\ 0.4e^{i2\pi(0.3)} \end{array} \right)$	$\left(\begin{array}{l} 0.49e^{i2\pi(0.50)} \\ 0.41e^{i2\pi(0.31)} \end{array} \right)$	$\left(\begin{array}{l} 0.55e^{i2\pi(0.66)} \\ 0.14e^{i2\pi(0.13)} \end{array} \right)$	$\left(\begin{array}{l} 0.45e^{i2\pi(0.36)} \\ 0.34e^{i2\pi(0.33)} \end{array} \right)$
\tilde{A}_2	$\left(\begin{array}{l} 0.53e^{i2\pi(0.63)} \\ 0.37e^{i2\pi(0.27)} \end{array} \right)$	$\left(\begin{array}{l} 0.47e^{i2\pi(0.55)} \\ 0.43e^{i2\pi(0.33)} \end{array} \right)$	$\left(\begin{array}{l} 0.51e^{i2\pi(0.61)} \\ 0.24e^{i2\pi(0.33)} \end{array} \right)$	$\left(\begin{array}{l} 0.15e^{i2\pi(0.16)} \\ 0.44e^{i2\pi(0.39)} \end{array} \right)$
\tilde{A}_3	$\left(\begin{array}{l} 0.56e^{i2\pi(0.66)} \\ 0.33e^{i2\pi(0.23)} \end{array} \right)$	$\left(\begin{array}{l} 0.45e^{i2\pi(0.46)} \\ 0.24e^{i2\pi(0.13)} \end{array} \right)$	$\left(\begin{array}{l} 0.25e^{i2\pi(0.26)} \\ 0.42e^{i2\pi(0.34)} \end{array} \right)$	$\left(\begin{array}{l} 0.59e^{i2\pi(0.69)} \\ 0.24e^{i2\pi(0.13)} \end{array} \right)$
\tilde{A}_4	$\left(\begin{array}{l} 0.59e^{i2\pi(0.69)} \\ 0.3e^{i2\pi(0.22)} \end{array} \right)$	$\left(\begin{array}{l} 0.5e^{i2\pi(0.6)} \\ 0.4e^{i2\pi(0.3)} \end{array} \right)$	$\left(\begin{array}{l} 0.52e^{i2\pi(0.63)} \\ 0.24e^{i2\pi(0.23)} \end{array} \right)$	$\left(\begin{array}{l} 0.5e^{i2\pi(0.16)} \\ ,0.4e^{i2\pi(0.34)} \end{array} \right)$
\tilde{A}_5	$\left(\begin{array}{l} 0.49e^{i2\pi(0.50)} \\ 0.41e^{i2\pi(0.31)} \end{array} \right)$	$\left(\begin{array}{l} 0.35e^{i2\pi(0.26)} \\ 0.14e^{i2\pi(0.23)} \end{array} \right)$	$\left(\begin{array}{l} 0.45e^{i2\pi(0.36)} \\ 0.34e^{i2\pi(0.33)} \end{array} \right)$	$\left(\begin{array}{l} 0.55e^{i2\pi(0.66)} \\ 0.14e^{i2\pi(0.13)} \end{array} \right)$

Then, the steps of the Algorithm 1 are implemented as follows:

Step 1: Using the similarity measures between the given numbers, we compute the support degrees as $T(R_{ij})$ as

$$T(R_{ij}) = \begin{bmatrix} 0.6735 & 0.5733 & 0.7449 & 0.8489 \\ 0.6664 & 0.5719 & 0.7664 & 0.8136 \\ 0.6434 & 0.5572 & 0.7285 & 0.8293 \\ 0.6557 & 0.5702 & 0.7682 & 0.8254 \\ 0.6587 & 0.5631 & 0.7660 & 0.8175 \end{bmatrix}$$

Step 2: Collect the valued of weighted $\zeta_{ij} = \frac{w_j(1+T(R_{ij}))}{\sum_{j=1}^n w_j(1+T(R_{ij}))} (j = 1, 2, \dots, n)$ as

$$\zeta_{ij} = \begin{bmatrix} 0.3015 & 0.3779 & 0.2096 & 0.1110 \\ 0.3006 & 0.3780 & 0.2124 & 0.1090 \\ 0.2998 & 0.3788 & 0.2102 & 0.1112 \\ 0.2991 & 0.3781 & 0.2129 & 0.1099 \\ 0.3002 & 0.3771 & 0.2132 & 0.1096 \end{bmatrix}$$

Step 3: Utilize the Cq-ROFPWA operator to obtain the collective values for each alternative and get

$$\begin{aligned} R_1 &= (0.5032e^{i2\pi(0.5661)}, 0.3182e^{i2\pi(0.2576)}), \\ R_2 &= (0.4842e^{i2\pi(0.5746)}, 0.3640e^{i2\pi(0.3164)}) \\ R_3 &= (0.4873e^{i2\pi(0.5614)}, 0.2970e^{i2\pi(0.1888)}), \\ R_4 &= (0.5350e^{i2\pi(0.6204)}, 0.3292e^{i2\pi(0.2620)}), \\ \text{and } R_5 &= (0.4483e^{i2\pi(0.4478)}, 0.2335e^{i2\pi(0.2552)}) \end{aligned}$$

while by Cq-ROFPWG operator, these values are obtained as

$$\begin{aligned} R_1 &= (0.5003e^{i2\pi(0.5398)}, 0.3699e^{i2\pi(0.2881)}), \\ R_2 &= (0.4378e^{i2\pi(0.5119)}, 0.3868e^{i2\pi(0.3229)}) \\ R_3 &= (0.4376e^{i2\pi(0.5093)}, 0.3221e^{i2\pi(0.2357)}), \\ R_4 &= (0.5298e^{i2\pi(0.5467)}, 0.3490e^{i2\pi(0.2730)}), \\ \text{and } R_5 &= (0.4292e^{i2\pi(0.3756)}, 0.3136e^{i2\pi(0.2781)}) \end{aligned}$$

Step 4: The score values corresponding to the aggregated numbers by Cq-ROFPWA operator are $S(R_1) = 0.2595$, $S(R_2) = 0.2233$, $S(R_3) = 0.2597$, $S(R_4) = 0.3382$ and $S(R_5) = 0.1506$. On the other hand, these score values for the values by Cq-ROFPWG operators are 0.3821, 0.2399, 0.3892, 0.4544 and 0.2132 respectively.

Step 5: The ranking order of the given numbers are $\tilde{A}_4 \geq \tilde{A}_3 \geq \tilde{A}_1 \geq \tilde{A}_2 \geq \tilde{A}_5$ by Cq-ROFPWA operator while $\tilde{A}_4 \geq \tilde{A}_3 \geq \tilde{A}_1 \geq \tilde{A}_2 \geq \tilde{A}_5$ by Cq-ROFPWG operator and hence conclude that \tilde{A}_4 is the best alternative.

To compare the performance of the proposed algorithm with the existing studies [48,49,55] based on the weighted averaging and geometric operators, we summarized their results in Table 2. From this table, we conclude that the optimal alternative remains the same as that of the proposed ones.

Table 2. Comparative analysis of the proposed method with the existing methods.

Methods	Operator	Score Values	Ranking
Garg and Rani [48]	Averaging	$S(\tilde{A}_1) = 0.4798, S(\tilde{A}_2) = 0.3531,$ $S(\tilde{A}_3) = 0.5279, S(\tilde{A}_4) = 0.5475,$ $S(\tilde{A}_5) = 0.3648$	$\tilde{A}_4 \geq \tilde{A}_3 \geq \tilde{A}_1$ $\geq \tilde{A}_5 \geq \tilde{A}_2$
	Geometric	$S(\tilde{A}_1) = 0.4127, S(\tilde{A}_2) = 0.2598,$ $S(\tilde{A}_3) = 0.4336, S(\tilde{A}_4) = 0.4727,$ $S(\tilde{A}_5) = 0.2633$	$\tilde{A}_4 \geq \tilde{A}_3 \geq \tilde{A}_1$ $\geq \tilde{A}_5 \geq \tilde{A}_2$
Rani and Garg [49]	Hamming	$S(\tilde{A}_1) = 0.3779, S(\tilde{A}_2) = 0.4490,$ $S(\tilde{A}_3) = 0.3602, S(\tilde{A}_4) = 0.3771,$ $S(\tilde{A}_5) = 0.3909$	$\tilde{A}_3 \geq \tilde{A}_4 \geq \tilde{A}_1$ $\geq \tilde{A}_5 \geq \tilde{A}_2$
	Euclidean	$S(\tilde{A}_1) = 0.1821, S(\tilde{A}_2) = 0.2564,$ $S(\tilde{A}_3) = 0.1697, S(\tilde{A}_4) = 0.2072,$ $S(\tilde{A}_5) = 0.1866$	$\tilde{A}_3 \geq \tilde{A}_1 \geq \tilde{A}_5$ $\geq \tilde{A}_4 \geq \tilde{A}_2$

Table 2. Cont.

Methods	Operator	Score Values	Ranking
Garg and Rani [55]	Averaging	$S(\tilde{A}_1) = 0.5402, S(\tilde{A}_2) = 0.5169,$ $S(\tilde{A}_3) = 0.5083, S(\tilde{A}_4) = 0.5536,$ $S(\tilde{A}_5) = 0.3706$	$\tilde{A}_4 \geq \tilde{A}_1 \geq \tilde{A}_2 \geq \tilde{A}_3 \geq \tilde{A}_5$
		$S(\tilde{A}_1) = 0.4108, S(\tilde{A}_2) = 0.2608,$ $S(\tilde{A}_3) = 0.4342, S(\tilde{A}_4) = 0.4747,$ $S(\tilde{A}_5) = 0.2586$	$\tilde{A}_4 \geq \tilde{A}_3 \geq \tilde{A}_1 \geq \tilde{A}_2 \geq \tilde{A}_5$
		$S(\tilde{A}_1) = 0.3993, S(\tilde{A}_2) = 0.3204,$ $S(\tilde{A}_3) = 0.4100, S(\tilde{A}_4) = 0.4852,$ $S(\tilde{A}_5) = 0.2624$	$\tilde{A}_4 \geq \tilde{A}_3 \geq \tilde{A}_1 \geq \tilde{A}_2 \geq \tilde{A}_5$
Proposed method for CPFS	Geometric	$S(\tilde{A}_1) = 0.3959, S(\tilde{A}_2) = 0.2498,$ $S(\tilde{A}_3) = 0.4120, S(\tilde{A}_4) = 0.4639,$ $S(\tilde{A}_5) = 0.2368$	$\tilde{A}_4 \geq \tilde{A}_3 \geq \tilde{A}_1 \geq \tilde{A}_2 \geq \tilde{A}_5$
		$S(\tilde{A}_1) = 0.2595, S(\tilde{A}_2) = 0.2233,$ $S(\tilde{A}_3) = 0.2597, S(\tilde{A}_4) = 0.3382,$ $S(\tilde{A}_5) = 0.1506$	$\tilde{A}_4 \geq \tilde{A}_3 \geq \tilde{A}_1 \geq \tilde{A}_2 \geq \tilde{A}_5$
		$S(\tilde{A}_1) = 0.3821, S(\tilde{A}_2) = 0.2399,$ $S(\tilde{A}_3) = 3892, S(\tilde{A}_4) = 0.4544,$ $S(\tilde{A}_5) = 0.2132$	$\tilde{A}_4 \geq \tilde{A}_3 \geq \tilde{A}_1 \geq \tilde{A}_2 \geq \tilde{A}_5$
Proposed method for Cq-ROFS	Averaging	$S(\tilde{A}_1) = 0.2595, S(\tilde{A}_2) = 0.2233,$ $S(\tilde{A}_3) = 0.2597, S(\tilde{A}_4) = 0.3382,$ $S(\tilde{A}_5) = 0.1506$	$\tilde{A}_4 \geq \tilde{A}_3 \geq \tilde{A}_1 \geq \tilde{A}_2 \geq \tilde{A}_5$
		$S(\tilde{A}_1) = 0.3821, S(\tilde{A}_2) = 0.2399,$ $S(\tilde{A}_3) = 3892, S(\tilde{A}_4) = 0.4544,$ $S(\tilde{A}_5) = 0.2132$	$\tilde{A}_4 \geq \tilde{A}_3 \geq \tilde{A}_1 \geq \tilde{A}_2 \geq \tilde{A}_5$
		$S(\tilde{A}_1) = 0.2595, S(\tilde{A}_2) = 0.2233,$ $S(\tilde{A}_3) = 0.2597, S(\tilde{A}_4) = 0.3382,$ $S(\tilde{A}_5) = 0.1506$	$\tilde{A}_4 \geq \tilde{A}_3 \geq \tilde{A}_1 \geq \tilde{A}_2 \geq \tilde{A}_5$

Example 4. Consider a decision making problem with four alternatives denoted by $X_i (i = 1, 2, 3, 4)$ and measured by four attributes A_1 (Anti-risk ability), A_2 (Growth ability), A_3 (Social impact) and A_4 (Environment impact), with weight vectors $w = (0.35, 0.25, 0.3, 0.1)$. The following steps of the Algorithm 2, based on VIKOR method, are implemented here to find the best alternative.

Step 1: The normalized decision matrix is summarized in Table 3 in terms of Cq-ROFNs.

Table 3. Decision matrix for Example 4.

	A_1	A_2	A_3	A_4
X_1	$\begin{pmatrix} 0.7e^{i2\pi(0.8)} \\ 0.66e^{i2\pi(0.7)} \end{pmatrix}$	$\begin{pmatrix} 0.59e^{i2\pi(0.70)} \\ 0.51e^{i2\pi(0.67)} \end{pmatrix}$	$\begin{pmatrix} 0.87e^{i2\pi(0.86)} \\ 0.5e^{i2\pi(0.8)} \end{pmatrix}$	$\begin{pmatrix} 0.75e^{i2\pi(0.7)} \\ 0.2e^{i2\pi(0.2)} \end{pmatrix}$
X_2	$\begin{pmatrix} 0.63e^{i2\pi(0.73)} \\ 0.5e^{i2\pi(0.68)} \end{pmatrix}$	$\begin{pmatrix} 0.57e^{i2\pi(0.75)} \\ 0.54e^{i2\pi(0.71)} \end{pmatrix}$	$\begin{pmatrix} 0.81e^{i2\pi(0.71)} \\ 0.5e^{i2\pi(0.68)} \end{pmatrix}$	$\begin{pmatrix} 0.6e^{i2\pi(0.62)} \\ 0.22e^{i2\pi(0.3)} \end{pmatrix}$
X_3	$\begin{pmatrix} 0.66e^{i2\pi(0.76)} \\ 0.55e^{i2\pi(0.67)} \end{pmatrix}$	$\begin{pmatrix} 0.55e^{i2\pi(0.88)} \\ 0.5e^{i2\pi(0.78)} \end{pmatrix}$	$\begin{pmatrix} 0.55e^{i2\pi(0.76)} \\ 0.5e^{i2\pi(0.71)} \end{pmatrix}$	$\begin{pmatrix} 0.79e^{i2\pi(0.9)} \\ 0.12e^{i2\pi(0.03)} \end{pmatrix}$
X_4	$\begin{pmatrix} 0.69e^{i2\pi(0.79)} \\ 0.6e^{i2\pi(0.6)} \end{pmatrix}$	$\begin{pmatrix} 0.62e^{i2\pi(0.71)} \\ 0.58e^{i2\pi(0.68)} \end{pmatrix}$	$\begin{pmatrix} 0.82e^{i2\pi(0.83)} \\ 0.5e^{i2\pi(0.7)} \end{pmatrix}$	$\begin{pmatrix} 0.65e^{i2\pi(0.72)} \\ 0.2e^{i2\pi(0.12)} \end{pmatrix}$

Step 2: The positive ideal x^* and the negative ideal x^- values under the attributes \tilde{A}_j are computed as

$$x^* = \left\{ \left(\begin{pmatrix} 0.7e^{i2\pi(0.8)} \\ 0.5e^{i2\pi(0.6)} \end{pmatrix}, \begin{pmatrix} 0.62e^{i2\pi(0.88)} \\ 0.5e^{i2\pi(0.67)} \end{pmatrix}, \begin{pmatrix} 0.87e^{i2\pi(0.86)} \\ 0.5e^{i2\pi(0.68)} \end{pmatrix}, \begin{pmatrix} 0.75e^{i2\pi(0.9)} \\ 0.12e^{i2\pi(0.03)} \end{pmatrix} \right) \right\}$$

and

$$x^- = \left\{ \begin{pmatrix} 0.63e^{i2\pi(0.73)}, \\ 0.66e^{i2\pi(0.7)} \end{pmatrix}, \begin{pmatrix} 0.55e^{i2\pi(0.70)}, \\ 0.58e^{i2\pi(0.78)} \end{pmatrix}, \begin{pmatrix} 0.55e^{i2\pi(0.71)}, \\ 0.5e^{i2\pi(0.8)} \end{pmatrix}, \begin{pmatrix} 0.6e^{i2\pi(0.62)}, \\ 0.22e^{i2\pi(0.3)} \end{pmatrix} \right\}$$

Step 3: Compute the values of group utility S_i and S'_i and their results are listed in Table 4.

Table 4. Values of the group utility.

Symbols	Values	Symbols	Values
S_1	0.35	S'_1	0.14
S_2	0.30	S'_2	0.12
S_3	0.24	S'_3	0.09
S_4	0.13	S'_4	0.04

Step 4: We compute the values of $Q_i, i = 1, 2, \dots, m$ and their results are given in Table 5.

Table 5. Ranking results.

Symbols	X_1	X_2	X_3	X_4	Ranking	Compromise Solution
S	0.35	0.30	0.24	0.13	$X_4 \geq X_3 \geq X_2 \geq X_1$	X_4
S'	0.14	0.12	0.09	0.04	$X_4 \geq X_3 \geq X_2 \geq X_1$	X_4
Q	0	1	0.85	0.39	$X_1 \geq X_4 \geq X_3 \geq X_2$	X_1
Compromise solution						X_1, X_4

Step 5: Using the values of S, S' and Q and ranking the alternatives, then we will obtain the compromise solution as X_1, X_4 .

Step 6: We obtain the compromise results using the condition 1 and condition 2, such that $Q(X_1) = 0$ and the second position is $Q(X_4) = 0.39$, then $MD = \frac{1}{m-1} = \frac{1}{4-1} = 0.333$, so $Q(X_4) - Q(X_1) = 0.39 > 0.333$ which holds the conditions $Q(X_4) - Q(X_1) \geq \frac{1}{4-1}$ but the alternative X_1 is the best ranked by S and S' , which holds the condition 1. By calculating, we get $Q(X_4) - Q(X_1) = 0.39 > 0.333$; $Q(X_2) - Q(X_1) = 1 > 0.333$ and $Q(X_3) - Q(X_1) = 0.85 > 0.333$. So, the condition 1 holds accurately, therefore by condition 1, X_1 and X_4 are the compromise solutions.

The comparison between proposed methods and existing methods for the numerical Example 4, are discussed in Table 6. Based on the VIKOR methods for the existing and proposed approaches, the best alternative is X_4 . It is clearly seen from this table that the VIKOR method under the CIFS and CPFS environments failed to rank the alternatives due to the violation of the constraint condition and hence these approaches are limited in nature. The VIKOR methods for the CIFS and CPFS are the special cases of the VIKOR methods for Cq-ROFS. When $q = 1$, the VIKOR methods for the Cq-ROFS reduce to the VIKOR methods for the CIFS. When $q = 2$, the VIKOR methods for the Cq-ROFS reduce to the VIKOR methods for the CPFS. Additionally, our approach is more flexible and decision makers can select different values of parameter q according to the risk attitude. According to the comparisons and analysis above, the VIKOR methods based on the Cq-ROFS proposed herein are better than the existing methods for aggregating complex intuitionistic fuzzy information and complex Pythagorean fuzzy information. Therefore, they are more suitable for solving difficult and complicated problems.

Table 6. Comparison between proposed approaches and existing approaches.

Methods	Values for S	Values for S'	Ranking for S and S'
VIKOR Method for CIFS	Cannot be Calculated	Cannot be Calculated	Cannot be Calculated
VIKOR Method for CPFS	Cannot be Calculated	Cannot be Calculated	Cannot be Calculated
VIKOR Method for Cq-ROFS	$X_1 = 0.35, X_2 = 0.30,$ $X_3 = 0.24, X_4 = 0.13$	$X_1 = 0.14, X_2 = 0.12,$ $X_3 = 0.09, X_4 = 0.04$	$X_4 \geq X_3 \geq X_2 \geq X_1$ $X_4 \geq X_3 \geq X_2 \geq X_1$

6.1. Advantages and Comparative Analysis

In the following, we provide some examples to confirm the utility as well as the effectiveness of the proposed MADM method and perform a comparison with the current studies [48,49,55]. In Reference [48], authors have presented the generalized complex aggregation operators based on the CIFS while power weighted aggregation operators are defined in Reference [49]. In Reference [55], the authors have presented the averaging and geometric operators for CIFS.

Example 5. Consider the five possible emerging technology enterprises $\tilde{A}_i (i = 1, 2, 3, 4, 5)$ to be examined under the four attributes, whose weight vector is $w = (0.3, 0.4, 0.2, 0.1)$. The expert has assessed the information by using the complex Pythagorean fuzzy information and the rating values are summarized in Table 7.

Table 7. Decision matrix for Example 5.

	G_1	G_2	G_3	G_4
\tilde{A}_1	$\left(\begin{array}{l} 0.7e^{j2\pi(0.8)} \\ 0.4e^{j2\pi(0.3)} \end{array} \right)$	$\left(\begin{array}{l} 0.59e^{j2\pi(0.70)} \\ 0.41e^{j2\pi(0.31)} \end{array} \right)$	$\left(\begin{array}{l} 0.87e^{j2\pi(0.86)} \\ 0.14e^{j2\pi(0.13)} \end{array} \right)$	$\left(\begin{array}{l} 0.75e^{j2\pi(0.7)} \\ 0.34e^{j2\pi(0.33)} \end{array} \right)$
\tilde{A}_2	$\left(\begin{array}{l} 0.63e^{j2\pi(0.73)} \\ 0.37e^{j2\pi(0.27)} \end{array} \right)$	$\left(\begin{array}{l} 0.57e^{j2\pi(0.75)} \\ 0.43e^{j2\pi(0.33)} \end{array} \right)$	$\left(\begin{array}{l} 0.81e^{j2\pi(0.71)} \\ 0.24e^{j2\pi(0.33)} \end{array} \right)$	$\left(\begin{array}{l} 0.6e^{j2\pi(0.62)} \\ 0.44e^{j2\pi(0.39)} \end{array} \right)$
\tilde{A}_3	$\left(\begin{array}{l} 0.66e^{j2\pi(0.76)} \\ 0.33e^{j2\pi(0.23)} \end{array} \right)$	$\left(\begin{array}{l} 0.55e^{j2\pi(0.88)} \\ 0.24e^{j2\pi(0.13)} \end{array} \right)$	$\left(\begin{array}{l} 0.55e^{j2\pi(0.76)} \\ 0.42e^{j2\pi(0.34)} \end{array} \right)$	$\left(\begin{array}{l} 0.79e^{j2\pi(0.9)} \\ 0.24e^{j2\pi(0.13)} \end{array} \right)$
\tilde{A}_4	$\left(\begin{array}{l} 0.69e^{j2\pi(0.79)} \\ 0.3e^{j2\pi(0.22)} \end{array} \right)$	$\left(\begin{array}{l} 0.62e^{j2\pi(0.71)} \\ 0.4e^{j2\pi(0.3)} \end{array} \right)$	$\left(\begin{array}{l} 0.82e^{j2\pi(0.83)} \\ 0.24e^{j2\pi(0.23)} \end{array} \right)$	$\left(\begin{array}{l} 0.65e^{j2\pi(0.72)} \\ 0.4e^{j2\pi(0.34)} \end{array} \right)$
\tilde{A}_5	$\left(\begin{array}{l} 0.59e^{j2\pi(0.70)} \\ 0.41e^{j2\pi(0.31)} \end{array} \right)$	$\left(\begin{array}{l} 0.85e^{j2\pi(0.78)} \\ 0.14e^{j2\pi(0.23)} \end{array} \right)$	$\left(\begin{array}{l} 0.65e^{j2\pi(0.76)} \\ 0.34e^{j2\pi(0.33)} \end{array} \right)$	$\left(\begin{array}{l} 0.87e^{j2\pi(0.89)} \\ 0.14e^{j2\pi(0.13)} \end{array} \right)$

By implementing the steps of the proposed Algorithm 1 for this considered information, then the final score values of the given alternatives are obtained by using Cq-ROFPWA operator as $S(\tilde{A}_1) = 0.8102$, $S(\tilde{A}_2) = 0.5949$, $S(\tilde{A}_3) = 0.7955$, $S(\tilde{A}_4) = 0.7447$, $S(\tilde{A}_5) = 0.8954$, while by Cq-ROFPWG operator as $S(\tilde{A}_1) = 0.7930$, $S(\tilde{A}_2) = 0.6454$, $S(\tilde{A}_3) = 0.8641$, $S(\tilde{A}_4) = 0.8182$, $S(\tilde{A}_5) = 0.8922$. From these computed score values, we can obtain the ranking order of the given alternatives as $\tilde{A}_5 \geq \tilde{A}_1 \geq \tilde{A}_3 \geq \tilde{A}_4 \geq \tilde{A}_2$ through Cq-ROFPWA operator and $\tilde{A}_5 \geq \tilde{A}_3 \geq \tilde{A}_4 \geq \tilde{A}_2 \geq \tilde{A}_1$ by Cq-ROFPWG operator. From both these ranking, we can conclude that the best alternative for the desired task is \tilde{A}_5 .

To compare the reliability of the proposed Algorithm 1 with the existing methods [48,49,55], we implemented their approaches on the considered information and their corresponding results are summarized in Table 8. Clearly, as seen from this table, the existing approaches [48,49,55] under the CIFS environment fail to classify the objects while the proposed method and the others can classify them. Also, we can see that the best alternative is \tilde{A}_5 .

Table 8. Comparison between proposed methods and existing methods.

Methods	Score Values	Ranking
Garg and Rani [48]	Cannot be calculated	No
Rani and Garg [49]	Cannot be calculated	No
Garg and Rani [55]	Cannot be calculated	No
Proposed method for CPFS	$S(\tilde{A}_1) = 0.9585, S(\tilde{A}_2) = 0.7272,$ $S(\tilde{A}_3) = 0.9591, S(\tilde{A}_4) = 0.8994,$ $S(\tilde{A}_5) = 1.0663$	$\tilde{A}_5 \geq \tilde{A}_3 \geq \tilde{A}_1$ $\geq \tilde{A}_1 \geq \tilde{A}_2.$
Proposed method for Cq-ROFS	$S(\tilde{A}_1) = 0.8102, S(\tilde{A}_2) = 0.5949,$ $S(\tilde{A}_3) = 0.7955, S(\tilde{A}_4) = 0.7447,$ $S(\tilde{A}_5) = 0.8954$	$\tilde{A}_5 \geq \tilde{A}_1 \geq \tilde{A}_3$ $\geq \tilde{A}_4 \geq \tilde{A}_2$

Example 6. Consider the five possible emerging technology enterprises $\tilde{A}_i (i = 1, 2, 3, 4, 5)$. are be examined under the four given attributes whose weight vector is $w = (0.3, 0.3, 0.3, 0.1)$. The expert utilize the complex PFS environment during assessing the information and hence we summarized their normalized rating values in Table 9.

Table 9. Decision matrix for Example 6.

	G_1	G_2	G_3	G_4
\tilde{A}_1	$\begin{pmatrix} 0.7e^{i2\pi(0.8)} \\ 0.4e^{i2\pi(0.3)} \end{pmatrix}$	$\begin{pmatrix} 0.59e^{i2\pi(0.70)} \\ 0.41e^{i2\pi(0.31)} \end{pmatrix}$	$\begin{pmatrix} 0.87e^{i2\pi(0.86)} \\ 0.14e^{i2\pi(0.13)} \end{pmatrix}$	$\begin{pmatrix} 0.75e^{i2\pi(0.7)} \\ 0.34e^{i2\pi(0.33)} \end{pmatrix}$
\tilde{A}_2	$\begin{pmatrix} 0.63e^{i2\pi(0.73)} \\ 0.37e^{i2\pi(0.27)} \end{pmatrix}$	$\begin{pmatrix} 0.57e^{i2\pi(0.75)} \\ 0.43e^{i2\pi(0.33)} \end{pmatrix}$	$\begin{pmatrix} 0.81e^{i2\pi(0.71)} \\ 0.24e^{i2\pi(0.33)} \end{pmatrix}$	$\begin{pmatrix} 0.6e^{i2\pi(0.62)} \\ 0.44e^{i2\pi(0.39)} \end{pmatrix}$
\tilde{A}_3	$\begin{pmatrix} 0.66e^{i2\pi(0.76)} \\ 0.33e^{i2\pi(0.23)} \end{pmatrix}$	$\begin{pmatrix} 0.55e^{i2\pi(0.88)} \\ 0.24e^{i2\pi(0.13)} \end{pmatrix}$	$\begin{pmatrix} 0.55e^{i2\pi(0.76)} \\ 0.42e^{i2\pi(0.34)} \end{pmatrix}$	$\begin{pmatrix} 0.79e^{i2\pi(0.9)} \\ 0.24e^{i2\pi(0.13)} \end{pmatrix}$
\tilde{A}_4	$\begin{pmatrix} 0.69e^{i2\pi(0.79)} \\ 0.3e^{i2\pi(0.22)} \end{pmatrix}$	$\begin{pmatrix} 0.62e^{i2\pi(0.71)} \\ 0.4e^{i2\pi(0.3)} \end{pmatrix}$	$\begin{pmatrix} 0.82e^{i2\pi(0.83)} \\ 0.24e^{i2\pi(0.23)} \end{pmatrix}$	$\begin{pmatrix} 0.65e^{i2\pi(0.72)} \\ 0.4e^{i2\pi(0.34)} \end{pmatrix}$
\tilde{A}_5	$\begin{pmatrix} 0.59e^{i2\pi(0.70)} \\ 0.41e^{i2\pi(0.31)} \end{pmatrix}$	$\begin{pmatrix} 0.85e^{i2\pi(0.78)} \\ 0.14e^{i2\pi(0.23)} \end{pmatrix}$	$\begin{pmatrix} 0.65e^{i2\pi(0.76)} \\ 0.34e^{i2\pi(0.33)} \end{pmatrix}$	$\begin{pmatrix} 0.87e^{i2\pi(0.89)} \\ 0.14e^{i2\pi(0.13)} \end{pmatrix}$

By implementing the steps of the proposed Algorithm 1 on this considered information, then the final score values of the given alternatives are obtained by using the Cq-ROFPWA operator as $S(\tilde{A}_1) = 0.89, S(\tilde{A}_2) = 0.63, S(\tilde{A}_3) = 0.76, S(\tilde{A}_4) = 0.79, S(\tilde{A}_5) = 0.85$, while by the Cq-ROFPWG operator as $S(\tilde{A}_1) = 0.76, S(\tilde{A}_2) = 0.56, S(\tilde{A}_3) = 0.69, S(\tilde{A}_4) = 0.73, S(\tilde{A}_5) = 0.73$. From these computed score values, we can obtain the ranking order of the given alternatives as $\tilde{A}_1 \geq \tilde{A}_5 \geq \tilde{A}_3 \geq \tilde{A}_4 \geq \tilde{A}_2$ through the Cq-ROFPWA operator and $\tilde{A}_1 \geq \tilde{A}_5 \geq \tilde{A}_4 \geq \tilde{A}_3 \geq \tilde{A}_2$ by the Cq-ROFPWG operator. From both these ranking, we can conclude that the best alternative for the desired task is \tilde{A}_1 .

The performance of this considered example with the existing studies [48,49,55] are executed and the results are tabulated in Table 10. Again, it is clearly seen from the table that the existing studies [48,49,55] under the CIFS environment fail to classify the objects while the presented approach and under CPFS, the best alternatives coincides with each other.

Table 10. Comparative analysis for Example 6.

Methods	Operator	Score Values	Ranking
Garg and Rani [48]	Averaging	Cannot be calculated	-
	Geometric	Cannot be calculated	-
Rani and Garg [49]	Hamming	Cannot be calculated	-
	Euclidean	Cannot be calculated	-
Garg and Rani [55]	Averaging	Cannot be calculated	-
	Geometric	Cannot be calculated	-
Proposed method for CPFS	Averaging	$S(\tilde{A}_1) = 1.0452, S(\tilde{A}_2) = 0.7663,$ $S(\tilde{A}_3) = 0.9266, S(\tilde{A}_4) = 0.9540,$ $S(\tilde{A}_5) = 1.0240$	$\tilde{A}_1 \geq \tilde{A}_5 \geq \tilde{A}_4$ $\geq \tilde{A}_3 \geq \tilde{A}_2$
		$S(\tilde{A}_1) = 0.8726, S(\tilde{A}_2) = 0.6821,$ $S(\tilde{A}_3) = 0.8408, S(\tilde{A}_4) = 0.8714,$ $S(\tilde{A}_5) = 0.8770$	$\tilde{A}_5 \geq \tilde{A}_1 \geq \tilde{A}_4$ $\geq \tilde{A}_3 \geq \tilde{A}_2$
		$S(\tilde{A}_1) = 0.8896, S(\tilde{A}_2) = 0.6287,$ $S(\tilde{A}_3) = 0.7683, S(\tilde{A}_4) = 0.7973,$ $S(\tilde{A}_5) = 0.8576$	$\tilde{A}_1 \geq \tilde{A}_5 \geq \tilde{A}_4$ $\geq \tilde{A}_3 \geq \tilde{A}_2$
	Geometric	$S(\tilde{A}_1) = 0.8531, S(\tilde{A}_2) = 0.6739,$ $S(\tilde{A}_3) = 0.8201, S(\tilde{A}_4) = 0.8620,$ $S(\tilde{A}_5) = 0.8562$	$\tilde{A}_4 \geq \tilde{A}_5 \geq \tilde{A}_1$ $\geq \tilde{A}_3 \geq \tilde{A}_2$

Example 7. Consider a MADM problem consists of the selection of the best contractor(s) from the given five builders $\tilde{A}_i (i = 1, 2, 3, 4, 5)$ which are evaluated under the four attributes $G_j (j = 1, 2, 3, 4)$ whose weight vector is 0.3, 0.4, 0.1 and 0.2. The decision matrix given by an expert to evaluate them is given in Table 11.

Table 11. Decision matrix for Example 7.

	G_1	G_2	G_3	G_4
\tilde{A}_1	$\begin{pmatrix} 0.7e^{i2\pi(0.80)} \\ 0.8e^{i2\pi(0.52)} \end{pmatrix}$	$\begin{pmatrix} 0.59e^{i2\pi(0.70)} \\ 0.51e^{i2\pi(0.71)} \end{pmatrix}$	$\begin{pmatrix} 0.87e^{i2\pi(0.86)} \\ 0.74e^{i2\pi(0.73)} \end{pmatrix}$	$\begin{pmatrix} 0.75e^{i2\pi(0.7)} \\ 0.64e^{i2\pi(0.63)} \end{pmatrix}$
\tilde{A}_2	$\begin{pmatrix} 0.63e^{i2\pi(0.73)} \\ 0.87e^{i2\pi(0.67)} \end{pmatrix}$	$\begin{pmatrix} 0.57e^{i2\pi(0.75)} \\ 0.53e^{i2\pi(0.63)} \end{pmatrix}$	$\begin{pmatrix} 0.81e^{i2\pi(0.71)} \\ 0.74e^{i2\pi(0.63)} \end{pmatrix}$	$\begin{pmatrix} 0.6e^{i2\pi(0.62)} \\ 0.84e^{i2\pi(0.59)} \end{pmatrix}$
\tilde{A}_3	$\begin{pmatrix} 0.66e^{i2\pi(0.76)} \\ 0.53e^{i2\pi(0.63)} \end{pmatrix}$	$\begin{pmatrix} 0.55e^{i2\pi(0.88)} \\ 0.54e^{i2\pi(0.73)} \end{pmatrix}$	$\begin{pmatrix} 0.55e^{i2\pi(0.76)} \\ 0.52e^{i2\pi(0.64)} \end{pmatrix}$	$\begin{pmatrix} 0.79e^{i2\pi(0.9)} \\ 0.74e^{i2\pi(0.83)} \end{pmatrix}$
\tilde{A}_4	$\begin{pmatrix} 0.69e^{i2\pi(0.79)} \\ 0.53e^{i2\pi(0.62)} \end{pmatrix}$	$\begin{pmatrix} 0.62e^{i2\pi(0.71)} \\ 0.54e^{i2\pi(0.53)} \end{pmatrix}$	$\begin{pmatrix} 0.82e^{i2\pi(0.83)} \\ 0.74e^{i2\pi(0.73)} \end{pmatrix}$	$\begin{pmatrix} 0.65e^{i2\pi(0.72)} \\ 0.54e^{i2\pi(0.74)} \end{pmatrix}$
\tilde{A}_5	$\begin{pmatrix} 0.59e^{i2\pi(0.70)} \\ 0.71e^{i2\pi(0.61)} \end{pmatrix}$	$\begin{pmatrix} 0.85e^{i2\pi(0.78)} \\ 0.74e^{i2\pi(0.63)} \end{pmatrix}$	$\begin{pmatrix} 0.65e^{i2\pi(0.76)} \\ 0.64e^{i2\pi(0.63)} \end{pmatrix}$	$\begin{pmatrix} 0.87e^{i2\pi(0.89)} \\ 0.74e^{i2\pi(0.73)} \end{pmatrix}$

By utilizing the proposed Cq-ROFPWA and Cq-ROFPWG operators to aggregate the given information and hence the score values of the aggregated numbers are obtained as 0.2837, 0.0255, 0.3542, 0.3378 and 0.3571 of each alternatives through the Cq-ROFPWA operators. On the other hand, if we aggregate the given information by the Cq-ROFPWG operator, then the final score values of the aggregated numbers corresponding to each alternative are computed as 0.0869, -0.0847, 0.1364,

0.2025 and 0.1419, respectively. Based on these score values, the ranking order of the given numbers are obtained as $\tilde{A}_5 \geq \tilde{A}_4 \geq \tilde{A}_3 \geq \tilde{A}_1 \geq \tilde{A}_2$ and $\tilde{A}_4 \geq \tilde{A}_5 \geq \tilde{A}_3 \geq \tilde{A}_1 \geq \tilde{A}_2$ through the Cq-ROFPWA and Cq-ROFWG operators, respectively. Hence, the best alternative by both the operators is \tilde{A}_5 .

To check the validity of the proposed method over the existing methods [48,49,55], the results are summarized in Table 12. It is seen from this table that all the existing approaches under the CIFS and CPFS environments fail to handle the decision making problems, while the proposed approach successfully overcomes the drawbacks of it. Thus, the applicability range of the considered method is wider than the existing approaches under the CIFS and CPFS environments.

Table 12. Comparative analysis for Example 7.

Methods	Score Values	Ranking
Garg and Rani [48]	Cannot be calculated	No
Rani and Garg [49]	Cannot be calculated	No
Garg and Rani [55]	Cannot be calculated	No
Proposed method for CPFS	Cannot be calculated	No
Proposed method for Cq-ROFS	$S(\tilde{A}_1) = 0.2837, S(\tilde{A}_2) = 0.0255,$ $S(\tilde{A}_3) = 0.3542, S(\tilde{A}_4) = 0.3378, S(\tilde{A}_5) = 0.3571$	$\tilde{A}_5 \geq \tilde{A}_4 \geq \tilde{A}_3$ $\geq \tilde{A}_1 \geq \tilde{A}_2$

6.2. Characteristic Comparison

From the above stated Examples 3–7, we conclude that the proposed VIKOR and Aggregation operators are successfully applied to solve the decision making problems while the existing methods fail. In order to compare the proposed concept Cq-ROFS with the various existing sets, we investigate their comparison through their characteristics. The results computed through their characteristic comparison are summarized in Table 13. From this investigation, we conclude that the proposed set performed better than the existing sets. Therefore, our method is superior and more effective for solving MADM problems.

Table 13. Characteristic comparison between the proposed methods with the existing methods.

Methods	Whether Flexibly to Show a Wide Range of Information	Whether Describe Information Using Periodical Features	Whether Have the Characteristic of Generalization	Whether Have More Powerful Characteristic
Xu and Yager [10]	No	No	No	No
Yager [7]	Yes	No	No	No
Yager [9]	Yes	No	Yes	No
Alkouri and Salleh [45]	No	Yes	Yes	No
Garg and Rani [48]	No	Yes	Yes	No
Rani and Garg [49]	No	Yes	Yes	No
Ullah et al. [53]	Yes	Yes	Yes	No
Proposed work	Yes	Yes	Yes	Yes

Apart from this, we also concluded that the several existing VIKOR methods and the aggregation operators are special cases of the proposed ones. The following are summarily obtained through the presented approach:

- (1) In the VIKOR method, if we consider $q = 1$, then it reduces to the VIKOR method for CIFS.
- (2) In the VIKOR method, if we consider the imaginary part is zero and the value of $q = 1$, then it reduces to the VIKOR method for IFS.
- (3) In the VIKOR method, if we set $q = 2$, it reduces to the VIKOR method for CPFS.
- (4) In the VIKOR method, if we set the imaginary part is zero and the value of $q = 2$, then it reduces to the VIKOR method for PFS.

- (5) In power aggregation operators, if we set $q = 1$, then proposed operator reduces to CIFS. On the other hand, if we set $q = 2$, then it becomes CPFS.
- (6) In power aggregation operators, if the imaginary part is considered to be zero and the value of $q = 1, q = 2$, the proposed methods are converted to IFS and PFS, respectively.

7. Conclusions

The key contributions of the work can be summarized below.

- (1) In the present paper, we explored the concept of Cq-ROFS, which is a generalization of the several existing fuzzy sets, to manage uncertain and unpredictable information in real-life problems. The major characteristic of the proposed Cq-ROFS is that they have a flexible parameter $q \geq 1$ integers to adjust the degree of membership and non-membership during aggregating the process. Also, in Cq-ROFS, the sum of squares of the real parts (imaginary parts) of the membership and non-membership degrees is less than or equal to one. The Cq-ROFS is more powerful and more general than the existing methods CIFS and CPFS.
- (2) To rank the given Cq-ROFNs, we presented some basic operational laws, score function and accuracy functions. The various properties of the stated functions have been discussed.
- (3) To aggregate the different preferences of the expert values, we presented the power weighted aggregation operators for a collection of Cq-ROFNs, namely, Cq-ROFPA operator, Cq-ROFPG operator, Cq-ROFPWA operator, Cq-ROFPWG operator, Cq-ROFPHA operator and Cq-ROFPHG operator. The salient features as well as properties of them are investigated in detail.
- (4) Furthermore, we discussed the concept of the VIKOR methods for Cq-ROFSs to solve decision making problems. Some special cases of the presented VIKOR method were presented.
- (5) Two algorithms based on the proposed aggregation operators and the VIKOR method were presented to solve the MADM problems. In these approaches, different values of the alternatives are aggregated by using weighted operators and then finally the obtained numbers are ranked by using the stated score functions.
- (6) To demonstrate the feasibility, reliability and effectiveness of the proposed approaches, several examples were given to compute their results based on the stated algorithm. The feasibility as well as superiority of the approaches is compared with the several existing approaches [48,49,55]. From these, it is concluded from this study that the proposed work gives more reasonable ways to handle the fuzzy information to solve practical problems.

In the future, we shall extend the applications of the presented method to diverse fuzzy environments as well as to different fields such as emerging decision problems, brain hemorrhage, risk evaluation and so forth. [60–62].

Author Contributions: Conceptualization, H.G., T.M. and Z.A.; Data curation, H.G.; Formal analysis, H.G., J.G.; Funding acquisition, J.G.; Methodology, H.G.; Writing—original draft, H.G., J.G., Writing—review, H.G., J.G. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by the Brain Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Science, ICT & Future Planning (NRF-2019M3C7A1020406) and the Basic Science Research Program through the NRF funded by the Ministry of Education (NRF-2017R1D1A1B03036423).

Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

The following abbreviations are used in the entire paper:

MADM	Multiple attribute decision making
AO	Aggregation Operator.
IFS	Intuitionistic fuzzy set
PFS	Pythagorean fuzzy set
CIFS	Complex Intuitionistic fuzzy set
CPFS	Complex Pythagorean fuzzy set
Cq-ROFS	Complex q-rung orthopair fuzzy set
Cq-ROFPA	Complex q-rung orthopair fuzzy power averaging
Cq-ROFPG	Complex q-rung orthopair fuzzy power geometric
Cq-ROFPWA	Complex q-rung orthopair fuzzy power weighted averaging
Cq-ROFWG	Complex q-rung orthopair fuzzy power weighted geometric
Cq-ROFPOWA	Complex q-rung orthopair fuzzy power ordered weighted averaging.
Cq-ROFWG	Complex q-rung orthopair fuzzy power ordered weighted geometric.
Cq-ROFPHA	Complex q-rung orthopair fuzzy power hybrid averaging.
Cq-ROFPHG	Complex q-rung orthopair fuzzy power hybrid geometric.
VIKOR	VlseKriterijumska Optimizacija I Kompromisno Resenje, meaning multicriteria optimization and compromise solution

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