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Fibonacci Numbers with a Prescribed Block of Digits

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Abstract: In this paper, we prove that $F_{22} = 17711$ is the largest Fibonacci number whose decimal expansion is of the form $ab \ldots bc \ldots c$. The proof uses lower bounds for linear forms in three logarithms of algebraic numbers and some tools from Diophantine approximation.

Keywords: Fibonacci numbers; digits; linear forms in logarithms; Baker–Davenport

MSC: primary 11A63, 11B39; secondary 11J86

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1. Introduction

Let $(F_n)_{n\geq 0}$ be the Fibonacci sequence given by second-order recurrence $F_{n+2} = F_{n+1} + F_n$, for $n \geq 0$, with initial conditions $F_0 = 0$ and $F_1 = 1$. A few terms of this sequence are

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, \ldots.$$

In the last decades, many results on Diophantine properties of Fibonacci numbers have been proved with the use of refined tools in number theory. For instance, Bugeaud, Mignotte, and Siksek [1] settled the problem of Fibonacci perfect power numbers (i.e., the equation $F_n = y^t$, for $t > 1$) by combing the powerful Baker's theory with the Modular method (used by Wiles in the proof of the Fermat Last Theorem). See a generalization of their result in [2].

We remark that digital problems involving Fibonacci numbers have received much attention in the literature. A first result in this direction was proved, in 2000, by F. Luca [3] who showed that the largest Fibonacci number with only one distinct digit is $F_{10} = 55$. After this, many authors worked on *repdigits* (i.e., integers having only one distinct digit in its decimal expansion) as expressions related to sum, product of terms of binary recurrences (see [4–13] and references therein). However, the related problem of finding all Fibonacci numbers with only two distinct digits remains open.

The aim of this paper is to continue this program. In fact, our main result searches for all Fibonacci numbers of the form $ab \ldots bc \ldots c$, which provides a generalization for Luca's result ($a = b = c$). More precisely, our main result is the following:

Theorem 1. *Let $a \in [1, 9]$ and $b, c \in [0, 9]$. The largest solution of the Diophantine equation*

$$F_n = a\underbrace{b \ldots b}_{m}\underbrace{c \ldots c}_{\ell} = a \cdot 10^{m+\ell} + b \cdot \left(\frac{10^m - 1}{9}\right) 10^{\ell} + c \cdot \left(\frac{10^{\ell} - 1}{9}\right), \tag{1}$$

in positive integers m, n, and ℓ, with $2 \leq \ell \leq m$, is $n = 22$. Explicitly, $F_{22} = 17711$.

Our proof combines two deep techniques in number theory, namely, the Baker's theory on linear forms in logarithms and some tools from Diophantine approximation (a reduction method due to Baker and Davenport).

2. Auxiliary Results

In this section, we shall present some results which will be useful in the proofs.

Let $\alpha = (1+\sqrt{5})/2$ and $\beta = -1/\alpha$. The Binet's formula asserts that $F_n = (\alpha^n - \beta^n)/\sqrt{5}$. From this formula, it is possible to deduce that the estimates

$$\alpha^{n-2} \leq F_n \leq \alpha^{n-1}, \tag{2}$$

hold for all $n \geq 1$. In addition, this Binet's formula allows us to manipulate our Diophantine equation to obtain upper bounds for some linear forms in three logarithms. Thus, in order to obtain lower bounds for these forms, we shall use the celebrated Baker's theory. Among these lower bounds, we decided to use one which was proved in ([1], Theorem 9.4).

Lemma 1. *Let $\gamma_1, \ldots, \gamma_t$ be real algebraic numbers and let $b_1, \ldots, b_t$ be nonzero rational integer numbers. Let D be the degree of the number field $\mathbb{Q}(\gamma_1, \ldots, \gamma_t)$ over $\mathbb{Q}$ and let A_j be a positive real number satisfying*

$$A_j \geq \max\{Dh(\gamma_j), |\log \gamma_j|, 0.16\},$$

where $j = 1, \ldots, t$. Assume that

$$B \geq \max\{|b_1|, \ldots, |b_t|\}.$$

If $\gamma_1^{b_1} \cdots \gamma_t^{b_t} \neq 1$, then

$$|\gamma_1^{b_1} \cdots \gamma_t^{b_t} - 1| \geq \exp(-1.4 \cdot 30^{t+3} \cdot t^{4.5} \cdot D^2(1+\log D)(1+\log B)A_1 \cdots A_t).$$

Here, the *logarithmic height* of an n-degree algebraic number α is defined as

$$h(\alpha) = \frac{1}{n}\left(\log|a| + \sum_{j=1}^{n} \log\max\{1, |\alpha^{(j)}|\}\right),$$

where a is the leading coefficient of the minimal primitive polynomial of α (over $\mathbb{Z}$) and $(\alpha^{(j)})_{1\leq j\leq n}$ are the (algebraic) conjugates of α.

With these lower and upper bounds, we shall obtain an upper bound for n which is, in general, very large and then the next step is to reduce it. For that, we shall use a reduction method which is originated from Diophantine approximation. Here, we shall use a result due to Dujella and Pethö [14] (which is a variant of a famous method due to Baker–Davenport). For a real number x we use $\| x \| = \min\{|x-n| : n \in \mathbb{Z}\}$ for the distance from x to the nearest integer (the so-called Nint function).

Lemma 2. *Let $M > 0$ be an integer and let γ, μ be real numbers, such that $\gamma \notin \mathbb{Q}$. Let p/q be a convergent of the continued fraction expansion of γ such that $q > 6M$ and $\epsilon := \| \mu q \| - M \| \gamma q \| > 0$. Then there is no solution to the Diophantine inequality*

$$0 < m\gamma - n + \mu < A \cdot B^{-m}$$

in positive integers m, n with

$$\frac{\log(Aq/\epsilon)}{\log B} \leq m < M.$$

After presetting these tools, we can now prove our main result.

3. The Proof of The Theorem

3.1. Finding a Bound on N

By the Binet's formula and the identity in Equation (1), we have

$$\frac{\alpha^n}{\sqrt{5}} - \left(a + \frac{b}{9}\right) 10^{m+\ell} = \left(\frac{c-b}{9}\right) 10^{\ell} + \frac{\beta^n}{\sqrt{5}} - \frac{c}{9}. \tag{3}$$

Thus,

$$\left| \frac{\alpha^n}{\sqrt{5}} - \left(a + \frac{b}{9}\right) 10^{m+\ell} \right| < 2 \cdot 10^{\ell}.$$

On dividing through by $(a + b/9)10^{m+\ell}$, we obtain

$$\left| \left(a + \frac{b}{9}\right)^{-1} \frac{\alpha^n}{\sqrt{5}} 10^{-(m+\ell)} - 1 \right| < \frac{2}{10^m}, \tag{4}$$

where we used the fact that $a + b/9 \geq 1$. Now, we are in a position to apply Lemma 1, but first we must prove that $|(\sqrt{5}(a + b/9))^{-1} 10^{-(m+\ell)} \alpha^n - 1| \neq 0$. Indeed, in the contrary case, we would get that $\alpha^{2n} \in \mathbb{Q}$ which is an absurd. Thus, let us take

$$\gamma_1 := \sqrt{5}(a + b/9), \ \gamma_2 := \alpha, \ \gamma_3 := 10, \ b_1 := -1, \ b_2 := n, \ b_3 := -(m + \ell).$$

Note that $\mathbb{Q}(\gamma_1, \gamma_2, \gamma_3) = \mathbb{Q}(\sqrt{5})$ and then $D = 2$. The conjugates of γ_1, γ_2, and γ_3 are $\gamma_1' = -\gamma_1, \ \gamma_2' = \beta, \ \gamma_3' = \gamma_3$, respectively. Surely, γ_2 and γ_3 are algebraic integers, while the minimal polynomial of γ_1 is $(X - \gamma_1)(X - \gamma_1') = X^2 - 5(a + b/9)^2$ which is a divisor of $81X^2 - 5(9a + b)^2$. Therefore,

$$h(\gamma_1) \leq \frac{1}{2} \left(\log 81 + 2 \log \left(10\sqrt{5}\right)\right) < 5.4.$$

(A more relaxed upper bound for $h(\gamma_1)$ could be found by using the well-known property that $h(x + yz) \leq h(x) + h(y) + h(z) + \log 2$). In addition, $h(\gamma_2) = (\log \alpha)/2 < 0.75$ and $h(\gamma_3) = \log 10 < 2.31$. Let us take $A_1 := 10.9, \ A_2 = 1.5$, and $A_3 = 4.7$. Of course, we can assume that $n > 14$. Thus,

$$\left| \left(a + \frac{b}{9}\right)^{-1} \frac{\alpha^n}{\sqrt{5}} 10^{-(m+\ell)} - 1 \right| > \exp\left(-7.6 \cdot 10^{13}(1 + \log B)\right), \tag{5}$$

where $B \geq \max\{n, m + \ell\}$

By combining the estimates in Equations (4) and (5), we get

$$m < 3.4 \cdot 10^{13}(1 + \log B).$$

Now, we have that F_n has $m + \ell + 1$ digits and so

$$m + \ell + 1 = \left\lfloor \frac{\log F_n}{\log 10} \right\rfloor + 1. \tag{6}$$

Since $m \geq \ell$ and $\lfloor x \rfloor + 1 > x$, we have

$$2m + 1 > \frac{\log F_n}{\log 10} \geq (n - 2)\frac{\log \alpha}{\log 10} > 0.2(n - 2)$$

so $10m + 6 \geq n$. Thus, we can take $B := 10m + 6$, which yields to

$$m < 3.4 \cdot 10^{13}(1 + \log(10m + 6))$$

and therefore $m \leq 1.3 \cdot 10^{15}$.

3.2. Reducing the Bound

Now, let us write $\Lambda := n \log \alpha - (m + \ell) \log 10 + \log((a + b/9)^{-1}(\sqrt{5})^{-1})$. We know that $e^x - 1 > x$, for all $x \in \mathbb{R}^+$. By supposing that $\Lambda > 0$ (the other case is completely similar, where we used the fact that $|e^x - 1| = 1 - e^{-|x|}$, if $x < 0$), we can rewrite Equation (4) as

$$0 < n \log \alpha - (m + \ell) \log 10 + \log \left((a + b/9)^{-1}(\sqrt{5})^{-1} \right) < 1.8 \cdot 10^{-m}.$$

Since $m > (0.1)n - 0.7$ (because $2m + 1 > 0.2(n - 2)$), we can divide the previous inequality by $\log 10$, to obtain

$$0 < n\gamma - (m + \ell) + \mu < 4 \cdot (1.2)^{-n}, \tag{7}$$

where $\gamma := \log \alpha / \log 10$ and $\mu := \log((a + b/9)^{-1}(\sqrt{5})^{-1}) / \log 10$.

Clearly γ is an irrational number (because α^k is irrational for any non-zero integer k). Let us denote p_n/q_n as the nth convergent of its continued fraction.

In order to reduce our bound on m, we shall use Lemma 2. Now, since $n \leq 10m + 6 < 1.4 \cdot 10^{16}$, we choose $M = 1.4 \cdot 10^{16}$. Thus,

$$\frac{p_{38}}{q_{38}} = \frac{14261348558663707 84}{6824015306170795931},$$

then $q_{38} \geq 6824015306170795931 > 8.4 \cdot 10^{16} = 6M$. Furthermore, we have $M \parallel q_{38}\gamma \parallel < 0.0013$. On the other hand, by computing $\parallel q_{38}\mu \parallel$, for $a \in [1, 9]$ and $b \in [0, 9]$, we have that the minimal value of this expression is obtained when $a = 8,\ b = 2$ and is > 0.0028. Hence,

$$\epsilon = \parallel q_{38}\mu \parallel - M \parallel q_{38}\gamma \parallel > 0.0015.$$

We notice the all the hypotheses of the Lemma 2 are fulfilled, where $A = 4$ and $B = 1.2$, so, by that lemma, there is no solution of the inequality in Equation (7) (and then for the Diophantine Equation (1)) for n in the range

$$[\lfloor \log(Aq_{38}/\epsilon) / \log B \rfloor + 1, M] = [281, 1.4 \cdot 10^{16}].$$

Since $n < M$, we get $n \leq 280$. By using Equation (6), we deduce that

$$m + \ell \leq (n - 1)\frac{\log \alpha}{\log 10} - 1 < 58.4$$

and so $m + \ell \leq 58$. Since $m + \ell + 1 \geq 5$, it is seen that F_n has at least 5 digits yielding $n \geq 21$. A simple search in the list of the Fibonacci numbers F_n in the range $n \in [21, 280]$ (see Table A1 in Appendix A), returns only $F_{22} = 17711$ with the required properties. This completes the proof. □

4. Conclusions

In this paper we have been interested in finding all Fibonacci numbers which are special concatenation of digits. In particular, we show that $F_{22} = 17711$ is the largest Fibonacci number whose decimal expansion is of the form $ab \ldots bc \ldots c$, where a, b, and c are decimal digits. Our approach to the proof is based on the combination of lower bounds for linear forms in logarithms (due to Baker) with reduction methods (due to Dujella–Pethö).

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Appendix A

Table A1. Values of $F(n)$ for n from 21 to 280.

n	F_n	n	F_n
21	10946	280	146910984068621881489442072459549121105480936013821976978 35
22	17711	279	90795981475102637178708944490299333694911317865144462661 46
23	28657	278	56115002593519244310733127969249787410569618148677514316 89
24	46368	277	34680978881583392867975816521049546284341699716466948344 57
25	75025	276	21434023711935851442757311448200241126227918432210565972 32
26	121393	275	13246955169647541425218505072849305158113781284256382372 25
27	196418	274	8187068542288310017538806375350935968114137147954183600 07
28	317811	273	5059886627359231407679698697498369189999644136302198772 18
29	514229	272	3127181914929078609859107677852566778114493011651984827 89
30	832040	271	1932704712430152797820591019645802411885151124650213944 29
31	1346269	270	1194477202498925812038516658206764366229341887001770883 60
32	2178309	269	738227509931226985782074361439038045655809237648443060 69
33	3524578	268	456249692567698826256442296767726320573532649353327822 91
34	5702887	267	281977817363528159525632064671311725082276588295115237 78
35	9227465	266	174271875204170666730810232096414595491256061058212585 13
36	14930352	265	107705942159357492794821832574897129591020527236902652 65
37	24157817	264	66565933044813173935988399521517465900235533821309932 48
38	39088169	263	41140009114544318858833433053379663690784993415592720 17
39	63245986	262	25425923930268855077154966468137802209450540405717212 31
40	102334155	261	15714085184275463781678466585241861481334453009875507 86
41	165580141	260	9711838745993391295476499882895940728116087395841704 45
42	267914296	259	6002246438282072486201966702345920753218365614033803 41
43	433494437	258	3709592307711318809274533180550019974897721781807901 04
44	701408733	257	2292654130570753676927433521795900778320643832225902 37
45	1134903170	256	1416938177140565132347099658754119196577077949581998 67
46	1836311903	255	875715953430188544580333863041781581743565882643903 70
47	2971215073	254	541222223710376587766765795712337614833512066938094 97
48	4807526976	253	334493729719811956813568067329443966910053815705808 73
49	7778742049	252	206728493990564630953197728382893647923458251232286 24
50	12586269025	251	127765235729247325860370338946550318986595564473522 49
51	20365011074	250	78963258261317305092827389436343328936862686758763 75
52	32951280099	249	48801977467930020767542949510206990049732877714758 74
53	53316291173	248	30161280793387284325284439926136338887129809044005 01
54	86267571272	247	18640696674542736442258509584070651162603068670753 73
55	139583862445	246	11520584118844547883025930342065687724526740373251 28
56	225851433717	245	7120112555698188559232579242004963438076328297502 45
57	365435296162	244	4400471563146359323793351100060724286450412075748 83
58	591286729879	243	2719640992551829235439228141944239151625916221753 62
59	956722026041	242	1680830570594530088354122958116485134824495853995 21
60	1548008755920	241	1038810421957299147085105183827754016801420367758 41
61	2504730781961	240	642020148637230941269017774288731118023075486236 80
62	4052739537881	239	396790273320068205816087409539022898778344881521 61
63	6557470319842	238	245229875317162735452930364749708219244730604715 19
64	10610209857723	237	151560398002905470363157044789314679533614276806 42
65	17167680177565	236	93669477314257265089773319960393539711116327908 77
66	27777890035288	235	57890920688648205273383724828921139822497948897 65
67	44945570212853	234	35778556625609059816389595131472399888618379011 12
68	72723460248141	233	22112364063039145456994129697448739933879569886 53
69	117669030460994	232	13666192562569914359395465434023659954738809124 59
70	190392490709135	231	8446171500469231097598664263425079979140760761 94

Table A1. *Cont.*

n	F_n	n	F_n
71	308061521170129	230	5220021062100683261796801170598579975598048362 65
72	498454011879264	229	3226150438368547835801863092826500003542712399 29
73	806515533049393	228	1993870623732135425994938077772079972055335963 36
74	1304969544928657	227	1232279814636412409806925015054420031487376435 93
75	2111485077978050	226	761590809095723016188013062717659940567959527 43
76	3416454622906707	225	470689005540689393618911952336760090919416908 50
77	5527939700884757	224	290901803555033622569101110380899849648542618 93
78	8944394323791464	223	179787201985655771049810841955860241270874289 57
79	14472334024676221	222	111114601569377851519290268425039608377668329 36
80	23416728348467685	221	68672600416277919530520573530820632893205960 21
81	37889062373143906	220	42442001153099931988769694894218975484462369 15
82	61305790721611591	219	26230599263177987541750878636601657408743591 06
83	99194853094755497	218	16211401889921944447018816257617318075718778 09
84	160500643816367088	217	10019197373256043094732062378984339333024812 97
85	259695496911122585	216	6192204516665901352286753878632978742693965 12
86	420196140727489673	215	3826992856590141742445308500351360590330847 85
87	679891637638612258	214	2365211660075759609841445378281618152363117 27
88	1100087778366101931	213	1461781196514382132603863122069742437967730 58
89	1779979416004714189	212	903430463561377477237582256211875714395386 69
90	2880067194370816120	211	558350732953004655366280865857866723572343 89
91	4660046610375530309	210	345079730608372821871301390354008990823042 80
92	7540113804746346429	209	213271002344631833494979475503857732749301 09
93	12200160415121876738	208	131808728263740988376321914850151258073741 71
94	19740274219868223167	207	81462274080890845118657560653706474675559 38
95	31940434634990099905	206	50346454182850143257664354196444783398182 33
96	51680708854858323072	205	31115819898040701860993206457261691277377 05
97	83621143489848422977	204	19230634284809441396671147739183092120805 28
98	135301852344706746049	203	11885185613231260464322058718078599156571 77
99	218922995834555169026	202	7345448671578180932349089021104492964233 51
100	354224848179261915075	201	4539736941653079531972969696974106192338 26
101	573147844013817084101	200	2805711729925101400376119324130386771895 25
102	927372692193078999176	199	1734025211727978131596850372843719420443 01
103	1500520536206896083277	198	1071686518197123268779268951286667351452 24
104	2427893228399975082453	197	662338693530854862817581421557052068990 77
105	3928413764606871165730	196	409347824666268405961687529729615282461 47
106	6356306993006846248183	195	252990868864586456855893891827436786529 30
107	10284720757613717413913	194	156356955801681949105793637902178495932 17
108	16641027750620563662096	193	96633913062904507750100253925258290597 13
109	26925748508234281076009	192	59723042738777441355693383976920205335 04
110	43566776258854844738105	191	36910870324127066394406869948338085262 09
111	70492524767089125814114	190	22812172414650374961286514028582120072 95
112	114059301025943970552219	189	14098697909476691433120355919755965189 14
113	184551825793033096366333	188	8713474505173683528166158108826154883 81
114	298611126818977066918552	187	5385223404303007904954197810929810305 33
115	483162952612010163284885	186	3328251100870675623211960297896344578 48
116	781774079430987230203437	185	2056972303432332281742237513033465726 85
117	1264937032042997393488322	184	1271278797438343341469722784862878851 63
118	2046711111473984623691759	183	785693505993988940272514728170586875 22
119	3311648143516982017180081	182	485585291444354401197208056692291976 41
120	5358359254990966640871840	181	300108214549634539075306671478294898 81
121	8670007398507948658051921	180	185477076894719862121901385213997077 60
122	14028366653498915298923761	179	114631137654914676953405286264297821 21
123	22698374052006863956975682	178	70845939239805185168496098949699256 39
124	36726740705505779255899443	177	43785198415109491784909187314598564 82
125	59425114757512643212875125	176	27060740824695693383586911635100691 57
126	96151855463018422468774568	175	16724457590413798401322275679497873 25
127	155576970220531065681649693	174	10336283234281894982264635955602818 32
128	251728825683549488150424261	173	6388174356131903419057639723895054 93
129	407305795904080553832073954	172	3948108878149991563206996231707763 39

Table A1. *Cont.*

n	F_n	n	F_n
130	659034621587630041982498215	171	244006547798191185585064349218729154
131	1066340417491710595814572169	170	150804340016807970735635273952047185
132	1725375039079340637797070384	169	93202207781383214849429075266681969
133	2791715456571051233611642553	168	57602132235424755886206198685365216
134	4517090495650391871408712937	167	35600075545958458963222876581316753
135	7308805952221443105020355490	166	22002056689466296922983322104048463
136	11825896447871834976429068427	165	13598018856492162040239554477268290
137	19134702400093278081449423917	164	8404037832974134882743767626780173
138	30960598847965113057878492344	163	5193981023518027157495786850488117
139	50095301248058391139327916261	162	3210056809456107725247980776292056
140	81055900096023504197206408605	161	1983924214061919432247806074196061
141	131151201344081895336534324866	160	1226132595394188293000174702095995
142	212207101440105399533740733471	159	757791618667731139247631372100066
143	343358302784187294870275058337	158	468340976726457153752543329995929
144	555565404224292694404015791808	157	289450641941273985495088042104137
145	898923707008479989274290850145	156	178890334785183168257455287891792
146	1454489111232772683678306641953	155	110560307156090817237632754212345
147	2353412818241252672952597492098	154	68330027629092351019822533679447
148	3807901929474025356630904134051	153	42230279526998466217810220532898
149	6161314747715278029583501626149	152	26099748102093884802012313146549
150	9969216677189303386214405760200	151	16130531424904581415797907386349

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