

Article

On the Solutions of a Class of Integral Equations Pertaining to Incomplete H -Function and Incomplete \overline{H} -Function

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Abstract: The main aim of this article is to study the Fredholm-type integral equation involving the incomplete H -function (IHF) and incomplete \overline{H} -function in the kernel. Firstly, we solve an integral equation associated with the IHF with the aid of the theory of fractional calculus and Mellin transform. Next, we examine an integral equation pertaining to the incomplete \overline{H} -function with the help of theory of fractional calculus and Mellin transform. Further, we indicate some known results by specializing the parameters of IHF and incomplete \overline{H} -function. The results computed in this article are very general in nature and capable of giving many new and known results connected with integral equations and their solutions hitherto scattered in the literature. The derived results are very useful in solving various real world problems.

Keywords: Fredholm-type integral equations; incomplete H -functions; incomplete \overline{H} -functions; Mellin transform; Weyl fractional integral

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1. Introduction

Several types of integral equation appear in the fields of engineering and science. They can be used to describe many physical problems. Integral equations have a variety of applications in fields including fluid mechanics, mathematical physics, quantum mechanics, queuing theory, etc. (for details, see [1–11]). Therefore, the study of integral equations and methods for solving them are very useful in science and engineering. Several authors introduced and studied the Fredholm integral equation involving special functions such as Legendre functions, hypergeometric function, and Fox H -function [12–20].

Incomplete special functions have variety of uses in the field of probability theory, communication theory, modeling of groundwater pumping, engineering, science, heat conduction, etc. Very recently, many authors work on these incomplete special functions (for details, see the works of Bansal et al. [21–25], Srivastava et al. [26], Kumar et al. [27], Bansal and Kumar [28], and Chaurasia and Kumar [29]).

Motivated by the work of Srivastava and Raina [18], we introduce the Fredholm-type integral equation involving the IHF and incomplete \bar{H} -function in the kernel.

2. Preliminaries

The classical gamma function $\Gamma(v)$ is expressed as follows:

$$\Gamma(v) = \begin{cases} \int_0^\infty e^{-u} u^{v-1} du, & (\Re(v) > 0) \\ \frac{\Gamma(v+\kappa)}{(v)_\kappa}, & (v \in \mathbb{C} \setminus \mathbb{Z}_0^-; \kappa \in \mathbb{N}_0). \end{cases} \quad (1)$$

In the above expression $(v)_\kappa$ indicates the Pochhammer symbol and is defined (for $v \in \mathbb{C}$, $\kappa \in \mathbb{N}_0$) as

$$(v)_\kappa = \frac{\Gamma(v+\kappa)}{\Gamma(v)} = \begin{cases} 1, & (\kappa = 0; v \in \mathbb{C} \setminus \{0\}) \\ v(v+1) \cdots (v+n-1), & (\kappa = n \in \mathbb{N}; v \in \mathbb{C}), \end{cases} \quad (2)$$

provided that the gamma quotient exists.

The well known IGFs $\gamma(v, y)$ and $\Gamma(v, y)$ are given in subsequent manner,

$$\gamma(v, y) = \int_0^y u^{v-1} e^{-u} du, \quad (\Re(v) > 0; y \geq 0) \quad (3)$$

and

$$\Gamma(v, y) = \int_y^\infty u^{v-1} e^{-u} du, \quad (y \geq 0; \Re(v) > 0 \text{ when } y = 0), \quad (4)$$

respectively and hold the decomposition formula given below:

$$\gamma(v, y) + \Gamma(v, y) = \Gamma(v), \quad (\Re(v) > 0). \quad (5)$$

Recently, IHFs $\Gamma_{p,q}^{m,n}(z)$ and $\gamma_{p,q}^{m,n}(z)$ were defined by Srivastava et al. ([26] Equations (2.1)–(2.4)) as follows:

$$\begin{aligned} \Gamma_{p,q}^{m,n}(z) &= \Gamma_{p,q}^{m,n} \left[z \left| \begin{matrix} (e_1, E_1, y), (e_j, E_j)_{2,p} \\ (f_j, F_j)_{1,q} \end{matrix} \right. \right] \\ &= \Gamma_{p,q}^{m,n} \left[z \left| \begin{matrix} (e_1, E_1, y), (e_2, E_2), \dots, (e_p, E_p) \\ (f_1, F_1), (f_2, F_2), \dots, (f_q, F_q) \end{matrix} \right. \right] \\ &= \frac{1}{2\pi i} \int_{\mathfrak{L}} \mathbb{F}(\xi, y) z^{-\xi} d\xi, \end{aligned} \quad (6)$$

where

$$\mathbb{F}(\xi, y) = \frac{\Gamma(1 - e_1 - E_1 \xi, y) \prod_{j=1}^m \Gamma(f_j + F_j \xi) \prod_{j=2}^n \Gamma(1 - e_j - E_j \xi)}{\prod_{j=m+1}^q \Gamma(1 - f_j - F_j \xi) \prod_{j=n+1}^p \Gamma(e_j + E_j \xi)} \quad (7)$$

and

$$\begin{aligned}\gamma_{p,q}^{m,n}(z) &= \gamma_{p,q}^{m,n} \left[z \left| \begin{matrix} (e_1, E_1, y), (e_j, E_j)_{2,p} \\ (f_j, F_j)_{1,q} \end{matrix} \right. \right] \\ &= \gamma_{p,q}^{m,n} \left[z \left| \begin{matrix} (e_1, E_1, y), (e_2, E_2), \dots, (e_p, E_p) \\ (f_1, F_1), (f_2, F_2), \dots, (f_q, F_q) \end{matrix} \right. \right] \\ &= \frac{1}{2\pi i} \int_{\mathcal{L}} \mathbb{G}(\xi, y) z^{-\xi} d\xi,\end{aligned}\quad (8)$$

where

$$\mathbb{G}(\xi, y) = \frac{\gamma(1 - e_1 - E_1\xi, y) \prod_{j=1}^m \Gamma(f_j + F_j\xi) \prod_{j=2}^n \Gamma(1 - e_j - E_j\xi)}{\prod_{j=m+1}^q \Gamma(1 - f_j - F_j\xi) \prod_{j=n+1}^p \Gamma(e_j + E_j\xi)}.\quad (9)$$

The IHFs $\gamma_{p,q}^{m,n}(z)$ and $\gamma_{p,q}^{m,n}(z)$ exist for all $y \geq 0$ under the set of conditions as given below:

The contour \mathcal{L} in the complex ξ -plane extends from $\gamma - i\infty$ to $\gamma + i\infty$, $\gamma \in \mathbb{R}$, and poles of the gamma functions $\Gamma(1 - e_j - E_j\xi)$, $j = \overline{1, n}$ do not exactly match with the poles of the gamma functions $\Gamma(f_j + F_j\xi)$, $j = \overline{1, m}$. The parameters m, n, p, q are nonnegative integers satisfying $0 \leq n \leq p$, $0 \leq m \leq q$. The coefficients E_j ($j = 1, \dots, p$) and F_j ($j = 1, \dots, q$) are positive real numbers and e_j and f_j are complex. All poles of $\mathbb{F}(\xi, y)$ and $\mathbb{G}(\xi, y)$ are assumed to be simple, and the empty product is taken as unity.

The Mellin–Barnes contour integral in Equation (6) converges absolutely and defines an analytic function for $|\arg(z)| < \frac{1}{2}\mathfrak{H}\pi$, where

$$\mathfrak{H} = \sum_{j=1}^n E_j + \sum_{j=1}^m F_j - \sum_{j=n+1}^p E_j - \sum_{j=m+1}^q F_j > 0.\quad (10)$$

Remark 1. Numerous special functions can be obtained by specializing the parameters of IHFs; some of them can be found in the articles given by Bansal and Choi [22] and Bansal et al. [23].

The incomplete \overline{H} -functions $\overline{\gamma}_{p,q}^{m,n}(z)$ and $\overline{\Gamma}_{p,q}^{m,n}(z)$ were defined by Srivastava et al. ([26] p. 134, Equations (7.5)–(7.8)) as follows:

$$\begin{aligned}\overline{\Gamma}_{p,q}^{m,n}(z) &= \overline{\Gamma}_{p,q}^{m,n} \left[z \left| \begin{matrix} (g_1, G_1; \in_1: y), (g_j, G_j; \in_j)_{2,n}, (g_j, G_j)_{n+1,p} \\ (h_j, H_j)_{1,m}, (h_j, H_j; \mathcal{K}_j)_{m+1,q} \end{matrix} \right. \right] \\ &:= \frac{1}{2\pi i} \int_{\mathcal{L}} \overline{f}(\xi, y) z^{-\xi} d\xi,\end{aligned}\quad (11)$$

where

$$\overline{f}(\xi, y) = \frac{[\Gamma(1 - g_1 - G_1\xi, y)]^{\in_1} \prod_{j=1}^m \Gamma(h_j + H_j\xi) \prod_{j=2}^n [\Gamma(1 - g_j - G_j\xi)]^{\in_j}}{\prod_{j=m+1}^q [\Gamma(1 - h_j - H_j\xi)]^{\mathcal{K}_j} \prod_{j=n+1}^p \Gamma(g_j + G_j\xi)}\quad (12)$$

and

$$\begin{aligned}\bar{\gamma}_{p,q}^{m,n}(z) &= \bar{\gamma}_{p,q}^{m,n} \left[z \left| \begin{array}{c} (g_1, G_1; \in_1: y), (g_j, G_j; \in_j)_{2,n}, (g_j, G_j)_{n+1,p} \\ (h_j, H_j)_{1,m}, (h_j, H_j; \mathcal{K}_j)_{m+1,q} \end{array} \right. \right] \\ &= \frac{1}{2\pi i} \int_{\mathfrak{L}} \bar{F}(\xi, y) z^{-\xi} d\xi,\end{aligned}\quad (13)$$

where

$$\bar{F}(\xi, y) = \frac{[\gamma(1 - g_1 - G_1\xi, y)]^{\in_1} \prod_{j=1}^m \Gamma(h_j + H_j\xi) \prod_{j=2}^n [\Gamma(1 - g_j - G_j\xi)]^{\in_j}}{\prod_{j=m+1}^q [\Gamma(1 - h_j - H_j\xi)]^{\mathcal{K}_j} \prod_{j=n+1}^p \Gamma(g_j + G_j\xi)}.\quad (14)$$

The conditions of incomplete \bar{H} -functions $\bar{\gamma}_{p,q}^{m,n}(z)$ and $\bar{\Gamma}_{p,q}^{m,n}(z)$ are stated in the articles given by Srivasatava et al. [26] and Inayat-Hussain [30].

Remark 2. A variety of special functions can be obtained by specializing the parameters of incomplete \bar{H} -functions; some of them can be found in the articles given by Kumar et al. [27].

Next, we give two important result, which are used in our main results.

1. The Mellin transform of IHF ([26] p.122, Equation (3.4)) is defined as follows:

$$\mathfrak{M} \left\{ \bar{\gamma}_{p,q}^{m,n} \left[zt^\sigma \left| \begin{array}{c} (e_1, E_1, y), (e_j, E_j)_{2,p} \\ (f_j, F_j)_{1,q} \end{array} \right. \right]; \xi \right\} = \frac{z^{-\frac{\xi}{\sigma}}}{\sigma} \mathbb{F} \left(\frac{\xi}{\sigma}, y \right),\quad (15)$$

where $\mathbb{F} \left(\frac{\xi}{\sigma}, y \right)$ is given in Equation (7) and the conditions are given in [26].

2. The Mellin transform of incomplete \bar{H} -function ([26] p.122, Equation (3.4)) is defined as follows:

$$\mathfrak{M} \left\{ \bar{\Gamma}_{p,q}^{m,n} \left[zt^\mu \left| \begin{array}{c} (g_1, G_1; \in_1: y), (g_j, G_j; \in_j)_{2,n}, (g_j, G_j)_{n+1,p} \\ (h_j, H_j)_{1,m}, (h_j, H_j; \mathcal{K}_j)_{m+1,q} \end{array} \right. \right]; \xi \right\} = \frac{z^{-\frac{\xi}{\mu}}}{\mu} \bar{f} \left(\frac{\xi}{\mu}, y \right),\quad (16)$$

where $\bar{f} \left(\frac{\xi}{\mu}, y \right)$ is given in Equation (12) and the conditions are given in [26].

Suppose \mathfrak{A} denotes the space of all functions f which are well defined on $\mathbb{R} = [0, \infty)$ and satisfy

- (i) $f \in \wp(\mathbb{R})$;
- (ii) $\lim_{z \rightarrow \infty} \{z^\alpha f^k(z)\} = 0, \quad (\forall \alpha, k \text{ is a nonnegative integers});$ and
- (iii) $f(z) = O(1), \quad z \rightarrow 0.$

For correspondence to the space of good functions defined on the whole real line $(-\infty, \infty)$, see the work of Lighthill [31].

The Weyl fractional integral operator of order α [32] is expressed as

$$W^{-\alpha} \{f(z)\} = \frac{1}{\Gamma(\alpha)} \int_z^\infty (\tau - z)^{\alpha-1} f(\tau) d\tau, \quad (\Re(\alpha) > 0, f \in \mathfrak{A}).\quad (17)$$

It should also be noted that, in all the lemmas and theorems, the factors t^{-k} and $t^{-\lambda}$ are absorbed in f , but they are there to keep the formulas as graceful as possible.

3. Solution of Fredholm-Type Integral Equation Involving Incomplete H-Functions

In this part, we give the solution of Fredholm-type integral equation involving the Incomplete H-functions with the help of the Mellin transform technique and the well known Weyl fractional operator.

Lemma 1. Let

- (i) M, N, P, Q are nonnegative integers such that (s.t.) $1 \leq M \leq Q$ and $0 \leq N \leq P$.
- (ii) $\Re(\lambda - k) > 0; \Re \left[k + \sigma \left(\frac{f_j}{F_j} \right) \right] > 0 \quad (j = 1, \dots, M)$.
- (iii) $y \geq 0, \sigma > 0$ and $\lambda \in \mathbb{C}$.
- (iv) $|\arg(A)| < \frac{1}{2}\mathfrak{H}\pi$, where \mathfrak{H} is given by Equation (10).

Then,

$$\begin{aligned} W^{k-\lambda} \left\{ t^{-\lambda} \Gamma_{P,Q}^{M,N} \left[A \left(\frac{x}{t} \right)^\sigma \left| \begin{array}{c} (e_1, E_1, y), (e_j, E_j)_{2,P} \\ (f_j, F_j)_{1,Q} \end{array} \right. \right] \right\} \\ = t^{-k} \Gamma_{P+1,Q+1}^{M,N+1} \left[A \left(\frac{x}{t} \right)^\sigma \left| \begin{array}{c} (e_1, E_1, y), (1-k, \sigma), (e_j, E_j)_{2,P} \\ (f_j, F_j)_{1,Q}, (1-\lambda, \sigma) \end{array} \right. \right]. \end{aligned} \quad (18)$$

Proof. To prove the assertion in Equation (18), we first employ the Weyl fractional integral presented in Equation (17), express the contour integral form of the IHF given in Equation (6), and then change the order of integrals (under the stated permissible conditions). Next, evaluating the Weyl fractional integral of power function and then reinterpreting the resulting Mellin–Barnes contour integral in terms of IHF, we easily achieve the required result. \square

Lemma 2. Let

- (i) M, N, P, Q are nonnegative integers s.t. $1 \leq M \leq Q$ and $0 \leq N \leq P$.
- (ii) $\Re(\lambda - k) > 0; \Re \left[k + \sigma \left(\frac{f_j}{F_j} \right) \right] > 0 \quad (j = 1, \dots, M)$.
- (iii) $y \geq 0, \sigma > 0$ and $\lambda \in \mathbb{C}$.
- (iv) $|\arg(A)| < \frac{1}{2}\mathfrak{H}\pi$, where \mathfrak{H} is given by Equation (10).

Then,

$$\begin{aligned} W^{k-\lambda} \left\{ t^{-\lambda} \gamma_{P,Q}^{M,N} \left[A \left(\frac{x}{t} \right)^\sigma \left| \begin{array}{c} (e_1, E_1, y), (e_j, E_j)_{2,P} \\ (f_j, F_j)_{1,Q} \end{array} \right. \right] \right\} \\ = t^{-k} \gamma_{P+1,Q+1}^{M,N+1} \left[A \left(\frac{x}{t} \right)^\sigma \left| \begin{array}{c} (e_1, E_1, y), (1-k, \sigma), (e_j, E_j)_{2,P} \\ (f_j, F_j)_{1,Q}, (1-\lambda, \sigma) \end{array} \right. \right]. \end{aligned} \quad (19)$$

Proof. To prove the assertion in Equation (19), we first utilize the Weyl fractional integral written in Equation (17), express the contour integral form of the IHF given in Equation (8), and then change the order of integrals (under the stated permissible conditions). Next, evaluating the Weyl fractional integral of power function and then reinterpreting the resulting Mellin–Barnes contour integral in terms of IHF, we easily achieve the required result. \square

Theorem 1. Let

- (i) M, N, P, Q are nonnegative integers s.t. $1 \leq M \leq Q$ and $0 \leq N \leq P$.

- (ii) $\Re \left(k + \sigma \left(\frac{e_j - 1}{E_j} \right) \right) < 0 \quad (j = 1, \dots, N); \Re \left[k + \sigma \left(\frac{f_j}{F_j} \right) \right] > 0 \quad (j = 1, \dots, M).$
 (iii) $y \geq 0, \sigma > 0, \text{ and } \lambda \in \mathbb{C}.$

Then, the following integral relation formula holds:

$$\begin{aligned} & \int_0^\infty t^{-k} \Gamma_{P+1, Q+1}^{M, N+1} \left[A \left(\frac{x}{t} \right)^\sigma \left| \begin{array}{c} (e_1, E_1, y), (1-k, \sigma), (e_j, E_j)_{2, P} \\ (f_j, F_j)_{1, Q}, (1-\lambda, \sigma) \end{array} \right. \right] f(t) dt \\ &= \int_0^\infty t^{-\lambda} \Gamma_{P, Q}^{M, N} \left[A \left(\frac{x}{t} \right)^\sigma \left| \begin{array}{c} (e_1, E_1, y), (e_j, E_j)_{2, P} \\ (f_j, F_j)_{1, Q} \end{array} \right. \right] D^{k-\lambda} \{f(t)\} dt. \end{aligned} \quad (20)$$

provided that $f \in \mathfrak{A}$ and $x > 0$.

Proof. Let Ω indicate the first member of the assertion in Equation (20) of Theorem 1. Then, by Lemma 1 and the definition in Equation (17), we have

$$\Omega = \int_0^\infty f(t) \left(\int_t^\infty \frac{(u-t)^{\lambda-k-1}}{\Gamma(\lambda-k)} u^{-\lambda} \Gamma_{P, Q}^{M, N} \left[A \left(\frac{x}{u} \right)^\sigma \left| \begin{array}{c} (e_1, E_1, y), (e_j, E_j)_{2, P} \\ (f_j, F_j)_{1, Q} \end{array} \right. \right] du \right) dt.$$

Next, changing the order of integration under the permissible conditions, we get

$$\Omega = \int_0^\infty u^{-\lambda} \Gamma_{P, Q}^{M, N} \left[A \left(\frac{x}{u} \right)^\sigma \left| \begin{array}{c} (e_1, E_1, y), (e_j, E_j)_{2, P} \\ (f_j, F_j)_{1, Q} \end{array} \right. \right] \left(\int_0^u \frac{(u-t)^{\lambda-k-1}}{\Gamma(\lambda-k)} f(t) dt \right) du.$$

Further, with the help of well known definition of Riemann–Liouville (RL) fractional derivative operator, we obtain

$$\Omega = \int_0^\infty u^{-\lambda} \Gamma_{P, Q}^{M, N} \left[A \left(\frac{x}{u} \right)^\sigma \left| \begin{array}{c} (e_1, E_1, y), (e_j, E_j)_{2, P} \\ (f_j, F_j)_{1, Q} \end{array} \right. \right] D^{k-\lambda} \{f(u)\} du,$$

which is the right-hand member of Equation (20). \square

Theorem 2. Let

- (i) M, N, P, Q are nonnegative integers s.t. $1 \leq M \leq Q$ and $0 \leq N \leq P$.
 (ii) $\Re \left(k + \sigma \left(\frac{e_j - 1}{E_j} \right) \right) < 0 \quad (j = 1, \dots, N); \Re \left[k + \sigma \left(\frac{f_j}{F_j} \right) \right] > 0 \quad (j = 1, \dots, M).$
 (iii) $y \geq 0, \sigma > 0, \text{ and } \lambda \in \mathbb{C}.$

Then, the following integral relation formula holds:

$$\begin{aligned} & \int_0^\infty t^{-k} \gamma_{P+1, Q+1}^{M, N+1} \left[A \left(\frac{x}{t} \right)^\sigma \left| \begin{array}{c} (e_1, E_1, y), (1-k, \sigma), (e_j, E_j)_{2, P} \\ (f_j, F_j)_{1, Q}, (1-\lambda, \sigma) \end{array} \right. \right] f(t) dt \\ &= \int_0^\infty t^{-\lambda} \gamma_{P, Q}^{M, N} \left[A \left(\frac{x}{t} \right)^\sigma \left| \begin{array}{c} (e_1, E_1, y), (e_j, E_j)_{2, P} \\ (f_j, F_j)_{1, Q} \end{array} \right. \right] D^{k-\lambda} \{f(t)\} dt. \end{aligned} \quad (21)$$

provided that $f \in \mathfrak{A}$ and $x > 0$.

Proof. We here omit the details since the proof is parallel to that of Theorem 1. \square

Theorem 3. Let

- (i) M, N, P, Q are nonnegative integers s.t. $1 \leq M \leq Q$ and $0 \leq N \leq P$.
- (ii) $\Re\left(\lambda + \sigma\left(\frac{e_j - 1}{E_j}\right)\right) < 0 \quad (j = 1, \dots, N); \Re\left[\lambda + \sigma\left(\frac{f_j}{F_j}\right)\right] > 0 \quad (j = 1, \dots, M).$
- (iii) $-\min_{1 \leq j \leq M}\left(\frac{\Re(f_j)}{F_j}\right) < \Re(\xi) < \min_{1 \leq j \leq N}\left(\frac{\Re(1 - e_j)}{E_j}\right).$
- (iv) $y \geq 0, \sigma > 0, f, g \in \mathfrak{A}$ and $\lambda \in \mathbb{C}.$

Then, the following integral equation

$$\int_0^\infty t^{-\lambda} \Gamma_{P,Q}^{M,N} \left[A\left(\frac{x}{t}\right)^\sigma \left| \begin{array}{c} (e_1, E_1, y), (e_j, E_j)_{2,P} \\ (f_j, F_j)_{1,Q} \end{array} \right. \right] f(t) dt = g(x) \quad (22)$$

has a solution given by

$$f(x) = \frac{\sigma x^{\lambda-1}}{2\pi i} \int_{\mathcal{L}} \left[\mathbb{F}\left(\frac{\xi}{\sigma}, y\right) \right]^{-1} x^{-\xi} \varphi(\xi) d\xi, \quad (23)$$

where

$$\varphi(\xi) = \int_0^\infty x^{\xi-1} g(x) dx$$

and $\mathbb{F}(\xi, y)$ is given in Equation (7).

Proof. To find the solution of integral equation in Equation (22), f is replaced by $D^{\lambda-k} f$ in Equation (20); we have

$$\int_0^\infty t^{-k} \Gamma_{P+1,Q+1}^{M,N+1} \left[A\left(\frac{x}{t}\right)^\sigma \left| \begin{array}{c} (e_1, E_1, y), (1-k, \sigma), (e_j, E_j)_{2,P} \\ (f_j, F_j)_{1,Q}, (1-\lambda, \sigma) \end{array} \right. \right] D^{\lambda-k} \{f(t)\} dt = g(x).$$

Multiplying both sides by $x^{\xi-1}$ and integrating with respect to x from 0 to ∞ then changing the order of integration under the permissible conditions, we have

$$\begin{aligned} \varphi(\xi) &= \int_0^\infty x^{\xi-1} g(x) dx = \int_0^\infty t^{-k} D^{\lambda-k} \{f(t)\} \\ &\quad \cdot \left(\int_0^\infty x^{\xi-1} \Gamma_{P+1,Q+1}^{M,N+1} \left[A\left(\frac{x}{t}\right)^\sigma \left| \begin{array}{c} (e_1, E_1, y), (1-k, \sigma), (e_j, E_j)_{2,P} \\ (f_j, F_j)_{1,Q}, (1-\lambda, \sigma) \end{array} \right. \right] dx \right) dt. \end{aligned}$$

Now, with the help of Equation (15), we get

$$\varphi(\xi) = \frac{\Gamma(k-\xi)}{\sigma \Gamma(\lambda-\xi)} A^{-\frac{\xi}{\sigma}} \mathbb{F}\left(\frac{\xi}{\sigma}, y\right) \int_0^\infty t^{\xi-k} D^{\lambda-k} \{f(t)\} dt.$$

Further, applying the well known Mellin inversion theorem, we get

$$D^{\lambda-k} \{f(t)\} = \frac{\sigma}{2\pi i} \int_{\mathcal{L}} t^{k-\xi-1} \frac{\Gamma(\lambda-\xi)}{\Gamma(k-\xi)} A^{\frac{\xi}{\sigma}} \left[\mathbb{F}\left(\frac{\xi}{\sigma}, y\right) \right]^{-1} \varphi(\xi) d\xi.$$

Next, operating both sides by $D^{k-\lambda}$, we have

$$f(t) = \frac{\sigma}{2\pi i} D^{k-\lambda} \left\{ \int_{\mathcal{L}} t^{k-\xi-1} \frac{\Gamma(\lambda-\xi)}{\Gamma(k-\xi)} A^{\frac{\xi}{\sigma}} \left[\mathbb{F} \left(\frac{\xi}{\sigma}, y \right) \right]^{-1} \varphi(\xi) d\xi \right\},$$

which finally yields

$$f(x) = \frac{\sigma x^{\lambda-1}}{2\pi i} \int_{\mathcal{L}} \left[\mathbb{F} \left(\frac{\xi}{\sigma}, y \right) \right]^{-1} x^{-\xi} \varphi(\xi) d\xi.$$

□

Theorem 4. Let

- (i) M, N, P, Q are nonnegative integers s.t. $1 \leq M \leq Q$ and $0 \leq N \leq P$.
- (ii) $\Re \left(\lambda + \sigma \left(\frac{e_j - 1}{E_j} \right) \right) < 0 \quad (j = 1, \dots, N); \Re \left[\lambda + \sigma \left(\frac{f_j}{F_j} \right) \right] > 0 \quad (j = 1, \dots, M).$
- (iii) $-\min_{1 \leq j \leq M} \left(\frac{\Re(f_j)}{F_j} \right) < \Re(\xi) < \min_{1 \leq j \leq N} \left(\frac{\Re(1-e_j)}{E_j} \right).$
- (iv) $y \geq 0, \sigma > 0, f, g \in \mathfrak{A}$ and $\lambda \in \mathbb{C}$.

Then, the following integral equation

$$\int_0^\infty t^{-\lambda} \gamma_{P,Q}^{M,N} \left[A \left(\frac{x}{t} \right)^\sigma \left| \begin{array}{c} (e_1, E_1, y), (e_j, E_j)_{2,P} \\ (f_j, F_j)_{1,Q} \end{array} \right. \right] f(t) dt = g(x) \quad (24)$$

has a solution given by

$$f(x) = \frac{\sigma x^{\lambda-1}}{2\pi i} \int_{\mathcal{L}} \left[\mathbb{G} \left(\frac{\xi}{\sigma}, y \right) \right]^{-1} x^{-\xi} \varphi(\xi) d\xi, \quad (25)$$

where

$$\varphi(\xi) = \int_0^\infty x^{\xi-1} g(x) dx$$

and $\mathbb{G}(\xi, y)$ is given in Equation (9).

Proof. We here omit the details since the proof is parallel to that of Theorem 3. □

4. Solution of Fredholm-Type Integral Equation Involving Incomplete \overline{H} -Function

In this portion, we give the solution of Fredholm-type integral equation involving the Incomplete \overline{H} -functions with the help of Mellin transform technique and well known Weyl fractional operator.

Lemma 3. Let

- (i) m, n, p, q are nonnegative integers s.t. $1 \leq m \leq q$ and $0 \leq n \leq p$.
- (ii) $\Re(\lambda - k) > 0; \Re \left[k + \sigma \left(\frac{h_j}{H_j} \right) \right] > 0 \quad (j = 1, \dots, m).$
- (iii) $y \geq 0, \sigma > 0$ and $\lambda \in \mathbb{C}$.

Then,

$$\begin{aligned}
 W^{k-\lambda} & \left\{ t^{-\lambda} \bar{\Gamma}_{p,q}^{m,n} \left[A \left(\frac{x}{t} \right)^\sigma \left| \begin{array}{c} (g_1, G_1; \in_1: y), (g_j, G_j; \in_j)_{2,n}, (g_j, G_j)_{n+1,p} \\ (h_j, H_j)_{1,m}, (h_j, H_j; \mathcal{K}_j)_{m+1,q} \end{array} \right. \right] \right\} \\
 & = t^{-k} \bar{\Gamma}_{p+1,q+1}^{m,n+1} \left[A \left(\frac{x}{t} \right)^\sigma \left| \begin{array}{c} (g_1, G_1; \in_1: y), (1-k, \sigma; 1), (g_j, G_j; \in_j)_{2,n}, (g_j, G_j)_{n+1,p} \\ (h_j, H_j)_{1,m}, (1-\lambda, \sigma; 1), (h_j, H_j; \mathcal{K}_j)_{m+1,q} \end{array} \right. \right]. \quad (26)
 \end{aligned}$$

Proof. To prove the assertion in Equation (26), we first employ the Weyl fractional integral written in Equation (17), express the contour integral form of the incomplete \bar{H} -function given in Equation (11), and then change the order of integrals (under the stated permissible conditions). Next, evaluating the Weyl fractional integral of power function and then reinterpreting the resulting Mellin–Barnes contour integral in terms of incomplete \bar{H} -function, we easily achieve the required result. \square

Lemma 4. Let

- (i) m, n, p, q are nonnegative integers s.t. $1 \leq m \leq q$ and $0 \leq n \leq p$.
- (ii) $\Re(\lambda - k) > 0; \Re \left[k + \sigma \left(\frac{h_j}{H_j} \right) \right] > 0 \quad (j = 1, \dots, m)$.
- (iii) $y \geq 0, \sigma > 0$ and $\lambda \in \mathbb{C}$.

Then,

$$\begin{aligned}
 W^{k-\lambda} & \left\{ t^{-\lambda} \bar{\gamma}_{p,q}^{m,n} \left[A \left(\frac{x}{t} \right)^\sigma \left| \begin{array}{c} (g_1, G_1; \in_1: y), (g_j, G_j; \in_j)_{2,n}, (g_j, G_j)_{n+1,p} \\ (h_j, H_j)_{1,m}, (h_j, H_j; \mathcal{K}_j)_{m+1,q} \end{array} \right. \right] \right\} \\
 & = t^{-k} \bar{\gamma}_{p+1,q+1}^{m,n+1} \left[A \left(\frac{x}{t} \right)^\sigma \left| \begin{array}{c} (g_1, G_1; \in_1: y), (1-k, \sigma; 1), (g_j, G_j; \in_j)_{2,n}, (g_j, G_j)_{n+1,p} \\ (h_j, H_j)_{1,m}, (1-\lambda, \sigma; 1), (h_j, H_j; \mathcal{K}_j)_{m+1,q} \end{array} \right. \right]. \quad (27)
 \end{aligned}$$

Proof. To prove the assertion in Equation (26), we first employ the definition of Weyl fractional integral expressed in Equation (17), express the contour integral form of the incomplete \bar{H} -function given in Equation (13), and then change the order of integrals (under the stated permissible conditions). Next, evaluating the Weyl fractional integral of power function and then reinterpreting the resulting Mellin–Barnes contour integral in terms of incomplete \bar{H} -function, we easily achieve the needed result. \square

Theorem 5. Let

- (i) m, n, p, q are nonnegative integers s.t. $1 \leq m \leq q$ and $0 \leq n \leq p$.
- (ii) $\Re \left(k + \sigma \left(\frac{g_j-1}{G_j} \right) \right) < 0 \quad (j = 1, \dots, n); \Re \left[k + \sigma \left(\frac{h_j}{H_j} \right) \right] > 0 \quad (j = 1, \dots, m)$.
- (iii) $y \geq 0, \sigma > 0$ and $\lambda \in \mathbb{C}$.

Then, the following integral relation formula holds:

$$\begin{aligned}
 & \int_0^\infty t^{-k} \bar{\Gamma}_{p+1,q+1}^{m,n+1} \left[A \left(\frac{x}{t} \right)^\sigma \left| \begin{array}{c} (g_1, G_1; \in_1: y), (1-k, \sigma; 1), (g_j, G_j; \in_j)_{2,n}, (g_j, G_j)_{n+1,p} \\ (h_j, H_j)_{1,m}, (1-\lambda, \sigma; 1), (h_j, H_j; \mathcal{K}_j)_{m+1,q} \end{array} \right. \right] f(t) dt \\
 & = \int_0^\infty t^{-\lambda} \bar{\Gamma}_{p,q}^{m,n} \left[A \left(\frac{x}{t} \right)^\sigma \left| \begin{array}{c} (g_1, G_1; \in_1: y), (g_j, G_j; \in_j)_{2,n}, (g_j, G_j)_{n+1,p} \\ (h_j, H_j)_{1,m}, (h_j, H_j; \mathcal{K}_j)_{m+1,q} \end{array} \right. \right] D^{k-\lambda} \{f(t)\} dt \quad (28)
 \end{aligned}$$

provided $f \in \mathfrak{A}$ and $x > 0$.

Proof. Let $\overline{\Omega}$ indicate the first member of the assertion in Equation (28) of Theorem 5. Then, by Lemma 3 and the definition in Equation (17), we have

$$\Omega = \int_0^\infty f(t) \int_t^\infty \frac{(u-t)^{\lambda-k-1}}{\Gamma(\lambda-k)} u^{-\lambda} \times \overline{\Gamma}_{p,q}^{m,n} \left[A\left(\frac{x}{u}\right)^\sigma \left| \begin{array}{c} (g_1, G_1; \in_1: y), (g_j, G_j; \in_j)_{2,n}, (g_j, G_j)_{n+1,p} \\ (h_j, H_j)_{1,m}, (h_j, H_j; \mathcal{K}_j)_{m+1,q} \end{array} \right. \right] du dt.$$

Next, changing the order of integration under the permissible conditions, we achieve the result below:

$$\Omega = \int_0^\infty u^{-\lambda} \overline{\Gamma}_{p,q}^{m,n} \left[A\left(\frac{x}{u}\right)^\sigma \left| \begin{array}{c} (g_1, G_1; \in_1: y), (g_j, G_j; \in_j)_{2,n}, (g_j, G_j)_{n+1,p} \\ (h_j, H_j)_{1,m}, (h_j, H_j; \mathcal{K}_j)_{m+1,q} \end{array} \right. \right] \times \left(\int_0^u \frac{(u-t)^{\lambda-k-1}}{\Gamma(\lambda-k)} f(t) dt \right) du.$$

Further, with the help of well known definition of RL fractional derivative operator, we obtain

$$\Omega = \int_0^\infty u^{-\lambda} \overline{\Gamma}_{p,q}^{m,n} \left[A\left(\frac{x}{u}\right)^\sigma \left| \begin{array}{c} (g_1, G_1; \in_1: y), (g_j, G_j; \in_j)_{2,n}, (g_j, G_j)_{n+1,p} \\ (h_j, H_j)_{1,m}, (h_j, H_j; \mathcal{K}_j)_{m+1,q} \end{array} \right. \right] D^{k-\lambda} \{f(u)\} du.$$

which is the right-hand member of Equation (20). \square

Theorem 6. Let

- (i) m, n, p, q are nonnegative integers s.t. $1 \leq m \leq q$ and $0 \leq n \leq p$.
- (ii) $\Re\left(k + \sigma\left(\frac{g_j-1}{G_j}\right)\right) < 0$ ($j = 1, \dots, n$); $\Re\left[k + \sigma\left(\frac{h_j}{H_j}\right)\right] > 0$ ($j = 1, \dots, m$).
- (iii) $y \geq 0$, $\sigma > 0$ and $\lambda \in \mathbb{C}$.

Then, the following integral relation formula holds:

$$\begin{aligned} & \int_0^\infty t^{-k} \overline{\gamma}_{p+1,q+1}^{m,n+1} \left[A\left(\frac{x}{t}\right)^\sigma \left| \begin{array}{c} (g_1, G_1; \in_1: y), (1-k, \sigma; 1), (g_j, G_j; \in_j)_{2,n}, (g_j, G_j)_{n+1,p} \\ (h_j, H_j)_{1,m}, (1-\lambda, \sigma; 1), (h_j, H_j; \mathcal{K}_j)_{m+1,q} \end{array} \right. \right] f(t) dt \\ &= \int_0^\infty t^{-\lambda} \overline{\gamma}_{p,q}^{m,n} \left[A\left(\frac{x}{t}\right)^\sigma \left| \begin{array}{c} (g_1, G_1; \in_1: y), (g_j, G_j; \in_j)_{2,n}, (g_j, G_j)_{n+1,p} \\ (h_j, H_j)_{1,m}, (h_j, H_j; \mathcal{K}_j)_{m+1,q} \end{array} \right. \right] D^{k-\lambda} \{f(t)\} dt. \quad (29) \end{aligned}$$

provided $f \in \mathfrak{A}$ and $x > 0$.

Proof. We here omit the details since the proof is parallel to that of Theorem 5. \square

Theorem 7. Let

- (i) m, n, p, q are nonnegative integers s.t. $1 \leq m \leq q$ and $0 \leq n \leq p$.
- (ii) $\Re\left(k + \sigma\left(\frac{g_j-1}{G_j}\right)\right) < 0$ ($j = 1, \dots, n$); $\Re\left[k + \sigma\left(\frac{h_j}{H_j}\right)\right] > 0$ ($j = 1, \dots, m$).

$$(iii) - \min_{1 \leq j \leq m} \left(\frac{\Re(h_j)}{H_j} \right) < \Re(\xi) < \min_{1 \leq j \leq n} \left(\frac{\Re(1-g_j)}{G_j} \right).$$

$$(iv) \ y \geq 0, \ \sigma > 0, \lambda \in \mathbb{C} \text{ and } f, g \in \mathfrak{A}.$$

Then, the following integral equation

$$\int_0^\infty t^{-\lambda} \bar{\Gamma}_{p,q}^{m,n} \left[A \left(\frac{x}{t} \right)^\sigma \left| \begin{matrix} (g_1, G_1; \in_1: y), (g_j, G_j; \in_j)_{2,n}, (g_j, G_j)_{n+1,p} \\ (h_j, H_j)_{1,m}, (h_j, H_j; \mathcal{K}_j)_{m+1,q} \end{matrix} \right. \right] f(t) dt = g(x) \quad (30)$$

has a solution given by

$$f(x) = \frac{\sigma x^{\lambda-1}}{2\pi i} \int_{\mathcal{L}} \left[\bar{f} \left(\frac{\xi}{\sigma}, y \right) \right]^{-1} x^{-\xi} \varphi(\xi) d\xi, \quad (31)$$

where

$$\varphi(\xi) = \int_0^\infty x^{\xi-1} g(x) dx$$

and $\bar{f}(\xi, y)$ is given in Equation (12).

Proof. To determine the solution of the integral equation in Equation (30), f is replaced by $D^{\lambda-k} f$ in Equation (28); we have

$$\int_0^\infty t^{-k} \bar{\Gamma}_{p+1,q+1}^{m,n+1} \left[A \left(\frac{x}{t} \right)^\sigma \left| \begin{matrix} (g_1, G_1; \in_1: y), (1-k, \sigma; 1), (g_j, G_j; \in_j)_{2,n}, (g_j, G_j)_{n+1,p} \\ (h_j, H_j)_{1,m}, (1-\lambda, \sigma; 1), (h_j, H_j; \mathcal{K}_j)_{m+1,q} \end{matrix} \right. \right] \times D^{\lambda-k} \{f(t)\} dt = g(x).$$

Multiplying both sides by $x^{\xi-1}$ and integrating with respect to x from 0 to ∞ then changing the order of integration under the permissible conditions, we have

$$\varphi(\xi) = \int_0^\infty x^{\xi-1} g(x) dx = \int_0^\infty t^{-k} D^{\lambda-k} \{f(t)\} \cdot \left(\int_0^\infty x^{\xi-1} \bar{\Gamma}_{p+1,q+1}^{m,n+1} \left[A \left(\frac{x}{t} \right)^\sigma \left| \begin{matrix} (g_1, G_1; \in_1: y), (1-k, \sigma; 1), (g_j, G_j; \in_j)_{2,n}, (g_j, G_j)_{n+1,p} \\ (h_j, H_j)_{1,m}, (1-\lambda, \sigma; 1), (h_j, H_j; \mathcal{K}_j)_{m+1,q} \end{matrix} \right. \right] dx \right) dt.$$

Now, with the help of Equation (16), we get

$$\varphi(\xi) = \frac{\Gamma(k-\xi)}{\sigma \Gamma(\lambda-\xi)} A^{-\frac{\xi}{\sigma}} \bar{f} \left(\frac{\xi}{\sigma}, y \right) \int_0^\infty t^{\xi-k} D^{\lambda-k} \{f(t)\} dt.$$

Further, applying the well known Mellin inversion theorem, we get

$$D^{\lambda-k} \{f(t)\} = \frac{\sigma}{2\pi i} \int_{\mathcal{L}} t^{k-\xi-1} \frac{\Gamma(\lambda-\xi)}{\Gamma(k-\xi)} A^{\frac{\xi}{\sigma}} \left[\bar{f} \left(\frac{\xi}{\sigma}, y \right) \right]^{-1} \varphi(\xi) d\xi$$

Next, operating both sides by $D^{k-\lambda}$, we have

$$f(t) = \frac{\sigma}{2\pi i} D^{k-\lambda} \left\{ \int_{\mathcal{L}} t^{k-\xi-1} \frac{\Gamma(\lambda-\xi)}{\Gamma(k-\xi)} A^{\frac{\xi}{\sigma}} \left[\bar{f} \left(\frac{\xi}{\sigma}, y \right) \right]^{-1} \varphi(\xi) d\xi \right\},$$

which finally yields

$$f(x) = \frac{\sigma x^{\lambda-1}}{2\pi i} \int_{\mathcal{L}} \left[\bar{f} \left(\frac{\xi}{\sigma}, y \right) \right]^{-1} x^{-\xi} \varphi(\xi) d\xi.$$

□

Theorem 8. Let

- (i) m, n, p, q are nonnegative integers s.t. $1 \leq m \leq q$ and $0 \leq n \leq p$.
- (ii) $\Re \left(k + \sigma \left(\frac{g_j - 1}{G_j} \right) \right) < 0 \quad (j = 1, \dots, n); \Re \left[k + \sigma \left(\frac{h_j}{H_j} \right) \right] > 0 \quad (j = 1, \dots, m).$
- (iii) $-\min_{1 \leq j \leq m} \left(\frac{\Re(h_j)}{H_j} \right) < \Re(\xi) < \min_{1 \leq j \leq n} \left(\frac{\Re(1-g_j)}{G_j} \right).$
- (iv) $y \geq 0, \sigma > 0, \lambda \in \mathbb{C}$ and $f, g \in \mathfrak{A}.$

Then, the following integral equation

$$\int_0^\infty t^{-\lambda} \bar{\gamma}_{p,q}^{m,n} \left[A \left(\frac{x}{t} \right)^\sigma \begin{matrix} (g_1, G_1; \in_1: y), (g_j, G_j; \in_j)_{2,n}, (g_j, G_j)_{n+1,p} \\ (h_j, H_j)_{1,m}, (h_j, H_j; \mathcal{K}_j)_{m+1,q} \end{matrix} \right] f(t) dt = g(x) \quad (32)$$

has a solution given by

$$f(x) = \frac{\sigma x^{\lambda-1}}{2\pi i} \int_{\mathcal{L}} \left[\bar{F} \left(\frac{\xi}{\sigma}, y \right) \right]^{-1} x^{-\xi} \varphi(\xi) d\xi, \quad (33)$$

where

$$\varphi(\xi) = \int_0^\infty x^{\xi-1} g(x) dx$$

and $\bar{F}(\xi, y)$ is given in Equation (14).

Proof. We here omit the details since the proof is parallel to that of Theorem 7. □

5. Applications

In this section, we present some already existing results which directly follow from our main result. Next, we establish a few interesting results which are obtained from our main result by specializing the parameters of incomplete H-functions and incomplete \bar{H} -functions.

- (i) If we reduce IHF into Fox H-function in the assertions in Equations (20) and (22), then the result obtained by Srivastava and Raina follows [18].
- (ii) If IHF is reduced into the Gauss hypergeometric function (${}_2F_1$) in Equations (20) and (22), then the result collected by Prabhakar and Kashyap follows [17].
- (iii) In Theorems 3 and 4, if IHFs are reduced into the incomplete Fox–Wright Ψ -functions ${}_p\Psi_Q^{(\Gamma)}$ and ${}_p\Psi_Q^{(\gamma)}$ (for details, see ([22] p.3, Equation (15))), then we get the solution of Fredholm integral equation involving the incomplete Fox–Wright Ψ -function ${}_p\Psi_Q^{(\Gamma)}$ and ${}_p\Psi_Q^{(\gamma)}$.
- (iv) In Theorems 7 and 8, if IHFs are reduced into incomplete Fox–Wright $\bar{\Psi}$ -functions ${}_p\bar{\Psi}_Q^{(\Gamma)}$ and ${}_p\bar{\Psi}_Q^{(\gamma)}$ (for details, see ([27] p.1216, Equation (1.7))), then we get the solution of Fredholm integral equation involving the incomplete Fox–Wright $\bar{\Psi}$ -function ${}_p\bar{\Psi}_Q^{(\Gamma)}$ and ${}_p\bar{\Psi}_Q^{(\gamma)}$.

Our main results are general in nature; thus, by specializing the parameters of incomplete H-functions and incomplete \bar{H} -functions, we obtain many Fredholm-type integral equations involving

different types of functions in the kernel having various applications in fields including fluid mechanics, mathematical physics, quantum mechanics, queuing theory, etc. (for details, see [1–11]).

6. Conclusions

Our derived results are very important in diverse fields. We can obtain many results by setting specific values of different parameters of IHFs and incomplete \overline{H} -functions. In this work, we solve the Fredholm-type integral equation involving the incomplete H-function and incomplete \overline{H} -function in the kernel. Next, we obtain the Melin transform of incomplete \overline{H} -function. Further, we indicate some known results by setting specific values of different parameters of incomplete H-function and incomplete \overline{H} -function. Thus, the results determined in this article would at once give numerous results involving a large variety of special functions appearing in different problems of science and engineering fields.

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