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A Novel Learning-Based Binarization Scheme Selector for Swarm Algorithms Solving Combinatorial Problems

José Lemus-Romani [1,*], Marcelo Becerra-Rozas [2], Broderick Crawford [2], Ricardo Soto [2], Felipe Cisternas-Caneo [2], Emanuel Vega [2], Mauricio Castillo [2], Diego Tapia [2], Gino Astorga [3], Wenceslao Palma [2], Carlos Castro [4] and José García [5]

1 Escuela de Construcción Civil, Pontificia Universidad Católica de Chile, Avenida Vicuña Mackenna 4860, Santiago 7820436, Chile
2 Escuela de Ingeniería Informática, Pontificia Universidad Católica de Valparaíso, Avenida Brasil 2241, Valparaíso 2362807, Chile; marcelo.becerra.r@mail.pucv.cl (M.B.-R.); broderick.crawford@pucv.cl (B.C.); ricardo.soto@pucv.cl (R.S.); felipe.cisternas.c@mail.pucv.cl (F.C.-C.); emanuel.vega.m@mail.pucv.cl (E.V.); mauricio.castillo.d@mail.pucv.cl (M.C.); diego.tapia.r@mail.pucv.cl (D.T.); wenceslao.palma@pucv.cl (W.P.)
3 Escuela de Negocios Internacionales, Universidad de Valparaíso, Alcalde Prieto Nieto 452, Viña del Mar, Valparaíso 2572048, Chile; gino.astorga@uv.cl
4 Departamento de Informática, Universidad Técnica Federico Santa María, Avenida España 1680, Valparaíso 2390123, Chile; Carlos.Castro@inf.utfsm.cl
5 Escuela de Ingeniería en Construcción, Pontificia Universidad Católica de Valparaíso, Avenida Brasil 2147, Valparaíso 2362804, Chile; jose.garcia@pucv.cl
* Correspondence: jose.lemus@uc.cl

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Abstract: Currently, industry is undergoing an exponential increase in binary-based combinatorial problems. In this regard, metaheuristics have been a common trend in the field in order to design approaches to successfully solve them. Thus, a well-known strategy includes the employment of continuous swarm-based algorithms transformed to perform in binary environments. In this work, we propose a hybrid approach that contains discrete smartly adapted population-based strategies to efficiently tackle binary-based problems. The proposed approach employs a reinforcement learning technique, known as SARSA (State–Action–Reward–State–Action), in order to utilize knowledge based on the run time. In order to test the viability and competitiveness of our proposal, we compare discrete state-of-the-art algorithms smartly assisted by SARSA. Finally, we illustrate interesting results where the proposed hybrid outperforms other approaches, thus, providing a novel option to tackle these types of problems in industry.

Keywords: combinatorial problems; metaheuristics; binarization scheme; SARSA; Q-learning; machine learning; discretization methods

1. Introduction

High complexity problems in binary domains are a common sight in industry, along with high digitalization and the incorporation of artificial intelligence. Among well-known problems with great complexity, we can find the Set Covering Problem (SCP) [1], Knapsack Problem [2], Set-Union Knapsack Problem [3], and Feature Selection [4]. In order to solve these kinds of problems, the employment of exact methods can be unmanageable within restricted resources, such as computational time. Thus, approximate methods, such as metaheuristics (MH), which do not guarantee the optimality, but do obtain solutions as close as possible to the optimal in a reasonable computational time, have been a recurrent answer from the scientific community.

In the literature, there exists designed MH capable to address them without the need for modifications. However, it has been demonstrated that MH designed to work in continuous domains assisted by a discretization scheme outperforms the classic binary-based approaches [5]. The classic design includes the transformation of domains through the two-step techniques. However, novel learning-based hybrids have been reported, which focus on improvements in the transformation process [2,6].

Hybrid methods have been designed as novel approaches that use multiple optimization tools. They have been a hot topic in the field, and several improvements over MH have been reported. Among the most relevant lines of research in the literature, four can be clearly distinguished, such as MH with "Mathematical Programming" [7]; hybridization between MH [8]; "Simheuristics", which interrelates MH with the simulation of problems [9]; and MH with Machine Learning (ML) [10–12].

In this work, we propose a hybrid approach composed of MH and ML, which includes continuous-based population algorithms supported by a learning-based binarization scheme. The novelty in the proposition concerns a multiple binarization scheme being balanced by a Reinforcement Learning (RL) technique, named SARSA, which is based on the run time. The main idea is to provide an adaptive binary-selector mechanism based on the knowledge generated by the processed dynamic data generated through the search, such as the diversity of solutions.

In RL approaches, the employment and management of rewards are well-known. In this work, five different types are considered in the reward system implemented: the global best, with a penalty, root adaptation, without penalty, and escalating adaptation. Regarding the population-based algorithms, in this work, the Sine–Cosine Algorithm (SCA), Harris Hawk Optimization (HHO), Whale Optimization Problem (WOA), and Grey Wolf Optimizer (GWO) are employed. This complete set of components profiting from the data generated on the run time by population-based algorithms motivated the challenge of proposing a learning-based approach with the capability to self-adapt and improve through the search.

In order to prove the competitiveness of the proposed hybrid algorithm, experimentation tests were carried out against multiple state-of-the-art binarization strategies solving the SCP. Lastly, we highlight the good performance illustrated by the proposed approach proving to be a good alternative to solving binary optimization problems.

The rest of this paper is organized as follows. In Section 2, we present a detailed description of all the implemented population-based MH, the state-of-the-art binarization scheme, how MH has been supported by ML, and the optimization problem tackled. The proposed hybrid is illustrated in Section 3, where we describe the designed learning model and the details employing SARSA with the reward system. Section 4 presents the results obtained together with their respective tables and figures. Finally, a proper analysis and discussions are illustrated in Section 5, followed by our conclusions and future lines of work.

2. Related Work

In this section, we present all the required concepts related to the proposal in order to understand the ideas and objectives behind the design.

2.1. Sine–Cosine Algorithm

This MH was designed by Mirjalili in 2016 [13] and took inspiration from the sine–cosine trigonometric functions. Sine–Cosine can be classified as a population-based algorithm where the population is randomly generated and subsequently perturbed by the following methodology:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^{t+1} = X_{i,j}^{t} + r_1 \cdot sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t|, & r_4 < 0.5 \\ X_{i,j}^{t+1} = X_{i,j}^{t} + r_1 \cdot cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t|, & r_4 \geq 0.5 \end{cases} \quad (1)$$

where parameter r_1 and uniform random numbers r_2, r_3, and r_4 are illustrated in Equations (2)–(5), respectively. In this regard, parameter r_1 determines the direction of the motion, that is towards or away from the best known solution. r_2 indicates the magnitude of the motion. r_3 gives how random the motion will be, thus, when $r_3 > 1$, it will be highly stochastic. r_4 determines which equation will be employed. In other words, it determines the phases of algorithm (exploration or exploitation) [14].

$$r_1 = a - t \cdot \frac{a}{T} \tag{2}$$

$$r_2 = 2 \cdot \pi \cdot Rand[0,1] \tag{3}$$

$$r_3 = 2 \cdot Rand[0,1] \tag{4}$$

$$r_4 = Rand[0,1] \tag{5}$$

2.2. Harris Hawk Optimizacion

This MH was designed by Hidari et al. in 2019 [15] and named Harris Hawk Optimization (HHO). This was inspired by the cooperative and hunting behavior of Harris Hawks over peregrines. In this regard, in each iteration, the best peregrine is assigned as the X_{rabbit} and is the objective for the rest of the population. Initially, for each falcon, their energy E and jump J are computed. E determines if exploitation or exploration is performed. This energy decreases over time and can be interpreted as the flock getting weaker after eluding the attacks of hawks. This situation can be mathematically modeled as follows.

$$\begin{aligned} &E = 2 \cdot E_0 \cdot (1 - \frac{t}{T}) \\ &t \in \{1, 2, ..., T\} \\ &T = \text{maximum iterations} \end{aligned} \tag{6}$$

In each iteration, the initial energy level E_0 is randomly adjusted by $[-1, 1]$. When E_0 decreases from 0 to 1, it demonstrates that the energy is running out for the flock; and when E_0 increases from 0 to 1, it means that the flock is gaining energy. However, as the iterations progress, the current energy E follows a decreasing trend. While $|E \geq 1|$, HHO performs exploration, and this situation changes to exploitation when $|E < 1|$. The exploration is mathematically modeled as follows.

$$X_i^{t+1} = \begin{cases} X_{rand}^t - r_1 \cdot |X_{rand}^t - 2 \cdot r_2 \cdot X_i^t| & q > 0.5 \\ (X_{rabbit}^t - X_m^t) - r_3 \cdot (LB + r_4(UB - LB)) & q < 0.5 \end{cases} \tag{7}$$

Thus, when $q > 0.5$, the scenario applied is for the hawks to randomly search the solution space. When $q < 0.5$, this represents a scenario where Peregrines perch around the flock.

Additionally, X_i^{t+1} corresponds to the updated position of the current falcon, X_{rand}^t is a randomly selected falcon, X_{rabbit}^t is the position of the best solution, and X_i^t is the current position of the falcon. r_1 to r_4, and q are uniform random numbers ranging between [0,1], while LB and UB are the limits of the search space, and the mean location of the population is X_m^t.

The exploitation strategies are carried out according to Equations (8), (9), (11) and (12). To decide what type of exploitation behavior is going to be used, the value of the current energy $|E|$ and r is used. In this context, r corresponds to a random number between [0.1], and when $|E| \geq 0.5$ and $r \geq 0.5$, we employ Equation (8).

$$\begin{aligned} &X_i^{t+1} = \Delta X_i^t - E \cdot |J \cdot X_{rabbit}^t - X_i^t| \\ &\Delta X_i^t = X_{rabbit}^t - X_i^t \\ &J = 2 \cdot (1 - r_5) \end{aligned} \tag{8}$$

where ΔX_i^t is the distance between the best position discovered thus far and the i-th hawk's present position. r_5 corresponds to a random number between [0,1] and represents the rabbit's erratic hop in an attempt to escape the predator. When $|E| \geq 0.5$ and $r < 0.5$, then we apply Equation (9).

$$
\begin{aligned}
X_i^{t+1} &= \begin{cases} Y & \text{If } f(Y) < f(x_i^t) \\ Z & \text{If } f(Z) < f(x_i^t) \end{cases} \\
Y &= X_{rabbit}^t - E \cdot |J \cdot X_{rabbit}^t - X_i^t| \\
Z &= Y + S \cdot LF(D)
\end{aligned}
\tag{9}
$$

where D and S are, respectively, the dimensions of the problem and a D-size vector containing random numbers, while $f(Y)$ and $f(Z)$ are values of the objective functions for the given vectors. LF represents the Lévy flight, which can be represented with the Equation (10).

$$
\begin{aligned}
LF(D) &= 0.01 \cdot \frac{\mu \cdot \sigma}{|v|^{\frac{1}{\beta}}} \\
\sigma &= \left(\frac{\Gamma(1+\beta) \cdot \sin(\frac{\pi\beta}{2})}{\Gamma(\frac{1+\beta}{2}) \cdot \beta 2^{\frac{\beta-1}{2}}} \right)
\end{aligned}
\tag{10}
$$

where μ and v are random numbers between [0,1], β is a constant with a value of 1.5. The value of 0.01 is used to control the step length, which can be changed to fit the problem landscape. When $|E| < 0.5$ and $r \geq 0.5$, we apply Equation (11).

$$
X_i^t = X_{rabbit}^t - E|\Delta X_i^t| \tag{11}
$$

Finally, if $|E| < 0.5$ and $r < 0.5$, we apply Equation (12).

$$
\begin{aligned}
X_i^{t+1} &= \begin{cases} Y & \text{If } f(Y) < f(x_i^t) \\ Z & \text{If } f(Z) < f(x_i^t) \end{cases} \\
Y &= X_{rabbit}^t - E \cdot |J \cdot X_{rabbit}^t - X_m^t| \\
Z &= Y + S \cdot LF(D)
\end{aligned}
\tag{12}
$$

2.3. Whale Optimization Algorithm

The Whale Optimization Algorithm (WOA) was designed by Mirjalili and Lewis in 2015 [16] and is inspired by humpback whale hunting behavior—particularly how they utilize a technique known as "bubble netting". WOA begins with a set of randomly generated solutions. The whales change their locations considering a randomly selected whale or the best solution found thus far at each iteration. In this context, when the Equation (13) has the value $|\overrightarrow{A}| \geq 1$, a new random whale is picked. However, when $|\overrightarrow{A}| < 1$, the best solution is chosen. On the other hand, through the parameter "p", WOA decides between a spiral and circular motion. In this regard, there are three motions that are critical:

1. Searching for prey ($p < 0.5$ and $|A| \geq 1$): The whales randomly search for prey based on the position of each prey. When the algorithm determines that $|\overrightarrow{A}| \geq 1$, we may say that it is exploring, allowing WOA to carry out a global search. This initial move is mathematically represented as follows:

$$
\begin{aligned}
\overrightarrow{X}_i^{t+1} &= \overrightarrow{X_{rand}^t} - \overrightarrow{A} \cdot \overrightarrow{D} \\
\overrightarrow{D} &= |\overrightarrow{C} \cdot \overrightarrow{X_{rand}^t} - \overrightarrow{X}_i^t|
\end{aligned}
\tag{13}
$$

where t indicates the current iteration, $\overrightarrow{A}$ and $\overrightarrow{C}$ are coefficient vectors, and $\overrightarrow{X_{rand}}$ is a randomly selected position vector (i.e., a random whale) from the current population. The vectors $\overrightarrow{A}$ and $\overrightarrow{C}$ may be calculated using the following Equation (14):

$$\begin{aligned} \overrightarrow{A} &= 2\overrightarrow{a} \cdot \overrightarrow{r} - \overrightarrow{a} \\ \overrightarrow{C} &= 2 \cdot \overrightarrow{r} \end{aligned} \tag{14}$$

where, $\overrightarrow{a}$ linearly decrease from 2 to 0 over iterations (in both the exploration and exploitation phases) and $\overrightarrow{r}$ is a uniform random vector of values between $[0, 1]$.

2. Encircling the prey ($p < 0.5$ and $|A| < 1$): When the whales find their target, they proceed to surround them. At the beginning, an optimal location is unknown; thus, each agent focuses on the nearest prey. After the best search agent has been identified, the other agents attempt to update their locations towards that agent. This movement is mathematically represented by Equation (15):

$$\begin{aligned} \overrightarrow{X}_i^{t+1} &= \overrightarrow{X^*}_i^t - \overrightarrow{A} \cdot \overrightarrow{D} \\ \overrightarrow{D} &= |\overrightarrow{C} \cdot \overrightarrow{X^*}_i^t - \overrightarrow{X}_i^t| \end{aligned} \tag{15}$$

where $\overrightarrow{X^*}$ is the position vector of the best solution found thus far, and $\overrightarrow{X}$ is the current position vector. Equation (14) is used to compute the vectors $\overrightarrow{A}$ and $\overrightarrow{C}$. It is worth noting that, if a better solution exists, $\overrightarrow{X}$ must be changed at each iteration.

3. Bubble net attack ($p \geq 0.5$): The "shrinking net method" is given by this movement. This behavior is accomplished by reducing the value of a in the Equation (14). As the whale spins, the bubble net decreases until the prey is captured. The following Equation (16) is used to represent this motion:

$$\begin{aligned} \overrightarrow{X}_i^{t+1} &= \overrightarrow{D'} \cdot e^{bl} \cdot \cos(2\pi l) + \overrightarrow{X^*}_i^t \\ \overrightarrow{D'} &= |\overrightarrow{X^*}_i^t - \overrightarrow{X}_i^t| \end{aligned} \tag{16}$$

where $\overrightarrow{D'}$ is the distance between the i-th whale and the prey (the best solution obtained thus far), b is a constant employed to define the form of the logarithmic spiral, and l is a random integer between $[-1, 1]$.

Moreover, humpback whales swim around their prey in a decreasing circle while also following a spiral trajectory. To simulate this behavior, there is a 50% chance of selecting either the encircling prey mechanism (2) or the spiral model (3) to update the location of the whales during optimization. Here is the mathematical model:

$$\overrightarrow{X}_i^{t+1} = \begin{cases} \overrightarrow{X^*}_i^t - \overrightarrow{A} \cdot \overrightarrow{D} & \text{If } p < 0.5 \\ \overrightarrow{D'} \cdot e^{bl} \cdot \cos(2\pi l) + \overrightarrow{X^*}_i^t & \text{If } p \geq 0.5 \end{cases} \tag{17}$$

2.4. Grey Wolf Optimizer

The Grey Wolf Optimizer (GWO) is inspired by the behavior of gray wolves. The hierarchy employed is led by the alpha wolf (α), which is followed by beta (β) wolves and delta wolves (δ). The remaining members of the pack are referred to as *omegas* [17]. The global optimum represents the location of the prey, and the alpha, beta, and delta wolves are the closest to the prey. The rest of the pack, formerly known as *omegas*, are updated in the search space based on the leaders. In GWO, in order to hunt prey, the following steps are required: encircle, stalk, raid, and search for prey.

1. Encircling the prey: The objective is for the pack to surround the prey, in order to carry out this movement; thus, each wolf will be moving toward the target.

$$\overrightarrow{X}(t+1) = \overrightarrow{X_p}(t) - \overrightarrow{A} \cdot \overrightarrow{D} \tag{18}$$

where t denotes the current iteration, $\overrightarrow{X_p}(t)$ denotes the prey's location in the t-th iteration, $\overrightarrow{X}(t)$ denotes the wolf's position, and $\overrightarrow{D}$ may be described as follows:

$$\overrightarrow{D} = \mid \overrightarrow{C} \cdot \overrightarrow{X_p}(t) - \overrightarrow{X}(t) \mid \tag{19}$$

Additionally, the coefficient vectors $\overrightarrow{A}$ and $\overrightarrow{C}$ of Equations (18) and (19) are computed as follows:

$$\overrightarrow{A} = 2a \cdot \overrightarrow{r_1} - a\,, \; \overrightarrow{C} = 2\overrightarrow{r_2} \tag{20}$$

where a is a parameter, $\overrightarrow{r1}$, and $\overrightarrow{r2}$ are uniform random vectors with values from 0 to 1.

2. Stalking the prey until it stops: This action is carried out by the whole pack based on information provided by the α, β, and δ wolves, who are supposed to be aware of the position of the prey. This action may be mathematically represented as follows:

$$\overrightarrow{X}(t+1) = \frac{\overrightarrow{X_1}(t) + \overrightarrow{X_3}(t) + \overrightarrow{X_3}(t)}{3} \tag{21}$$

where $\overrightarrow{X_1}(t)$, $\overrightarrow{X_2}(t)$, and $\overrightarrow{X_3}(t)$ are defined as illustrated in Equation (18). $\overrightarrow{X_1}(t)$ replaces $\overrightarrow{X_p}(t)$, $\overrightarrow{A}$ and $\overrightarrow{D}$ by $\overrightarrow{X_\alpha}(t)$, $\overrightarrow{A_1}$ and $\overrightarrow{D_\alpha}$, respectively. $\overrightarrow{X_2}(t)$ replaces $\overrightarrow{X_p}(t)$, $\overrightarrow{A}$, and $\overrightarrow{D}$ by $\overrightarrow{X_\beta}(t)$, $\overrightarrow{A_2}$, and $\overrightarrow{D_\beta}$, respectively. Lastly, $\overrightarrow{X_3}(t)$ replaces $\overrightarrow{X_p}(t)$, $\overrightarrow{A}$, and $\overrightarrow{D}$ by $\overrightarrow{X_\delta}(t)$, $\overrightarrow{A_1}$, and $\overrightarrow{D_\delta}$, respectively.
On the other hand, $\overrightarrow{X\alpha}$, $\overrightarrow{X\beta}$, and $\overrightarrow{X_\delta}$ are the iteration's first three best answers. They define $\overrightarrow{A1}$, $\overrightarrow{A2}$, and $\overrightarrow{A3}$ as in the Equation (20). Finally, $\overrightarrow{D_\alpha}$, $\overrightarrow{D_\beta}$, and $\overrightarrow{D_\delta}$ as the Equation (19), where $\overrightarrow{D_\alpha}$ replaces $\overrightarrow{C}$ and $\overrightarrow{X_p}(t)$ by $\overrightarrow{C_1}$ and $\overrightarrow{X_\alpha}(t)$, respectively. $\overrightarrow{D_\beta}$ replace $\overrightarrow{C}$ and $\overrightarrow{X_p}(t)$ by $\overrightarrow{C_2}$ and $\overrightarrow{X_\beta}(t)$, respectively. $\overrightarrow{D_\delta}$ replaces $\overrightarrow{C}$ and $\overrightarrow{X_p}(t)$ by $\overrightarrow{C_3}$ and $\overrightarrow{X_\delta}(t)$, respectively. $\overrightarrow{C1}$, $\overrightarrow{C2}$, and $\overrightarrow{C3}$ are specified in the equation for vector $\overrightarrow{C}$ (Equation (20)).

3. Attack the prey: the main parameter in this movements is a, which manages the exploration or exploitation performed by GWO, i.e., moving closer to or further away from the prey. In this regard, a is defined between [0,2] and is mathematically illustrated as follows:

$$a = 2 - t\frac{2}{T} \tag{22}$$

where t is the current iteration and T is the total amount of iterations. According to the corresponding author, the range of possible values for a enables a seamless transition between exploration and exploitation. Thus, when a is close to 0, the wolves are attacking the prey or rather, the MH is exploiting the search space.

4. Search for prey: In order to hunt down their prey, wolves disperse. This behavior is mimicked by setting the parameter a to a value closer to 2. It is worth noting that every wolf can discover a more suitable (ideal) prey. If a wolf approaches the prey, it becomes the new alpha, and the remaining wolves are classified as beta, delta, or omegas according to their distance from the prey.

The four metaheuristics described above were created to solve continuous optimization problems. In order for continuous metaheuristics to solve discrete optimization problems, a transfer of solutions is necessary.

2.5. Two-Step Binarization Scheme

The methodology behind binarization techniques for continuous MH consists in transferring the values of the continuous domain of the MH to a binary domain; this is done to preserve the quality movements that have continuous MH in order to generate quality binary solutions. Although there are MH that work in binary domains without the need to incorporate a binary scheme, continuous MH assisted by a binary scheme has proven to achieve great performance in multiple combinatorial NP-Hard problems—for instance, the Binary Bat Algorithm [18], Particle Swarm Optimization [19], Binary Salp Swarm Algorithm [20], Binary Dragonfly [21], and Binary Magnetic Optimization Algorithm [22].

In the literature, among the binary schemes, two large groups can be defined. First, the operators that do not provoke alterations in operations related to different elements of the MH. In this regard, the two-step techniques stand out, as they are the most used in the last decade [5] and the Angle Modulation technique [23]. The second group includes the methods that alter the normal functioning of a MH. For instance, Quantum Binary [24] and Set-Based Approaches, in addition to the techniques based on clustering [2,6].

In the scientific community, the two-step binary schemes are of great relevance. They have been employed to tackle multiple types of problems [25]. This binarization scheme, as the name implicates, it is composed of two steps. The first step is the transfer function [19], which transfers the values generated by the continuous MH to a continuous interval between 0 and 1. The second step involves binarization, which consists in transferring the number between that interval in a binary value, Figure 1.

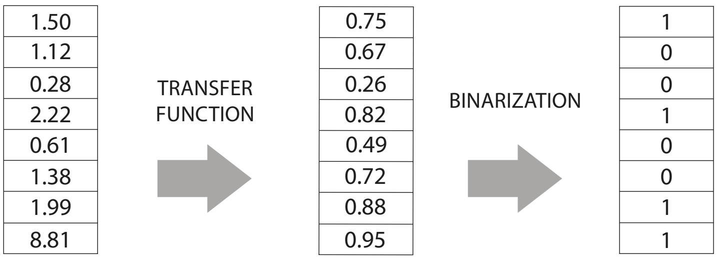

Figure 1. Example of classic Binarization Scheme.

2.5.1. First Step: Transfer Functions

Kennedy et al. in 1997 [26] introduced transfer functions to the optimization field. Their main advantage is the delivery of a probability between 0 and 1 at a low computational cost. There are two types of functions, the S-Shaped [19,27] and the V-Shaped [28], which are illustrated in the Figure 2. For each type of function, four variations were proposed, Table 1.

Table 1. Transfer functions [27].

Transfer Function	Type
$T(d_w^j) = \frac{1}{1+e^{-2d_w^j}}$	S1
$T(d_w^j) = \frac{1}{1+e^{-d_w^j}}$	S2
$T(d_w^j) = \frac{1}{1+e^{\frac{-d_w^j}{2}}}$	S3
$T(d_w^j) = \frac{1}{1+e^{\frac{-d_w^j}{3}}}$	S4
$T(d_w^j) = \left\lvert erf\left(\frac{\sqrt{\pi}}{2} d_w^j\right)\right\rvert$	V1
$T(d_w^j) = \left\lvert tanh(d_w^j)\right\rvert$	V2
$T(d_w^j) = \left\lvert \frac{d_w^j}{\sqrt{1+(d_w^j)^2}}\right\rvert$	V3
$T(d_w^j) = \left\lvert \frac{2}{\pi} arctan\left(\frac{\pi}{2} d_w^j\right)\right\rvert$	V4

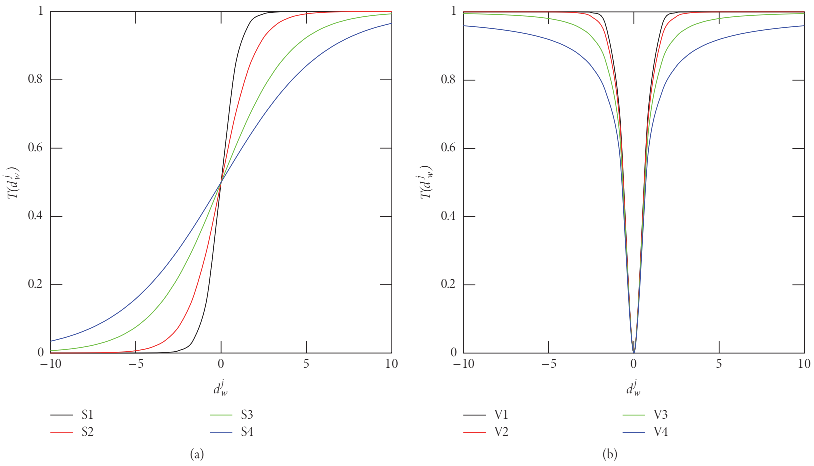

Figure 2. (**a**) S and (**b**) V transfer functions.

2.5.2. Second Step: Binarization

The binarization functions discretize the probability obtained from the transfer function and deliver a binary value. For this step, there are different techniques in the literature [29], such as those exemplified in Table 2.

Table 2. Techniques of binarization [5].

Binarization	Type
$X^j_{new} = \begin{cases} 1 & if\ rand \leq T(d^j_w) \\ 0 & else. \end{cases}$	Standard
$X^j_{new} = \begin{cases} X^j_w & if\ rand \leq T(d^j_w) \\ 0 & else. \end{cases}$	Complement
$X^j_{new} = \begin{cases} 0 & if\ T(d^j_w) \leq \alpha \\ X^j_w & if\ \alpha < T(d^j_w) \leq \frac{1}{2}(1+\alpha) \\ 1 & if\ T(d^j_w) \geq \frac{1}{2}(1+\alpha) \end{cases}$	Static Probability
$X^j_{new} = \begin{cases} X^j_{Best} & if\ rand < T(d^j_w) \\ 0 & else. \end{cases}$	Elitist
$X^j_{new} = \begin{cases} P[X^j_{new} = \zeta_j] = \frac{f(\zeta)}{\sum_{\delta \in Q_g} f(\delta)} & if\ rand \leq T(d^j_w) \\ P[X^j_{new} = 0] = 1 & else. \end{cases}$	Elitist Roulette

2.6. Set Covering Problem

The SCP is defined as a binary matrix (A) of m-rows and n-columns, where $a_{i,j} \in \{0,1\}$ is the value of each cell in the matrix A, where i and j are the size of the m-rows and n-columns, respectively:

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} & \dots & a_{1,n} \\ a_{2,1} & a_{2,2} & \dots & a_{2,n} \\ \dots & \dots & \dots & \dots \\ a_{m,1} & a_{m,2} & \dots & a_{m,n} \end{pmatrix} \tag{23}$$

Defining the column j satisfies a row i if a_{ij} is equal to 1, and this is the contrary case if this is 0. In addition, there is an associated cost $c_j \in C$, where $C = \{c_1, c_2, \dots c_n\}$, together with that $I = \{1, 2, \dots, m\}$ and $J = \{1, 2, \dots, n\}$ are the sets of rows and columns, respectively.

3. The Proposal: Binarization Scheme Selector

This paper proposes a binarization scheme selector that incorporates multiple transfer functions and discretization methods. The main objective includes the smart selection, employment, and correct balance of them led by SARSA.

This novel binarization approach is based on the behavior of hyperheuristics, which have been proven to be effective for several issues [30]. The proposed design determines which types of binarization are more appropriate to apply in each iteration. The decision is based on the knowledge processed from dynamic data generated through the search in the run time. In this context, more adequate binarization methods can be applied with a higher probability to achieve good results. Figure 3 and Algorithm 1 illustrate the proposed design for the binary scheme selector, where a key element is depicted as Δ, which represents the dimensional perturbation on each byte in the solution vector, in other words, we can represent the perturbations performed by the MHs.

Algorithm 1 Data-driven dynamic discretization framework

```
Initialize a random swarm
Initialize Q-Table
for iteration (t) do
    Select action a_t for s_t from the Q-Table
    for solution (i) do
        for dimension (d) do
            X_{i,d}^{t+1} = X_{i,d}^{t} + Δ(a_t)
        end for
    end for
    Get immediate reward r_t
    Get the maximum Q value for the next state s_{t+1}
    Update Q-Table
    Update the current state s_t = s_{t+1}
end for
```

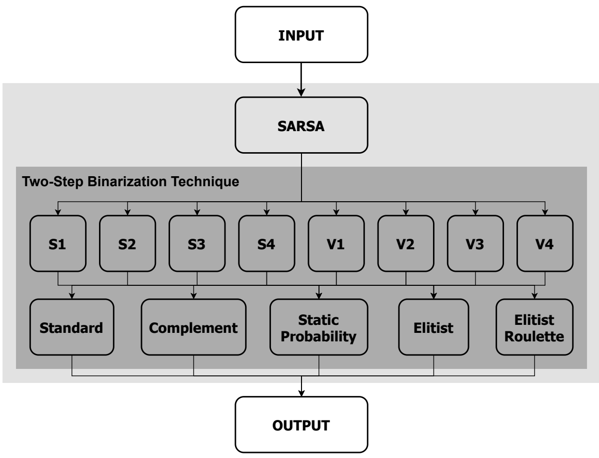

Figure 3. Binary scheme selector with SARSA as a smart operator.

3.1. SARSA

Temporal Difference (TD) algorithms are well-known RL approaches that focus on the study of the environment, generate knowledge, and update the current state [31]. The difference between the present assessment of a state's worth, the discounted value of the future state, and the reward are displayed by the TD algorithms. These algorithms concentrate on state-to-state transitions and state-learned values.

Among the TD algorithms, there is SARSA, an online control algorithm and on-policy method [31]. In other words, SARSA algorithms are online algorithms because they perform the updates of the action-value function estimation at the end of each step without waiting for the term condition. Due to this, the Q-value is available to be used in the next state. They are control algorithms since they perform actions to achieve their purpose, which is the state–action optimal value function estimation.

On the other hand, they are on-policy, which means that the agent learns the value of the state–action pair based on the action performed and, thus, evaluating the current policy, unlike other techniques, such as Q-learning, which performs one policy and evaluates another.

These kinds of policies allow agents to learn to act optimally by experiencing the consequences of their actions without having to develop domain maps. The "environment" includes the current "states" in which the agent interacts and makes decisions, and there are several recognized states. In this context, each agent has a set of actions that cause a modification in the "reward" as well as in the subsequent state.

Thus, when the value reached is equal to one, the state is modified. In addition, when the agent selects an action to perform, he receives a reward for his decision. Rewards are delayed, and the agents must learn from the system to receive them. The value of the state–action pair is learned by the agent as a function of the action performed. Thus, when the value of the current state–action is updated, the next action a_{t+1} is performed.

In Figure 4, the state-to-state transitions are considered, and the values of each have been learned. To understand the algorithm, let us consider the transitions as a pair of values, state–action to state–action, where the values of the state–action pairs are learned. Formally, these cases are identical: both are Markov chains with a reward process. The theorems ensuring the convergence of state values under TD are also applied to the corresponding algorithm for the action values. The update performed by the state–action can be defined as follows, Equation (24):

$$Q(s_t, a_t) \longleftarrow Q(s_t, a_t) + \alpha \cdot [r_{t+1} + \gamma \cdot Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t)] \tag{24}$$

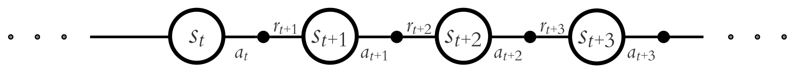

Figure 4. A SARSA algorithm sequence.

After each transition, the state is updated, until a terminal state is reached. When a state s_{t+1} is terminal, then $Q(s_{t+1}, a_{t+1})$ is defined as zero. Each transition process is composed of five events: $s_t, a_t, r_{t+1}, s_{t+1}$, and a_{t+1} (State–Action–Reward–State–Action); providing the name for the SARSA algorithm. Algorithm 2 of the algorithm is shown below:

Algorithm 2 SARSA: on-policy TD control

```
Algorithm parameters: α ϵ [0,1]
Initialize Q(s,a)
while t ≤ Maximum number of iterations do
    Initialize s
    Choose a of s using the policy obtained from Q
    while s ≠ s_terminal do
        Choose action a, Note r
        Create s′
        Choose a′ of s′ using the policy obtained from Q
        Q(s,a) ⟵ Q(s,a) + α · [r + γ · Q(s′,a′) − Q(s,a)]
    end while
end while
```

3.2. Rewards

The rewards in RL algorithms are a critical component in the performance. It is such an important issue that several methods have been proposed in the literature [32–35]. The value assigned to r in the generic SARSA is determined by the type of reward from the chosen metrics as illustrated in Figure 5.

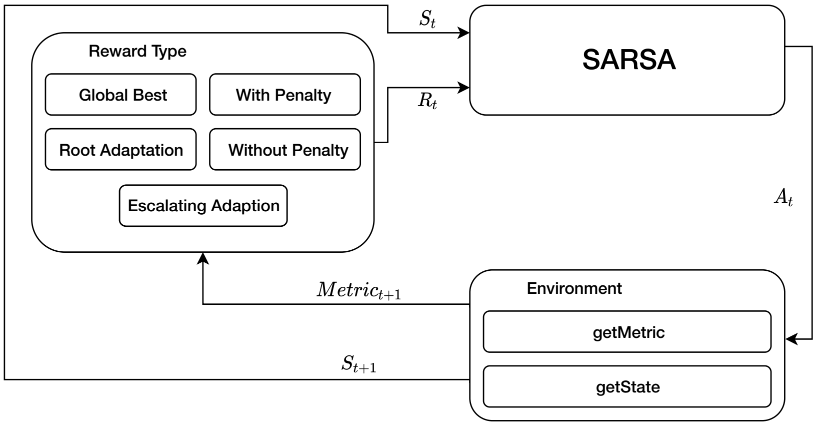

Figure 5. A SARSA scheme for different rewards.

In Table 3, we illustrate detailed information about the rewards employed by SARSA. First, we use the penalty employed by Xu Yue and Song Shin Siang in [32,33], respectively. This reward applies a fixed increment or reduction in value for actions that result in an improvement or absence of it in the overall fitness. Regarding the reward without penalty, employed by Abed-alguni [34]. In this reward, no penalty is attached to the action committed. On the other hand, we have three additional sorts of incentives, which were reviewed and presented by Nareyek Alexander in [35].

Table 3. Reward types.

Reward Types	Mathematical Formula
With Penalty	$r_n = \begin{cases} +1, & \text{If fitness improves} \\ -1, & \text{Otherwise} \end{cases}$
Without Penalty	$r_n = \begin{cases} +1, & \text{If fitness improves} \\ 0, & \text{Otherwise} \end{cases}$
Global Best	$r_n = \begin{cases} \frac{W}{BestFitness}, & \text{If fitness improves} \\ 0, & \text{Otherwise} \end{cases}$
Root Adaption	$r_n = \begin{cases} \sqrt{BestFitness}, & \text{If fitness improves} \\ 0, & \text{Otherwise} \end{cases}$
Scalating Adaption	$r_n = \begin{cases} W \cdot BestFitness, & \text{If fitness improves} \\ 0, & \text{Otherwise} \end{cases}$

3.3. Metric (getMetric)

Several metrics have been proposed in the literature. In this work, the fitness improvement metric is employed. The objective is directly related to the following scenarios, if the fitness function improves, SARSA will reward; if the fitness function remains unchanged, SARSA will apply a penalty as indicated by the type of reward.

3.4. State Determination (getState)

As is well known, metaheuristics have two phases that allow them to perform the optimization process. The phases are the exploration of the search space to find tentative regions with good solutions and the exploitation phase where the search for the best regions to find better solutions is intensified. Our proposal will have, as states for both Q-Learning and SARSA, the exploration and exploitation phases. To use these phases, we need to measure the exploration and exploitation of our algorithms. One of the techniques that stands out is the use of diversity metrics.

There are numerous methods for determining diversity [36]. In this work, diversity is computed by using the method proposed by Hussain Kashif et al. [37], which is expressed mathematically as:

$$Div = \frac{1}{l \cdot n} \sum_{d=1}^{l} \sum_{i=1}^{n} |\bar{x}^d - x_i^d|, \tag{25}$$

where Div represents the diversity status determination, $\bar{x}^d$ denotes the mean values of the individuals in dimension d, x_i^d denotes the value of the i-th individual in dimension d, n denotes the population size, and l denotes the size of the individuals' dimension.

We consider the exploration and exploitation percentages to be $XPL\%$ and $XPT\%$, respectively. The percentages $XPL\%$ and $XPT\%$ are computed from the study of Morales-Castañeda et al. [38] as follows:

$$XPL\% = \frac{Div}{Div_{max}} \cdot 100, \tag{26}$$

$$XPT\% = \frac{|Div - Div_{max}|}{Div_{max}} \cdot 100. \tag{27}$$

where Div represents the diversity state determined by Equation (25), and Div_{max} denotes the maximum value of the diversity state discovered throughout the optimization process.

4. Experimental

In order to determine if the integration of SARSA as a binary scheme selector improves the performance of a MH, five versions of SARSA corresponding to different types of rewards are implemented and compared against Q-Learning [39], Table 4 illustrates the details corresponding to the name assigned to each approach and the type employed.

Table 4. SARSA and Q-Learning implementations.

Name	Reward Types
SA1	With Penalty
SA2	Without Penalty
SA3	Global Best
SA4	Root Adaption
SA5	Scalating Adaption
QL1	With Penalty
QL2	Without Penalty
QL3	Global Best
QL4	Root Adaption
QL5	Scalating Adaption

In this work, the performance comparison is carried out by analyzing five versions of SARSA, five versions of Q-Learning, and two well-known and recommended binarization strategies, Table 5. The 12 approaches described were applied on HHO, GWO, WOA, and SCA solving the SCP, as shown in Table 6, in order to demonstrate the robustness of our hybridization proposal.

Table 5. Recommended binarization schemes in the literature.

Name	Binarization	Transfer Function	Cite
BCL	Elitist	V4	[29]
MIR	Complement	V4	[19]

Table 6. Configuration details from SCP instances employed in this work.

Instance Set	m	n	Cost Range	Density (%)
4	200	1000	[1,100]	2
5	200	2000	[1,100]	2
6	200	1000	[1,100]	5
A	300	3000	[1,100]	2
B	300	3000	[1,100]	5
C	400	4000	[1,100]	2
D	400	4000	[1,100]	5

The configuration of the parameters of the four metaheuristics was carried out based on the original authors of each one of them.

4.1. Experimental Results

In Tables 7–14, the achieved results are illustrated. The detailed information presented in each table can be described as follows: the first column corresponds to the name of the Beasley's instances (45 in total) [40], the second column is the best value known to date, the next three columns (*Best*, *Avg*, and *RPD*) present the best value reached, the averages, and the RPD obtained from the independent runs. The RPD corresponds to the Relative Percentage Deviation as defined in Equation (28). These three columns mentioned above are repeated for all versions (*BCL, MIR, QL1, SA1, QL2, SA2, QL3, SA3, QL4, SA4, QL5, and SA5*).

Finally, the last row corresponds to the mean of each column, and we highlight in bold the best values reached. For each MH implemented, the population size employed was 40, and 1000 iterations were performed per run. With this, the stopping condition was at 40,000 evaluations of the objective function as employed in [29]. The implementation was developed in Python 3.8.5 and processed using the free Google Colaboratory services [41]. The parameter settings for SARSA and QL algorithms were as follows: $\gamma = 0.4$ and $\alpha = 0.1$.

Table 7. Result comparison with WOA employing the approaches BCL, MIR, QL1, SA1, QL2, and SA2.

		BCL			MIR			QL1			SA1			QL2			SA2		
Inst.	**Opt.**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**
41	429	543	582.82	26.57	664	751.74	54.78	521	529.17	21.45	519	521.0	20.98	530	532.4	23.54	521	524.1	21.45
42	512	554	581.72	8.2	699	762.29	36.52	543	548.67	6.05	521	533.4	1.76	538	546.44	5.08	523	535.6	2.15
43	516	565	597.22	9.5	717	798.68	38.95	539	546.89	4.46	522	527.6	1.16	537	543.78	4.07	525	532.7	1.74
44	494	541	559.89	9.51	635	694.42	28.54	513	522.89	3.85	502	510.2	1.62	519	526.33	5.06	497	508.0	0.61
45	512	565	591.0	10.35	700	773.87	36.72	535	541.43	4.49	522	525.8	1.95	537	541.89	4.88	523	528.4	2.15
46	560	593	626.22	5.89	745	874.68	33.04	579	584.44	3.39	565	568.3	0.89	573	580.33	2.32	567	570.3	1.25
47	430	455	482.17	5.81	540	613.32	25.58	444	446.29	3.26	435	436.0	1.16	440	445.29	2.33	435	439.5	1.16
48	492	536	566.67	8.94	732	779.1	48.78	505	509.5	2.64	498	501.7	1.22	505	507.83	2.64	496	499.6	0.81
49	641	717	751.42	11.86	946	1013.35	47.58	680	689.0	6.08	658	672.8	2.65	686	690.8	7.02	671	677.6	4.68
410	514	543	582.82	5.64	664	751.74	29.18	521	529.17	1.36	519	521.0	0.97	530	532.4	3.11	521	524.1	1.36
51	253	288	298.33	13.83	369	416.77	45.85	276	277.0	9.09	266	271.0	5.14	277	278.33	9.49	267	273.1	5.53
52	302	346	368.33	14.57	456	521.03	50.99	329	332.83	8.94	316	325.7	4.64	326	332.17	7.95	319	325.3	5.63
53	226	240	251.42	6.19	323	351.81	42.92	232	233.67	2.65	229	229.9	1.33	232	233.5	2.65	230	230.8	1.77
54	242	267	275.67	10.33	330	362.45	36.36	251	252.67	3.72	245	248.0	1.24	250	252.5	3.31	247	249.4	2.07
55	211	223	236.92	5.69	274	294.9	29.86	217	218.33	2.84	213	214.4	0.95	216	218.83	2.37	212	214.5	0.47
56	213	237	255.08	11.27	311	343.97	46.01	224	228.33	5.16	214	220.4	0.47	227	229.0	6.57	218	221.3	2.35
57	293	330	337.75	12.63	403	450.94	37.54	306	311.83	4.44	296	300.4	1.02	311	313.2	6.14	302	304.6	3.07
58	288	306	328.43	6.25	408	445.03	41.67	298	298.5	3.47	288	291.6	0.0	298	299.33	3.47	291	293.9	1.04
59	279	307	322.82	10.04	403	443.06	44.44	287	289.8	2.87	281	283.3	0.72	284	287.4	1.79	282	284.3	1.08
510	265	288	298.33	8.68	369	416.77	39.25	276	277.0	4.15	266	271.0	0.38	277	278.33	4.53	267	273.1	0.75
61	138	161	170.4	16.67	336	368.0	143.48	143	147.23	3.62	141	142.6	2.17	144	146.68	4.35	141	144.1	2.17
62	146	164	193.55	12.33	415	506.68	184.25	155	156.17	6.16	148	150.8	1.37	154	155.83	5.48	147	152.3	0.68
63	145	172	194.5	18.62	390	474.71	168.97	149	150.33	2.76	145	147.8	0.0	149	150.4	2.76	147	148.4	1.38
64	131	136	151.0	3.82	262	318.9	100.0	134	134.83	2.29	131	132.3	0.0	132	134.17	0.76	131	133.1	0.0
65	161	188	209.17	16.77	379	514.0	135.4	178	181.83	10.56	161	167.9	0.0	180	181.5	11.8	163	172.2	1.24
a1	253	284	300.8	12.25	583	626.6	130.43	261	268.38	3.16	260	262.3	2.77	263	266.84	3.95	260	263.2	2.77
a2	252	284	306.12	12.7	553	615.9	119.44	271	271.67	7.54	255	262.1	1.19	266	269.83	5.56	261	264.0	3.57
a3	232	276	284.75	18.97	505	568.9	117.67	242	246.5	4.31	239	242.8	3.02	244	245.6	5.17	240	243.4	3.45
a4	234	282	308.67	20.51	518	568.48	121.37	245	249.0	4.7	238	242.0	1.71	251	251.8	7.26	238	242.5	1.71
a5	236	262	283.88	11.02	531	570.32	125.0	246	247.5	4.24	241	243.2	2.12	242	247.33	2.54	241	244.2	2.12
b1	69	90	104.2	30.43	549	592.4	695.65	71	71.55	2.9	69	69.9	0.0	70	71.68	1.45	69	70.5	0.0
b2	76	94	118.25	23.68	487	587.03	540.79	79	80.0	3.95	76	77.0	0.0	78	79.5	2.63	76	77.2	0.0
b3	80	110	134.17	37.5	662	766.94	727.5	82	82.67	2.5	81	81.3	1.25	82	82.17	2.5	81	81.7	1.25
b4	79	101	123.92	27.85	617	683.74	681.01	83	83.83	5.06	79	81.2	0.0	83	83.83	5.06	79	81.3	0.0
b5	72	82	116.42	13.89	521	603.65	623.61	73	73.83	1.39	72	72.5	0.0	73	74.33	1.39	72	72.9	0.0

Table 7. *Cont.*

Inst.	Opt.	BCL Best	BCL Avg	BCL RPD	MIR Best	MIR Avg	MIR RPD	QL1 Best	QL1 Avg	QL1 RPD	SA1 Best	SA1 Avg	SA1 RPD	QL2 Best	QL2 Avg	QL2 RPD	SA2 Best	SA2 Avg	SA2 RPD
c1	227	266	280.4	17.18	707	732.6	211.45	243	248.27	7.05	235	238.2	3.52	243	247.81	7.05	234	238.4	3.08
c2	219	264	280.5	20.55	703	799.94	221.0	236	239.83	7.76	225	230.1	2.74	234	238.83	6.85	229	232.3	4.57
c3	243	287	322.2	18.11	798	930.16	228.4	255	259.67	4.94	246	249.4	1.23	258	260.83	6.17	249	254.2	2.47
c4	219	261	283.58	19.18	721	788.58	229.22	232	233.83	5.94	221	226.0	0.91	232	233.83	5.94	227	229.0	3.65
c5	215	262	288.83	21.86	692	765.71	221.86	227	231.0	5.58	218	222.0	1.4	229	231.33	6.51	221	225.4	2.79
d1	60	99	135.4	65.0	781	869.4	1201.67	62	64.61	3.33	60	61.7	0.0	63	64.97	5.0	61	62.3	1.67
d2	66	84	119.58	27.27	902	988.87	1266.67	69	69.0	4.55	67	67.6	1.52	68	69.0	3.03	67	67.4	1.52
d3	72	93	139.58	29.17	907	1082.39	1159.72	77	78.33	6.94	74	75.2	2.78	76	77.33	5.56	73	74.8	1.39
d4	62	78	128.5	25.81	760	880.65	1125.81	63	63.67	1.61	62	62.5	0.0	62	63.4	0.0	62	62.2	0.0
d5	61	87	115.4	42.62	777	877.1	1173.77	64	65.17	4.92	61	62.4	0.0	63	64.33	3.28	61	62.6	0.0
		286.91	310.86	17.01	572.09	643.15	276.64	267.02	270.36	4.94	**259.56**	**263.21**	**1.78**	267.38	270.29	4.9	260.98	264.66	2.28

Table 8. Result comparison with WOA employing the approaches QL3, SA3, QL4, SA4, QL5, and SA5.

Inst.	Opt.	QL3 Best	QL3 Avg	QL3 RPD	SA3 Best	SA3 Avg	SA3 RPD	QL4 Best	QL4 Avg	QL4 RPD	SA4 Best	SA4 Avg	SA4 RPD	QL5 Best	QL5 Avg	QL5 RPD	SA5 Best	SA5 Avg	SA5 RPD
41	429	524	530.0	22.14	518	522.4	20.75	530	532.5	23.54	518	522.8	20.75	526	531.64	22.61	518	523.6	20.75
42	512	543	548.0	6.05	528	534.9	3.12	534	544.44	4.3	525	534.1	2.54	524	547.0	2.34	525	536.3	2.54
43	516	533	540.33	3.29	519	531.8	0.58	535	540.78	3.68	522	529.8	1.16	536	544.11	3.88	524	533.5	1.55
44	494	516	524.11	4.45	502	508.3	1.62	513	524.22	3.85	504	511.3	2.02	517	525.55	4.66	508	514.4	2.83
45	512	540	545.78	5.47	521	525.8	1.76	537	544.78	4.88	521	531.0	1.76	531	545.0	3.71	524	532.6	2.34
46	560	577	583.11	3.04	568	570.6	1.43	577	584.78	3.04	563	569.9	0.54	573	582.35	2.32	566	571.1	1.07
47	430	444	447.14	3.26	434	436.8	0.93	438	444.67	1.86	435	438.7	1.16	438	445.0	1.86	434	439.0	0.93
48	492	506	510.17	2.85	497	500.1	1.02	505	509.0	2.64	495	500.3	0.61	504	508.91	2.44	495	501.2	0.61
49	641	680	684.25	6.08	662	674.6	3.28	680	690.0	6.08	667	673.1	4.06	672	689.04	4.84	664	674.1	3.59
410	514	524	530.0	1.95	518	522.4	0.78	530	532.5	3.11	518	522.8	0.78	526	531.64	2.33	518	523.6	0.78
51	253	276	279.33	9.09	269	272.8	6.32	274	278.0	8.3	268	273.1	5.93	273	277.48	7.91	270	272.3	6.72
52	302	330	332.83	9.27	319	324.2	5.63	327	331.33	8.28	317	324.0	4.97	325	331.96	7.62	319	324.7	5.63
53	226	233	234.5	3.1	230	231.8	1.77	231	233.67	2.21	229	230.5	1.33	231	233.96	2.21	229	230.4	1.33
54	242	246	250.0	1.65	246	249.2	1.65	250	251.5	3.31	247	249.8	2.07	249	252.35	2.89	247	250.0	2.07
55	211	218	219.33	3.32	213	215.3	0.95	217	218.33	2.84	212	215.7	0.47	215	218.22	1.9	212	215.4	0.47
56	213	225	227.0	5.63	215	222.2	0.94	228	229.5	7.04	218	220.8	2.35	223	228.04	4.69	218	221.0	2.35

Table 8. *Cont.*

Inst.	Opt.	QL3 Best	QL3 Avg	QL3 RPD	SA3 Best	SA3 Avg	SA3 RPD	QL4 Best	QL4 Avg	QL4 RPD	SA4 Best	SA4 Avg	SA4 RPD	QL5 Best	QL5 Avg	QL5 RPD	SA5 Best	SA5 Avg	SA5 RPD
57	293	307	310.2	4.78	301	304.0	2.73	303	311.0	3.41	298	303.3	1.71	307	311.81	4.78	302	306.4	3.07
58	288	297	298.0	3.12	291	292.7	1.04	295	297.83	2.43	289	293.5	0.35	294	297.87	2.08	290	292.8	0.69
59	279	284	289.17	1.79	281	284.8	0.72	287	290.5	2.87	282	283.6	1.08	284	289.57	1.79	281	284.8	0.72
510	265	276	279.33	4.15	269	272.8	1.51	274	278.0	3.4	268	273.1	1.13	273	277.48	3.02	270	272.3	1.89
61	138	144	146.74	4.35	141	143.5	2.17	142	146.39	2.9	142	143.8	2.9	143	146.61	3.62	142	143.9	2.9
62	146	152	156.0	4.11	148	153.3	1.37	156	157.33	6.85	149	152.3	2.05	154	156.65	5.48	150	152.9	2.74
63	145	148	149.17	2.07	148	149.0	2.07	149	150.33	2.76	146	147.9	0.69	147	149.96	1.38	147	148.4	1.38
64	131	134	134.67	2.29	131	132.9	0.0	131	134.5	0.0	131	133.1	0.0	133	135.04	1.53	132	133.9	0.76
65	161	176	179.5	9.32	168	174.5	4.35	177	179.17	9.94	164	170.7	1.86	175	180.0	8.7	165	172.9	2.48
a1	253	264	266.97	4.35	261	263.7	3.16	264	266.87	4.35	261	263.4	3.16	264	267.22	4.35	260	263.2	2.77
a2	252	265	270.4	5.16	260	265.1	3.17	269	271.0	6.75	261	264.6	3.57	266	270.83	5.56	259	264.1	2.78
a3	232	242	246.0	4.31	240	242.1	3.45	243	245.5	4.74	241	242.7	3.88	240	246.17	3.45	240	243.4	3.45
a4	234	246	246.6	5.13	240	243.1	2.56	249	250.0	6.41	238	242.8	1.71	244	249.04	4.27	240	244.1	2.56
a5	236	241	248.17	2.12	241	244.4	2.12	246	248.17	4.24	240	242.5	1.69	243	248.74	2.97	241	244.4	2.12
b1	69	70	71.87	1.45	69	70.3	0.0	69	71.68	0.0	69	70.6	0.0	71	71.65	2.9	69	69.9	0.0
b2	76	78	79.17	2.63	76	77.0	0.0	78	79.5	2.63	76	77.3	0.0	78	79.87	2.63	76	77.0	0.0
b3	80	82	82.67	2.5	81	81.4	1.25	82	82.0	2.5	81	81.6	1.25	81	82.26	1.25	81	81.5	1.25
b4	79	83	83.5	5.06	80	81.9	1.27	83	84.0	5.06	80	81.6	1.27	83	83.87	5.06	80	81.4	1.27
b5	72	74	74.5	2.78	72	73.0	0.0	73	74.33	1.39	72	73.1	0.0	73	74.18	1.39	72	73.0	0.0
c1	227	241	247.48	6.17	233	239.6	2.64	241	247.29	6.17	236	241.0	3.96	243	247.75	7.05	237	240.6	4.41
c2	219	238	240.17	8.68	230	233.3	5.02	238	239.6	8.68	229	232.5	4.57	232	239.81	5.94	229	232.4	4.57
c3	243	261	261.8	7.41	248	253.4	2.06	258	261.33	6.17	248	253.9	2.06	256	260.61	5.35	247	251.8	1.65
c4	219	228	234.17	4.11	226	229.2	3.2	230	233.5	5.02	223	227.4	1.83	229	233.09	4.57	227	229.4	3.65
c5	215	229	231.0	6.51	221	224.0	2.79	223	228.67	3.72	222	224.8	3.26	226	231.0	5.12	222	224.7	3.26
d1	60	64	65.06	6.67	61	62.7	1.67	64	64.79	6.67	61	62.2	1.67	64	65.13	6.67	61	62.6	1.67
d2	66	67	68.75	1.52	66	67.4	0.0	68	68.83	3.03	67	67.8	1.52	68	69.04	3.03	67	67.8	1.52
d3	72	76	77.33	5.56	74	75.6	2.78	76	77.33	5.56	73	75.0	1.39	77	77.7	6.94	74	75.8	2.78
d4	62	63	63.8	1.61	62	62.6	0.0	63	63.67	1.61	62	62.1	0.0	62	63.43	0.0	62	62.4	0.0
d5	61	63	65.0	3.28	62	62.5	1.64	65	65.67	6.56	61	62.9	0.0	63	65.3	3.28	61	62.2	0.0
		266.84	277	4.75	260.89	264.51	2.38	266.71	270.2	4.77	260.64	264.42	2.25	265.24	270.31	4.27	261.22	264.96	2.49

Table 9. Result comparison with SCA employing the approaches BCL, MIR, QL1, SA1, QL2, and SA2.

Inst.	Opt.	BCL Best	BCL Avg	BCL RPD	MIR Best	MIR Avg	MIR RPD	QL1 Best	QL1 Avg	QL1 RPD	SA1 Best	SA1 Avg	SA1 RPD	QL2 Best	QL2 Avg	QL2 RPD	SA2 Best	SA2 Avg	SA2 RPD
41	429	557	580.0	29.84	545	734.48	27.04	533	538.0	24.24	515	520.93	20.05	530	537.83	23.54	516	519.2	20.28
42	512	573	605.78	11.91	550	725.1	7.42	548	552.89	7.03	524	531.38	2.34	537	551.11	4.88	527	535.9	2.93
43	516	557	598.83	7.95	559	766.84	8.33	548	552.67	6.2	524	529.0	1.55	543	554.44	5.23	524	529.0	1.55
44	494	533	557.06	7.89	547	688.48	10.73	519	531.22	5.06	500	510.38	1.21	530	533.78	7.29	502	514.2	1.62
45	512	563	591.5	9.96	565	751.35	10.35	540	549.22	5.47	518	527.69	1.17	537	551.67	4.88	524	530.3	2.34
46	560	594	635.22	6.07	591	840.42	5.54	578	587.33	3.21	564	568.1	0.71	577	589.89	3.04	564	571.45	0.71
47	430	449	483.44	4.42	456	586.97	6.05	440	448.11	2.33	434	439.5	0.93	442	448.25	2.79	435	440.1	1.16
48	492	515	565.67	4.67	518	727.23	5.28	507	514.6	3.05	494	498.25	0.41	512	516.0	4.07	497	501.9	1.02
49	641	713	759.75	11.23	698	964.68	8.89	689	695.67	7.49	657	674.0	2.5	696	700.83	8.58	665	676.9	3.74
410	514	557	580.0	8.37	545	734.48	6.03	533	538.0	3.7	515	520.93	0.19	530	537.83	3.11	516	519.2	0.39
51	253	289	303.33	14.23	282	396.55	11.46	276	281.0	9.09	270	272.83	6.72	279	282.17	10.28	270	272.8	6.72
52	302	346	366.92	14.57	335	486.87	10.93	333	334.5	10.26	316	322.83	4.64	334	336.0	10.6	318	326.27	5.3
53	226	246	258.17	8.85	238	331.74	5.31	233	235.5	3.1	230	230.91	1.77	231	235.67	2.21	230	231.1	1.77
54	242	257	276.5	6.2	253	338.84	4.55	255	256.0	5.37	247	249.33	2.07	253	255.67	4.55	247	250.5	2.07
55	211	227	237.92	7.58	226	289.03	7.11	216	221.0	2.37	213	215.18	0.95	218	221.0	3.32	213	216.36	0.95
56	213	244	258.58	14.55	234	324.77	9.86	223	230.67	4.69	214	220.15	0.47	221	230.17	3.76	218	221.8	2.35
57	293	323	342.75	10.24	313	427.1	6.83	317	319.6	8.19	296	303.83	1.02	310	314.4	5.8	297	305.27	1.37
58	288	320	333.3	11.11	302	444.35	4.86	298	299.33	3.47	291	294.18	1.04	300	301.8	4.17	290	294.55	0.69
59	279	312	326.92	11.83	298	414.26	6.81	290	293.67	3.94	281	285.18	0.72	291	294.4	4.3	283	284.7	1.43
510	265	289	303.33	9.06	282	396.55	6.42	276	281.0	4.15	270	272.83	1.89	279	282.17	5.28	270	272.8	1.89
61	138	152	165.2	10.14	348	369.8	152.17	141	145.77	2.17	142	145.45	2.9	144	148.16	4.35	142	145.36	2.9
62	146	170	196.17	16.44	161	484.97	10.27	157	159.83	7.53	146	152.63	0.0	158	159.83	8.22	151	153.0	3.42
63	145	156	179.75	7.59	151	436.71	4.14	149	151.33	2.76	145	150.65	0.0	150	151.67	3.45	148	149.5	2.07
64	131	139	155.25	6.11	137	303.0	4.58	135	136.33	3.05	131	133.53	0.0	136	136.17	3.82	131	133.6	0.0
65	161	193	215.25	19.88	185	450.06	14.91	177	183.17	9.94	161	169.72	0.0	178	183.67	10.56	165	171.82	2.48
a1	253	286	302.8	13.04	272	596.8	7.51	262	267.13	3.56	261	264.73	3.16	266	269.42	5.14	260	263.9	2.77
a2	252	289	304.2	14.68	281	577.52	11.51	271	273.83	7.54	258	264.82	2.38	272	273.67	7.94	254	266.2	0.79
a3	232	266	283.44	14.66	250	555.52	7.76	245	248.6	5.6	242	259.5	4.31	246	249.0	6.03	238	245.5	2.59
a4	234	271	289.3	15.81	256	544.71	9.4	250	253.0	6.84	239	247.91	2.14	248	253.6	5.98	241	246.3	2.99
a5	236	266	286.86	12.71	253	513.9	7.2	249	250.67	5.51	243	249.64	2.97	248	252.83	5.08	242	244.6	2.54
b1	69	81	108.6	17.39	527	585.0	663.77	70	71.74	1.45	69	99.0	0.0	71	72.68	2.9	69	70.9	0.0
b2	76	93	110.33	22.37	81	529.32	6.58	78	80.33	2.63	76	79.53	0.0	80	82.0	5.26	76	77.8	0.0
b3	80	90	117.08	12.5	84	687.06	5.0	82	83.33	2.5	81	82.84	1.25	83	83.83	3.75	80	84.4	0.0
b4	79	96	116.42	21.52	84	582.87	6.33	83	84.0	5.06	80	86.13	1.27	83	84.83	5.06	82	83.3	3.8
b5	72	83	104.09	15.28	75	573.1	4.17	74	75.0	2.78	72	75.53	0.0	75	75.33	4.17	72	74.27	0.0

Table 9. *Cont.*

Inst.	Opt.	BCL Best	BCL Avg	BCL RPD	MIR Best	MIR Avg	MIR RPD	QL1 Best	QL1 Avg	QL1 RPD	SA1 Best	SA1 Avg	SA1 RPD	QL2 Best	QL2 Avg	QL2 RPD	SA2 Best	SA2 Avg	SA2 RPD
c1	227	269	302.8	18.5	254	536.6	11.89	240	245.55	5.73	235	241.1	3.52	241	251.32	6.17	237	240.5	4.41
c2	219	264	284.0	20.55	243	715.52	10.96	235	242.5	7.31	226	236.58	3.2	241	244.17	10.05	230	235.7	5.02
c3	243	273	306.25	12.35	265	745.74	9.05	261	263.17	7.41	248	258.7	2.06	259	263.17	6.58	252	254.5	3.7
c4	219	251	280.67	14.61	235	669.42	7.31	236	237.0	7.76	225	243.67	2.74	233	235.83	6.39	228	231.6	4.11
c5	215	239	271.17	11.16	232	569.45	7.91	228	232.33	6.05	218	226.33	1.4	233	234.33	8.37	222	225.9	3.26
d1	60	89	93.2	48.33	67	701.4	11.67	62	64.42	3.33	61	67.62	1.67	64	66.0	6.67	62	64.6	3.33
d2	66	81	105.83	22.73	69	802.45	4.55	69	70.33	4.55	67	71.35	1.52	69	69.5	4.55	67	73.27	1.52
d3	72	81	109.75	12.5	79	845.29	9.72	78	78.67	8.33	74	77.17	2.78	78	79.0	8.33	75	81.8	4.17
d4	62	68	91.42	9.68	64	675.35	3.23	63	64.0	1.61	62	74.31	0.0	63	64.5	1.61	62	64.6	0.0
d5	61	77	101.0	26.23	67	767.1	9.84	65	66.6	6.56	61	67.4	0.0	66	67.0	8.2	62	69.27	1.64
		284.16	307.68	13.94	290.16	581.97	26.03	269.16	273.08	5.55	**259.91**	266.96	2.04	269.67	273.92	6.01	261.2	265.92	2.62

Table 10. Result comparison with SCA employing the approaches QL3, SA3, QL4, SA4, QL5, and SA5.

Inst.	Opt.	QL3 Best	QL3 Avg	QL3 RPD	SA3 Best	SA3 Avg	SA3 RPD	QL4 Best	QL4 Avg	QL4 RPD	SA4 Best	SA4 Avg	SA4 RPD	QL5 Best	QL5 Avg	QL5 RPD	SA5 Best	SA5 Avg	SA5 RPD
41	429	534	537.5	24.48	517	521.85	20.51	533	536.83	24.24	518	523.75	20.75	530	535.17	23.54	518	522.32	20.75
42	512	547	552.67	6.84	524	533.64	2.34	552	556.89	7.81	526	534.3	2.73	537	552.37	4.88	521	534.52	1.76
43	516	540	555.0	4.65	522	529.42	1.16	535	550.22	3.68	524	534.17	1.55	536	547.05	3.88	523	531.2	1.36
44	494	518	531.33	4.86	500	510.57	1.21	512	532.56	3.64	501	509.08	1.42	511	532.84	3.44	499	510.43	1.01
45	512	544	552.56	6.25	520	526.2	1.56	541	552.67	5.66	526	531.6	2.73	542	549.79	5.86	522	529.45	1.95
46	560	577	588.22	3.04	567	571.79	1.25	584	592.56	4.29	566	571.0	1.07	568	589.3	1.43	565	569.58	0.89
47	430	447	450.5	3.95	434	437.62	0.93	447	452.88	3.95	434	438.58	0.93	439	451.95	2.09	434	438.13	0.93
48	492	509	513.0	3.46	494	500.42	0.41	508	515.83	3.25	496	502.3	0.81	507	515.04	3.05	493	499.67	0.2
49	641	692	696.83	7.96	664	678.69	3.59	688	694.83	7.33	666	674.31	3.9	684	698.48	6.71	658	672.58	2.65
410	514	534	537.5	3.89	517	521.85	0.58	533	536.83	3.7	518	523.75	0.78	530	535.17	3.11	518	522.32	0.78
51	253	277	282.5	9.49	268	271.92	5.93	278	282.33	9.88	267	274.73	5.53	274	282.39	8.3	270	273.52	6.72
52	302	332	336.5	9.93	322	327.62	6.62	328	334.5	8.61	320	326.4	5.96	329	336.28	8.94	313	324.5	3.64
53	226	235	236.17	3.98	230	230.54	1.77	236	236.83	4.42	230	231.18	1.77	230	236.13	1.77	229	231.1	1.33
54	242	253	255.17	4.55	248	249.91	2.48	252	256.17	4.13	246	250.0	1.65	251	254.96	3.72	243	249.14	0.41
55	211	218	222.33	3.32	212	215.92	0.47	221	222.0	4.74	212	215.69	0.47	217	221.09	2.84	212	215.43	0.47
56	213	231	232.8	8.45	213	221.27	0.0	228	233.2	7.04	218	222.45	2.35	224	231.2	5.16	217	222.22	1.88

Table 10. *Cont.*

			QL3			SA3			QL4			SA4			QL5			SA5	
Inst.	**Opt.**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**
57	293	313	317.33	6.83	298	303.75	1.71	317	321.17	8.19	297	304.08	1.37	314	317.84	7.17	296	303.68	1.02
58	288	298	301.83	3.47	291	294.91	1.04	298	300.0	3.47	290	295.55	0.69	297	301.39	3.12	289	293.0	0.35
59	279	289	293.5	3.58	281	286.91	0.72	292	293.67	4.66	280	284.0	0.36	286	293.17	2.51	281	284.82	0.72
510	265	277	282.5	4.53	268	271.92	1.13	278	282.33	4.91	267	274.73	0.75	274	282.39	3.4	270	273.52	1.89
61	138	144	148.42	4.35	141	143.88	2.17	146	148.29	5.8	141	144.42	2.17	146	148.65	5.8	141	144.72	2.17
62	146	157	159.17	7.53	148	152.0	1.37	155	158.0	6.16	150	153.83	2.74	154	158.26	5.48	149	152.25	2.05
63	145	151	151.83	4.14	148	149.38	2.07	150	151.67	3.45	148	150.14	2.07	149	151.65	2.76	145	148.52	0.0
64	131	134	135.67	2.29	131	134.14	0.0	135	136.2	3.05	131	134.38	0.0	134	136.52	2.29	131	133.86	0.0
65	161	182	183.67	13.04	162	170.92	0.62	179	183.17	11.18	161	171.83	0.0	175	182.96	8.7	163	170.43	1.24
a1	253	263	268.81	3.95	260	263.5	2.77	263	268.9	3.95	262	265.18	3.56	265	269.68	4.74	262	264.44	3.56
a2	252	273	275.0	8.33	263	266.36	4.37	271	272.83	7.54	262	267.4	3.97	270	274.33	7.14	261	265.59	3.57
a3	232	249	252.0	7.33	241	243.36	3.88	251	251.33	8.19	242	247.0	4.31	242	248.74	4.31	240	243.18	3.45
a4	234	249	252.2	6.41	240	243.1	2.56	250	252.2	6.84	237	243.64	1.28	247	253.5	5.56	240	244.82	2.56
a5	236	245	250.33	3.81	242	244.4	2.54	247	251.5	4.66	243	246.1	2.97	248	251.22	5.08	241	244.44	2.12
b1	69	72	72.9	4.35	69	72.07	0.0	72	72.87	4.35	69	70.42	0.0	72	73.03	4.35	69	72.8	0.0
b2	76	80	81.5	5.26	76	77.75	0.0	78	80.67	2.63	76	80.54	0.0	78	81.39	2.63	76	77.68	0.0
b3	80	82	83.67	2.5	81	82.5	1.25	82	84.0	2.5	81	82.07	1.25	81	83.35	1.25	81	81.58	1.25
b4	79	82	84.33	3.8	81	82.93	2.53	83	84.83	5.06	81	83.91	2.53	84	85.26	6.33	79	82.43	0.0
b5	72	74	74.83	2.78	72	75.92	0.0	74	75.0	2.78	72	74.25	0.0	74	74.78	2.78	72	74.38	0.0
c1	227	244	251.03	7.49	234	239.09	3.08	239	251.0	5.29	234	237.5	3.08	244	250.94	7.49	237	242.5	4.41
c2	219	241	243.33	10.05	229	231.93	4.57	236	241.8	7.76	225	231.75	2.74	236	242.17	7.76	226	232.41	3.2
c3	243	260	262.8	7.0	247	253.09	1.65	259	262.83	6.58	252	260.1	3.7	256	263.87	5.35	248	255.41	2.06
c4	219	236	237.83	7.76	227	233.82	3.65	234	236.67	6.85	225	235.31	2.74	234	237.13	6.85	225	231.59	2.74
c5	215	232	233.83	7.91	222	225.0	3.26	227	231.83	5.58	219	224.71	1.86	226	233.62	5.12	221	225.47	2.79
d1	60	64	66.0	6.67	61	63.73	1.67	64	65.81	6.67	61	64.08	1.67	63	66.06	5.0	60	64.16	0.0
d2	66	69	69.8	4.55	67	71.27	1.52	69	70.0	4.55	67	70.42	1.52	68	70.13	3.03	67	69.14	1.52
d3	72	78	79.0	8.33	73	77.27	1.39	78	79.33	8.33	73	80.17	1.39	78	79.09	8.33	73	78.0	1.39
d4	62	63	63.83	1.61	62	65.3	0.0	64	64.4	3.23	62	64.73	0.0	63	64.3	1.61	62	64.15	0.0
d5	61	64	66.17	4.92	61	67.14	0.0	65	65.83	6.56	61	64.6	0.0	64	66.35	4.92	61	64.74	0.0
		277	273.86	6.08	260.62	265.26	2.27	269.6	273.89	5.94	260.82	266.0	2.29	267.36	273.58	5.1	262	**265.11**	**2.02**

Table 11. Result comparison with GWO employing the approaches BCL, MIR, QL1, SA1, QL2, and SA2.

			BCL			MIR			QL1			SA1			QL2			SA2	
Inst.	Opt.	Best	Avg	RPD	Best	Avg	RPD	Best	Avg	RPD	Best	Avg	RPD	Best	Avg	RPD	Best	Avg	RPD
41	429	517	519.7	20.51	524	529.42	22.14	528	530.6	23.08	516	521.43	20.28	523	529.83	21.91	518	522.87	20.75
42	512	515	525.88	0.59	533	541.23	4.1	537	541.43	4.88	519	527.67	1.37	536	543.88	4.69	522	531.27	1.95
43	516	517	523.44	0.19	536	541.58	3.88	537	542.0	4.07	518	526.29	0.39	531	541.11	2.91	522	532.06	1.16
44	494	496	503.06	0.4	514	519.84	4.05	516	521.33	4.45	500	513.06	1.21	512	520.0	3.64	500	511.94	1.21
45	512	514	523.44	0.39	536	543.35	4.69	537	543.22	4.88	521	529.83	1.76	537	542.25	4.88	520	531.15	1.56
46	560	562	567.28	0.36	572	579.87	2.14	567	574.89	1.25	563	568.93	0.54	571	576.78	1.96	565	572.08	0.89
47	430	433	434.67	0.7	437	442.65	1.63	438	442.83	1.86	434	439.24	0.93	438	441.86	1.86	435	438.38	1.16
48	492	494	498.92	0.41	498	505.16	1.22	505	507.83	2.64	494	499.29	0.41	506	507.67	2.85	494	500.85	0.41
49	641	651	662.25	1.56	674	685.39	5.15	682	686.83	6.4	665	673.88	3.74	683	686.5	6.55	666	678.44	3.9
410	514	517	519.7	0.58	524	529.42	1.95	528	530.6	2.72	516	521.43	0.39	523	529.83	1.75	518	522.87	0.78
51	253	265	268.73	4.74	269	277.06	6.32	271	276.5	7.11	266	271.44	5.14	272	276.33	7.51	269	273.0	6.32
52	302	318	322.36	5.3	324	330.87	7.28	322	325.6	6.62	314	324.24	3.97	328	330.5	8.61	316	325.92	4.64
53	226	229	229.8	1.33	230	233.0	1.77	231	232.67	2.21	228	231.06	0.88	230	233.17	1.77	229	230.93	1.33
54	242	242	246.83	0.0	248	251.26	2.48	250	251.67	3.31	245	249.89	1.24	248	251.5	2.48	247	250.25	2.07
55	211	212	213.67	0.47	214	217.42	1.42	215	217.67	1.9	212	214.72	0.47	216	217.17	2.37	212	216.45	0.47
56	213	213	216.57	0.0	221	225.65	3.76	222	225.8	4.23	215	221.94	0.94	228	228.5	7.04	216	220.83	1.41
57	293	302	302.0	3.07	307	311.19	4.78	310	312.5	5.8	296	302.22	1.02	308	309.67	5.12	298	303.36	1.71
58	288	291	291.0	1.04	292	296.61	1.39	297	297.0	3.12	288	292.07	0.0	292	296.25	1.39	291	294.07	1.04
59	279	280	281.57	0.36	284	287.65	1.79	284	285.25	1.79	281	285.07	0.72	287	288.83	2.87	280	283.43	0.36
510	265	265	268.73	0.0	269	277.06	1.51	271	276.5	2.26	266	271.44	0.38	272	276.33	2.64	269	273.0	1.51
61	138	140	143.0	1.45	143	145.8	3.62	140	144.9	1.45	141	144.06	2.17	143	145.61	3.62	141	143.94	2.17
62	146	146	149.83	0.0	152	155.55	4.11	155	156.0	6.16	148	151.8	1.37	150	156.33	2.74	146	152.15	0.0
63	145	145	147.83	0.0	147	149.61	1.38	147	148.83	1.38	146	149.94	0.69	147	150.0	1.38	146	148.84	0.69
64	131	131	132.0	0.0	132	134.26	0.76	131	133.5	0.0	131	133.55	0.0	134	135.17	2.29	131	133.81	0.0
65	161	161	167.33	0.0	172	178.9	6.83	166	176.17	3.11	161	173.29	0.0	168	175.83	4.35	161	169.86	0.0
a1	253	256	260.2	1.19	266	266.75	5.14	260	264.42	2.77	259	264.25	2.37	262	265.97	3.56	262	264.08	3.56
a2	252	257	261.67	1.98	267	271.06	5.95	270	272.8	7.14	260	264.58	3.17	264	269.2	4.76	260	267.91	3.17
a3	232	238	240.57	2.59	241	247.03	3.88	247	247.25	6.47	242	244.5	4.31	244	246.0	5.17	241	248.45	3.88
a4	234	237	239.83	1.28	246	250.29	5.13	246	249.2	5.13	237	244.31	1.28	245	249.33	4.7	240	245.43	2.56
a5	236	240	242.33	1.69	245	248.65	3.81	241	246.67	2.12	239	246.2	1.27	247	248.5	4.66	242	244.25	2.54
b1	69	69	70.0	0.0	71	72.0	2.9	69	74.1	0.0	69	71.41	0.0	69	71.65	0.0	69	72.61	0.0
b2	76	76	76.45	0.0	79	81.13	3.95	77	79.17	1.32	76	79.38	0.0	78	79.5	2.63	76	77.62	0.0
b3	80	80	81.09	0.0	82	84.29	2.5	81	82.17	1.25	80	84.95	0.0	82	83.5	2.5	80	81.74	0.0
b4	79	79	80.78	0.0	82	85.06	3.8	81	83.67	2.53	80	82.39	1.27	81	83.17	2.53	79	82.81	0.0
b5	72	72	72.5	0.0	74	75.1	2.78	73	74.0	1.39	72	73.69	0.0	73	74.4	1.39	72	74.04	0.0

Table 11. *Cont.*

		BCL			MIR			QL1			SA1			QL2			SA2		
Inst.	**Opt.**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**
c1	227	234	238.4	3.08	249	251.2	9.69	231	242.29	1.76	237	241.1	4.41	239	246.61	5.29	235	240.77	3.52
c2	219	227	227.5	3.65	238	242.81	8.68	234	238.25	6.85	225	239.0	2.74	236	238.33	7.76	226	234.5	3.2
c3	243	245	249.5	0.82	258	264.23	6.17	259	262.2	6.58	245	257.33	0.82	255	258.33	4.94	249	253.64	2.47
c4	219	223	226.92	1.83	229	235.39	4.57	232	233.8	5.94	224	234.83	2.28	231	233.67	5.48	223	229.86	1.83
c5	215	220	221.5	2.33	230	234.58	6.98	221	228.67	2.79	220	225.07	2.33	225	227.75	4.65	223	226.09	3.72
d1	60	61	61.6	1.67	66	67.2	10.0	61	70.87	1.67	60	67.61	0.0	61	64.16	1.67	61	64.32	1.67
d2	66	66	67.33	0.0	70	72.84	6.06	68	69.0	3.03	67	72.33	1.52	67	68.25	1.52	66	68.75	0.0
d3	72	72	73.78	0.0	77	80.35	6.94	77	78.0	6.94	74	80.29	2.78	75	76.67	4.17	74	76.83	2.78
d4	62	62	62.83	0.0	64	66.42	3.23	63	63.33	1.61	62	67.0	0.0	62	63.33	0.0	62	64.29	0.0
d5	61	61	62.1	0.0	64	67.32	4.92	62	64.17	1.64	61	64.71	0.0	62	64.17	1.64	61	62.68	0.0
		258.47	**261.7**	**1.46**	265.56	278	4.61	265.33	269.03	3.9	259.4	265.39	1.79	265.36	268.96	3.96	260.29	265.39	2.05

Table 12. Result comparison with GWO employing the approaches QL3, SA3, QL4, SA4, QL5, and SA5.

		QL3			SA3			QL4			SA4			QL5			SA5		
Inst.	**Opt.**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**
41	429	526	529.6	22.61	517	523.43	20.51	522	528.33	21.68	516	523.65	20.28	522	528.79	21.68	516	523.59	20.28
42	512	538	543.67	5.08	524	533.74	2.34	535	542.22	4.49	526	538.16	2.73	534	541.32	4.3	516	531.35	0.78
43	516	535	541.67	3.68	524	533.84	1.55	524	538.67	1.55	518	529.8	0.39	537	541.05	4.07	520	531.04	0.78
44	494	516	519.5	4.45	498	508.5	0.81	508	518.89	2.83	502	511.06	1.62	510	520.95	3.24	500	510.11	1.21
45	512	532	542.22	3.91	521	529.19	1.76	535	542.78	4.49	519	527.29	1.37	529	540.47	3.32	522	528.69	1.95
46	560	571	575.78	1.96	566	571.0	1.07	571	576.14	1.96	564	569.38	0.71	568	576.8	1.43	565	571.19	0.89
47	430	438	443.25	1.86	434	438.55	0.93	438	441.33	1.86	436	438.79	1.4	434	442.23	0.93	433	437.52	0.7
48	492	503	505.5	2.24	493	500.94	0.2	498	504.0	1.22	494	499.82	0.41	496	505.65	0.81	493	500.29	0.2
49	641	677	683.0	5.62	657	677.45	2.5	675	684.5	5.3	656	675.0	2.34	666	682.43	3.9	654	676.38	2.03
410	514	526	529.6	2.33	517	523.43	0.58	522	528.33	1.56	516	523.65	0.39	522	528.79	1.56	516	523.59	0.39
51	253	271	275.17	7.11	266	272.57	5.14	274	278.0	8.3	269	273.65	6.32	273	277.48	7.91	267	273.2	5.53
52	302	329	331.17	8.94	316	325.37	4.64	327	332.0	8.28	318	326.15	5.3	325	330.21	7.62	318	326.12	5.3
53	226	230	232.8	1.77	229	231.06	1.33	231	233.0	2.21	229	231.71	1.33	230	232.96	1.77	229	231.04	1.33
54	242	251	252.5	3.72	247	248.9	2.07	247	252.0	2.07	246	250.12	1.65	247	251.61	2.07	244	249.73	0.83
55	211	214	216.67	1.42	212	215.12	0.47	214	216.2	1.42	212	215.52	0.47	212	217.17	0.47	212	215.2	0.47
56	213	226	227.5	6.1	215	220.36	0.94	220	225.5	3.29	213	220.58	0.0	219	226.46	2.82	215	221.8	0.94

Table 12. *Cont.*

Inst.	Opt.	QL3 Best	QL3 Avg	QL3 RPD	SA3 Best	SA3 Avg	SA3 RPD	QL4 Best	QL4 Avg	QL4 RPD	SA4 Best	SA4 Avg	SA4 RPD	QL5 Best	QL5 Avg	QL5 RPD	SA5 Best	SA5 Avg	SA5 RPD
57	293	309	311.0	5.46	298	303.94	1.71	307	308.0	4.78	299	304.0	2.05	303	310.3	3.41	297	303.0	1.37
58	288	294	296.33	2.08	289	293.89	0.35	295	297.17	2.43	289	293.47	0.35	289	296.57	0.35	288	293.48	0.0
59	279	283	285.83	1.43	281	283.79	0.72	286	287.33	2.51	281	285.06	0.72	283	287.38	1.43	280	284.11	0.36
510	265	271	275.17	2.26	266	272.57	0.38	274	278.0	3.4	269	273.65	1.51	273	277.48	3.02	267	273.2	0.75
61	138	144	145.87	4.35	141	143.81	2.17	142	145.74	2.9	141	144.86	2.17	142	145.52	2.9	140	143.73	1.45
62	146	153	156.0	4.79	146	152.0	0.0	151	154.17	3.42	149	153.13	2.05	147	154.42	0.68	146	153.17	0.0
63	145	149	150.33	2.76	145	149.0	0.0	149	150.33	2.76	147	150.94	1.38	148	149.43	2.07	148	149.08	2.07
64	131	133	134.17	1.53	131	133.09	0.0	133	134.33	1.53	132	134.0	0.76	131	134.22	0.0	131	133.17	0.0
65	161	175	179.33	8.7	161	171.4	0.0	177	179.0	9.94	164	171.19	1.86	168	177.87	4.35	161	171.77	0.0
a1	253	263	266.26	3.95	261	267.17	3.16	262	266.65	3.56	259	264.4	2.37	261	266.35	3.16	258	263.15	1.98
a2	252	265	267.75	5.16	262	265.36	3.97	262	270.2	3.97	261	266.07	3.57	264	270.12	4.76	259	264.75	2.78
a3	232	243	244.83	4.74	240	244.36	3.45	241	245.2	3.88	239	243.33	3.02	243	246.38	4.74	238	243.05	2.59
a4	234	247	248.67	5.56	240	245.92	2.56	249	251.0	6.41	242	244.92	3.42	244	248.74	4.27	238	243.36	1.71
a5	236	241	248.0	2.12	242	245.09	2.54	240	244.33	1.69	242	245.07	2.54	240	247.17	1.69	241	244.05	2.12
b1	69	69	71.55	0.0	69	70.88	0.0	69	71.84	0.0	69	71.17	0.0	69	71.68	0.0	69	70.87	0.0
b2	76	78	80.0	2.63	76	78.04	0.0	78	79.5	2.63	76	78.0	0.0	77	79.04	1.32	76	78.12	0.0
b3	80	81	81.83	1.25	81	82.0	1.25	81	82.0	1.25	81	84.06	1.25	81	82.61	1.25	80	81.42	0.0
b4	79	82	84.0	3.8	80	82.45	1.27	81	82.83	2.53	80	83.72	1.27	82	83.83	3.8	80	81.73	1.27
b5	72	74	74.8	2.78	72	73.53	0.0	74	74.4	2.78	72	74.12	0.0	72	74.61	0.0	72	75.48	0.0
c1	227	232	248.13	2.2	235	242.82	3.52	239	247.45	5.29	236	239.57	3.96	237	246.9	4.41	236	241.38	3.96
c2	219	236	239.0	7.76	225	233.29	2.74	238	239.25	8.68	227	231.88	3.65	227	237.72	3.65	224	233.14	2.28
c3	243	259	260.67	6.58	250	257.91	2.88	259	261.17	6.58	249	255.12	2.47	248	259.72	2.06	248	254.73	2.06
c4	219	230	233.83	5.02	224	230.87	2.28	228	233.67	4.11	225	232.06	2.74	228	233.38	4.11	224	228.54	2.28
c5	215	226	227.67	5.12	219	224.75	1.86	224	227.0	4.19	220	224.82	2.33	226	230.33	5.12	219	226.23	1.86
d1	60	62	64.48	3.33	60	63.19	0.0	62	64.26	3.33	61	64.33	1.67	61	63.9	1.67	61	63.14	1.67
d2	66	68	69.0	3.03	67	69.21	1.52	68	69.0	3.03	67	68.12	1.52	67	69.48	1.52	67	68.68	1.52
d3	72	76	77.5	5.56	73	76.29	1.39	76	77.0	5.56	73	77.71	1.39	73	77.39	1.39	73	75.57	1.39
d4	62	64	64.0	3.23	62	63.06	0.0	62	63.67	0.0	62	65.29	0.0	62	64.15	0.0	62	63.23	0.0
d5	61	62	64.6	1.64	61	64.92	0.0	64	65.17	4.92	62	67.07	1.64	63	65.3	3.28	61	62.88	0.0
		265.6	268.89	4.26	259.84	265.29	1.92	264.71	268.7	4.01	260.18	265.45	2.19	262.96	268.81	3.07	259.2	264.87	1.76

Table 13. Result comparison with HHO employing the approaches BCL, MIR, QL1, SA1, QL2, and SA2.

			BCL			MIR			QL1			SA1			QL2			SA2	
Inst.	**Opt.**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**
41	429	520	522.64	21.21	523	530.35	21.91	527	529.0	22.84	514	523.17	19.81	524	528.33	22.14	516	522.5	20.28
42	512	528	535.71	3.12	537	544.1	4.88	538	541.78	5.08	517	530.94	0.98	536	541.78	4.69	519	535.0	1.37
43	516	526	530.67	1.94	538	544.1	4.26	531	540.0	2.91	523	530.36	1.36	537	540.38	4.07	522	531.91	1.16
44	494	503	511.56	1.82	510	521.35	3.24	513	519.38	3.85	500	508.65	1.21	514	521.0	4.05	500	508.5	1.21
45	512	520	530.73	1.56	539	546.45	5.27	536	541.22	4.69	518	530.36	1.17	534	541.56	4.3	523	529.54	2.15
46	560	565	570.71	0.89	573	581.71	2.32	569	576.0	1.61	567	572.14	1.25	574	577.71	2.5	566	573.33	1.07
47	430	436	437.8	1.4	438	444.03	1.86	438	442.57	1.86	435	437.19	1.16	440	443.5	2.33	436	438.08	1.4
48	492	497	501.83	1.02	504	508.58	2.44	498	504.0	1.22	496	501.47	0.81	498	503.83	1.22	494	499.33	0.41
49	641	664	672.0	3.59	677	685.45	5.62	677	679.67	5.62	657	677.35	2.5	682	684.33	6.4	663	678.42	3.43
410	514	520	522.64	1.17	523	530.35	1.75	527	529.0	2.53	514	523.17	0.0	524	528.33	1.95	516	522.5	0.39
51	253	269	272.82	6.32	272	278.03	7.51	273	276.17	7.91	267	272.0	5.53	279	279.0	10.28	270	272.08	6.72
52	302	318	323.73	5.3	327	331.94	8.28	326	328.6	7.95	314	324.94	3.97	330	330.6	9.27	316	325.08	4.64
53	226	230	231.0	1.77	232	233.61	2.65	232	233.0	2.65	230	231.21	1.77	230	232.0	1.77	228	231.0	0.88
54	242	246	248.92	1.65	249	252.03	2.89	249	251.33	2.89	246	249.38	1.65	250	251.5	3.31	247	249.82	2.07
55	211	214	215.7	1.42	214	217.55	1.42	215	217.0	1.9	212	215.75	0.47	217	218.0	2.84	214	216.31	1.42
56	213	220	221.25	3.29	222	227.29	4.23	227	227.33	6.57	213	219.5	0.0	218	221.67	2.35	216	221.58	1.41
57	293	303	304.33	3.41	308	312.32	5.12	309	310.67	5.46	298	303.29	1.71	310	310.67	5.8	297	304.25	1.37
58	288	292	293.8	1.39	296	297.74	2.78	296	297.0	2.78	291	294.5	1.04	292	296.0	1.39	292	296.42	1.39
59	279	284	285.22	1.79	283	288.35	1.43	285	286.67	2.15	280	283.25	0.36	286	287.67	2.51	282	284.17	1.08
510	265	269	272.82	1.51	272	278.03	2.64	273	276.17	3.02	267	272.0	0.75	279	279.0	5.28	270	272.08	1.89
61	138	141	142.6	2.17	146	146.4	5.8	143	144.84	3.62	141	144.96	2.17	143	145.32	3.62	141	145.58	2.17
62	146	149	151.58	2.05	155	156.84	6.16	151	154.67	3.42	146	152.73	0.0	152	155.17	4.11	146	150.67	0.0
63	145	147	147.91	1.38	147	149.42	1.38	147	149.5	1.38	145	148.08	0.0	149	149.67	2.76	148	149.71	2.07
64	131	131	132.58	0.0	133	134.29	1.53	133	134.0	1.53	131	133.62	0.0	132	133.4	0.76	131	133.67	0.0
65	161	168	171.71	4.35	173	178.58	7.45	176	178.0	9.32	162	169.09	0.62	171	176.67	6.21	163	171.46	1.24
a1	253	262	263.4	3.56	263	266.5	3.95	261	265.19	3.16	262	265.91	3.56	261	265.1	3.16	260	264.91	2.77
a2	252	259	263.83	2.78	265	270.74	5.16	267	268.8	5.95	260	264.92	3.17	266	269.0	5.56	258	264.5	2.38
a3	232	241	243.0	3.88	243	245.94	4.74	244	245.17	5.17	239	243.67	3.02	243	244.83	4.74	241	244.82	3.88
a4	234	244	244.33	4.27	244	248.65	4.27	244	245.33	4.27	239	242.9	2.14	244	247.0	4.27	239	243.6	2.14
a5	236	241	242.33	2.12	245	248.58	3.81	247	248.0	4.66	243	247.2	2.97	244	245.6	3.39	242	245.55	2.54
b1	69	69	70.0	0.0	71	72.0	2.9	70	70.81	1.45	69	70.71	0.0	69	70.9	0.0	69	70.6	0.0
b2	76	76	77.09	0.0	78	80.16	2.63	78	78.83	2.63	76	79.48	0.0	77	78.33	1.32	76	79.0	0.0
b3	80	81	81.27	1.25	82	82.52	2.5	81	81.83	1.25	81	82.55	1.25	82	82.17	2.5	80	82.43	0.0
b4	79	81	82.0	2.53	82	83.94	3.8	80	82.5	1.27	79	83.5	0.0	83	83.33	5.06	80	83.54	1.27
b5	72	72	72.2	0.0	74	74.52	2.78	73	73.67	1.39	72	75.32	0.0	73	73.83	1.39	72	73.92	0.0

Table 13. *Cont.*

			BCL			MIR			QL1			SA1			QL2			SA2	
Inst.	**Opt.**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**
c1	227	237	239.25	4.41	246	249.6	8.37	237	244.65	4.41	233	240.42	2.64	241	245.94	6.17	235	242.1	3.52
c2	219	234	234.0	6.85	237	240.74	8.22	237	238.33	8.22	222	236.42	1.37	238	238.0	8.68	229	234.09	4.57
c3	243	252	253.33	3.7	258	261.71	6.17	256	258.67	5.35	245	257.77	0.82	258	258.67	6.17	250	256.09	2.88
c4	219	227	228.86	3.65	231	234.68	5.48	230	231.75	5.02	225	234.85	2.74	231	232.17	5.48	225	232.27	2.74
c5	215	223	223.0	3.72	228	232.29	6.05	225	228.0	4.65	221	230.77	2.79	227	227.33	5.58	221	227.18	2.79
d1	60	62	62.6	3.33	65	65.8	8.33	62	64.06	3.33	61	64.05	1.67	63	64.19	5.0	61	62.17	1.67
d2	66	67	67.83	1.52	69	69.61	4.55	68	68.67	3.03	67	75.56	1.52	68	68.67	3.03	67	68.27	1.52
d3	72	74	75.17	2.78	77	78.29	6.94	76	76.67	5.56	74	77.79	2.78	75	76.33	4.17	74	76.42	2.78
d4	62	62	62.67	0.0	63	63.77	1.61	62	63.0	0.0	62	65.63	0.0	62	62.67	0.0	62	66.92	0.0
d5	61	61	62.57	0.0	65	66.0	6.56	63	64.6	3.28	61	63.8	0.0	64	64.6	4.92	62	63.77	1.64
		261.89	**264.47**	2.8	266.16	270.11	4.75	265.56	268.14	4.2	**259.44**	265.61	**1.84**	266.0	268.35	4.37	260.42	265.45	2.23

Table 14. Result comparison with HHO employing the approaches QL3, SA3, QL4, SA4, QL5, and SA5.

			QL3			SA3			QL4			SA4			QL5			SA5	
Inst.	**Opt.**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**	**Best**	**Avg**	**RPD**
41	429	528	528.5	23.08	519	524.54	20.98	525	528.6	22.38	519	524.4	20.98	522	527.26	21.68	514	522.29	19.81
42	512	537	543.14	4.88	517	534.27	0.98	537	540.89	4.88	524	537.36	2.34	535	542.32	4.49	520	532.79	1.56
43	516	536	539.67	3.88	523	531.62	1.36	530	538.33	2.71	523	531.31	1.36	534	539.68	3.49	523	532.39	1.36
44	494	519	521.75	5.06	501	510.44	1.42	512	516.89	3.64	504	514.08	2.02	512	518.37	3.64	503	509.5	1.82
45	512	533	539.78	4.1	518	529.43	1.17	526	536.33	2.73	519	529.5	1.37	529	539.05	3.32	516	529.81	0.78
46	560	575	577.57	2.68	566	570.93	1.07	574	578.88	2.5	563	569.64	0.54	573	578.91	2.32	565	571.75	0.89
47	430	443	443.0	3.02	434	440.38	0.93	440	441.67	2.33	435	439.33	1.16	438	442.39	1.86	435	439.0	1.16
48	492	501	505.5	1.83	496	502.75	0.81	502	505.17	2.03	498	501.81	1.22	501	505.04	1.83	498	502.62	1.22
49	641	684	687.33	6.71	659	678.87	2.81	685	686.0	6.86	664	677.22	3.59	669	683.96	4.37	659	675.17	2.81
410	514	528	528.5	2.72	519	524.54	0.97	525	528.6	2.14	519	524.4	0.97	522	527.26	1.56	514	522.29	0.0
51	253	275	277.33	8.7	267	273.0	5.53	276	278.0	9.09	269	273.12	6.32	272	276.65	7.51	267	272.37	5.53
52	302	327	329.67	8.28	319	323.93	5.63	323	327.0	6.95	317	325.0	4.97	320	329.75	5.96	318	326.11	5.3
53	226	232	232.5	2.65	228	230.75	0.88	232	232.67	2.65	230	231.21	1.77	231	232.87	2.21	229	231.31	1.33
54	242	250	251.5	3.31	247	249.67	2.07	249	251.17	2.89	245	248.71	1.24	247	250.78	2.07	245	249.72	1.24
55	211	218	218.0	3.32	212	215.6	0.47	216	217.33	2.37	212	215.64	0.47	216	217.28	2.37	213	216.18	0.95
56	213	221	225.55	3.76	215	220.53	0.94	227	227.5	6.57	213	220.79	0.0	223	226.3	4.69	215	221.0	0.94

Table 14. *Cont.*

Inst.	Opt.	QL3 Best	QL3 Avg	QL3 RPD	SA3 Best	SA3 Avg	SA3 RPD	QL4 Best	QL4 Avg	QL4 RPD	SA4 Best	SA4 Avg	SA4 RPD	QL5 Best	QL5 Avg	QL5 RPD	SA5 Best	SA5 Avg	SA5 RPD
57	293	309	311.0	5.46	299	304.43	2.05	309	310.0	5.46	297	305.07	1.37	304	310.38	3.75	297	303.25	1.37
58	288	295	296.5	2.43	289	293.56	0.35	292	296.0	1.39	288	292.85	0.0	292	296.41	1.39	290	292.71	0.69
59	279	287	288.0	2.87	280	284.64	0.36	283	285.67	1.43	281	286.31	0.72	283	287.09	1.43	281	284.47	0.72
510	265	275	277.33	3.77	267	273.0	0.75	276	278.0	4.15	269	273.12	1.51	272	276.65	2.64	267	272.37	0.75
61	138	143	144.97	3.62	141	144.56	2.17	142	144.84	2.9	141	144.71	2.17	141	144.52	2.17	142	146.0	2.9
62	146	153	155.0	4.79	147	150.74	0.68	152	154.67	4.11	146	151.94	0.0	151	154.74	3.42	149	152.67	2.05
63	145	149	149.67	2.76	147	149.22	1.38	147	149.33	1.38	148	150.0	2.07	147	149.39	1.38	147	149.0	1.38
64	131	133	134.0	1.53	131	133.87	0.0	134	134.17	2.29	131	133.5	0.0	131	133.83	0.0	131	133.64	0.0
65	161	170	175.67	5.59	164	172.53	1.86	175	178.5	8.7	165	172.79	2.48	171	177.22	6.21	167	172.67	3.73
a1	253	261	265.1	3.16	260	264.58	2.77	262	265.42	3.56	259	263.87	2.37	261	264.94	3.16	260	265.38	2.77
a2	252	267	269.2	5.95	261	267.83	3.57	267	270.2	5.95	260	266.54	3.17	263	268.52	4.37	258	266.53	2.38
a3	232	244	245.33	5.17	240	243.75	3.45	245	245.6	5.6	239	243.92	3.02	240	244.96	3.45	240	243.5	3.45
a4	234	246	247.0	5.13	241	245.64	2.99	246	247.33	5.13	240	244.67	2.56	240	246.68	2.56	238	242.86	1.71
a5	236	246	247.0	4.24	240	244.85	1.69	241	244.33	2.12	241	244.79	2.12	243	246.39	2.97	240	243.35	1.69
b1	69	69	70.84	0.0	69	70.32	0.0	70	71.16	1.45	69	73.44	0.0	70	71.23	1.45	70	71.58	1.45
b2	76	78	78.5	2.63	76	78.6	0.0	76	78.17	0.0	76	79.79	0.0	77	78.78	1.32	77	78.54	1.32
b3	80	82	82.0	2.5	80	81.8	0.0	82	82.0	2.5	81	81.71	1.25	81	82.0	1.25	81	82.27	1.25
b4	79	82	82.83	3.8	80	83.08	1.27	83	83.2	5.06	81	83.47	2.53	80	83.12	1.27	79	82.75	0.0
b5	72	74	74.0	2.78	72	73.65	0.0	73	73.67	1.39	73	74.69	1.39	72	73.39	0.0	73	76.25	1.39
c1	227	241	246.06	6.17	238	240.18	4.85	239	245.16	5.29	232	239.77	2.2	238	245.42	4.85	235	239.12	3.52
c2	219	233	236.33	6.39	225	234.17	2.74	233	236.67	6.39	227	234.86	3.65	234	237.04	6.85	229	233.69	4.57
c3	243	253	256.67	4.12	252	256.27	3.7	256	258.83	5.35	250	258.17	2.88	254	258.09	4.53	246	254.0	1.23
c4	219	230	231.5	5.02	229	235.4	4.57	228	230.8	4.11	226	229.93	3.2	229	232.15	4.57	225	231.24	2.74
c5	215	228	228.67	6.05	222	226.55	3.26	226	228.0	5.12	221	223.86	2.79	225	228.34	4.65	221	224.67	2.79
d1	60	63	64.19	5.0	61	63.23	1.67	63	64.32	5.0	60	63.5	0.0	62	64.09	3.33	61	64.64	1.67
d2	66	68	68.67	3.03	67	69.7	1.52	68	68.75	3.03	67	68.36	1.52	68	68.65	3.03	67	68.18	1.52
d3	72	76	76.67	5.56	72	76.74	0.0	77	77.33	6.94	74	77.45	2.78	75	76.85	4.17	74	78.36	2.78
d4	62	62	62.33	0.0	62	63.76	0.0	63	63.0	1.61	62	65.45	0.0	62	63.04	0.0	62	64.0	0.0
d5	61	64	64.75	4.92	62	63.28	1.64	63	64.8	3.28	62	67.36	1.64	63	64.56	3.28	62	65.08	1.64
		266.4	268.42	4.5	260.31	265.58	2.16	265.56	268.11	4.3	260.44	265.76	2.22	263.84	268.14	3.48	267	265.27	2.23

In Tables 7 and 8, the approach SA1 leads with mean values for the columns Best, Avg, and RPD with 259.56, 263.21, and 1.78, respectively. Nevertheless, in terms of the computed performance, other SA approaches follow right behind SA1. In Tables 9 and 10, the lead in performance is shared by SA1 with the best mean value for the column Best with 259.91 and SA5 with the best computed mean values for the columns Avg and RPD with 265.11 and 2.02, respectively. Here, we can observe more robustness in the performance by SA5 and some inconsistency by SA1.

In Tables 11 and 12, the approach BCL leads the overall performance with the minimum mean values for the columns Best, Avg, and RPD, followed by SA5 and SA1. In Tables 13 and 14, the approach SA1 leads the best values reached (Best) with the smallest RPD values with 259.44 and 1.84, respectively. The approach with the best mean for the column Avg is BCL with 264.47. Nevertheless, approaches employing SARSA follow close to the leaders in the performance, which proves the effectiveness of the proposal.

$$\mathrm{RPD} = \frac{100 \cdot (Best - Opt)}{Opt}. \tag{28}$$

4.2. Statistical Results

A p-value lesser than 0.05 means that the difference between the techniques is statistically significant, and thus the comparison of their averages is valid.

The results obtained are grouped in Tables 15–18. For each table, a matrix of the averages obtained from the 45 instances is illustrated. The description for each table is as follows: the first row and first columns present the 11 versions of the MH to be compared. We can read the information by row as follows: obtaining a p-value less than 0.05 means that the version in the row obtained a better performance over the version located in the column for the SCP and that the difference between the results is statistically significant.

p-values greater than 0.05 have been replaced by ">0.05" to facilitate the reading of the comparison matrix. In this context, significant differences in the performance can be observed in Tables 15 and 16. First, all the hybrid approaches employing learning-based components outperformed the classic approach employing two-step transformation (BCL). The performances between approaches employing the same RL techniques, such as Q-learning (QL1-QL5 vs. QL1-QL5) and SARSA (SA1-SA5 vs. SA1-SA5), performed equally.

Lastly, approaches employing the RL technique SARSA in almost all instances significantly outperformed the ones employing Q-learning. On the other hand, an interesting phenomenon can be observed in Tables 17 and 18. The only major differences in performance were observed between BCL against the approaches employing Q-learning (QL1–QL5) and SA4–SA5. The performances between approaches employing the same and different RL techniques were not significantly different.

Table 15. The average Wilcoxon–Mann–Whitney test of WOA.

Ver.	BCL	QL1	QL2	QL3	QL4	QL5	SA1	SA2	SA3	SA4	SA5
BCL	-	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05
QL1	**0.00**	-	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05
QL2	**0.00**	≥0.05	-	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05
QL3	**0.00**	≥0.05	≥0.05	-	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05
QL4	**0.00**	≥0.05	≥0.05	≥0.05	-	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05
QL5	**0.00**	≥0.05	≥0.05	≥0.05	≥0.05	-	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05
SA1	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	-	≥0.05	≥0.05	≥0.05	≥0.05
SA2	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	≥0.05	-	≥0.05	≥0.05	≥0.05
SA3	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	≥0.05	≥0.05	-	≥0.05	≥0.05
SA4	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	≥0.05	≥0.05	≥0.05	-	≥0.05
SA5	**0.00**	**0.00**	**0.00**	**0.01**	**0.00**	**0.01**	≥0.05	≥0.05	≥0.05	≥0.05	-

Table 16. The average Wilcoxon–Mann–Whitney test of SCA.

Ver.	BCL	QL1	QL2	QL3	QL4	QL5	SA1	SA2	SA3	SA4	SA5
BCL	-	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05
QL1	**0.00**	-	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05
QL2	**0.00**	≥0.05	-	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05
QL3	**0.00**	≥0.05	≥0.05	-	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05
QL4	**0.00**	≥0.05	≥0.05	≥0.05	-	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05
QL5	**0.00**	≥0.05	≥0.05	≥0.05	≥0.05	-	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05
SA1	**0.00**	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	-	≥0.05	≥0.05	≥0.05	≥0.05
SA2	**0.00**	**0.02**	**0.02**	**0.02**	**0.01**	**0.02**	≥0.05	-	≥0.05	≥0.05	≥0.05
SA3	**0.00**	**0.02**	**0.02**	**0.02**	**0.02**	**0.02**	≥0.05	≥0.05	-	≥0.05	≥0.05
SA4	**0.00**	**0.04**	**0.04**	**0.03**	**0.03**	**0.03**	≥0.05	≥0.05	≥0.05	-	≥0.05
SA5	**0.00**	≥0.05	**0.04**	**0.04**	**0.03**	**0.04**	≥0.05	≥0.05	≥0.05	≥0.05	-

Table 17. The average Wilcoxon–Mann–Whitney test of GWO.

Ver.	BCL	QL1	QL2	QL3	QL4	QL5	SA1	SA2	SA3	SA4	SA5
BCL	-	**0.01**	**0.00**	**0.01**	**0.00**	**0.00**	≥0.05	≥0.05	≥0.05	**0.04**	**0.03**
QL1	≥0.05	-	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05
QL2	≥0.05	≥0.05	-	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05
QL3	≥0.05	≥0.05	≥0.05	-	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05
QL4	≥0.05	≥0.05	≥0.05	≥0.05	-	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05
QL5	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	-	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05
SA1	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	-	≥0.05	≥0.05	≥0.05	≥0.05
SA2	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	-	≥0.05	≥0.05	≥0.05
SA3	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	-	≥0.05	≥0.05
SA4	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	-	≥0.05
SA5	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	-

Table 18. The average Wilcoxon–Mann–Whitney test of HHO.

Ver.	BCL	QL1	QL2	QL3	QL4	QL5	SA1	SA2	SA3	SA4	SA5
BCL	-	**0.02**	**0.01**	**0.01**	**0.01**	**0.01**	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05
QL1	≥0.05	-	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05
QL2	≥0.05	≥0.05	-	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05
QL3	≥0.05	≥0.05	≥0.05	-	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05
QL4	≥0.05	≥0.05	≥0.05	≥0.05	-	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05
QL5	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	-	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05
SA1	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	-	≥0.05	≥0.05	≥0.05	≥0.05
SA2	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	-	≥0.05	≥0.05	≥0.05
SA3	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	-	≥0.05	≥0.05
SA4	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	-	≥0.05
SA5	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	≥0.05	-

4.3. Action Charts

The charts of average actions performed are illustrated in Figures 6–13. They represent the graphical representation of the average choice of Q-Learning or SARSA during the iterative process. They generate a weight system in order to properly select the binarization schemes, and the objective is the identification of preferences according to the state of exploitation or exploration of the environment using the run time. The graphs are composed in the x-axis by the average number of times the action was selected for the respective state, while the 40 possible actions to be taken for the binarization scheme selector are in the y-axis.

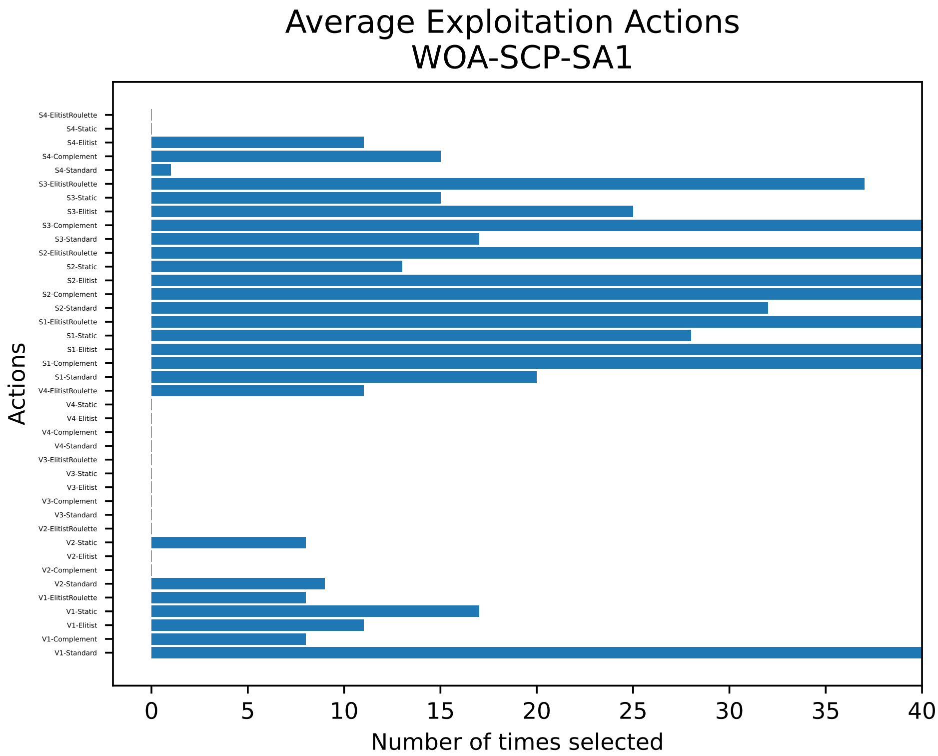

Figure 6. WOA-SA1—The average number of actions in the exploitation state.

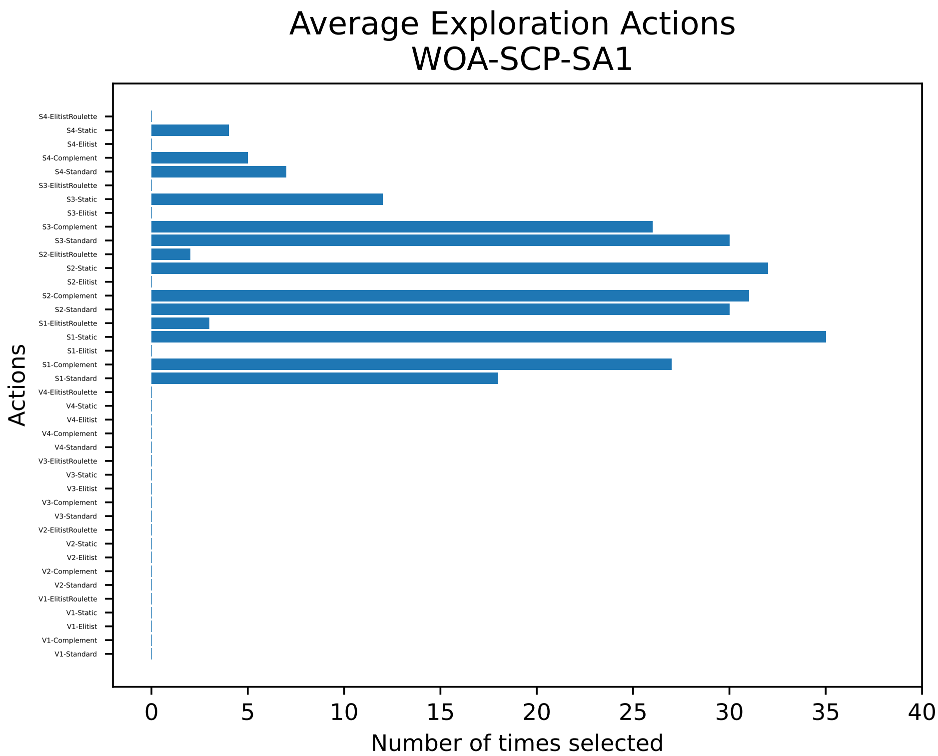

Figure 7. WOA-SA1—The average number of actions in the exploration state.

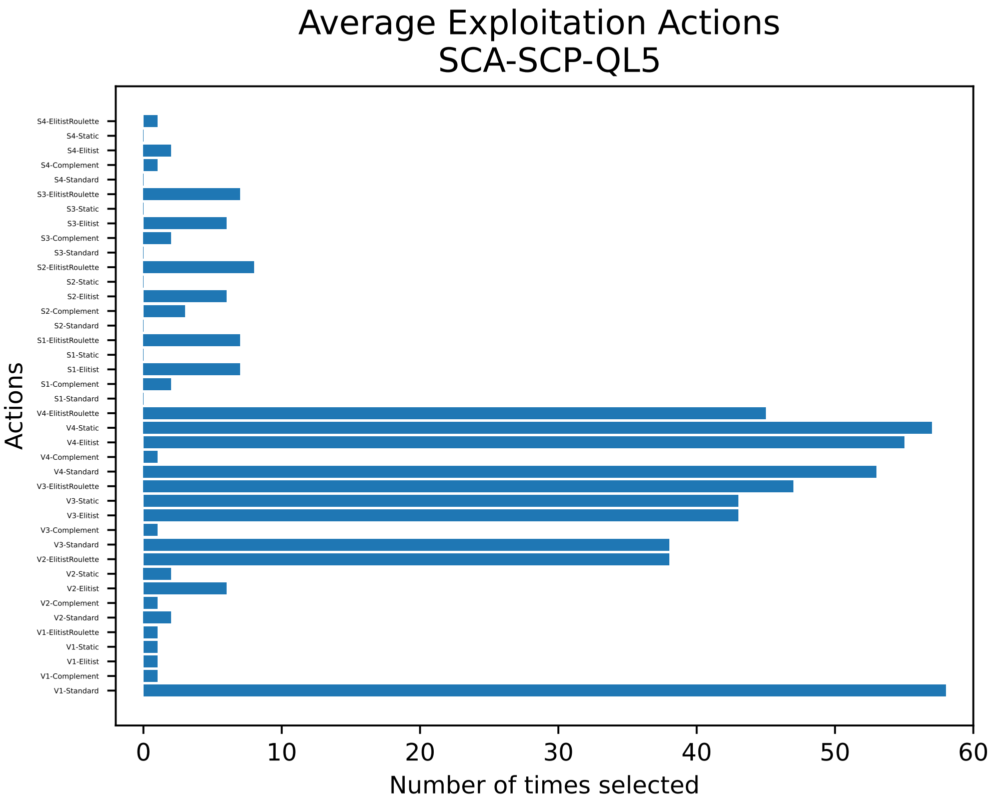

Figure 8. SCA-QL5—The average number of actions in the exploitation state.

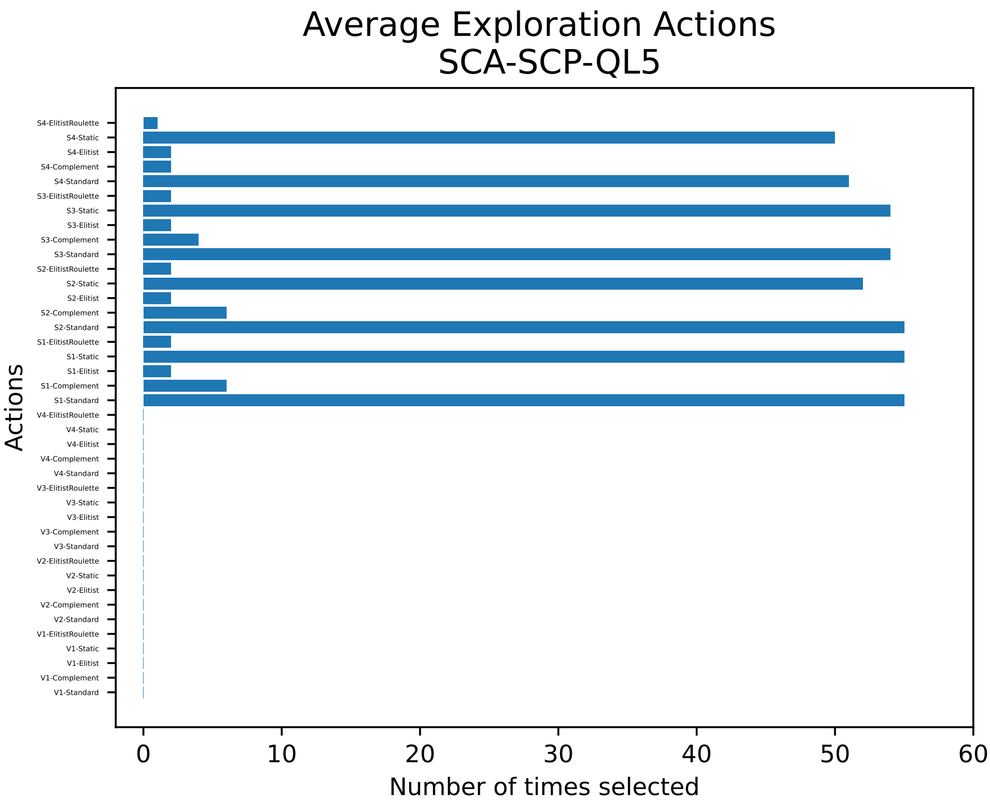

Figure 9. SCA-QL5—The average number of actions in the exploration state.

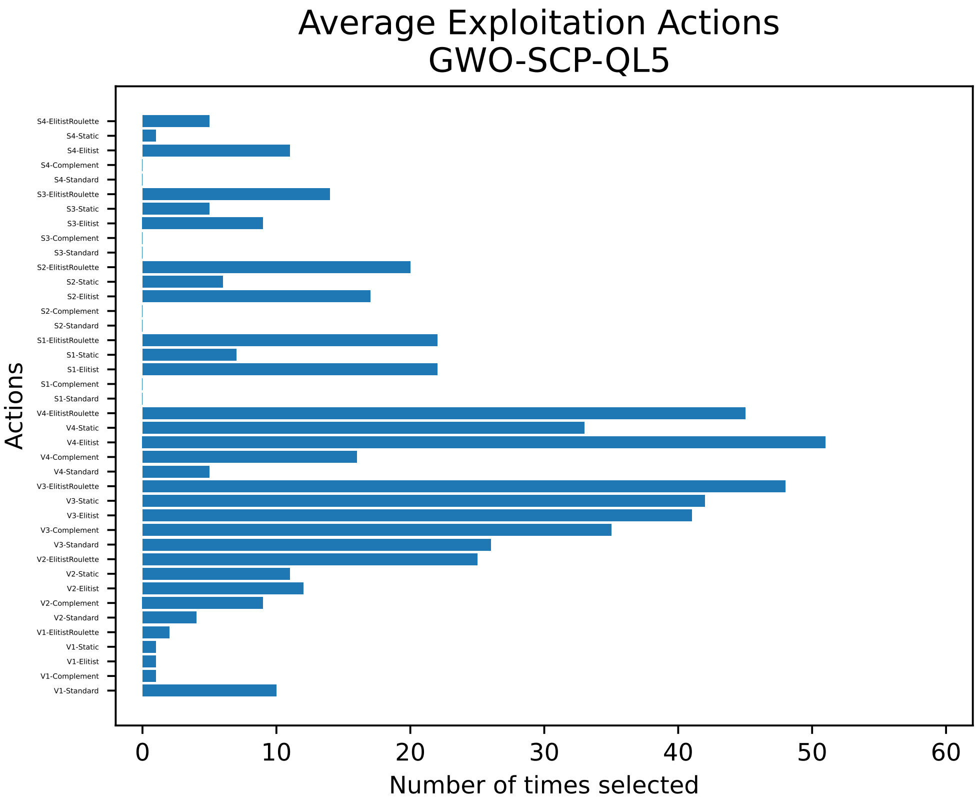

Figure 10. GWO-QL5—The average number of actions in the exploitation state.

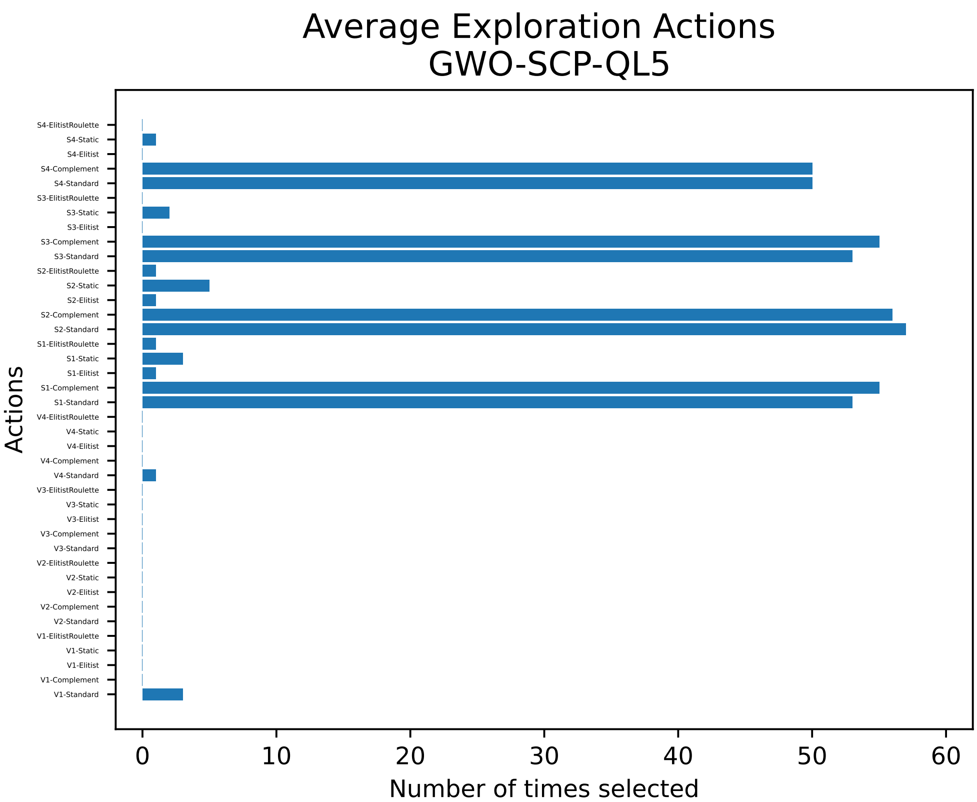

Figure 11. GWO-QL5—The average number of actions in the exploration state.

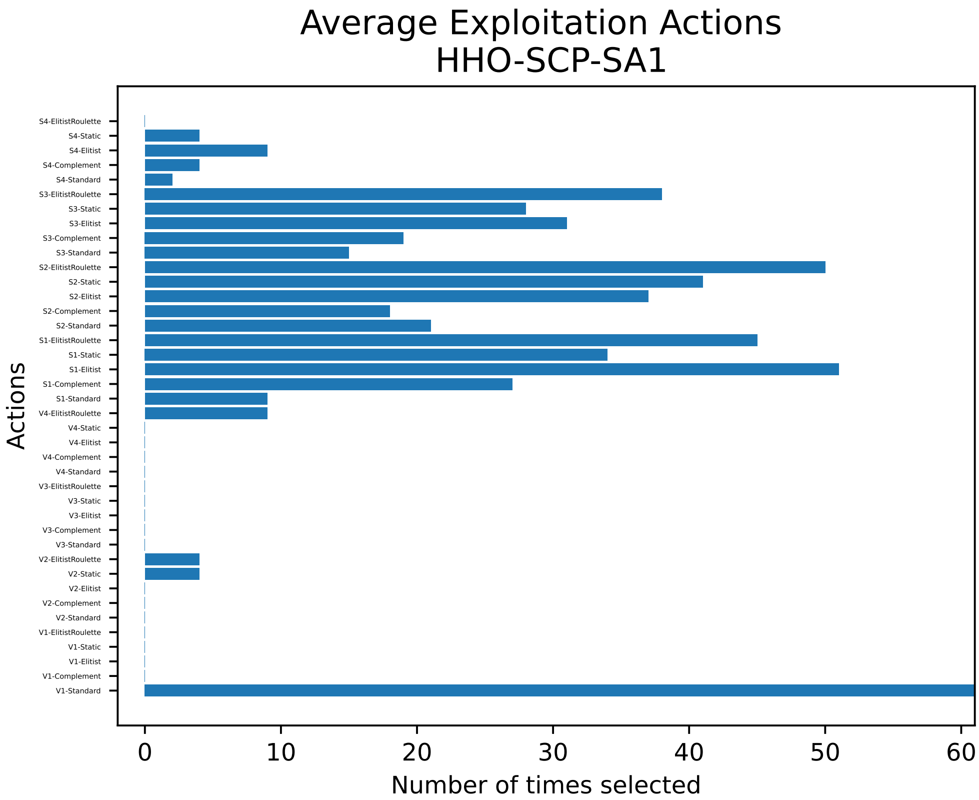

Figure 12. HHO-SA1—The average number of actions in the exploitation state.

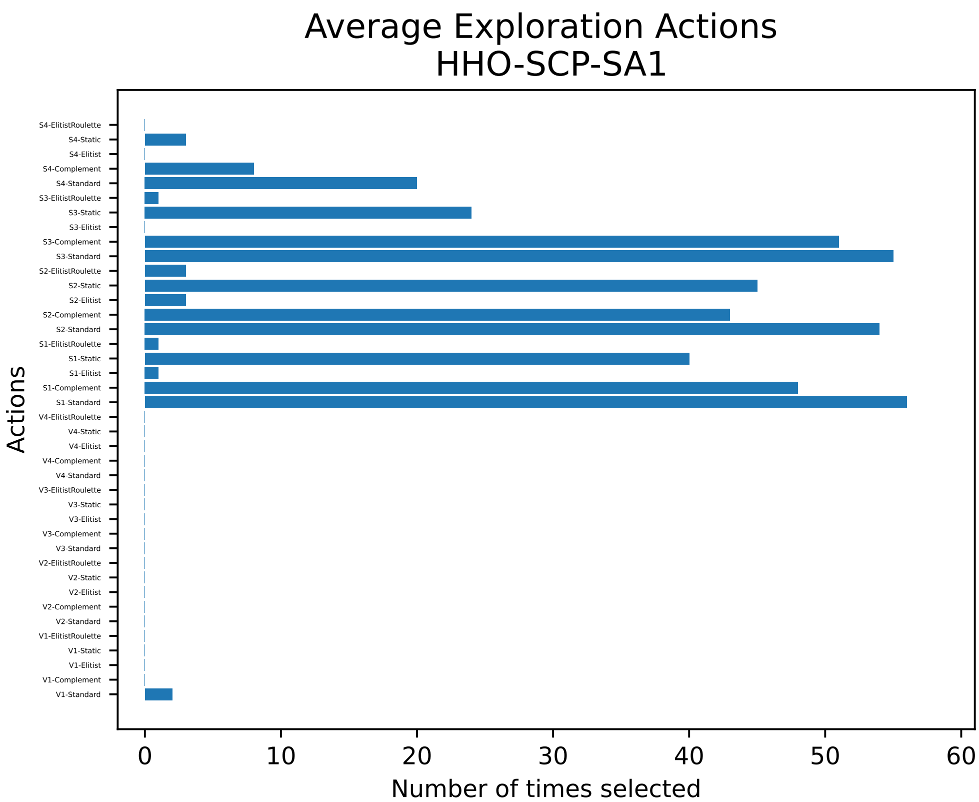

Figure 13. HHO-SA1 —The average number of actions in the exploration state.

4.4. Exploration and Exploitation Charts

The visualization of MH metrics is fundamental to understanding their behavior through the search, and thus the exploration and exploitation graphs presented in Morales-Castañeda et al. [38] are of great contribution to the analysis of exploration and exploitation in terms of diversity among the solutions. In this work, the decision-making of the state was calculated by means of the Dimensional-Hussain diversity (Section 3.4). In the exploration and exploitation plots illustrated in Figures 14–21, the x-axis is the number of total iterations, while the y-axis is the percentage of exploration and exploitation. They are measured by Equations (26) and (27)—the results obtained from the representative instances for each MH.

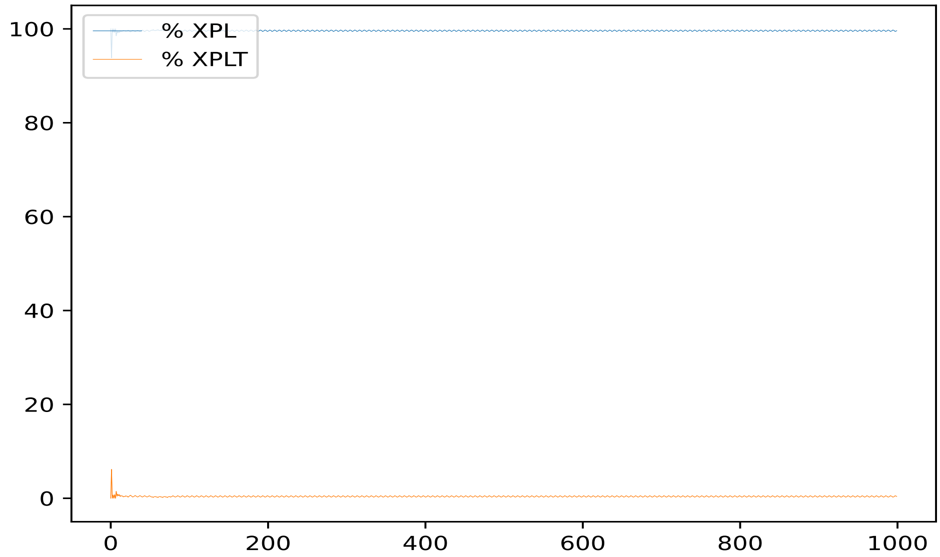

Figure 14. WOA-MIR-510 Dimensional Hussain.

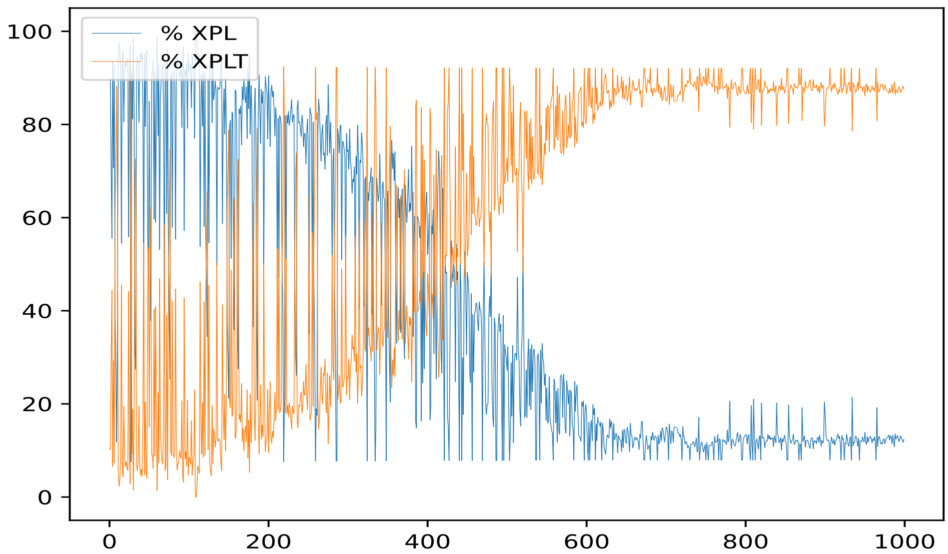

Figure 15. WOA-SA1-510 Dimensional Hussain.

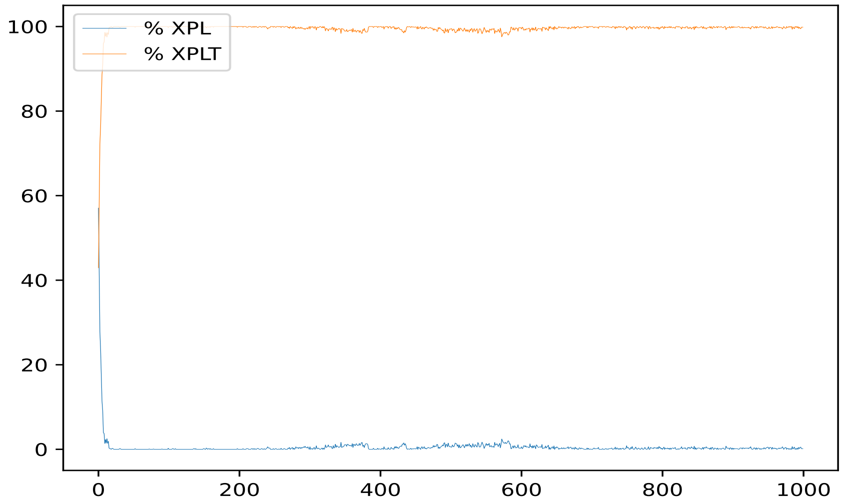

Figure 16. SCA-BCL-65 Dimensional Hussain.

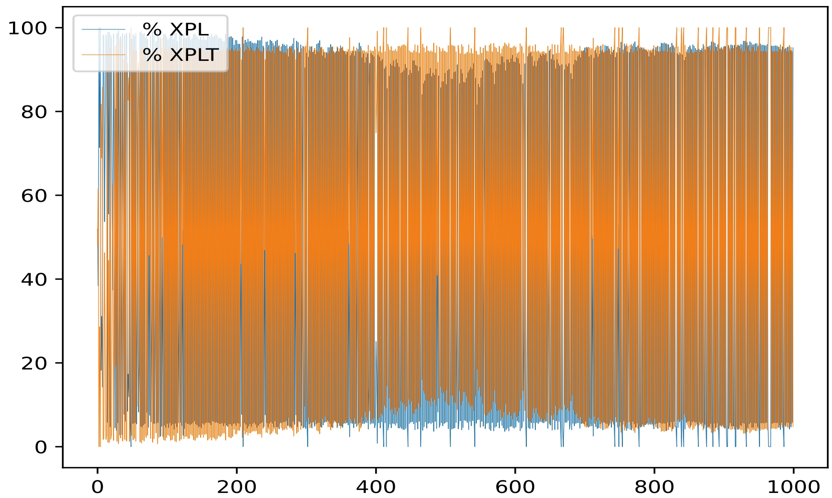

Figure 17. SCA-QL5-65 Dimensional Hussain.

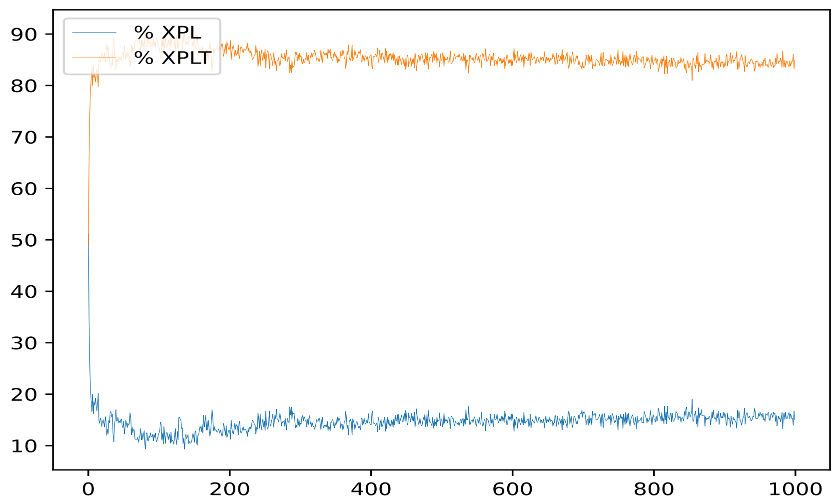

Figure 18. GWO-BCL-b5 Dimensional Hussain.

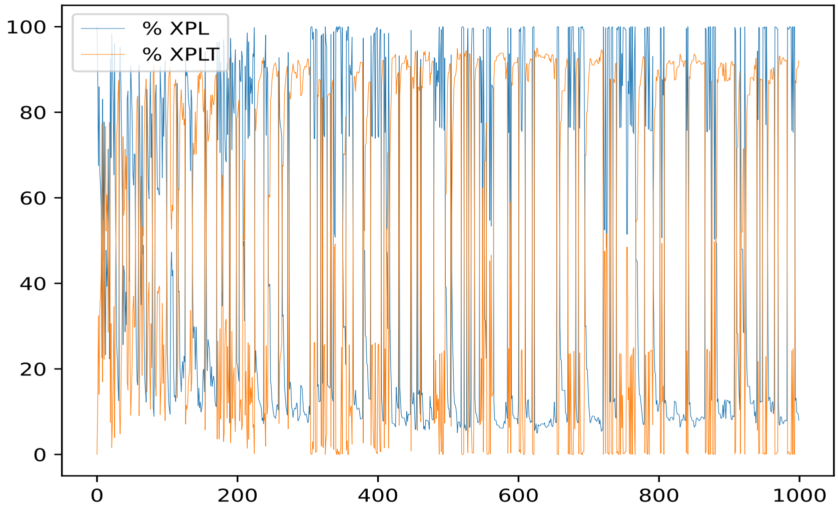

Figure 19. GWO-QL5-b5 Dimensional Hussain.

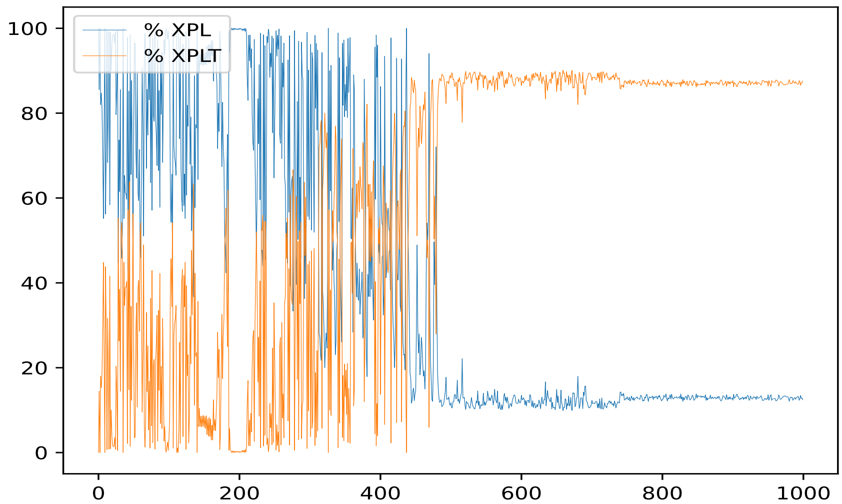

Figure 20. HHO-MIR-d5 Dimensional Hussain.

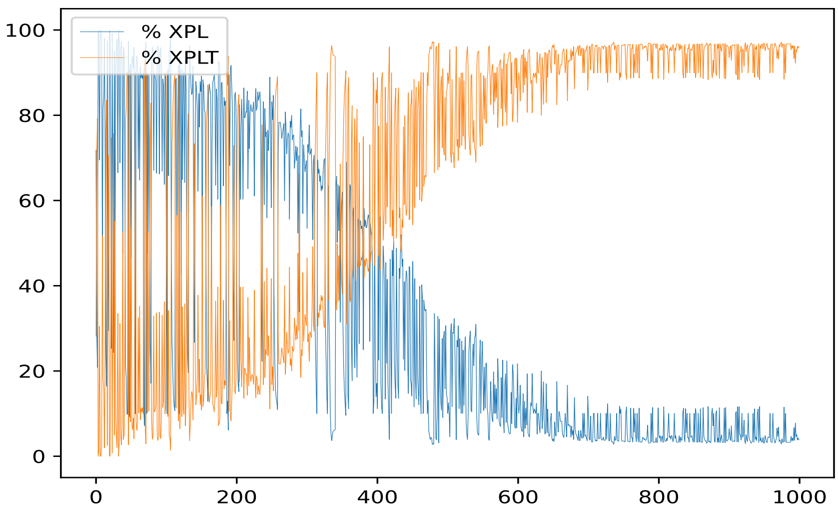

Figure 21. HHO-SA1-d5 Dimensional Hussain.

5. Discussions and Analysis

In this section, a discussion and detailed analysis of the results achieved in Section 4 are presented.

5.1. Best, Average, and RDP

First, by observing the results obtained and summarized in Figures 22–25, the following ideas are highlighted:

- In SCA, the approach SA1 obtained the best performance regarding the best values reached (Best). However, if we observe the values computed for average, and RPD, the best performance was achieved by the approach SA5. In WOA, the approach SA1 obtained the best performance all-around reported for the values Best, average, and RPD.
- In GWO, the approach BCL obtained the best performance when comparing the computed values for Best, average, and RPD.
- In HHO, the approach SA1 obtained the best performance when comparing Best and RPD. However, if we observe the values computed for average, the best performance was achieved by the BCL version.
- In SCA and WOA, all versions with a binarization scheme selector obtained better performance against versions with static binarization schemes when comparing the Best, average, and RPD.
- In SCA, WOA, GWO, and HHO, the best performance overall was observed for Best, average, and RPD was achieved by the proposed SARSA versions against the ones employing Q-Learning. Nevertheless, the approach SA5 was the only version employing SARSA that was outperformed by QL5.
- In SCA, WOA, GWO, and HHO, observing the overall performance achieved by the two well-known static binarization methods, BCL led the results and greatly outperformed the MIR approaches.

While conducting a comparison of behaviors of the MH, we can observe that the SCA and WOA, achieved improvements in their performance when using Q-Learning and SARSA. On the other hand, regarding HHO and GWO, statistically significant improvements were not obtained. In this context, one of the reasons for this behavior lies in the movement operators of each MH. In HHO and GWO, we observe operators of higher complexity, which follow different logic according to the behavior of their internal parameters.

For instance, E in the case of HHO, the energy is decreasing during the iterations, thus, influencing the motion operator employed. This is based on the logic that, the first iterations should be explored and the last ones exploited. In the case of SCA and WOA, we observe simpler movement operators, where the use of the exploration or exploitation operators depends on random decisions. For instance, SCA with parameter r_4 and p in the case of WOA.

5.2. Average Wilcoxon Test

The Wilcoxon–Mann–Whitney test is the non-parametric test we used to compare two independent samples. In Tables 15–18, the average p-values obtained are presented in order to simplify their visualization. From these tables, the following is observed:

- All the MIR versions obtained a worse performance compared to the rest of the versions, in addition to the fact that their difference is statistically significant, and thus it was eliminated from the Tables 15–18, in order to facilitate the comparison.
- For WOA and SCA, there was no statistically significant difference between BCL versus Q-Learning versions.
- For GWO and HHO, there was a statistically significant difference between the versions of BCL and Q-Learning.

- For WOA, SCA, GWO, and HHO, there was no statistically significant difference between the versions of BCL and SARSA, except for GWO with the versions SA4 and SA5.
- For WOA, there was a statistically significant difference between the versions of SARSA versus Q-Learning.
- For SCA, there was a statistically significant difference between the SARSA versions versus the Q-Learning versions, except for the SA1 version versus the QL1, QL2, and QL3 versions.
- For GWO and HHO, there was no statistically significant difference between the SARSA versions versus the Q-Learning versions.

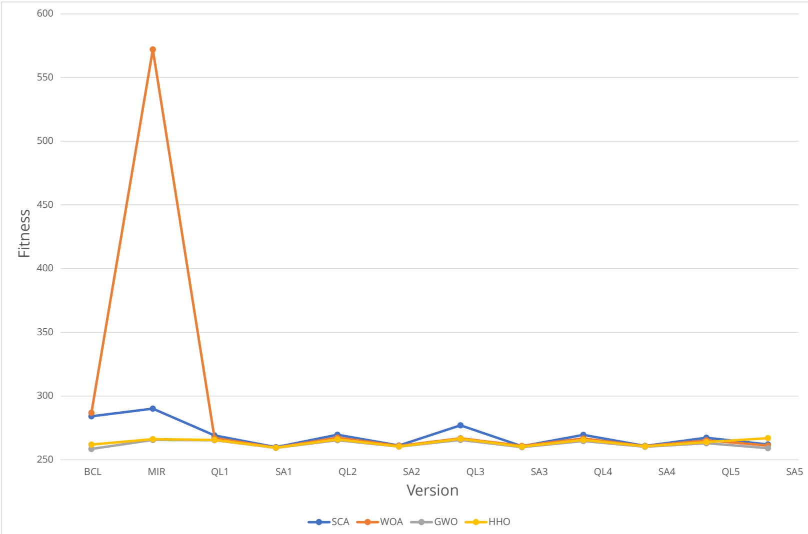

Figure 22. The average Best fitness.

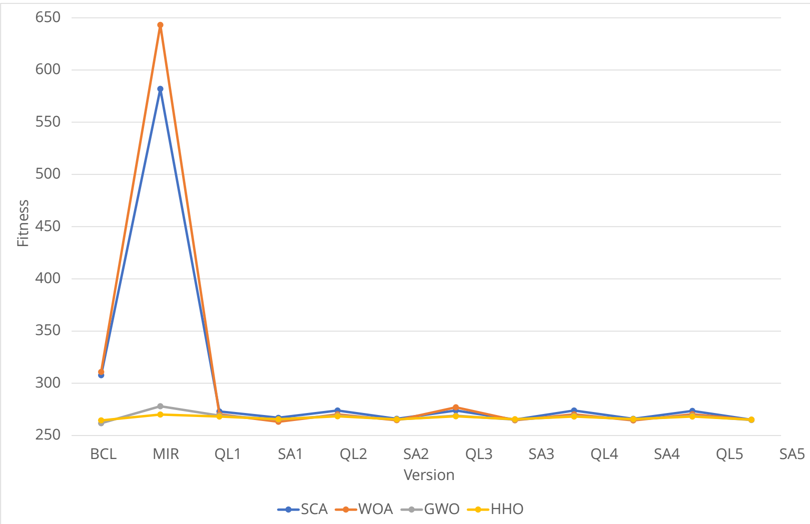

Figure 23. The average Avg fitness.

5.3. Choice of Binarization Schemes

It is known from the literature that binarization schemes have a strong impact on the performance of the MH [5], and thus Figures 6–12 give us detailed information related to the employment of these during the exploration and exploitation processes, which can be described as follows:

- The action corresponding to MIR (V4 and Complement), had the lowest selection rate for actions by both Q-Learning and SARSA versions.
- For both Q-Learning and SARSA binarization scheme selectors, when in a scanning state, the preference observed was for S-type transfer functions.
- For both Q-Learning and SARSA binarization scheme selectors, when in a scanning state, there was a preference for Standard and Static binarization followed by Complement.
- In the selectors of binarization schemes with Q-Learning, when in an exploitation state, there was a preference for V-type transfer functions.
- In the selectors of binarization schemes with SARSA, when in an operational state, there was a preference for the transfer function types S1, S2, S3, and V1.
- For both Q-Learning and SARSA binarization scheme selectors, the Elitist and Elitist Roulette binarization were mainly preferred when in an operational state.

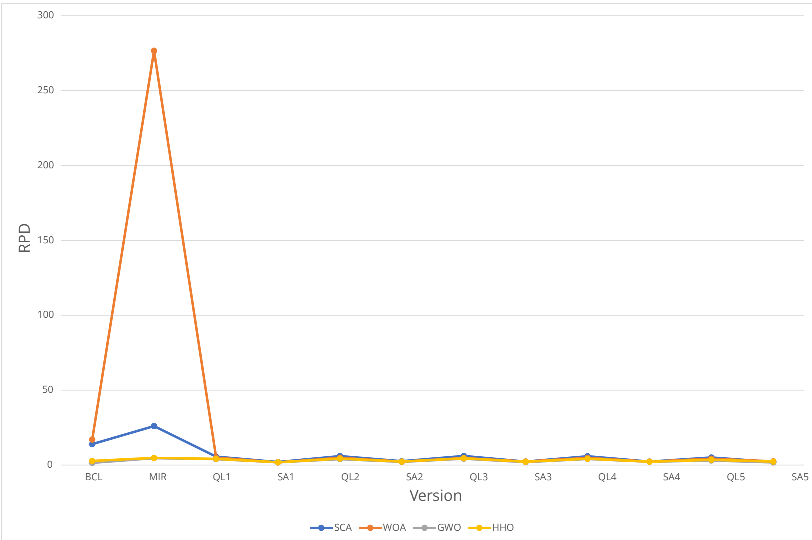

Figure 24. The average RPD.

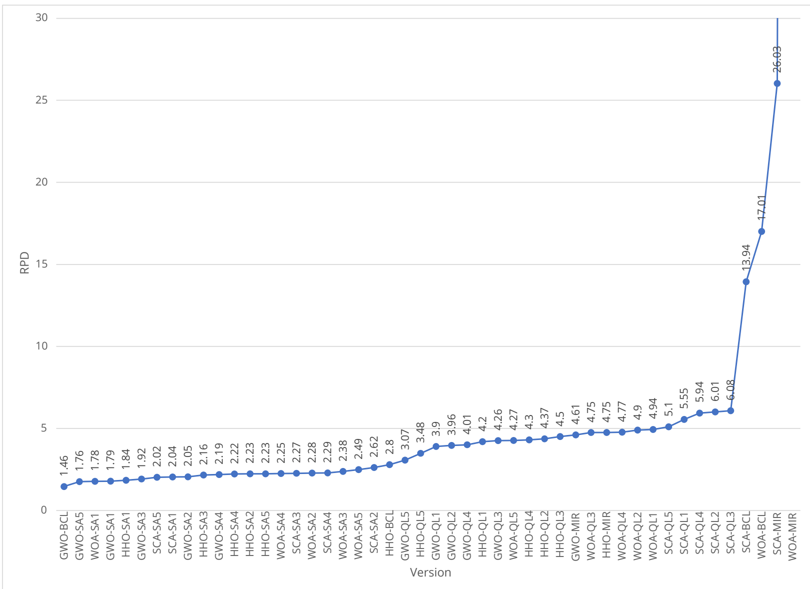

Figure 25. The average RPD Ranking.

5.4. Exploration and Exploitation

The exploration and exploitation measurements proposed in [38] provide detailed information related to the behaviors observed through the exploration and exploitation metrics during the iterative process. This process allows the proper observation of the influence of the binarization schemes employed between different versions, Figures 14–21 illustrate the following information:

- In SCA, WOA, GWO, and HHO, the BCL version presented a sharp increase in the exploitation percentage in the initial iterations and remained mostly constant.
- In SCA and WOA, the approaches based in MIR presented high exploitation values during the whole search process. This behavior, according to Morales-Castañera, can be attributed to random search behaviors, which are related to the fact of achieving the worst performance among all the versions compared by RDP.
- In GWO, the approach based in MIR, presented a slight increase in the exploitation percentage performed. However, the values reached were low in quality. This observation was equally done over the approaches employing BCL but with a worse performance when compared with the RPD.
- In HHO, the approach based in MIR, obtained higher exploration values in the early runs compared to the approaches employing static schemes. This can be explained by the similarities to the Q-Learning and SARSA approaches, where the movements are focused on exploration, which is well-known of HHO.
- In WOA and GWO, the approach SA1 presented a behavior similar to the one obtained by MIR in GWO. However, greater amplitude in variations were observed as recurrent results of Q-Learning and SARSA in the experimentation phase. They were the third and fourth approaches with better performances among the versions with a binarization scheme selector compared by RPD.
- In SCA and GWO, within Q-Learning and SARSA versions, exploration and exploitation graphs with constant changes in each iteration were presented.
- In HHO, the Q-Learning version presented an equal amount of variations as the ones observed in SCA and GWO. However, a change during the second half of the iterative process was observed, a common change of movements in HHO.

Along with this, we can observe a different influence of the binarization schemes to each MH. In the literature, the recommendations for the case of BCL were V4, which is associated with exploration, and Elitist, which is associated with exploitation. On the other hand, concerning MIR, both V4 and Complement were associated with exploration. The different behaviors observed in the performance and balance of exploration and exploitation opens the following question, Are MH more susceptible to binarization schemes?. In this regard, future works will focus on exposing this relationship and building scientific evidence regarding this issue.

6. Conclusions

In this work, a novel learning-based binarization scheme selector was proposed. In this context, novel approaches have proven to be highly efficient in tackling hard optimization problems. The designed learning-based method employed a Reinforcement Learning technique, named SARSA, which utilizes the dynamic data generated through the search by continuous populated-based algorithms. The main objective behind the proposed approach was to design a balanced binarization scheme selector.

Regarding the results achieved, the five different versions of SARSA demonstrated competitive performances. The experimentation solving the SCP illustrated that WOA and SCA assisted by Q-Learning and SARSA obtained statistically significant better results. However, regarding HHO and GWO, the opposite phenomenon was observed for the version applying Q-Learning. In this regard, the implementations employing static binarization schemes (V4 and Elitist), presented better performance in most of the 45 instances. Nevertheless, the implementations applying SARSA maintained good overall performance.

On the other hand, observing the real profits given by the employed rewards with Q-Learning, we could not demonstrate a significant difference. The results achieved were similar, and thus it cannot be concluded that, for the solved problems, the type of reward used directly impacts the quality of the solutions. In the case of SARSA, an equal phenomenon was observed. No statistically significant differences were determined. However, comparing the versions of Q-Learning and SARSA, the latter achieved significance for SCA and WOA.

Within future works, along with answering the question in Section 5.3, the option of evaluating other MH with exploration and exploitation behaviors must be pursued in order to further exploit the benefits and continue building solid evidence using the improvements of learning-based models. Likewise, different diversities can be evaluated to determine if there are significant differences in their results and the possibility of grouping them under another classification according to the exploration and exploitation percentages they generate.

This is an area of great interest due to a large number of methods for calculating diversity. Other future works can contemplate the increase of actions for the proposed selection scheme, i.e., to add more transfer and binarization functions, such as O-Shaped [42], Z-Shaped [43], Q-Shaped [44], and U-Shaped [45]. In addition to evaluating other techniques of Temporal Difference, it is possible to explore new options, such as using techniques focused on large multi-dimensional variable sizes. This context includes the "Deep Q-Network" and others from deep learning.

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Conflicts of Interest: The authors declare no conflict of interest.

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