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Observer-Based Control for Nonlinear Time-Delayed Asynchronously Switching Systems: A New LMI Approach

Amin Taghieh [1], Ardashir Mohammadzadeh [2,*], Jafar Tavoosi [3], Saleh Mobayen [4,*], Thaned Rojsiraphisal [5], Jihad H. Asad [6] and Anton Zhilenkov [7]

1 Faculty of Electrical and Computer Engineering, University of Tabriz, Tabriz 5166616471, Iran; amin.taghieh@yahoo.com
2 Electrical Engineering Department, University of Bonab, Bonab 5551395133, Iran
3 Department of Electrical Engineering, Ilam University, Ilam 69315516, Iran; j.tavoosi@ilam.ac.ir
4 Future Technology Research Center, National Yunlin University of Science and Technology, Douliu 64002, Taiwan
5 Advanced Research Center for Computational Simulation, Department of Mathematics, Faculty of Science, Chiang Mai University, Chiang Mai 50200, Thailand; thaned@gmail.com
6 Department of Physics, Faculty of Applied Sciences, Palestine Technical University, Tulkarm P.O. Box 7, Palestine; j.asad@ptuk.edu.ps
7 Department of Cyber-Physical Systems, St. Petersburg State Marine Technical University, 190121 Saint-Petersburg, Russia; zhilenkovanton@gmail.com
* Correspondence: a.mzadeh@ubonab.ac.ir (A.M.); mobayens@yuntech.edu.tw (S.M.)

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Abstract: This paper designs an observer-based controller for switched systems (SSs) with nonlinear dynamics, exogenous disturbances, parametric uncertainties, and time-delay. Based on the multiple Lyapunov–Krasovskii and average dwell time (DT) approaches, some conditions are presented to ensure the robustness and investigate the effect of time-delay, uncertainties, and lag issues between switching times. The control parameters are determined through solving the established linear matrix inequalities (LMIs) under asynchronous switching. A novel LMI-based conditions are suggested to guarantee the H_∞ performance. Finally, the accuracy of the designed observer-based controller is examined by simulations on practical case-study plants.

Keywords: observer-based controller; time-delay; parametric uncertainties; multiple Lyapunov–Krasovskii; average dwell time method; LMI; asynchronous switching

1. Introduction

Switched systems (SSs) are a class of hybrid systems due to their strong potential applications [1]. Various physical problems can be modeled and represented by SSs. Moreover, SSs consist of different modes with a switching signal to determine the active mode within any time intervals. The main challenges are the effect of time-delay, uncertain parameters, and the effect of switching instants [2].

In recent years, controller design of SSs has been one of the most significant aspects of the research in the literature. For example, by the use of the DT method, the state-output feedback control systems are studied in [3]. The Lyapunov–Krasovskii approach is developed in [4,5], to investigate the effect of time-delay. As is well known, parameter variations or an error in the measurement of system parameters may cause uncertainty in many dynamical systems. Constructing a piecewise linear Lyapunov function, the robust stability conditions are obtained for linear SSs subject to polytopic uncertainties [6]. In [7], both state/output feedback schemes are developed in the framework of LMI to analyze the uncertain singular SS. The robust H_∞ control approach is investigated in [8] to stabilize a discrete-time SS against polytopic uncertainties. In [9], the stability of SSs with time-delayed switchings and bounded uncertainties is investigated based on generalized polyhedral cells. The problem of polytopic uncertainties under time-delay condition has been studied in [10,11]. The effect of affine parametric uncertainties is investigated in [12,13] and the upper bounds of uncertainties are determined via computational algorithms. It should be noted that, in polytopic uncertain problems, a great number of LMIs should be solved to cope with uncertain parameters. Therefore, the computational cost of the problem is high and needs more studies. The computational characteristics are studied in [14], and it is shown that less conservative analysis is required in comparison with upper bound computations of uncertainties. The stabilization of SSs in a discrete-time form is investigated in [15], and the effect of uncertainties and time-delay is analyzed by the use of output feedback control system. Similarly, the state feedback control system is developed in [16] for discrete-time SSs and the stabilization conditions are acquired. The predictive control approach is suggested in [17] to ensure the robustness of discrete-time SSs with variable switching laws and time-delays.

The asynchronous switching (AS) problem is another challenging issue in SSs. The lag among the switching times and associated controllers commonly cause the asynchronous switching problem [18]. This problem has been rarely studied for SSs under time-delay condition. For instance, a Lyapunov method is suggested in [19] to compute L_2 gains of SSs with exogenous perturbations and AS problem. An asynchronous dynamic control technique is addressed for time-delayed SSs in [20]. The AS problem is taken into account in [21], and the finite-time stability criteria and state feedback control scheme are suggested to investigate the stabilization problem. The tracking accuracy of linear SSs subject to time-delays and AS is studied in [22].

Utilizing the average DT approach (ADT), the boundedness of nonlinear SSs is studied in [23], and the input delay and AS effects are investigated. The H_∞ problem is studied in [24] for discrete-time SSs, and the AS problem is analyzed using non-fragile controllers. The Lyapunov–Krasovskii approach is formulated in [25], and the asynchronous stabilization of neutral SSs is analyzed. The free-weighting matrices technique on the basis of the ADT method is developed in [26] to design a stable state feedback controller for linear SSs subject to the AS problem. The zonotope method is employed in [27] to acquire the appropriate estimations and then the computational complexity of the robust H_∞ scheme is analyzed.

The robust control of nonlinear SSs under unmeasurable states, time-delay, uncertainties, and the AS problem has not been completely studied. The main contributions are:

- Unlike the reviewed research, besides the time-delay and asynchronous switching, the parametric uncertainties are also considered in this study to ensure the robust stabilization problem.
- Due to the inaccessibility of all state variables in many actual operations, an observer-based control system is developed to reconstruct the state variables of SSs under asynchronous switching.
- The asynchronous H_∞ problem is investigated and novel LMI-based conditions are presented as a feasibility problem to design the observer-based controller and to compute the prescribed performance index in the under exogenous disturbances.
- The AS problem is addressed by a simple observer-based method such that the computational complexity is reduced.
- The average DT technique is developed by the Lyapunov–Krasovskii method, and some stabilization conditions are derived.
- The robustness against time-delay, AS, and uncertainties is analyzed via LMIs and the singular-value decomposition (SVD) approach.

2. Problem Formulation and Preliminaries

Consider the SSs as (1):

$$\begin{cases} \dot{\chi}(t) = (\mathcal{A}^n_{\sigma(t)} + \Delta\mathcal{A}_{\sigma(t)})\chi(t) + (\mathcal{B}^n_{\sigma(t)} + \Delta\mathcal{B}_{\sigma(t)})\chi(t-\iota_\tau) \\ +(\mathcal{D}^n_{\sigma(t)} + \Delta\mathcal{D}_{\sigma(t)})u_{\tilde{\sigma}(t)}(t) + \Im_{\sigma(t)}(t,\chi(t)) + (\mathcal{W}^n_{\sigma(t)} + \Delta\mathcal{W}_{\sigma(t)})\tilde{d}(t) \\ y(t) = \mathcal{C}\chi(t) \\ \chi(s) = \varphi(s), \quad s \in [-\iota_\tau, 0] \end{cases} \tag{1}$$

where $\chi(t) \in \mathbb{R}^{n_\chi}$ denotes state vector, $u_{\tilde{\sigma}(t)}(t) \in \mathbb{R}^{n_u}$ represents control signal, $y(t) \in \mathbb{R}^{n_y}$ is output, $\Im_{\sigma(t)}(t,\chi(t))$ is the nonlinear function, $\tilde{d}(t)$ is the exogenous disturbance which belongs to $l_2 \in [0,\infty)$, ι_τ denotes the time-delay, and $\varphi(s)$ is a function which specifies the initial state. The switching signal $\sigma(t) : [t_0,\infty) \to M = \{1,2,\ldots,l\}$ is a piecewise function, where l designates the number of system modes. Associated to the function $\sigma(t)$, the switching sequence $\sigma(t) : \{(t_0,\sigma(t_0)),(t_1,\sigma(t_1)),\ldots,(t_k,\sigma(t_k)),\ldots|\sigma(t_k) \in M, k = 1,2,\ldots,N\}$ is determined, and k denotes the switching number. Ideally, the observers and controllers alter simultaneously with the system modes, which we can write $\sigma(t) = \tilde{\sigma}(t)$, but in actual operation, since it takes some times to perceive the active subsystem and apply the matched observer and control signals, the switchings of the observers and control signals $\tilde{\sigma}(t)$ lag behind system modes, which means that $\tilde{\sigma}(t) : \{(t_0,\sigma(t_0)),(t_1 + \omega_1,\sigma(t_1)),\ldots,(t_k+\omega_k,\sigma(t_k)),\ldots|\sigma(t_k) \in M\}$. For each subsystem, the length of the lag time ϖ_k is unknown, but it is assumed that $\omega_k < \varpi, \forall k = 1,2,\ldots,N$, where ϖ is a known constant.

On the other hand, since all of the states are not measurable in some actual operations, the following switched observer is suggested (1):

$$\begin{cases} \dot{\hat{\chi}}(t) = \mathcal{A}^n_{\sigma(t)}\hat{\chi}(t) + \mathcal{B}^n_{\sigma(t)}\hat{\chi}(t-\iota_\tau) + \mathcal{D}^n_{\sigma(t)}u_{\tilde{\sigma}(t)}(t) \\ +\Im_{\sigma(t)}(t,\hat{\chi}(t)) + \mathcal{L}_{\tilde{\sigma}(t)}(y(t) - \hat{y}(t)) \\ \hat{y}(t) = \mathcal{C}\hat{\chi}(t) \\ \hat{\chi}(s) = 0, \quad s \in [-\iota_\tau, 0] \end{cases} \tag{2}$$

where $\hat{\chi}(t) \in \mathbb{R}^{n_\chi}$ is observer states, and $\mathcal{L}_{\tilde{\sigma}(t)}$ is the observer gain that has to be designed. Moreover, a controller is updated, i.e., $u_{\tilde{\sigma}(t)}(t) = \mathcal{K}_{\tilde{\sigma}(t)}\hat{\chi}(t)$. To analyze the AS problem, the entire operation time of the system is written as $[t_{k-1}+\omega_{k-1}, t_k)$, $k = 1,2,\ldots,N$, with $\omega_0 = 0$ and mismatched intervals $[t_k, t_k+\omega_k), k = 1,2,\ldots,N$. Without loss of generality, assume that the ith subsystem is active at $t_{k-1}, \sigma(t_{k-1}) = i$, and the jth subsystem is active at $t_k, \sigma(t_k) = j$, then the corresponding observer and control signals are identified at $t_{k-1} + \omega_{k-1}$ and $t_k + \omega_k$. Moreover, the state estimation error is defined as $e(t) = \chi(t) - \hat{\chi}(t)$, and the switched observer dynamic (2) can be rewritten as

$$\begin{cases} \dot{\hat{\chi}}(t) &= (\mathcal{A}^n_i + \mathcal{D}^n_i\mathcal{K}_i)\hat{\chi}(t) + \mathcal{B}^n_i\hat{\chi}(t-\iota_\tau) + \Im_i(t,\hat{\chi}(t)) \\ &+\mathcal{L}_i\mathcal{C}e(t), \quad t \in [t_{k-1}+\omega_{k-1}, t_k) \\ \dot{\hat{\chi}}(t) &= (\mathcal{A}^n_j + \mathcal{D}^n_j\mathcal{K}_i)\hat{\chi}(t) + \mathcal{B}^n_j\hat{\chi}(t-\iota_\tau) + \Im_j(t,\hat{\chi}(t)) \\ &+\mathcal{L}_i\mathcal{C}e(t), \quad t \in [t_k, t_k+\omega_k) \end{cases} \tag{3}$$

Additionally, estimation error dynamics are written as:

$$\begin{cases} \dot{e}(t) = (\mathcal{A}^n_i - \mathcal{L}_i\mathcal{C} + \Delta\mathcal{A}_i)e(t) + (\Delta\mathcal{A}_i + \Delta\mathcal{D}_i\mathcal{K}_i)\hat{\chi}(t) + (\mathcal{B}^n_i + \Delta\mathcal{B}_i)e(t-\iota_\tau) \\ +\Delta\mathcal{B}_i\hat{\chi}(t-\iota_\tau) + \Im_i(t,\chi(t)) - \Im_i(t,\hat{\chi}(t)) + (\mathcal{W}^n_i + \Delta\mathcal{W}_i)\tilde{d}(t), t \in [t_{k-1}+\omega_{k-1}, t_k) \\ \dot{e}(t) = (\mathcal{A}^n_j - \mathcal{L}_i\mathcal{C} + \Delta\mathcal{A}_j)e(t) + (\Delta\mathcal{A}_j + \Delta\mathcal{D}_j\mathcal{K}_i)\hat{\chi}(t) + (\mathcal{B}^n_j + \Delta\mathcal{B}_j)e(t-\iota_\tau) \\ +\Delta\mathcal{B}_j\hat{\chi}(t-\iota_\tau) + \Im_j(t,\chi(t)) - \Im_j(t,\hat{\chi}(t)) + (\mathcal{W}^n_j + \Delta\mathcal{W}_j)\tilde{d}(t), t \in [t_k, t_k+\omega_k) \end{cases} \tag{4}$$

From (3) and (4), the augmented switched system under AS is achieved as follows:

$$\begin{cases} \dot{\xi}(t) = \bar{\mathcal{A}}_{ii}\xi(t) + \bar{\mathcal{B}}_i\xi(t-\iota_\tau) + \bar{\Im}_i(t) + \bar{\mathcal{W}}_i^n\tilde{d}(t), & t \in [t_{k-1}+\omega_{k-1}, t_k) \\ \dot{\xi}(t) = \bar{\mathcal{A}}_{ji}\xi(t) + \bar{\mathcal{B}}_j\xi(t-\iota_\tau) + \bar{\Im}_j(t) + \bar{\mathcal{W}}_j^n\tilde{d}(t), & t \in [t_k, t_k+\omega_k) \end{cases} \tag{5}$$

where $\xi(t) = [\hat{\chi}^T(t), e^T(t)]^T$, and

$$\bar{\mathcal{A}}_{ii} = \begin{bmatrix} \mathcal{A}_i^n + \mathcal{D}_i^n\mathcal{K}_i & \mathcal{L}_i\mathcal{C} \\ \Delta\mathcal{A}_i + \Delta\mathcal{D}_i\mathcal{K}_i & \mathcal{A}_i^n - \mathcal{L}_i\mathcal{C} + \Delta\mathcal{A}_i \end{bmatrix}$$

$$\bar{\mathcal{A}}_{ji} = \begin{bmatrix} \mathcal{A}_j^n + \mathcal{D}_j^n\mathcal{K}_i & \mathcal{L}_i\mathcal{C} \\ \Delta\mathcal{A}_j + \Delta\mathcal{D}_j\mathcal{K}_i & \mathcal{A}_j^n - \mathcal{L}_i\mathcal{C} + \Delta\mathcal{A}_j \end{bmatrix}$$

$$\bar{\mathcal{B}}_i = \begin{bmatrix} \mathcal{B}_i^n & 0 \\ \Delta\mathcal{B}_i & \mathcal{B}_i^n + \Delta\mathcal{B}_i \end{bmatrix}, \qquad \bar{\mathcal{B}}_j = \begin{bmatrix} \mathcal{B}_j^n & 0 \\ \Delta\mathcal{B}_j & \mathcal{B}_j^n + \Delta\mathcal{B}_j \end{bmatrix}$$

$$\bar{\Im}_i(t) = \begin{bmatrix} \Im_i(t,\hat{\chi}(t)) \\ \Im_i(t,\chi(t)) - \Im_i(t,\hat{\chi}(t)) \end{bmatrix}, \qquad \bar{\Im}_j(t) = \begin{bmatrix} \Im_j(t,\hat{\chi}(t)) \\ \Im_j(t,\chi(t)) - \Im_j(t,\hat{\chi}(t)) \end{bmatrix}$$

$$\bar{\mathcal{W}}_i(t) = \begin{bmatrix} 0 \\ \mathcal{W}_i^n + \Delta\mathcal{W}_i \end{bmatrix}, \qquad \bar{\mathcal{W}}_j(t) = \begin{bmatrix} 0 \\ \mathcal{W}_j^n + \Delta\mathcal{W}_j \end{bmatrix}$$

Note that parametric uncertain matrices $\Delta\mathcal{A}_i, \Delta\mathcal{B}_i, \Delta\mathcal{D}_i$, and $\Delta\mathcal{W}_i$ $\forall i \in M$ have the following structure:

$$\Delta_i : \begin{cases} \Delta\mathcal{A}_i = \sum_{\ell=1}^{q} \delta_{\theta_\ell}\mathcal{E}_\ell^i \\ \Delta\mathcal{B}_i = \sum_{\ell=1}^{q} \delta_{\theta_\ell}\mathcal{F}_\ell^i \\ \Delta\mathcal{D}_i = \sum_{\ell=1}^{q} \delta_{\theta_\ell}\mathcal{G}_\ell^i \\ \Delta\mathcal{W}_i = \sum_{\ell=1}^{q} \delta_{\theta_\ell}\mathcal{T}_\ell^i \end{cases} \tag{6}$$

where $\delta_{\theta_\ell} \in [-\tilde{\delta}_{\theta_\ell}, \tilde{\delta}_{\theta_\ell}], \ell = 1, \dots, q$ are the uncertain parameters. $\mathcal{E}_\ell^i, \mathcal{F}_\ell^i, \mathcal{G}_\ell^i$ and $\mathcal{T}_\ell^i$ are the uncertainty matrices to determine the dependency of $\Delta\mathcal{A}_i, \Delta\mathcal{B}_i, \Delta\mathcal{D}_i, \Delta\mathcal{W}_i$ on δ_{θ_ℓ}.

Assumption 1. *Consider a full-row rank matrix $\Psi_{n_y \times n_\chi}$; From this assumption, the SVD of Ψ is written as*

$$\Psi_{n_y \times n_\chi} = \Xi_{n_y \times n_y} \begin{bmatrix} \Psi_{0_{n_y \times n_y}} & 0 \end{bmatrix} \Upsilon_{n_\chi \times n_\chi}^T \tag{7}$$

where Ξ/Υ and Ψ_0 are unitary and diagonal matrices, respectively.

Lemma 1 ([28])**.** *Consider a full rank matrix $\Psi_{n_y \times n_\chi}$ and symmetric matrix $\mathcal{X}_{n_\chi \times n_\chi}$; There exists $\Omega_{n_y \times n_y}$ such that $\Psi\mathcal{X} = \Omega\Psi$, if and only if:*

$$\mathcal{X} = \Upsilon \begin{bmatrix} \mathcal{X}_1 & 0 \\ 0 & \mathcal{X}_2 \end{bmatrix} \Upsilon^T \tag{8}$$

where dimensions of $\mathcal{X}_1$, $\mathcal{X}_2$ are as $n_y \times n_y$ and $(n_\chi - n_y) \times (n_\chi - n_y)$, respectively. Υ is defined in (7).

Assumption 2. *Consider that, for each $i \in M$ and any vectors $\eta_1(t), \eta_2(t)$, the nonlinear vector-valued function $\Im_i(.)$ is Lipschitz such that*

$$\|\Im_i(t,\eta_1(t)) - \Im_i(t,\eta_2(t))\| \le \bar{\pi}\|\mathcal{H}_i(\eta_1(t) - \eta_2(t))\| \tag{9}$$

where M is defined in (1), $\mathcal{H}_i$'s are real weighting Lipschitz matrices, and $\bar{\pi}$ is a Lipschitz constant.

Definition 1. *(a) The system (5) is exponentially stable in the presence of switchings $\sigma(.)$ and when $\tilde{d}(t) = 0$, if the following condition for $\varepsilon \geq 1$ and $\lambda > 0$ is satisfied:*

$$\begin{cases} \|\xi(t)\| \leq \varepsilon \|\xi(t_0)\|_{\aleph} e^{-\lambda(t-t_0)}, \quad \forall t \geq t_0 \\ \|\xi(t_0)\|_{\aleph} = sup_{-\iota_\tau \leq \phi \leq 0} \|\xi(t_0 + \phi)\| \end{cases} \tag{10}$$

(b) If the system (5) satisfies the following condition for $\tilde{d}(t) \neq 0$, $\tilde{d}(t) \in l_2[0, \infty)$, and positive constants λ_α, ψ

$$\int_{t_0}^{\infty} e^{-\lambda_\alpha(s-t_0)} y^T(s) y(s) \mathrm{d}s \leq \int_{t_0}^{\infty} \psi^2 \tilde{d}^T(s) \tilde{d}(s) \mathrm{d}s \tag{11}$$

Then, (5) has H_∞ performance.

Definition 2 ([29])**.** *For any scalars k_1, k_2, let $\mathcal{N}_\sigma(k_1, k_2)$ denote the switchings $\sigma(.)$ over (k_1, k_2). If*

$$\mathcal{N}_\sigma(k_1, k_2) \leq \mathcal{N}_0 + \frac{k_2 - k_1}{\tau_a} \tag{12}$$

holds for $\tau_a > 0, \mathcal{N}_0 \geq 0$, then τ_a is called the average dwell time.

Lemma 2 ([15])**.** *For a scalar $\epsilon > 0$, any matrices $\mathcal{X}_i \in \mathbb{R}^{x \times y}, i = 1, \ldots, q$, and positive-definite matrices $\mathcal{P} \in \mathbb{R}^{x \times x}$, the following inequality holds:*

$$(\mathcal{X}_1 + \cdots + \mathcal{X}_q)^T \mathcal{P} (\mathcal{X}_1 + \cdots + \mathcal{X}_q) \leq \epsilon (\mathcal{X}_1^T \mathcal{P} \mathcal{X}_1 + \cdots + \mathcal{X}_q^T \mathcal{P} \mathcal{X}_q) \tag{13}$$

Lemma 3 ([30])**.** *let $\vartheta, D, F, E, \zeta$ be some vectors or matrices with appropriate dimensions. As a result, the inequality*

$$2\vartheta^T D F E \zeta \leq \epsilon \vartheta^T D D^T \vartheta + \epsilon^{-1} \zeta^T E^T E \zeta \tag{14}$$

holds for $F^T F \leq I$ and a real positive parameter ϵ.

3. Main Results

In this section, an observer-based control technique is designed for the switched system (5) in the presence of the uncertain parameters $\delta_{\theta_\ell} \in [-\tilde{\delta}_{\theta_\ell}, \tilde{\delta}_{\theta_\ell}], \ell = 1, \ldots, q$. The design methods for the augmented switched system (5) with $\tilde{d}(t) = 0$ and $\tilde{d}(t) \neq 0$ are investigated in Theorems 1 and 2, respectively.

Theorem 1. *Consider system (5) under uncertain parameters $\delta_{\theta_\ell} \in [-\tilde{\delta}_{\theta_\ell}, \tilde{\delta}_{\theta_\ell}]$. For given positive scalars $\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4, \epsilon_5, \epsilon_6$, and parameters $\iota_\tau, \varpi, \lambda_\alpha, \lambda_\beta, \bar{\pi}, \mu \geq 1$, suppose that there exist symmetric matrices $\mathcal{X}_{\hat{\chi}i} > 0, \mathcal{X}_{ei} > 0, \mathcal{Y}_{\hat{\chi}i} > 0, \mathcal{Y}_{ei} > 0$ and matrices $\mathcal{Z}_{\hat{\chi}i}, \mathcal{Z}_{ei}$, for any $i, j \in M, i \neq j$, such that:*

$$\mathcal{X}_{\hat{\chi}j} \leq \mu \mathcal{X}_{\hat{\chi}i}, \quad \mathcal{X}_{ej} \leq \mu \mathcal{X}_{ei}, \quad \mathcal{Y}_{\hat{\chi}j} \leq \mu \mathcal{Y}_{\hat{\chi}i}, \quad \mathcal{Y}_{ej} \leq \mu \mathcal{Y}_{ei} \tag{15}$$

$$\Psi_i = \begin{bmatrix} \Psi_{i1} & \Psi_{i2} & \Psi_{i3} & \Psi_{i4} & \Psi_{i5} & \Psi_{i6} & \Psi_{i7} & \Psi_{i8} & \Psi_{i9} \\ * & -\mathbb{I} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\mathbb{I} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\mathbb{I} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\mathbb{I} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & * & -\mathcal{Y}_{\hat{\chi}i} & 0 & 0 \\ * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & -\mathcal{Y}_{ei} \end{bmatrix} < 0 \tag{16}$$

$$\Theta_i = \begin{bmatrix} \Theta_{i1} & \Theta_{i2} & \Theta_{i3} & \Theta_{i4} & \Theta_{i5} & \Theta_{i6} & \Theta_{i7} & \Theta_{i8} & \Theta_{i9} \\ * & -\mathbb{I} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\mathbb{I} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\mathbb{I} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\mathbb{I} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & * & -\mathcal{Y}_{\hat{\chi}i} & 0 & 0 \\ * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & -\mathcal{Y}_{ei} \end{bmatrix} < 0 \tag{17}$$

where

$$\Psi_{i1} = \begin{bmatrix} \Psi_{i1_{(1,1)}} & \mathcal{B}_i^n \mathcal{Y}_{\hat{\chi}i} & \mathcal{Z}_{ei}\mathcal{C} & 0 \\ * & -e^{-\lambda_\alpha \iota_\tau}\mathcal{Y}_{\hat{\chi}i} & 0 & 0 \\ * & * & \Psi_{i1_{(3,3)}} & \mathcal{B}_i^n \mathcal{Y}_{ei} \\ * & * & * & -e^{-\lambda_\alpha \iota_\tau}\mathcal{Y}_{ei} \end{bmatrix}$$

$$\Psi_{i3} = \begin{bmatrix} 0 & \cdots & 0 \\ \bar{\epsilon}_5 \mathcal{Y}_{\hat{\chi}i}(\bar{\mathcal{F}}_1^i)^T & \cdots & \bar{\epsilon}_5 \mathcal{Y}_{\hat{\chi}i}(\bar{\mathcal{F}}_q^i)^T \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{bmatrix}$$

$$\Psi_{i2} = \begin{bmatrix} \bar{\epsilon}_3\left(\mathcal{X}_{\hat{\chi}i}(\bar{\mathcal{E}}_1^i)^T + \mathcal{Z}_{\hat{\chi}i}^T(\bar{\mathcal{G}}_1^i)^T\right) & \cdots & \bar{\epsilon}_3\left(\mathcal{X}_{\hat{\chi}i}(\bar{\mathcal{E}}_q^i)^T + \mathcal{Z}_{\hat{\chi}i}^T(\bar{\mathcal{G}}_q^i)^T\right) \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{bmatrix}$$

$$\Psi_{i4} = \begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ \bar{\epsilon}_2 \mathcal{X}_{ei}(\bar{\mathcal{E}}_1^i)^T & \cdots & \bar{\epsilon}_2 \mathcal{X}_{ei}(\bar{\mathcal{E}}_q^i)^T \\ 0 & \cdots & 0 \end{bmatrix}, \Psi_{i5} = \begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ \bar{\epsilon}_4 \mathcal{Y}_{ei}(\bar{\mathcal{F}}_1^i)^T & \cdots & \bar{\epsilon}_4 \mathcal{Y}_{ei}(\bar{\mathcal{F}}_q^i)^T \end{bmatrix}$$

$$\Psi_{i6} = \begin{bmatrix} \bar{\epsilon}_1 \gamma \mathcal{X}_{\hat{\chi}i}\mathcal{H}_i^T \\ 0 \\ 0 \\ 0 \end{bmatrix}, \Psi_{i7} = \begin{bmatrix} \mathcal{X}_{\hat{\chi}i} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \Psi_{i8} = \begin{bmatrix} 0 \\ 0 \\ \bar{\epsilon}_6 \gamma \mathcal{X}_{ei}\mathcal{H}_i^T \\ 0 \end{bmatrix}, \Psi_{i9} = \begin{bmatrix} 0 \\ 0 \\ \mathcal{X}_{ei} \\ 0 \end{bmatrix}$$

$$\begin{aligned} \Psi_{i1_{(1,1)}} &= \mathcal{A}_i^n \mathcal{X}_{\hat{\chi}i} + \mathcal{X}_{\hat{\chi}i}(\mathcal{A}_i^n)^T + \mathcal{D}_i^n \mathcal{Z}_{\hat{\chi}i} + \mathcal{Z}_{\hat{\chi}i}^T(\mathcal{D}_i^n)^T + \epsilon_1^{-1} I + \lambda_\alpha \mathcal{X}_{\hat{\chi}i} \\ \Psi_{i1_{(3,3)}} &= \mathcal{A}_i^n \mathcal{X}_{ei} + \mathcal{X}_{ei}(\mathcal{A}_i^n)^T - \mathcal{Z}_{ei}\mathcal{C} - \mathcal{C}^T \mathcal{Z}_{ei}^T \\ &\quad + \left(\epsilon_2^{-1} + \epsilon_3^{-1} + \epsilon_4^{-1} + \epsilon_5^{-1} + \epsilon_6^{-1}\right) I + \lambda_\alpha \mathcal{X}_{ei} \\ \bar{\epsilon}_1 &= \sqrt{\epsilon_1}, \bar{\epsilon}_2 = \sqrt{\epsilon_2 \times q}, \bar{\epsilon}_3 = \sqrt{\epsilon_3 \times q}, \bar{\epsilon}_4 = \sqrt{\epsilon_4 \times q}, \bar{\epsilon}_5 = \sqrt{\epsilon_5 \times q}, \bar{\epsilon}_6 = \sqrt{\epsilon_6} \\ \bar{\mathcal{E}}_\ell^i &= \tilde{\delta}_{\theta_\ell}\mathcal{E}_\ell^i, \quad \bar{\mathcal{F}}_\ell^i = \tilde{\delta}_{\theta_\ell}\mathcal{F}_\ell^i, \quad \bar{\mathcal{G}}_\ell^i = \tilde{\delta}_{\theta_\ell}\mathcal{G}_\ell^i \end{aligned}$$

and

$$\Theta_{i1} = \begin{bmatrix} \Theta_{i1_{(1,1)}} & \mathcal{B}_j^n \mathcal{Y}_{\hat{\chi}i} & \mathcal{Z}_{ei}\mathcal{C} & 0 \\ * & -\mathcal{Y}_{\hat{\chi}i} & 0 & 0 \\ * & * & \Theta_{i1_{(3,3)}} & \mathcal{B}_j^n \mathcal{Y}_{ei} \\ * & * & * & -\mathcal{Y}_{ei} \end{bmatrix}, \Theta_{i3} = \begin{bmatrix} 0 & \cdots & 0 \\ \bar{\epsilon}_5 \mathcal{Y}_{\hat{\chi}i}(\bar{\mathcal{F}}_1^j)^T & \cdots & \bar{\epsilon}_5 \mathcal{Y}_{\hat{\chi}i}(\bar{\mathcal{F}}_q^j)^T \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{bmatrix}$$

$$\Theta_{i2} = \begin{bmatrix} \bar{\epsilon}_3\left(\mathcal{X}_{\hat{\chi}i}(\bar{\mathcal{E}}_1^j)^T + \mathcal{Z}_{\hat{\chi}i}^T(\bar{\mathcal{G}}_1^j)^T\right) & \cdots & \bar{\epsilon}_3\left(\mathcal{X}_{\hat{\chi}i}(\bar{\mathcal{E}}_q^j)^T + \mathcal{Z}_{\hat{\chi}i}^T(\bar{\mathcal{G}}_q^j)^T\right) \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{bmatrix}$$

$$\Theta_{i4} = \begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ \bar{\epsilon}_2\mathcal{X}_{ei}(\bar{\mathcal{E}}_1^j)^T & \cdots & \bar{\epsilon}_2\mathcal{X}_{ei}(\bar{\mathcal{E}}_q^j)^T \\ 0 & \cdots & 0 \end{bmatrix}, \Theta_{i5} = \begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ \bar{\epsilon}_4\mathcal{Y}_{ei}(\bar{\mathcal{F}}_1^j)^T & \cdots & \bar{\epsilon}_4\mathcal{Y}_{ei}(\bar{\mathcal{F}}_q^j)^T \end{bmatrix}$$

$$\Theta_{i6} = \begin{bmatrix} \bar{\epsilon}_1\gamma\mathcal{X}_{\hat{\chi}i}\mathcal{H}_j^T \\ 0 \\ 0 \\ 0 \end{bmatrix}, \Theta_{i7} = \begin{bmatrix} \mathcal{X}_{\hat{\chi}i} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \Theta_{i8} = \begin{bmatrix} 0 \\ 0 \\ \bar{\epsilon}_6\gamma\mathcal{X}_{ei}\mathcal{H}_j^T \\ 0 \end{bmatrix}, \Theta_{i9} = \begin{bmatrix} 0 \\ 0 \\ \mathcal{X}_{ei} \\ 0 \end{bmatrix}$$

$$\begin{aligned} \Theta_{i1_{(1,1)}} &= \mathcal{A}_j^n\mathcal{X}_{\hat{\chi}i} + \mathcal{X}_{\hat{\chi}i}\left(\mathcal{A}_j^n\right)^T + \mathcal{D}_j^n\mathcal{Z}_{\hat{\chi}i} + \mathcal{Z}_{\hat{\chi}i}^T\left(\mathcal{D}_j^n\right)^T + \epsilon_1^{-1}I - \lambda_\beta\mathcal{X}_{\hat{\chi}i} \\ \Theta_{i1_{(3,3)}} &= \mathcal{A}_j^n\mathcal{X}_{ei} + \mathcal{X}_{ei}\left(\mathcal{A}_j^n\right)^T - \mathcal{Z}_{ei}\mathcal{C} - \mathcal{C}^T\mathcal{Z}_{ei}^T \\ &\quad + \left(\epsilon_2^{-1} + \epsilon_3^{-1} + \epsilon_4^{-1} + \epsilon_5^{-1} + \epsilon_6^{-1}\right)I - \lambda_\beta\mathcal{X}_{ei} \end{aligned}$$

Then, system (5) is exponentially stable with switchings that satisfy $\tau_a > \tau_a^* = \frac{\ln\mu + (\lambda_\alpha + \lambda_\beta)(\iota_\tau + \varpi)}{\lambda_\alpha}$. *Moreover, the gains* $\mathcal{K}_i, \forall i \in M$ *and* $\mathcal{L}_i, \forall i \in M$ *are achieved as follows:*

$$\mathcal{K}_i = \mathcal{Z}_{\hat{\chi}i}\mathcal{X}_{\hat{\chi}i}^{-1} \tag{18}$$

$$\mathcal{L}_i = \mathcal{Z}_{ei}\mathcal{U}\mathcal{C}_0\mathcal{X}_{e1i}^{-1}\mathcal{C}_0^{-1}\mathcal{U}^T \tag{19}$$

Proof. From (5), when at time $t \in [t_{k-1} + \omega_{k-1}, t_k)$, the ith subsystem is activated and the corresponding switching controller $\mathcal{K}_i$ and observer $\mathcal{L}_i$ are identified. During $t \in [t_{k-1} + \omega_{k-1}, t_k)$, consider:

$$V_{1i}(t) = \xi^T(t)P_i\xi(t) + \int_{t-\iota_\tau}^{t} e^{-\lambda_\alpha(t-s)}\xi^T(s)Q_i\xi(s)\mathrm{d}s \tag{20}$$

where $\xi(t) = [\hat{\chi}^T(t), e^T(t)]^T$, $P_i = \mathrm{diag}\{P_{\hat{\chi}i}, P_{ei}\}$, and $Q_i = \mathrm{diag}\{Q_{\hat{\chi}i}, Q_{ei}\}$. The (20) can be rewritten as

$$\begin{aligned} V_{1i}(t) &= \hat{\chi}^T(t)P_{\hat{\chi}i}\hat{\chi}(t) + e^T(t)P_{ei}e(t) \\ &\quad + \int_{t-\iota_\tau}^{t} e^{-\lambda_\alpha(t-s)}\hat{\chi}^T(s)Q_{\hat{\chi}i}\hat{\chi}(s)\mathrm{d}s + \int_{t-\iota_\tau}^{t} e^{-\lambda_\alpha(t-s)}e^T(s)Q_{ei}e(s)\mathrm{d}s \end{aligned} \tag{21}$$

Considering (3), the derivative of V_{1i} leads to

$$\begin{aligned} \dot{V}_{1i}(t) &= 2\hat{\chi}^T(t)P_{\hat{\chi}i}\dot{\hat{\chi}}(t) + 2e^T(t)P_{ei}\dot{e}(t) + \hat{\chi}^T(t)Q_{\hat{\chi}i}\hat{\chi}(t) - e^{-\lambda_\alpha\iota_\tau}\hat{\chi}^T(t-\iota_\tau)Q_{\hat{\chi}i}\hat{\chi}(t-\iota_\tau) \\ &\quad - \lambda_\alpha\int_{t-\iota_\tau}^{t} e^{-\lambda_\alpha(t-s)}\hat{\chi}^T(s)Q_{\hat{\chi}i}\hat{\chi}(s)\mathrm{d}s + e^T(t)Q_{ei}e(t) \\ &\quad - e^{-\lambda_\alpha\iota_\tau}e^T(t-\iota_\tau)Q_{ei}e(t-\iota_\tau) - \lambda_\alpha\int_{t-\iota_\tau}^{t} e^{-\lambda_\alpha(t-s)}e^T(s)Q_{ei}e(s)\mathrm{d}s \end{aligned} \tag{22}$$

According to (3), it can be concluded that

$$\begin{aligned} 2\hat{\chi}^T(t)P_{\hat{\chi}i}\dot{\hat{\chi}}(t) &= \hat{\chi}^T(t)\left(P_{\hat{\chi}i}(\mathcal{A}_i^n + \mathcal{D}_i^n\mathcal{K}_i) + (\mathcal{A}_i^n + \mathcal{D}_i^n\mathcal{K}_i)^TP_{\hat{\chi}i}\right)\hat{\chi}(t) \\ &\quad + 2\hat{\chi}^T(t)P_{\hat{\chi}i}\mathcal{B}_i^n\hat{\chi}(t-\iota_\tau) + 2\hat{\chi}^T(t)P_{\hat{\chi}i}\mathcal{L}_i\mathcal{C}e(t) + 2\hat{\chi}^T(t)P_{\hat{\chi}i}\Im_i(t,\hat{\chi}(t)) \end{aligned} \tag{23}$$

From Lemma 3, one can attain

$$\begin{aligned} 2\hat{\chi}^T(t)P_{\hat{\chi}i}\dot{\hat{\chi}}(t) &\le \hat{\chi}^T(t)\left(P_{\hat{\chi}i}(\mathcal{A}_i^n + \mathcal{D}_i^n\mathcal{K}_i) + (\mathcal{A}_i^n + \mathcal{D}_i^n\mathcal{K}_i)^TP_{\hat{\chi}i}\right)\hat{\chi}(t) \\ &+ 2\hat{\chi}^T(t)P_{\hat{\chi}i}\mathcal{B}_i^n\hat{\chi}(t-\iota_\tau) + \epsilon_1^{-1}\hat{\chi}^T(t)P_{\hat{\chi}i}P_{\hat{\chi}i}\hat{\chi}(t) + \epsilon_1\Im_i(t,\hat{\chi}(t))^T\Im_i(t,\hat{\chi}(t)) \end{aligned} \tag{24}$$

Considering the estimation error dynamic (4) and parametric uncertain matrices (6), one has

$$\begin{aligned}2e^T(t)P_{ei}\dot{e}(t) &= e^T(t)\Big(P_{ei}(\mathcal{A}_i^n-\mathcal{L}_i\mathcal{C})+(\mathcal{A}_i^n-\mathcal{L}_i\mathcal{C})^T P_{ei}\Big)e(t)\\ &+2e^T(t)P_{ei}\left(\sum_{\ell=1}^{q}\delta_{\theta_\ell}\mathcal{E}_\ell^i\right)e(t)+2e^T(t)P_{ei}\left(\sum_{\ell=1}^{q}\delta_{\theta_\ell}\mathcal{E}_\ell^i+\delta_{\theta_\ell}\mathcal{G}_\ell^i\mathcal{K}_i\right)\hat{\chi}(t)\\ &+2e^T(t)P_{ei}\mathcal{B}_i^n e(t-\iota_\tau)+2e^T(t)P_{ei}\left(\sum_{\ell=1}^{q}\delta_{\theta_\ell}\mathcal{F}_\ell^i\right)e(t-\iota_\tau)\\ &+2e^T(t)P_{ei}\left(\sum_{\ell=1}^{q}\delta_{\theta_\ell}\mathcal{F}_\ell^i\right)\hat{\chi}(t-\iota_\tau)+2e^T(t)P_{ei}(\Im_i(t,\chi(t))-\Im_i(t,\hat{\chi}(t)))\end{aligned} \tag{25}$$

Regarding Lemmas 2 and 3, and considering $\delta_{\theta_\ell}\in[-\tilde{\delta}_{\theta_\ell},\tilde{\delta}_{\theta_\ell}]$, $\ell=1,\ldots,q$, it is derived that

$$\begin{aligned}&2e^T(t)P_{ei}\dot{e}(t)\le e^T(t)\Big(P_{ei}(\mathcal{A}_i^n-\mathcal{L}_i\mathcal{C})+(\mathcal{A}_i^n-\mathcal{L}_i\mathcal{C})^T P_{ei}\Big)e(t)+2e^T(t)P_{ei}\mathcal{B}_i^n e(t-\iota_\tau)\\ &+\epsilon_2^{-1}e^T(t)P_{ei}P_{ei}e(t)+(\epsilon_2\times q)e^T(t)\left(\sum_{\ell=1}^{q}(\tilde{\delta}_{\theta_\ell})^2(\mathcal{E}_\ell^i)^T\mathcal{E}_\ell^i\right)e(t)\\ &+\epsilon_3^{-1}e^T(t)P_{ei}P_{ei}e(t)+(\epsilon_3\times q)\hat{\chi}^T(t)\left(\sum_{\ell=1}^{q}(\tilde{\delta}_{\theta_\ell})^2(\mathcal{E}_\ell^i+\mathcal{G}_\ell^i\mathcal{K}_i)^T(\mathcal{E}_\ell^i+\mathcal{G}_\ell^i\mathcal{K}_i)\right)\hat{\chi}(t)\\ &+\epsilon_4^{-1}e^T(t)P_{ei}P_{ei}e(t)+(\epsilon_4\times q)e^T(t-\iota_\tau)\left(\sum_{\ell=1}^{q}(\tilde{\delta}_{\theta_\ell})^2(\mathcal{F}_\ell^i)^T\mathcal{F}_\ell^i\right)e(t-\iota_\tau)\\ &+\epsilon_5^{-1}e^T(t)P_{ei}P_{ei}e(t)+(\epsilon_5\times q)\hat{\chi}^T(t-\iota_\tau)\left(\sum_{\ell=1}^{q}(\tilde{\delta}_{\theta_\ell})^2(\mathcal{F}_\ell^i)^T\mathcal{F}_\ell^i\right)\hat{\chi}(t-\iota_\tau)\\ &+\epsilon_6^{-1}e^T(t)P_{ei}P_{ei}e(t)+\epsilon_6(\Im_i(t,\chi(t))-\Im_i(t,\hat{\chi}(t)))^T(\Im_i(t,\chi(t))-\Im_i(t,\hat{\chi}(t)))\end{aligned} \tag{26}$$

Now, from (22), (24), (26), considering Assumption 2, and defining $\bar{\mathcal{E}}_\ell^i=\tilde{\delta}_{\theta_\ell}\mathcal{E}_\ell^i$, $\bar{\mathcal{F}}_\ell^i=\tilde{\delta}_{\theta_\ell}\mathcal{F}_\ell^i$, $\bar{\mathcal{G}}_\ell^i=\tilde{\delta}_{\theta_\ell}\mathcal{G}_\ell^i$, we have:

$$\dot{V}_{1i}(t)+\lambda_\alpha V_{1i}(t)\le\zeta^T(t)\Pi_i\zeta(t) \tag{27}$$

where $\zeta(t)=[\hat{\chi}^T(t),\hat{\chi}^T(t-\iota_\tau),e^T(t),e^T(t-\iota_\tau)]^T$, and

$$\Pi_i=\begin{bmatrix}\Pi_{i(1,1)} & P_{\hat{\chi}i}\mathcal{B}_i^n & P_{\hat{\chi}i}\mathcal{L}_i\mathcal{C} & 0\\ * & \Pi_{i(2,2)} & 0 & 0\\ * & * & \Pi_{i(3,3)} & P_{ei}\mathcal{B}_i^n\\ * & * & * & \Pi_{i(4,4)}\end{bmatrix}<0 \tag{28}$$

with

$$\begin{aligned}\Pi_{i(1,1)}&=P_{\hat{\chi}i}(\mathcal{A}_i^n+\mathcal{D}_i^n\mathcal{K}_i)+(\mathcal{A}_i^n+\mathcal{D}_i^n\mathcal{K}_i)^T P_{\hat{\chi}i}\\ &+\epsilon_1^{-1}P_{\hat{\chi}i}P_{\hat{\chi}i}+\epsilon_1\bar{\pi}^2\mathcal{H}_i^T\mathcal{H}_i+Q_{\hat{\chi}i}+\lambda_\alpha P_{\hat{\chi}i}\\ &+(\epsilon_3\times q)\sum_{\ell=1}^{q}\left(\bar{\mathcal{E}}_\ell^i+\bar{\mathcal{G}}_\ell^i\mathcal{K}_i\right)^T\left(\bar{\mathcal{E}}_\ell^i+\bar{\mathcal{G}}_\ell^i\mathcal{K}_i\right)\\ \Pi_{i(2,2)}&=-e^{-\lambda_\alpha\iota_\tau}Q_{\hat{\chi}i}+(\epsilon_5\times q)\sum_{\ell=1}^{q}(\bar{\mathcal{F}}_\ell^i)^T\bar{\mathcal{F}}_\ell^i\end{aligned}$$

$$\begin{aligned}\Pi_{i(3,3)} &= P_{ei}(\mathcal{A}_i^n - \mathcal{L}_i\mathcal{C}) + (\mathcal{A}_i^n - \mathcal{L}_i\mathcal{C})^T P_{ei} \\ &\left(\epsilon_2^{-1} + \epsilon_3^{-1} + \epsilon_4^{-1} + \epsilon_5^{-1} + \epsilon_6^{-1}\right) P_{ei}P_{ei} + \epsilon_6 \bar{\pi}^2 \mathcal{H}_i^T \mathcal{H}_i + Q_{ei} + \lambda_\alpha P_{ei} \\ &+ (\epsilon_2 \times q) \sum_{\ell=1}^{q} (\bar{\mathcal{E}}_\ell^i)^T \bar{\mathcal{E}}_\ell^i \\ \Pi_{i(4,4)} &= -e^{-\lambda_\alpha \iota_\tau} Q_{ei} + (\epsilon_4 \times q) \sum_{\ell=1}^{q} (\bar{\mathcal{F}}_\ell^i)^T \bar{\mathcal{F}}_\ell^i\end{aligned}$$

Using the Schur lemma, (28) is written as:

$$\Gamma_i = \begin{bmatrix} \Gamma_{i1} & \Gamma_{i2} & \Gamma_{i3} & \Gamma_{i4} & \Gamma_{i5} \\ * & -\mathbb{I} & 0 & 0 & 0 \\ * & * & -\mathbb{I} & 0 & 0 \\ * & * & * & -\mathbb{I} & 0 \\ * & * & * & * & -\mathbb{I} \end{bmatrix} < 0 \tag{29}$$

in which

$$\begin{aligned}\Gamma_{i1} &= \begin{bmatrix} \Gamma_{i1_{(1,1)}} & P_{\hat{\chi}i}\mathcal{B}_i^n & P_{\hat{\chi}i}\mathcal{L}_i\mathcal{C} & 0 \\ * & -e^{-\lambda_\alpha \iota_\tau} Q_{\hat{\chi}i} & 0 & 0 \\ * & * & \Gamma_{i1_{(3,3)}} & P_{ei}\mathcal{B}_i^n \\ * & * & * & -e^{-\lambda_\alpha \iota_\tau} Q_{ei} \end{bmatrix} \\ \Gamma_{i2} &= \begin{bmatrix} \bar{\epsilon}_3(\bar{\mathcal{E}}_1^i + \bar{\mathcal{G}}_1^i \mathcal{K}_i)^T & \cdots & \bar{\epsilon}_3\left(\bar{\mathcal{E}}_q^i + \bar{\mathcal{G}}_q^i \mathcal{K}_i\right)^T \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{bmatrix} \\ \Gamma_{i3} &= \begin{bmatrix} 0 & \cdots & 0 \\ \bar{\epsilon}_5(\bar{\mathcal{F}}_1^i)^T & \cdots & \bar{\epsilon}_5(\bar{\mathcal{F}}_q^i)^T \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{bmatrix}, \Gamma_{i4} = \begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ \bar{\epsilon}_2(\bar{\mathcal{E}}_1^i)^T & \cdots & \bar{\epsilon}_2(\bar{\mathcal{E}}_q^i)^T \\ 0 & \cdots & 0 \end{bmatrix} \\ \Gamma_{i5} &= \begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ \bar{\epsilon}_4(\bar{\mathcal{F}}_1^i)^T & \cdots & \bar{\epsilon}_4(\bar{\mathcal{F}}_q^i)^T \end{bmatrix} \\ \bar{\epsilon}_2 &= \sqrt{\epsilon_2 \times q}, \quad \bar{\epsilon}_3 = \sqrt{\epsilon_3 \times q}, \quad \bar{\epsilon}_4 = \sqrt{\epsilon_4 \times q}, \quad \bar{\epsilon}_5 = \sqrt{\epsilon_5 \times q} \\ \Gamma_{i1_{(1,1)}} &= P_{\hat{\chi}i}(\mathcal{A}_i^n + \mathcal{D}_i^n \mathcal{K}_i) + (\mathcal{A}_i^n + \mathcal{D}_i^n \mathcal{K}_i)^T P_{\hat{\chi}i} \\ &+ \epsilon_1^{-1} P_{\hat{\chi}i} P_{\hat{\chi}i} + \epsilon_1 \bar{\pi}^2 \mathcal{H}_i^T \mathcal{H}_i + Q_{\hat{\chi}i} + \lambda_\alpha P_{\hat{\chi}i} \\ \Gamma_{i1_{(3,3)}} &= P_{ei}(\mathcal{A}_i^n - \mathcal{L}_i\mathcal{C}) + (\mathcal{A}_i^n - \mathcal{L}_i\mathcal{C})^T P_{ei} \\ &\left(\epsilon_2^{-1} + \epsilon_3^{-1} + \epsilon_4^{-1} + \epsilon_5^{-1} + \epsilon_6^{-1}\right) P_{ei}P_{ei} \\ &+ \epsilon_6 \bar{\pi}^2 \mathcal{H}_i^T \mathcal{H}_i + Q_{ei} + \lambda_\alpha P_{ei}\end{aligned}$$

From Lemma 1, if we could find matrix $\mathcal{V}$ such that

$$\mathcal{X}_{ei} = \mathcal{V} \begin{bmatrix} \mathcal{X}_{e1i} & 0 \\ 0 & \mathcal{X}_{e2i} \end{bmatrix} \mathcal{V}^T \tag{30}$$

could be established, the condition $\mathcal{C}\mathcal{X}_{ei} = \mathcal{R}_i\mathcal{C}$ holds. Now, let $P_{\hat{\chi}i}^{-1} = \mathcal{X}_{\hat{\chi}i}, P_{ei}^{-1} = \mathcal{X}_{ei}$, $Q_{\hat{\chi}i}^{-1} = \mathcal{Y}_{\hat{\chi}i}, Q_{ei}^{-1} = \mathcal{Y}_{ei}, \mathcal{K}_i\mathcal{X}_{\hat{\chi}i} = \mathcal{Z}_{\hat{\chi}i}, \mathcal{L}_i\mathcal{R}_i = \mathcal{Z}_{ei}$, applying the congruent transformation

$$\mathrm{diag}\left\{\mathcal{X}_{\hat{\chi}i}, \mathcal{Y}_{\hat{\chi}i}, \mathcal{X}_{ei}, \mathcal{Y}_{ei}, \underbrace{I,\ldots,I}_{r}, \underbrace{I,\ldots,I}_{r}, \underbrace{I,\ldots,I}_{r}, \underbrace{I,\ldots,I}_{r}\right\}$$

to the LMI (29), results in the LMI (16). Furthermore, based on Assumption 1, Equation (30), and the condition $\mathcal{C}\mathcal{X}_{ei} = \mathcal{R}_i\mathcal{C}$, the matrices $\mathcal{R}_i$ can be computed as follows:

$$\begin{aligned} &\mathcal{U}[\mathcal{C}_0 \quad 0]\mathcal{V}^T\mathcal{V}\begin{bmatrix}\mathcal{X}_{e1i} & 0\\ 0 & \mathcal{X}_{e2i}\end{bmatrix}\mathcal{V}^T = \mathcal{R}_i\mathcal{U}[\mathcal{C}_0 \quad 0]\mathcal{V}^T \\ &\mathcal{U}\mathcal{C}_0\mathcal{X}_{e1i} = \mathcal{R}_i\mathcal{U}\mathcal{C}_0 \Longrightarrow \mathcal{R}_i = \mathcal{U}\mathcal{C}_0\mathcal{X}_{e1i}(\mathcal{U}\mathcal{C}_0)^{-1} \end{aligned} \tag{31}$$

Therefore, observer gains can be computed through $\mathcal{L}_i = \mathcal{Z}_{ei}\mathcal{U}\mathcal{C}_0\mathcal{X}_{e1i}^{-1}\mathcal{C}_0^{-1}\mathcal{U}^T$. Furthermore, according to $\mathcal{K}_i\mathcal{X}_{\hat{\chi}i} = \mathcal{Z}_{\hat{\chi}i}$, it can be deduced that $\mathcal{K}_i = \mathcal{Z}_{\hat{\chi}i}\mathcal{X}_{\hat{\chi}i}^{-1}$. As a result, when $t \in [t_{k-1} + \omega_{k-1}, t_k)$, one can achieve that

$$\dot{V}_{1i}(t) + \lambda_\alpha V_{1i}(t) \le \zeta^T(t)\Psi_i\zeta(t) \tag{32}$$

From LMI (16), $\dot{V}_{1i}(t) + \lambda_\alpha V_{1i}(t) \le 0$ holds, and it means that, when $t \in [t_{k-1} + \omega_{k-1}, t_k)$, one has

$$V_{1i}(t) \le V_{1i}(t_{k-1} + \omega_{k-1})e^{-\lambda_\alpha(t-t_{k-1}-\omega_{k-1})} \tag{33}$$

Furthermore, according to (5), it is obvious that, at time $t \in [t_k, t_k + \omega_k)$, the jth subsystem is activated, but the control signal $\mathcal{K}_i$ and the observer $\mathcal{L}_i$ are not varied. Therefore, consider:

$$\begin{aligned} V_{2i}(t) &= \xi^T(t)P_i\xi(t) + \int_{t-\iota_\tau}^{t} e^{\lambda_\beta(t-s)}\xi^T(s)Q_i\xi(s)\mathrm{d}s \\ &= \hat{\chi}^T(t)P_{\hat{\chi}i}\hat{\chi}(t) + e^T(t)P_{ei}e(t) \\ &+ \int_{t-\iota_\tau}^{t} e^{\lambda_\beta(t-s)}\hat{\chi}^T(s)Q_{\hat{\chi}i}\hat{\chi}(s)\mathrm{d}s + \int_{t-\iota_\tau}^{t} e^{\lambda_\beta(t-s)}e^T(s)Q_{ei}e(s)\mathrm{d}s \end{aligned} \tag{34}$$

Similarly, we can write:

$$\dot{V}_{2i}(t) - \lambda_\beta V_{2i}(t) \le \zeta^T(t)\Theta_i\zeta(t) \tag{35}$$

Moreover, LMI (17) implies that $\dot{V}_{2i}(t) - \lambda_\beta V_{2i}(t) \le 0$. Integrating both sides of this inequality, one can acquire that

$$V_{2i}(t) \le V_{2i}(t_k)e^{\lambda_\beta(t-t_k)} \tag{36}$$

On the other hand, one can obtain the following inequalities:

$$\begin{aligned} \int_{t-\iota_\tau}^{t} e^{\lambda_\beta(t-s)}\xi^T(s)Q_i\xi(s)\mathrm{d}s &\le e^{\lambda_\beta\iota_\tau}\int_{t-\iota_\tau}^{t} \xi^T(s)Q_i\xi(s)\mathrm{d}s \\ &\le e^{(\lambda_\alpha+\lambda_\beta)\iota_\tau}\int_{t-\iota_\tau}^{t} e^{-\lambda_\alpha(t-s)}\xi^T(s)Q_i\xi(s)\mathrm{d}s \end{aligned} \tag{37}$$

From (15), (33), (36), (37), and similar to [31], one can derive the following results:

$$\begin{aligned} V(t) \leq & [e^{(\lambda_\alpha+\lambda_\beta)\iota_\tau}]^{N_\sigma(t_k,t)} \mu^{N_{\tilde{\sigma}}(t_k,t)} V(t_k) e^{\lambda_\beta \mathfrak{T}^+(t_k,t)-\lambda_\alpha \mathfrak{T}^-(t_k,t)} \\ \leq & [e^{(\lambda_\alpha+\lambda_\beta)\iota_\tau}]^{N_\sigma(t_{k-1},t)} \mu^{N_{\tilde{\sigma}}(t_{k-1},t)} V(t_{k-1}) e^{\lambda_\beta \mathfrak{T}^+(t_{k-1},t)-\lambda_\alpha \mathfrak{T}^-(t_{k-1},t)} \\ \leq & \cdots \leq \ [e^{(\lambda_\alpha+\lambda_\beta)\iota_\tau}]^{N_\sigma(t_0,t)} \mu^{N_{\tilde{\sigma}}(t_0,t)} V(t_0) e^{\lambda_\beta \mathfrak{T}^+(t_0,t)-\lambda_\alpha \mathfrak{T}^-(t_0,t)} \\ \leq & [e^{(\lambda_\alpha+\lambda_\beta)\iota_\tau}]^{N_\sigma(t_0,t)} \mu^{N_{\tilde{\sigma}}(t_0,t)} V(t_0) e^{(\lambda_\alpha+\lambda_\beta)\mathfrak{T}^+(t_0,t)-\lambda_\alpha(t-t_0)} \\ \leq & [e^{(\lambda_\alpha+\lambda_\beta)\iota_\tau}]^{N_\sigma(t_0,t)} \mu^{N_{\tilde{\sigma}}(t_0,t)} V(t_0) e^{\varpi(\lambda_\alpha+\lambda_\beta)N_\sigma(t_0,t)-\lambda_\alpha(t-t_0)} \end{aligned} \tag{38}$$

where $\mathfrak{T}^-(t_0,t)$ indicates the total matched intervals and $\mathfrak{T}^+(t_0,t)$ stands for (t_0,t), and $N_\sigma(t_0,t)$ is written as:

$$\begin{cases} N_\sigma(t_0,t) = N_{\tilde{\sigma}}(t_0,t), & t \in [t_{k-1}+\omega_{k-1}, t_k), \forall k = 1,2,\ldots,N. \\ N_\sigma(t_0,t) = N_{\tilde{\sigma}}(t_0,t)+1, & t \in [t_k, t_k+\omega_k), \forall k = 1,2,\ldots,N. \end{cases} \tag{39}$$

Furthermore, from Definition 2 and (39), we have:

$$\begin{aligned} V(t) \leq & \left[\mu e^{(\lambda_\alpha+\lambda_\beta)\iota_\tau}\right]^{\left(\frac{t-t_0}{\tau_a}\right)} V(t_0) e^{\varpi\left(\lambda_\alpha+\lambda_\beta\right)\left(\frac{t-t_0}{\tau_a}\right)-\lambda_\alpha(t-t_0)} \\ = & V(t_0) e^{-\left(\lambda_\alpha - \frac{\ln\left(\mu e^{(\lambda_\alpha+\lambda_\beta)\iota_\tau}\right)+\varpi\left(\lambda_\alpha+\lambda_\beta\right)}{\tau_a}\right)(t-t_0)} \end{aligned} \tag{40}$$

In addition, from (20) and (34), one can further achieve that

$$V(t) \geq \kappa_1 \|\xi(t)\|^2, \quad V(t_0) \leq \kappa_2 \|\xi(t_0)\|^2_{\aleph}. \tag{41}$$

Then, the solution of (5) exists globally and is satisfied as follows:

$$\|\xi(t)\|^2 \leq \frac{\kappa_2}{\kappa_1} \|\xi(t_0)\|^2_{\aleph} e^{-\left(\lambda_\alpha - \frac{\ln\left(\mu e^{(\lambda_\alpha+\lambda_\beta)\iota_\tau}\right)+\varpi\left(\lambda_\alpha+\lambda_\beta\right)}{\tau_a}\right)(t-t_0)} \tag{42}$$

where

$$\begin{aligned} \kappa_1 &= \min_{i\in M} \lambda_{\min}(P_i) \\ \kappa_2 &= \max_{i\in M} \lambda_{\max}(P_i) + \iota_\tau \max_{i\in M} \lambda_{\max}(Q_i) \end{aligned}$$

□

Remark 1. *It is worth nothing that the results of Theorem 1 can be extended to investigate the asynchronous H_∞ observer-based control problem and the prescribed performance index for the system (5) subject to the parametric uncertainties and the exogenous disturbance $\tilde{d}(t)$ can be obtained using the same method in [22] and Definition 1. In this regard, the following results can be achieved:*

$$\begin{cases} \dot{V}_{1i}(t) + \lambda_\alpha V_{1i}(t) \leq -y^T(t)y(t) + \vartheta^2 \tilde{d}^T(t)\tilde{d}(t), & t \in [t_{k-1}+\omega_{k-1}, t_k) \\ \dot{V}_{2i}(t) - \lambda_\beta V_{2i}(t) \leq -y^T(t)y(t) + \vartheta^2 \tilde{d}^T(t)\tilde{d}(t), & t \in [t_k, t_k+\omega_k) \end{cases} \tag{43}$$

Now, defining $\Upsilon(t) = y^T(t)y(t) - \vartheta^2 \tilde{d}^T(t)\tilde{d}(t)$ *and integrating from* $t_{k-1} + \omega_{k-1}$ *to* t *and* t_k *to* t *lead to the following inequalities:*

$$\begin{cases} V_{1i}(t) \le V_{1i}(t_{k-1} + \omega_{k-1})e^{-\lambda_\alpha(t-t_{k-1}-\omega_{k-1})} - \int_{t_{k-1}+\omega_{k-1}}^{t} \Upsilon(s)e^{-\lambda_\alpha(t-s)}\mathrm{d}s, \\ t \in [t_{k-1} + \omega_{k-1}, t_k) \\ V_{2i}(t) \le V_{2i}(t_k)e^{\lambda_\beta(t-t_k)} - \int_{t_k}^{t} \Upsilon(s)e^{\lambda_\beta(t-s)}\mathrm{d}s, \\ t \in [t_k, t_k + \omega_k) \end{cases} \tag{44}$$

Considering (38) *and* (46)*, the following results can be achieved:*

$$\begin{aligned} V(t) \le & [e^{(\lambda_\alpha+\lambda_\beta)\iota_\tau}]^{N_\sigma(t_k,t)}\mu^{N_{\tilde{\sigma}}(t_k,t)}V(t_k)e^{\lambda_\beta \mathfrak{T}^+(t_k,t)-\lambda_\alpha\mathfrak{T}^-(t_k,t)} \\ & - \int_{t_k}^{t} \mu^{N_{\tilde{\sigma}}(s,t)}[e^{(\lambda_\alpha+\lambda_\beta)\iota_\tau}]^{N_\sigma(s,t)}\Upsilon(s)e^{\lambda_\beta\mathfrak{T}^+(s,t)-\lambda_\alpha\mathfrak{T}^-(s,t)}\mathrm{d}s \\ \le & [e^{(\lambda_\alpha+\lambda_\beta)\iota_\tau}]^{N_\sigma(t_{k-1},t)}\mu^{N_{\tilde{\sigma}}(t_{k-1},t)}V(t_{k-1})e^{\lambda_\beta\mathfrak{T}^+(t_{k-1},t)-\lambda_\alpha\mathfrak{T}^-(t_{k-1},t)} \\ & - \int_{t_{k-1}}^{t} \mu^{N_{\tilde{\sigma}}(s,t)}[e^{(\lambda_\alpha+\lambda_\beta)\iota_\tau}]^{N_\sigma(s,t)}\Upsilon(s)e^{\lambda_\beta\mathfrak{T}^+(s,t)-\lambda_\alpha\mathfrak{T}^-(s,t)}\mathrm{d}s \\ \le & \cdots \le \ [e^{(\lambda_\alpha+\lambda_\beta)\iota_\tau}]^{N_\sigma(t_0,t)}\mu^{N_{\tilde{\sigma}}(t_0,t)}V(t_0)e^{\lambda_\beta\mathfrak{T}^+(t_0,t)-\lambda_\alpha\mathfrak{T}^-(t_0,t)} \\ & - \int_{t_0}^{t} \mu^{N_{\tilde{\sigma}}(s,t)}[e^{(\lambda_\alpha+\lambda_\beta)\iota_\tau}]^{N_\sigma(s,t)}\Upsilon(s)e^{\lambda_\beta\mathfrak{T}^+(s,t)-\lambda_\alpha\mathfrak{T}^-(s,t)}\mathrm{d}s \\ & = V(t_0)e^{\lambda_\beta\mathfrak{T}^+(t_0,t)-\lambda_\alpha\mathfrak{T}^-(t_0,t)+N_{\tilde{\sigma}}(t_0,t)\ln\mu+N_\sigma(t_0,t)\ln[e^{(\lambda_\alpha+\lambda_\beta)\iota_\tau}]} \\ & - \int_{t_0}^{t} \Upsilon(s)e^{\lambda_\beta\mathfrak{T}^+(s,t)-\lambda_\alpha\mathfrak{T}^-(s,t)+N_{\tilde{\sigma}}(s,t)\ln\mu+N_\sigma(s,t)\ln[e^{(\lambda_\alpha+\lambda_\beta)\iota_\tau}]}\mathrm{d}s \end{aligned} \tag{45}$$

Furthermore, utilizing some simplifications, for all $\tilde{d}(t) \in l_2[0,\infty)$*, one can get*

$$\int_{t_0}^{\infty} e^{-\lambda_\alpha(s-t_0)}y^T(s)y(s)\mathrm{d}s \le \int_{t_0}^{\infty} \psi^2\tilde{d}^T(s)\tilde{d}(s)\mathrm{d}s \tag{46}$$

in which the prescribed performance index ψ *is computed as*

$$\psi = \sqrt{\frac{\mu\vartheta^2}{1 - \frac{(\lambda_\alpha+\lambda_\beta)\varpi}{\ln\mu+(\iota_\tau+\varpi)(\lambda_\alpha+\lambda_\beta)}}} \tag{47}$$

Furthermore, ψ *can be minimized via searching the optimal value for the scalar* ϑ *in the elicited stabilization conditions. The following Theorem gives the sufficient conditions to design the observer-based controller and the prescribed performance index* ψ *for the system* (5)*.*

Theorem 2. *Consider system* (5) *under uncertain parameters* $\delta_{\theta_\ell} \in [-\tilde{\delta}_{\theta_\ell}, \tilde{\delta}_{\theta_\ell}]$*. For given positive scalars* $\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4, \epsilon_5, \epsilon_6, \epsilon_7$*, and parameters* $\iota_\tau, \varpi, \lambda_\alpha, \lambda_\beta, \bar{\pi}, \vartheta, \mu \ge 1$*, suppose that there exist symmetric matrices* $\mathcal{X}_{\hat{\chi}i} > 0, \mathcal{X}_{ei} > 0, \mathcal{Y}_{\hat{\chi}i} > 0, \mathcal{Y}_{ei} > 0$ *and matrices* $\mathcal{Z}_{\hat{\chi}i}, \mathcal{Z}_{ei}$*, for any* $i, j \in M, i \ne j$*, such that:*

$$\mathcal{X}_{\hat{\chi}j} \le \mu\mathcal{X}_{\hat{\chi}i}, \quad \mathcal{X}_{ej} \le \mu\mathcal{X}_{ei}, \quad \mathcal{Y}_{\hat{\chi}j} \le \mu\mathcal{Y}_{\hat{\chi}i}, \quad \mathcal{Y}_{ej} \le \mu\mathcal{Y}_{ei} \tag{48}$$

$$\Psi_i = \begin{bmatrix} \Psi_{i1} & \Psi_{i2} & \Psi_{i3} & \Psi_{i4} & \Psi_{i5} & \Psi_{i6} & \Psi_{i7} & \Psi_{i8} & \Psi_{i9} & \Psi_{i11} \\ * & -\mathbb{I} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\mathbb{I} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\mathbb{I} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\mathbb{I} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\mathbb{I} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & * & \Psi_{i10} & 0 \\ * & * & * & * & * & * & * & * & * & \Psi_{i12} \end{bmatrix} < 0 \tag{49}$$

$$\Theta_i = \begin{bmatrix} \Theta_{i1} & \Theta_{i2} & \Theta_{i3} & \Theta_{i4} & \Theta_{i5} & \Theta_{i6} & \Theta_{i7} & \Theta_{i8} & \Theta_{i9} & \Theta_{i11} \\ * & -\mathbb{I} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\mathbb{I} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\mathbb{I} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\mathbb{I} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\mathbb{I} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & * & \Theta_{i10} & 0 \\ * & * & * & * & * & * & * & * & * & \Theta_{i12} \end{bmatrix} < 0 \tag{50}$$

where

$$\Psi_{i1} = \begin{bmatrix} \Psi_{i1_{(1,1)}} & \mathcal{B}_i^n \mathcal{Y}_{\hat{\chi}i} & \mathcal{Z}_{ei}\mathcal{C} & 0 & 0 \\ * & -e^{-\lambda_\alpha \iota_\tau}\mathcal{Y}_{\hat{\chi}i} & 0 & 0 & 0 \\ * & * & \Psi_{i1_{(3,3)}} & \mathcal{B}_i^n \mathcal{Y}_{ei} & \mathcal{W}_i^n \\ * & * & * & -e^{-\lambda_\alpha \iota_\tau}\mathcal{Y}_{ei} & 0 \\ * & * & * & * & -\vartheta^2 I \end{bmatrix}$$

$$\Psi_{i2} = \begin{bmatrix} \bar{\epsilon}_3\left(\mathcal{X}_{\hat{\chi}i}(\bar{\mathcal{E}}_1^i)^T + \mathcal{Z}_{\hat{\chi}i}^T(\bar{\mathcal{G}}_1^i)^T\right) & \cdots & \bar{\epsilon}_3\left(\mathcal{X}_{\hat{\chi}i}(\bar{\mathcal{E}}_q^i)^T + \mathcal{Z}_{\hat{\chi}i}^T(\bar{\mathcal{G}}_q^i)^T\right) \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{bmatrix}$$

$$\Psi_{i3} = \begin{bmatrix} 0 & \cdots & 0 \\ \bar{\epsilon}_5 \mathcal{Y}_{\hat{\chi}i}(\bar{\mathcal{F}}_1^i)^T & \cdots & \bar{\epsilon}_5 \mathcal{Y}_{\hat{\chi}i}(\bar{\mathcal{F}}_q^i)^T \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{bmatrix}, \Psi_{i4} = \begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ \bar{\epsilon}_2 \mathcal{X}_{ei}(\bar{\mathcal{E}}_1^i)^T & \cdots & \bar{\epsilon}_2 \mathcal{X}_{ei}(\bar{\mathcal{E}}_q^i)^T \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{bmatrix}$$

$$\Psi_{i5} = \begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ \bar{\epsilon}_4 \mathcal{Y}_{ei}(\bar{\mathcal{F}}_1^i)^T & \cdots & \bar{\epsilon}_4 \mathcal{Y}_{ei}(\bar{\mathcal{F}}_q^i)^T \\ 0 & \cdots & 0 \end{bmatrix}, \Psi_{i6} = \begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ \bar{\epsilon}_7 (\bar{\mathcal{T}}_1^i)^T & \cdots & \bar{\epsilon}_7 \left(\bar{\mathcal{T}}_q^i\right)^T \end{bmatrix}$$

$$\Psi_{i7} = \begin{bmatrix} \bar{\epsilon}_1 \gamma \mathcal{X}_{\hat{\chi}i} \mathcal{H}_i^T \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \Psi_{i8} = \begin{bmatrix} 0 \\ 0 \\ \bar{\epsilon}_6 \gamma \mathcal{X}_{ei} \mathcal{H}_i^T \\ 0 \\ 0 \end{bmatrix}, \Psi_{i9} = \begin{bmatrix} \mathcal{X}_{\hat{\chi}i} & 0 \\ 0 & 0 \\ 0 & \mathcal{X}_{ei} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Psi_{i10} = \begin{bmatrix} -\mathcal{Y}_{\hat{\chi}i} & 0 \\ * & -\mathcal{Y}_{ei} \end{bmatrix}, \Psi_{i11} = \begin{bmatrix} \sqrt{3}\mathcal{X}_{\hat{\chi}i}\mathcal{C}^T & 0 \\ 0 & 0 \\ 0 & \sqrt{3}\mathcal{X}_{ei}\mathcal{C}^T \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \Psi_{i12} = \begin{bmatrix} -I & 0 \\ * & -I \end{bmatrix}$$

$$\begin{aligned}
\Psi_{i1_{(1,1)}} &= \mathcal{A}_i^n \mathcal{X}_{\hat{\chi}i} + \mathcal{X}_{\hat{\chi}i}(\mathcal{A}_i^n)^T + \mathcal{D}_i^n \mathcal{Z}_{\hat{\chi}i} + \mathcal{Z}_{\hat{\chi}i}^T(\mathcal{D}_i^n)^T + \epsilon_1^{-1} I + \lambda_\alpha \mathcal{X}_{\hat{\chi}i} \\
\Psi_{i1_{(3,3)}} &= \mathcal{A}_i^n \mathcal{X}_{ei} + \mathcal{X}_{ei}(\mathcal{A}_i^n)^T - \mathcal{Z}_{ei}\mathcal{C} - \mathcal{C}^T \mathcal{Z}_{ei}^T \\
&\quad + \left(\epsilon_2^{-1} + \epsilon_3^{-1} + \epsilon_4^{-1} + \epsilon_5^{-1} + \epsilon_6^{-1} + \epsilon_7^{-1}\right) I + \lambda_\alpha \mathcal{X}_{ei} \\
\bar{\epsilon}_1 &= \sqrt{\epsilon_1}, \bar{\epsilon}_2 = \sqrt{\epsilon_2 \times q}, \bar{\epsilon}_3 = \sqrt{\epsilon_3 \times q}, \bar{\epsilon}_4 = \sqrt{\epsilon_4 \times q}, \bar{\epsilon}_5 = \sqrt{\epsilon_5 \times q}, \\
\bar{\epsilon}_6 &= \sqrt{\epsilon_6}, \bar{\epsilon}_7 = \sqrt{\epsilon_7 \times q}, \bar{\mathcal{E}}_\ell^i = \tilde{\delta}_{\theta_\ell}\mathcal{E}_\ell^i, \bar{\mathcal{F}}_\ell^i = \tilde{\delta}_{\theta_\ell}\mathcal{F}_\ell^i, \bar{\mathcal{G}}_\ell^i = \tilde{\delta}_{\theta_\ell}\mathcal{G}_\ell^i, \bar{\mathcal{T}}_\ell^i = \tilde{\delta}_{\theta_\ell}\mathcal{T}_\ell^i
\end{aligned}$$

and

$$\Theta_{i1} = \begin{bmatrix} \Theta_{i1_{(1,1)}} & \mathcal{B}_j^n \mathcal{Y}_{\hat{\chi}i} & \mathcal{Z}_{ei}\mathcal{C} & 0 & 0 \\ * & -\mathcal{Y}_{\hat{\chi}i} & 0 & 0 & 0 \\ * & * & \Theta_{i1_{(3,3)}} & \mathcal{B}_j^n \mathcal{Y}_{ei} & \mathcal{W}_j^n \\ * & * & * & -\mathcal{Y}_{ei} & 0 \\ * & * & * & * & -\vartheta^2 I \end{bmatrix}$$

$$\Theta_{i2} = \begin{bmatrix} \bar{\epsilon}_3\left(\mathcal{X}_{\hat{\chi}i}(\bar{\mathcal{E}}_1^j)^T + \mathcal{Z}_{\hat{\chi}i}^T(\bar{\mathcal{G}}_1^j)^T\right) & \cdots & \bar{\epsilon}_3\left(\mathcal{X}_{\hat{\chi}i}(\bar{\mathcal{E}}_q^j)^T + \mathcal{Z}_{\hat{\chi}i}^T(\bar{\mathcal{G}}_q^j)^T\right) \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{bmatrix}$$

$$\Theta_{i3} = \begin{bmatrix} 0 & \cdots & 0 \\ \bar{\epsilon}_5\mathcal{Y}_{\hat{\chi}i}(\bar{\mathcal{F}}_1^j)^T & \cdots & \bar{\epsilon}_5\mathcal{Y}_{\hat{\chi}i}(\bar{\mathcal{F}}_q^j)^T \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{bmatrix}, \Theta_{i4} = \begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ \bar{\epsilon}_2\mathcal{X}_{ei}(\bar{\mathcal{E}}_1^j)^T & \cdots & \bar{\epsilon}_2\mathcal{X}_{ei}(\bar{\mathcal{E}}_q^j)^T \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{bmatrix}$$

$$\Theta_{i5} = \begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ \bar{\epsilon}_4\mathcal{Y}_{ei}(\bar{\mathcal{F}}_1^j)^T & \cdots & \bar{\epsilon}_4\mathcal{Y}_{ei}(\bar{\mathcal{F}}_q^j)^T \\ 0 & \cdots & 0 \end{bmatrix}, \Theta_{i6} = \begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ \bar{\epsilon}_7\left(\bar{\mathcal{T}}_1^j\right)^T & \cdots & \bar{\epsilon}_7\left(\bar{\mathcal{T}}_q^j\right)^T \end{bmatrix}$$

$$\Theta_{i7} = \begin{bmatrix} \bar{\epsilon}_1\gamma\mathcal{X}_{\hat{\chi}i}\mathcal{H}_j^T \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \Theta_{i8} = \begin{bmatrix} 0 \\ 0 \\ \bar{\epsilon}_6\gamma\mathcal{X}_{ei}\mathcal{H}_j^T \\ 0 \\ 0 \end{bmatrix}, \Theta_{i9} = \begin{bmatrix} \mathcal{X}_{\hat{\chi}i} & 0 \\ 0 & 0 \\ 0 & \mathcal{X}_{ei} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Theta_{i10} = \begin{bmatrix} -\mathcal{Y}_{\hat{\chi}i} & 0 \\ * & -\mathcal{Y}_{ei} \end{bmatrix}, \Theta_{i11} = \begin{bmatrix} \sqrt{3}\mathcal{X}_{\hat{\chi}i}\mathcal{C}^T & 0 \\ 0 & 0 \\ 0 & \sqrt{3}\mathcal{X}_{ei}\mathcal{C}^T \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \Theta_{i12} = \begin{bmatrix} -I & 0 \\ * & -I \end{bmatrix}$$

$$\begin{aligned}
\Theta_{i1_{(1,1)}} &= \mathcal{A}_j^n \mathcal{X}_{\hat{\chi}i} + \mathcal{X}_{\hat{\chi}i}\left(\mathcal{A}_j^n\right)^T + \mathcal{D}_j^n \mathcal{Z}_{\hat{\chi}i} + \mathcal{Z}_{\hat{\chi}i}^T\left(\mathcal{D}_j^n\right)^T + \epsilon_1^{-1} I - \lambda_\beta \mathcal{X}_{\hat{\chi}i} \\
\Theta_{i1_{(3,3)}} &= \mathcal{A}_j^n \mathcal{X}_{ei} + \mathcal{X}_{ei}\left(\mathcal{A}_j^n\right)^T - \mathcal{Z}_{ei}\mathcal{C} - \mathcal{C}^T \mathcal{Z}_{ei}^T \\
&\quad + \left(\epsilon_2^{-1} + \epsilon_3^{-1} + \epsilon_4^{-1} + \epsilon_5^{-1} + \epsilon_6^{-1} + \epsilon_7^{-1}\right) I - \lambda_\beta \mathcal{X}_{ei}
\end{aligned}$$

Then, system (5) *is exponentially stable under switchings* $\tau_a > \tau_a^* = \frac{\ln\mu + (\lambda_\alpha+\lambda_\beta)(\iota_\tau+\varpi)}{\lambda_\alpha}$. *Moreover, the controller gains, the observer gains, and the performance index* ψ *can be designed via* (18), (19), *and* (47), *respectively.*

Proof. Considering the conditions (43) and utilizing the same method in Theorem 1, LMIs (48)–(50) can be achieved. □

Remark 2. *In this paper, the robustness against time-delay, AS, and uncertainties are studied. For future studies, the designed scheme can be can be extended by employing the concept of impulsive stabilization [32–34].*

Remark 3. *Although static output feedback control method has been presented in [31] to control SSs, the switched observer is designed in this paper to estimate the states of the system. The proposed switched observer-based controller enables us to reconstruct the system states and steer them to zero in the presence of the AS problem and uncertain parameters of the system. Note that AS among system/observer modes is a challenging problem investigated in this paper. In this regard, the gains of the observer have switched asynchronously with the modes of the system; therefore, the suggested switched observer can tackle the lag between the switching instants of the system/observer. Furthermore, unlike the results of [31], the effects of external disturbances on the system are studied in this paper. For this purpose, a prescribed* H_∞ *performance level is considered, and a novel set of LMI-based conditions is achieved based on the multiple Lyapunov–Krasovskii functionals and an ADT approach. Therefore, the robustness of the system is guaranteed in the presence of external disturbances.*

4. Simulation Results

The numerical/practical examples are given to evaluate theoretical accomplishments in Theorems 1 and 2. In particular, the mass-springer plant with a switching dynamic [23,31] and the $F-18$ aircraft system [31,35] are provided to examine the designed controller under AS problem, affine parametric uncertainty, time-delay, and exogenous disturbances.

Example 1. *Considering the SS of the form* (1) *subject to the parametric uncertain matrices* (6) *and time-delay AS problem, the matrices are*

$$
\begin{aligned}
\mathcal{A}_1 &= \begin{bmatrix} 1.2+\delta_{\theta_1} & 0.4 \\ 1 & -1+\delta_{\theta_1} \end{bmatrix}, & \mathcal{A}_2 &= \begin{bmatrix} 1.6+\delta_{\theta_1} & 0.1 \\ 0.6 & -0.5+\delta_{\theta_1} \end{bmatrix} \\
\mathcal{B}_1 &= \begin{bmatrix} 0.2+\delta_{\theta_1} & 0.1 \\ 0.1 & -0.1+\delta_{\theta_1} \end{bmatrix}, & \mathcal{B}_2 &= \begin{bmatrix} 0.2+\delta_{\theta_1} & 0.1 \\ 0.1 & 0.1+\delta_{\theta_1} \end{bmatrix} \\
\mathcal{D}_1 &= \begin{bmatrix} -0.3+\delta_{\theta_1} & 0.6 \\ 0.5 & 0.9+\delta_{\theta_1} \end{bmatrix}, & \mathcal{D}_2 &= \begin{bmatrix} 0.1+\delta_{\theta_1} & 1.1 \\ 0.8 & -0.3+\delta_{\theta_1} \end{bmatrix} \\
\mathcal{W}_1 &= \begin{bmatrix} 0.2+\delta_{\theta_1} & 0.1 \\ 0.1 & 0.2+\delta_{\theta_1} \end{bmatrix}, & \mathcal{W}_2 &= \begin{bmatrix} 0.1+\delta_{\theta_1} & 0.2 \\ 0.2 & 0.2+\delta_{\theta_1} \end{bmatrix} \\
\Im_1(t,\chi(t)) &= \begin{bmatrix} 0.3\chi_1(t)\sin(\chi_2(t)) \\ 0.3\chi_1(t)\cos(\chi_2(t)) \end{bmatrix}, & \Im_2(t,\chi(t)) &= \begin{bmatrix} 0.5\chi_2(t)\cos(\chi_1(t)) \\ 0.5\chi_2(t)\sin(\chi_1(t)) \end{bmatrix} \\
\tilde{d}(t) &= \begin{bmatrix} 2\sin(4\pi t) \\ \frac{1}{1+t} \end{bmatrix}, & \mathcal{C} &= \begin{bmatrix} 0.4 & 0.6 \\ 0.6 & 0.4 \end{bmatrix}
\end{aligned}
\tag{51}
$$

The uncertainty of the system belongs to the interval as $[-0.1, 0.1]$. Then,

$$\begin{aligned} \mathcal{A}_1^n &= \begin{bmatrix} 1.2 & 0.4 \\ 1 & -1 \end{bmatrix}, & \mathcal{A}_2^n &= \begin{bmatrix} 1.6 & 1.1 \\ 0.6 & -0.5 \end{bmatrix} \\ \mathcal{B}_1^n &= \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & -0.1 \end{bmatrix}, & \mathcal{B}_2^n &= \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.1 \end{bmatrix} \\ \mathcal{D}_1^n &= \begin{bmatrix} -0.3 & 0.6 \\ 0.5 & 0.9 \end{bmatrix}, & \mathcal{D}_2^n &= \begin{bmatrix} 0.1 & 1.1 \\ 0.8 & -0.3 \end{bmatrix} \\ \mathcal{W}_1^n &= \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}, & \mathcal{W}_2^n &= \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 0.2 \end{bmatrix} \end{aligned} \tag{52}$$

Furthermore, the uncertain matrices are $\mathcal{E}_1^1 = \mathcal{E}_1^2 = \mathcal{F}_1^1 = \mathcal{F}_1^2 = \mathcal{G}_1^1 = \mathcal{G}_1^2 = \mathcal{T}_1^1 = \mathcal{T}_1^2 = I_{2\times 2}$. In addition, the time-delay and the upper-bound of lag time are selected as $\iota_\tau = 1$ (s) and $\varpi = 500$ (ms), respectively. Let $\lambda_\alpha = 0.3$, $\lambda_\beta = 0.2$, $\varepsilon_1 : \varepsilon_7 = 1$, and $\mu = 1.1$, then, the ADT is computed via $\tau_a > \tau_a^* = \frac{\ln \mu + (\lambda_\alpha + \lambda_\beta)(\iota_\tau + \varpi)}{\lambda_\alpha} = 2.8177$, and, from (47) the given value for the prescribed performance index is $\psi = 0.25$. In the case of the observer-based controller, using LMI Toolbox of MATLAB to solve the LMIs (48)–(50), the controller and observer gains are achieved as follows:

$$\mathcal{K}_1 = \begin{bmatrix} 2.0316 & -7.0247 \\ -15.5929 & -5.4244 \end{bmatrix}, \qquad \mathcal{K}_2 = \begin{bmatrix} 0.7716 & -7.2377 \\ -14.7551 & -4.7179 \end{bmatrix} \tag{53}$$

$$\mathcal{L}_1 = \begin{bmatrix} -8.3063 & 28.7817 \\ 7.3309 & 7.3658 \end{bmatrix}, \qquad \mathcal{L}_2 = \begin{bmatrix} -8.1690 & 28.9192 \\ 7.6978 & 6.9693 \end{bmatrix} \tag{54}$$

Figure 1 displays the switchings of the simulations. The time responses of the first and the second state variables with their estimations are represented in Figures 2 and 3, respectively. Figure 4 shows the time history of the estimation error while the system's output is demonstrated in Figure 5. Furthermore, the time response of control law is depicted in Figure 6. It can be viewed from simulations that utilizing the suggested observer-based control method leads to the stable estimates and states of the system with robust performance against the time-delay, parametric uncertainty, and exogenous disturbances. Moreover, variation of the designed control signal is acceptable.

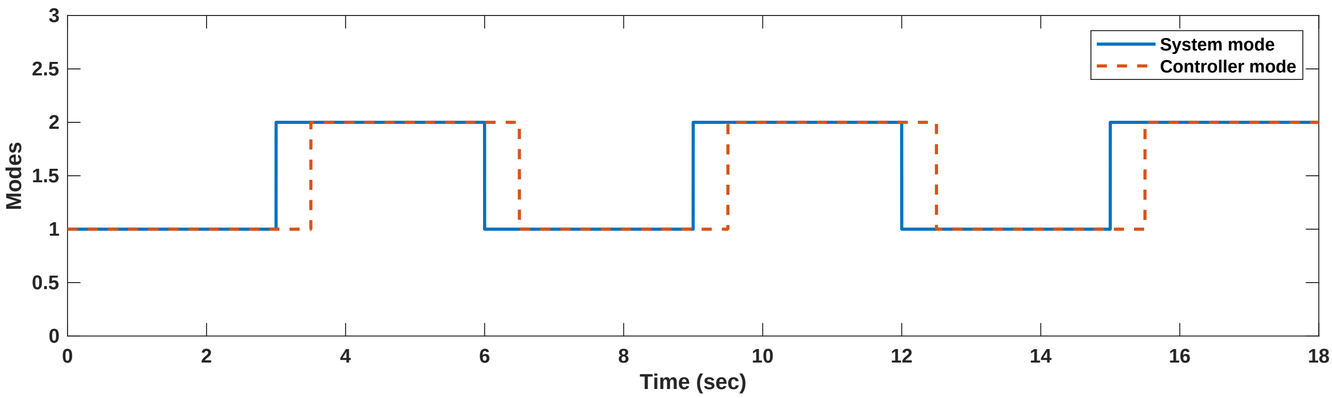

Figure 1. Example 1: Switching signal.

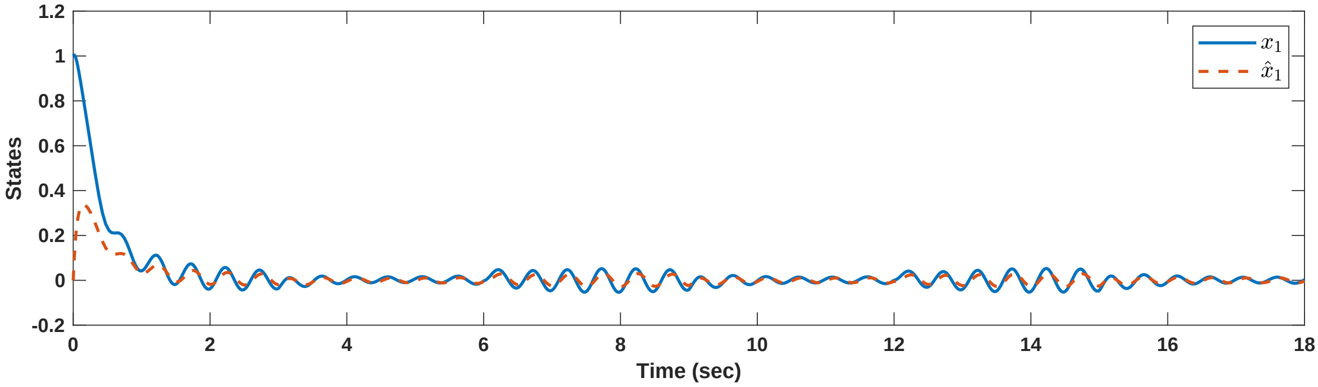

Figure 2. Example 1: First state variable and its corresponding estimation.

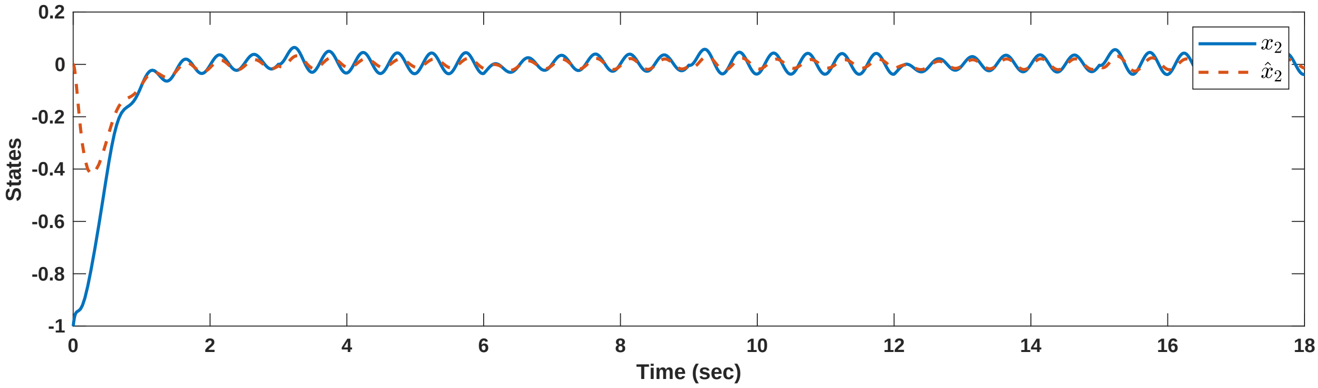

Figure 3. Example 1: Second state variable and its corresponding estimation.

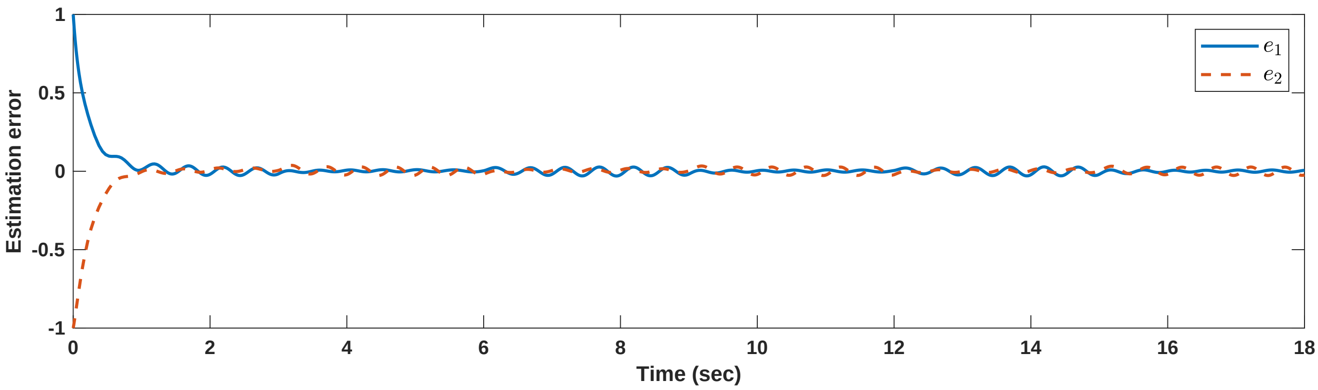

Figure 4. Example 1: Estimation error.

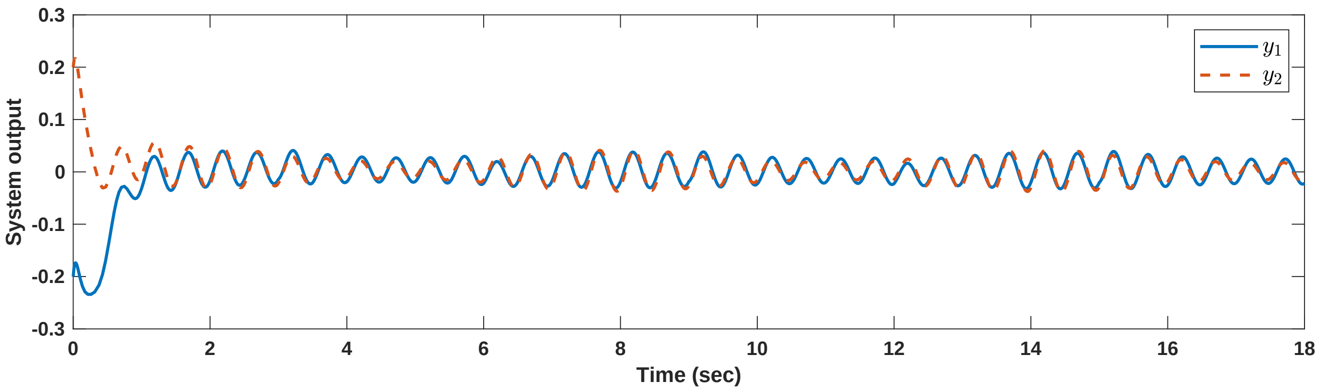

Figure 5. Example 1: Output signal.

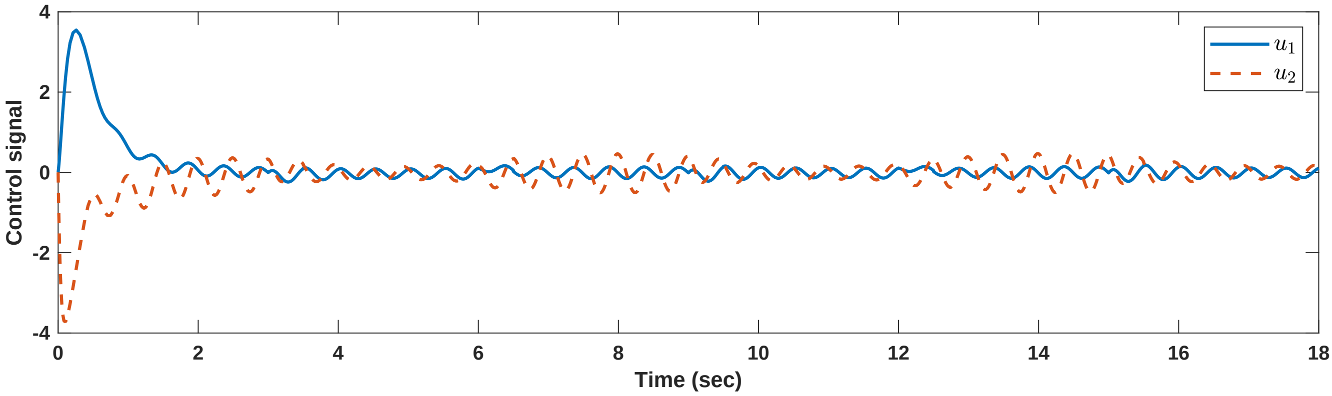

Figure 6. Example 1: Control signal.

Example 2. *Consider the mass-springer mechanical system (see [31] for more detail) with the following parameters:*

$$\begin{aligned}
\mathcal{A}_1 &= \begin{bmatrix} -1 & 1 \\ -1 & 2 \end{bmatrix}, & \mathcal{A}_2 &= \begin{bmatrix} -1 & 1 \\ -2-\delta_{\theta_1} & 3+\delta_{\theta_1} \end{bmatrix} \\
\mathcal{B}_1 &= \begin{bmatrix} 0 & 0 \\ -\delta_{\theta_1} & 1+\delta_{\theta_1} \end{bmatrix}, & \mathcal{B}_2 &= \begin{bmatrix} 0 & 0 \\ -1-\delta_{\theta_1} & 2+\delta_{\theta_1} \end{bmatrix} \\
\mathcal{D}_1 &= \begin{bmatrix} 0 \\ 1+\delta_{\theta_1} \end{bmatrix}, & \mathcal{D}_2 &= \begin{bmatrix} 0 \\ 1+\delta_{\theta_1} \end{bmatrix} \\
\mathcal{W}_1 &= \begin{bmatrix} 0.2+\delta_{\theta_1} & 0.1 \\ 0.1 & 0.2+\delta_{\theta_1} \end{bmatrix}, & \mathcal{W}_2 &= \begin{bmatrix} 0.1+\delta_{\theta_1} & 0.2 \\ 0.2 & 0.2+\delta_{\theta_1} \end{bmatrix} \\
\Im_1(t,\chi(t)) &= \begin{bmatrix} 0.1\sin(\chi_1(t)) \\ 0.1\cos(\chi_2(t)) \end{bmatrix}, & \Im_2(t,\chi(t)) &= \begin{bmatrix} 0.1\sin(\chi_2(t)) \\ 0.1\cos(\chi_1(t)) \end{bmatrix} \\
\tilde{d}(t) &= \begin{bmatrix} 0.7\sin(6\pi t) \\ 0.9\sin(2\pi t) \end{bmatrix}, & \mathcal{C} &= \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}
\end{aligned} \tag{55}$$

The nominal matrices are

$$\begin{aligned}
\mathcal{A}_1^n &= \begin{bmatrix} -1 & 1 \\ -1 & 2 \end{bmatrix}, & \mathcal{A}_2^n &= \begin{bmatrix} -1 & 1 \\ -2 & 3 \end{bmatrix} \\
\mathcal{B}_1^n &= \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, & \mathcal{B}_2^n &= \begin{bmatrix} 0 & 0 \\ -1 & 2 \end{bmatrix} \\
\mathcal{D}_1^n &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}, & \mathcal{D}_2^n &= \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\
\mathcal{W}_1^n &= \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 0.1 \end{bmatrix}, & \mathcal{W}_2^n &= \begin{bmatrix} 0.1 & 0.1 \\ 0 & 0.1 \end{bmatrix}
\end{aligned} \tag{56}$$

Accordingly, we have:

$$\begin{aligned}
\mathcal{E}_1^1 &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, & \mathcal{E}_1^2 &= \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix} \\
\mathcal{F}_1^1 &= \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix}, & \mathcal{F}_1^2 &= \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix} \\
\mathcal{G}_1^1 &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}, & \mathcal{G}_1^2 &= \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\
\mathcal{T}_1^1 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, & \mathcal{T}_1^2 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\end{aligned} \tag{57}$$

Let $\iota_\tau = 2$ (s) and $\varpi = 1$ (s). For simulations, choose $\lambda_\alpha = 0.4, \lambda_\beta = 0.2, \varepsilon_1 : \varepsilon_7 = 1$, and $\mu = 1.1$. As a result, the ADT is $\tau_a > \tau_a^* = \frac{\ln\mu + (\lambda_\alpha + \lambda_\beta)(\iota_\tau + \varpi)}{\alpha} = 4.7383$ and the given value for the prescribed index is $\psi = 0.2537$. Furthermore, by the use of LMI Toolbox of MATLAB to solve (48)–(50), the observer and controller parameters are acquired as follows:

$$\mathcal{K}_1 = \begin{bmatrix} -2.1619 & -25.7037 \end{bmatrix}, \qquad \mathcal{K}_2 = \begin{bmatrix} -2.8005 & -37.8457 \end{bmatrix} \tag{58}$$

$$\mathcal{L}_1 = \begin{bmatrix} 17.2409 & -4.5831 \\ -214.1548 & 196.4140 \end{bmatrix}, \qquad \mathcal{L}_2 = \begin{bmatrix} 27.5308 & -13.8381 \\ -334.9659 & 301.0917 \end{bmatrix} \tag{59}$$

Figure 7 depicts the switchings. The time responses and their approximations are illustrated in Figures 8 and 9. The estimation error is demonstrated in Figure 10 while the system's output is demonstrated in Figure 11. From the simulations, it is clear that the suggested observer-based control system can ensure the robustness under the parametric uncertainty, time-delay, exogenous disturbance, and the AS problem. Therefore, the designed controller, which is displayed in Figure 12, is persuasive.

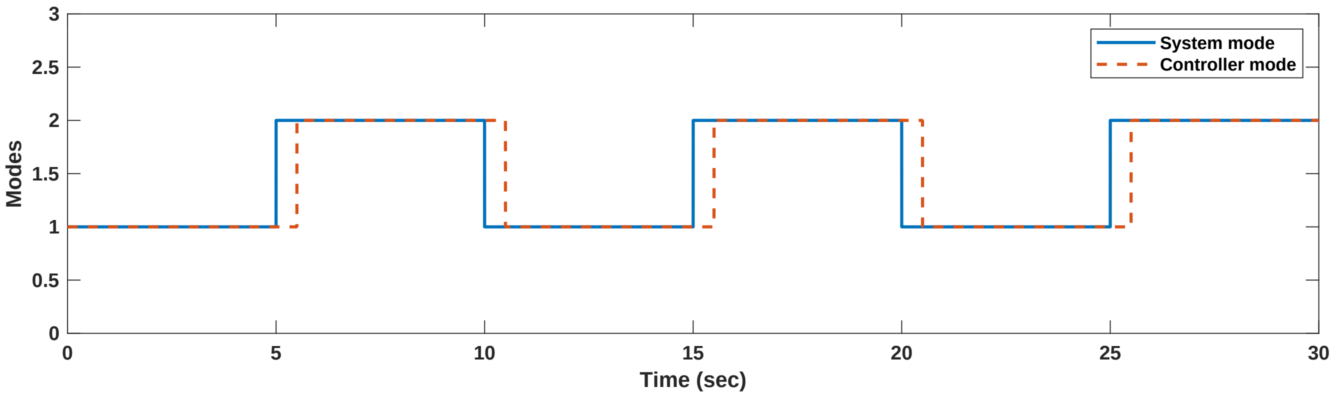

Figure 7. Example 2: Switching signal.

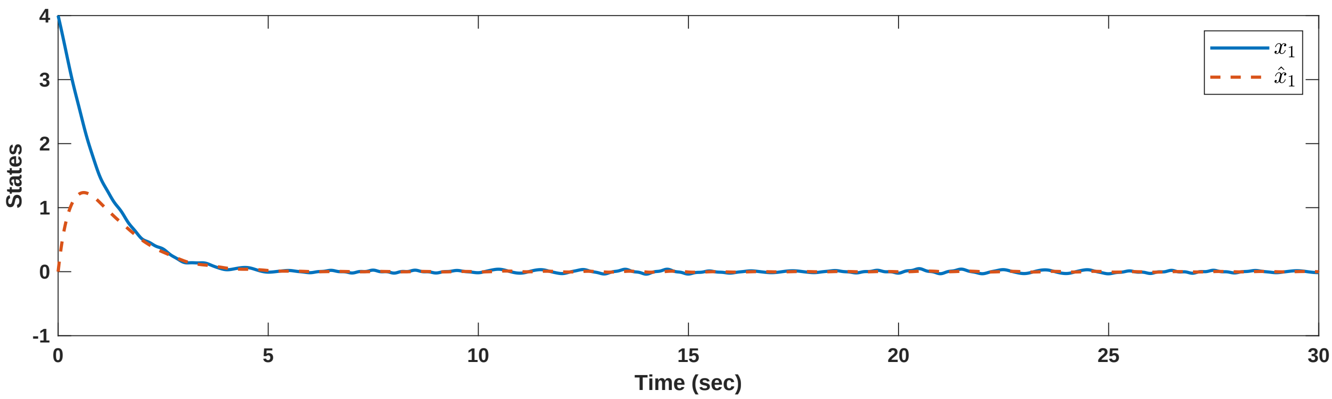

Figure 8. Example 2: First state variable and its estimation.

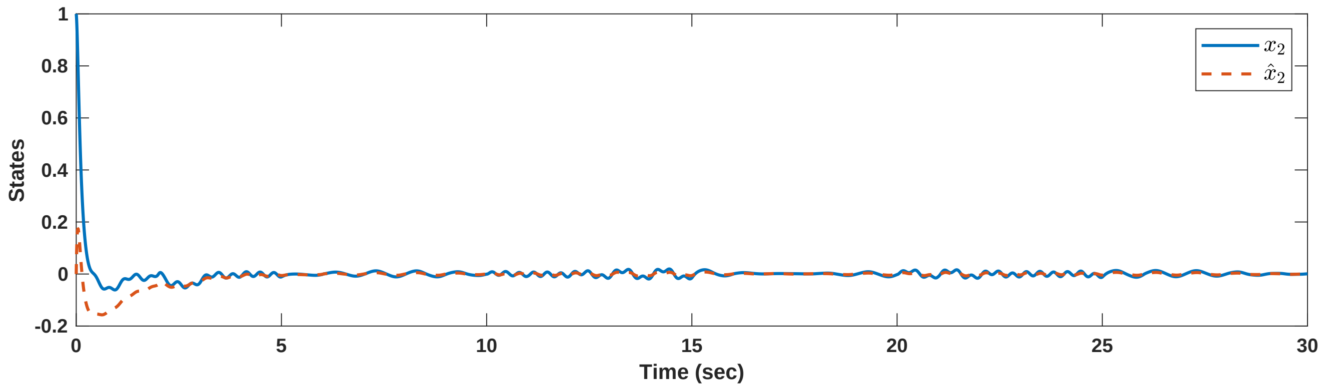

Figure 9. Example 2: Second state variable and its estimation.

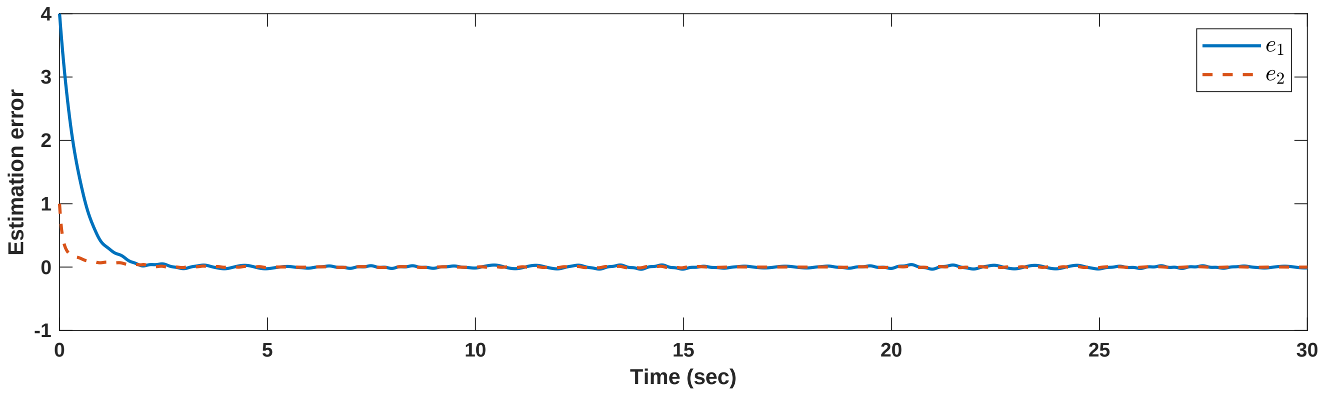

Figure 10. Example 2: Time response of the estimation error.

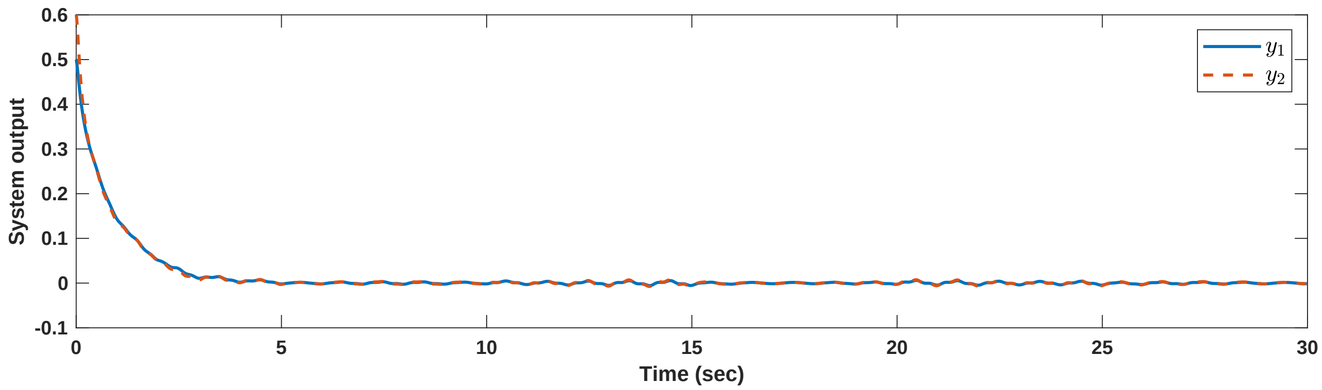

Figure 11. Example 2: Time response of output.

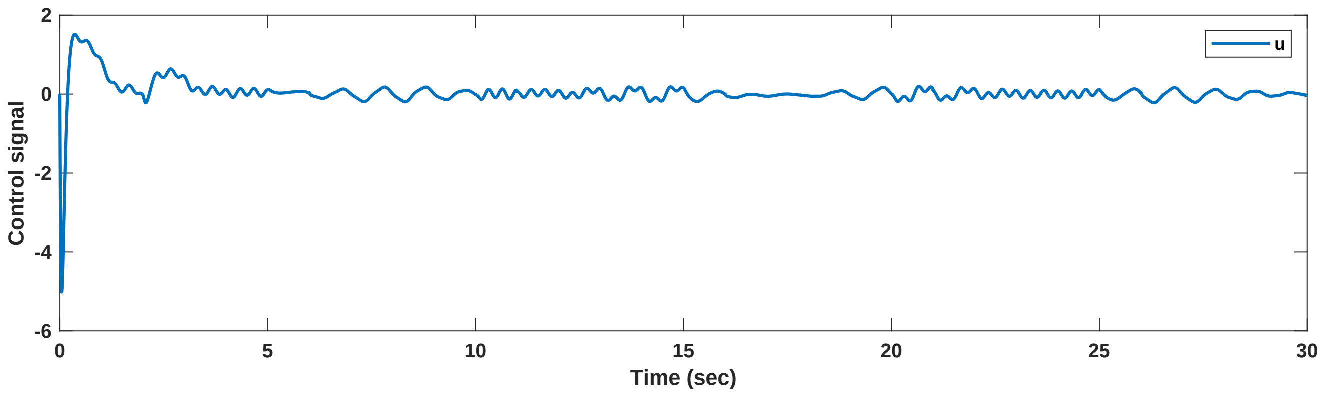

Figure 12. Example 2: Control signal.

Example 3. *The designed observer-based control scheme is examined to an $F-18$ aircraft (see [31] for more details) with the dynamic model of the form (3) and the parameters as follows:*

$$
\begin{aligned}
\mathcal{A}_1 &= \mathcal{A}_{\text{long}}^{\text{m5h40}} = \begin{bmatrix} -0.2423+0.1\delta_{\theta_1} & 0.9964+0.5\delta_{\theta_1} \\ -2.342+\delta_{\theta_1} & -0.1737+0.1\delta_{\theta_1} \end{bmatrix} \\
\mathcal{A}_2 &= \mathcal{A}_{\text{long}}^{\text{m6h30}} = \begin{bmatrix} -0.0416+0.05\delta_{\theta_1} & -0.01141+0.05\delta_{\theta_1} \\ -2.595+\delta_{\theta_1} & -0.8161+0.5\delta_{\theta_1} \end{bmatrix} \\
\mathcal{B}_1 &= \mathcal{B}_{\text{long}}^{\text{m5h40}} = \begin{bmatrix} 0.161+0.1\delta_{\theta_1} & 0.387+0.1\delta_{\theta_1} \\ -1.144+\delta_{\theta_1} & -0.06+0.05\delta_{\theta_1} \end{bmatrix} \\
\mathcal{B}_2 &= \mathcal{B}_{\text{long}}^{\text{m6h30}} = \begin{bmatrix} 0.017+0.05\delta_{\theta_1} & 0.001+0.05\delta_{\theta_1} \\ -1.817+\delta_{\theta_1} & -0.336+0.1\delta_{\theta_1} \end{bmatrix} \\
\mathcal{D}_1 &= \mathcal{D}_{\text{long}}^{\text{m5h40}} = \begin{bmatrix} -0.2423+0.1\delta_{\theta_1} & 0.4978+0.1\delta_{\theta_1} \\ -1.8420+0.1\delta_{\theta_1} & -0.0877 \end{bmatrix} \\
\mathcal{D}_2 &= \mathcal{D}_{\text{long}}^{\text{m6h30}} = \begin{bmatrix} 0.5088+0.1\delta_{\theta_1} & 0.0107+0.05\delta_{\theta_1} \\ 0.1310+0.1\delta_{\theta_1} & 0.6219+0.1\delta_{\theta_1} \end{bmatrix} \\
\mathcal{W}_1 &= \begin{bmatrix} 0.2+0.2\delta_{\theta_1} & 0.1 \\ 0.1 & 0.2+0.2\delta_{\theta_1} \end{bmatrix}, \quad \mathcal{W}_2 = \begin{bmatrix} 0.2+0.2\delta_{\theta_1} & 0.1 \\ 0.1 & 0.2+0.2\delta_{\theta_1} \end{bmatrix} \\
\Im_1(t,\chi(t)) &= \begin{bmatrix} 0.1\sin(\chi_2(t)) \\ 0.1\sin(\chi_1(t)) \end{bmatrix}, \quad \Im_2(t,\chi(t)) = \begin{bmatrix} 0.1\sin(\chi_1(t)) \\ 0.1\sin(\chi_2(t)) \end{bmatrix} \\
\tilde{d}(t) &= \begin{bmatrix} e^{(-0.1t)}\sin(0.6\pi t) \\ \sin(0.6\pi t) \end{bmatrix}, \quad \mathcal{C} = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.6 \end{bmatrix}
\end{aligned} \tag{60}
$$

where the uncertain parameter is $\delta_{\theta_1} = 0.6\sin(t)$, and one can obtain

$$
\begin{aligned}
\mathcal{A}_1^n &= \begin{bmatrix} -0.2423 & 0.9964 \\ -2.342 & -0.1737 \end{bmatrix}, & \mathcal{A}_2^n &= \begin{bmatrix} -0.0416 & -0.01141 \\ -2.595 & -0.8161 \end{bmatrix} \\
\mathcal{B}_1^n &= \begin{bmatrix} 0.161 & 0.387 \\ -1.144 & -0.06 \end{bmatrix}, & \mathcal{B}_2^n &= \begin{bmatrix} 0.017 & 0.001 \\ -1.817 & -0.336 \end{bmatrix} \\
\mathcal{D}_1^n &= \begin{bmatrix} -0.2423 & 0.4978 \\ -1.8420 & -0.0877 \end{bmatrix}, & \mathcal{D}_2^n &= \begin{bmatrix} 0.5088 & 0.0107 \\ 0.1310 & 0.6219 \end{bmatrix} \\
\mathcal{W}_1^n &= \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}, & \mathcal{W}_2^n &= \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}
\end{aligned} \tag{61}
$$

The uncertainty matrices are

$$
\begin{aligned}
\mathcal{E}_1^1 &= \begin{bmatrix} 0.1 & 0.5 \\ 1 & 0.1 \end{bmatrix}, & \mathcal{E}_1^2 &= \begin{bmatrix} 0.05 & 0.05 \\ 1 & 0.5 \end{bmatrix} \\
\mathcal{F}_1^1 &= \begin{bmatrix} 0.1 & 0.1 \\ 1 & 0.05 \end{bmatrix}, & \mathcal{F}_1^2 &= \begin{bmatrix} 0.05 & 0.05 \\ 1 & 0.1 \end{bmatrix} \\
\mathcal{G}_1^1 &= \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0 \end{bmatrix}, & \mathcal{G}_1^2 &= \begin{bmatrix} 0.1 & 0.05 \\ 0.1 & 0.1 \end{bmatrix} \\
\mathcal{T}_1^1 &= \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, & \mathcal{T}_1^2 &= \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}
\end{aligned} \tag{62}
$$

Furthermore, it is presumed that $\iota_\tau = 2$ (s) and $\varpi = 500$ (ms). With $\lambda_\alpha = 0.4, \lambda_\beta = 0.2$, $\varepsilon_1 : \varepsilon_7 = 1$, and $\mu = 1.1$, the ADT is 3.9883 and the prescribed performance index is $\psi = 0.2328$. Furthermore, solving (48)–(50) yields:

$$\mathcal{K}_1 = \begin{bmatrix} -13.0352 & 9.7596 \\ -49.4119 & -31.3771 \end{bmatrix}, \quad \mathcal{K}_2 = \begin{bmatrix} -17.2132 & 6.7025 \\ -53.4946 & -38.5073 \end{bmatrix} \tag{63}$$

$$\mathcal{L}_1 = \begin{bmatrix} 198.4849 & -158.3565 \\ 130.5456 & -63.4251 \end{bmatrix}, \quad \mathcal{L}_2 = \begin{bmatrix} 198.3110 & -159.4725 \\ 72.5886 & -24.3458 \end{bmatrix} \tag{64}$$

The switching signal is shown in Figure 13, and the estimated signals and trajectories controlled by the designed observer-based controller are illustrated in Figures 14 and 15. Moreover, the estimation error and system's output are represented in Figures 16 and 17, respectively. It is perceivable that the observer-based controller is properly operating under the AS problem and with respect to time-delay, uncertainties, and external perturbations.

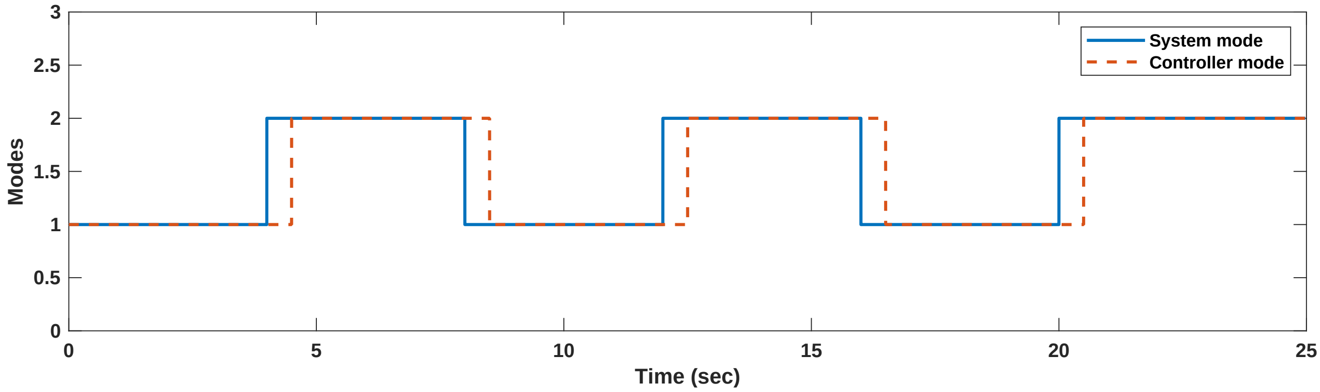

Figure 13. Example 3: Switching signal.

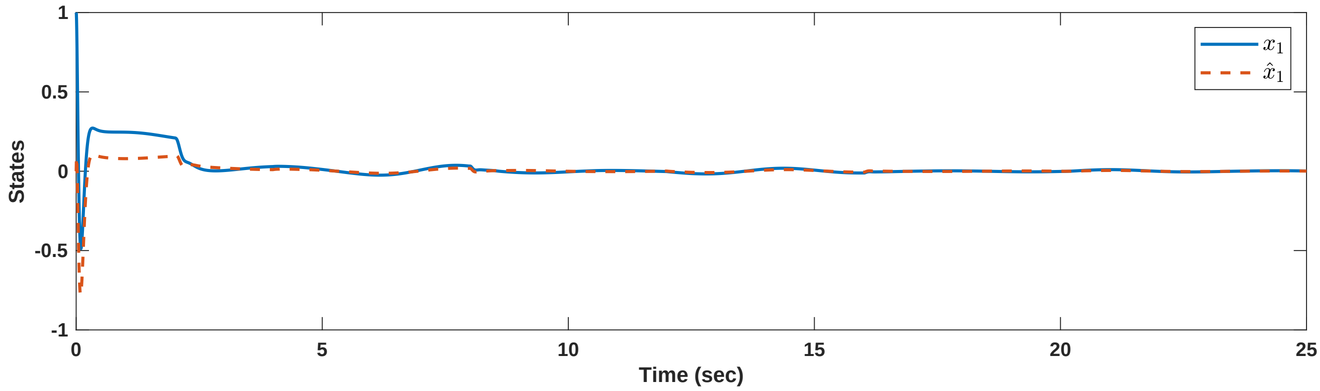

Figure 14. Example 3: First state and its estimation.

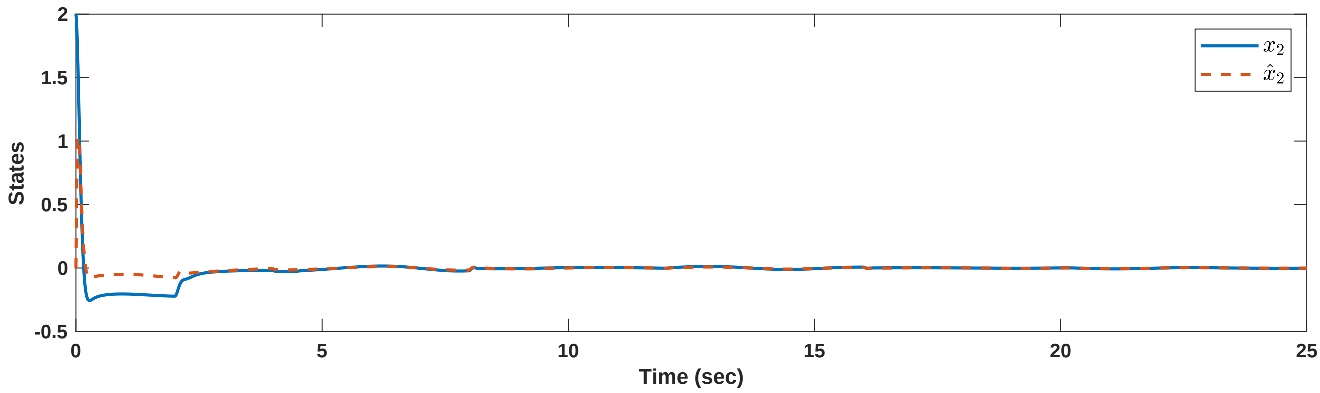

Figure 15. Example 3: Second state and its estimation.

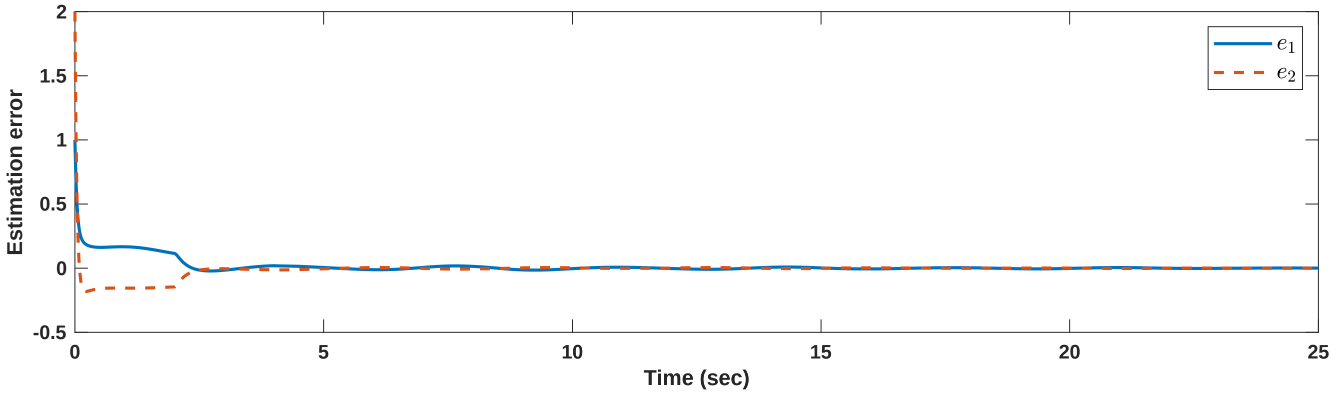

Figure 16. Example 3: Estimation error.

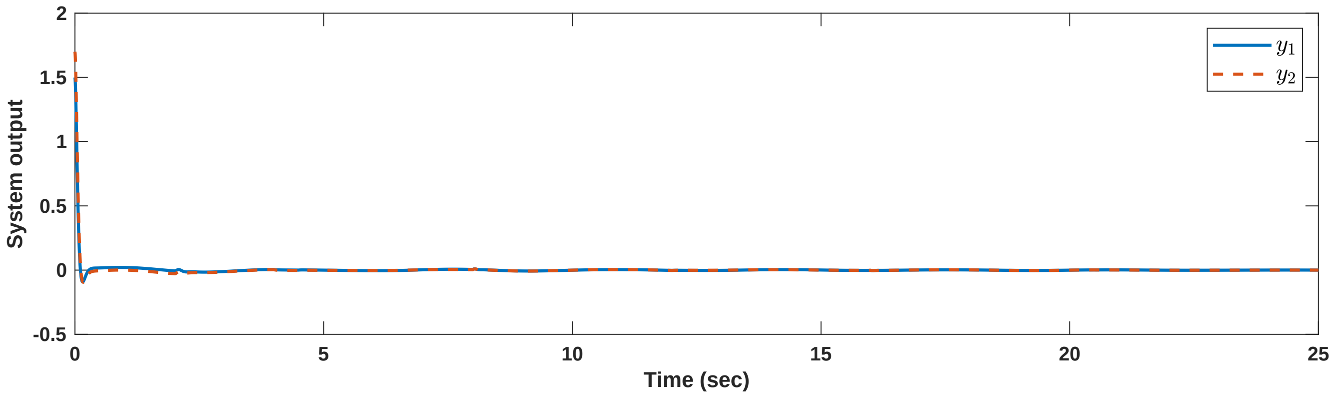

Figure 17. Example 3: Time response of the $F-18$ aircraft system's output.

5. Conclusions

In this study, a control technique was presented for nonlinear SSs under time-delay, uncertainties, and AS problems. In this regard, switched Lyapunov–Krasovskii techniques and the ADT approach were utilized to obtain sufficient stabilization conditions. To derive the observer/controller gains, the obtained conditions were converted into LMIs, and an observer-based control policy was developed to reconstruct the system states and stabilize the closed-loop system. Stabilization conditions were proposed as a feasibility problem that depends on the value of time-delay, upper bounds of the uncertainties and lag time, and Lipschitz constants. Furthermore, proposing a novel set of LMI-based conditions, the H_∞ observer-based control problem was investigated for the AS problem against exogenous disturbances. Finally, simulations have demonstrated the superiority of the suggested observer-based control law.

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