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A Robust Share-of-Choice Model

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Abstract: In this paper, we propose an approach to take into account, in a robust way, part-worth uncertainty in a share-of-choice (SOC) model. More precisely, we extend the method proposed by Wang and Curry by endogenously including competition. Indeed in their approach, competition is described exogenously and the model cannot take into account part-worth uncertainty for the competition's products. Our extension permits us to take into account all effects of part-worth uncertainty, even those relative to the competition, and therefore improve substantially Wang and Curry's approach.

Keywords: share-of-choice model; robust optimisation; conjoint analysis

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1. Introduction

More than sixty years ago, the development of mathematical psychology [1–3] allowed a significant breakthrough in the modelling of consumers' behaviour, which led to the development of conjoint analysis. Since the seminal paper of Green and Rao [4], conjoint analysis has become an inescapable tool in marketing science (see [5–11] for literature reviews). However, the accuracy of part-worths estimation is one of the main issues concerning conjoint analysis (see for instance [12–15] for studies conducted to assess different conjoint analysis methods) and handling these inaccuracies in an efficient manner is a crucial point.

Robust optimisation is a powerful method to handle parameters' uncertainties in optimisation models. Soyster [16] proposes a linear programming model that takes into account data uncertainty in a robust way. However this approach is in practice barely useful as it is far too conservative. Other approaches that are less conservative have been proposed by Ben-Tal and Nemirovskii [17,18] and El Ghaoui et al. [19]. These approaches rely on convex problems that can be computationally challenging to solve. Bertsimas and Sim [20] propose an alternative approach that combines both the linearity advantage and the control of conservatism. Indeed their approach use a parameter (T) to control the number of coefficient prone to uncertainty. The modeller can thus vary this number to obtain a compromise between robustness (high T) and less over-conservatism (low T).

Zufryden [21] seems to be the first one to propose a product design optimiser using the paradigm of conjoint analysis. His model, later named share-of-choice (SOC), aims at finding the product configuration that will maximise the share of preference and is formulated as a mixed-integer linear program. The SOC model permitted, among others, the development of product line design (PLD) models (see for instance [22,23]) or the development of production models (see for instance [24–26]).

Following Bertsimas and Sim approach,Wang and Curry [27] were the first to propose a SOC model that takes into account part-worth uncertainty using a robust approach (later, Bertsimas and Mišić [28] will propose a more general method for PLD). However, their SOC model does not take competition into account in an endogenous way. Indeed, competition is modelled through hurdle utilities that are treated as exogenous parameters. The problem with this approach lies in the fact that the robust model cannot take into account part-worth uncertainty for the competition's products. In other words, this model takes into account only a part of the effects of part-worth uncertainty.

The present paper proposes to overcome this problem. Concretely, it proposes an extension of Wang and Curry's model in order to take into account part-worth uncertainty also for the competition's products. To do so, we have to make two changes in Wang and Curry's model. First, the hurdle utilities are replaced with the utilities of competition's products and are computed endogenously using part-worths. But doing so, i.e., introducing explicitly competition utilities in a robust model, we create a new issue. Indeed, in this case, attributes that have ordinal levels with respect to utility must be handled with care. For instance, the area of an hotel room is such an attribute, as for everybody, the larger the better. For these attributes, the part-worth of a given level cannot be lower than the part-worth of a less desired level and these constraints must be introduced in the model in order to avoid distortions. Taking into account these constraints is the second change made to Wang and Curry's model.

The present paper is organised as follows. In Section 2, we present the deterministic version of the SOC model. Then, in Section 3, we develop a robust version of this deterministic model. In Section 4 a numerical example is presented. This example serves to illustrate the feasibility of the method as well as to explain how to use the software provided as Supplementary Material. Finally, we conclude in Section 5.

2. The Deterministic Model

In this section, we present the deterministic version of the SOC model from which our robust model will be built. Roughly speaking, the SOC model aims at finding the product configuration that will maximise the share of preference taking into account the products proposed by competition. Throughout the rest of the paper, K, C, I and J_i represent the sets of respondents, competitors, attributes and attribute levels, respectively. The variable k, c, i and j, represent a respondent, a competitor, an attribute and an attribute level, respectively. Configuring a product consists in choosing a level for each attribute. Let $X(i,j)$ be the variable describing the product configuration. We have $X(i,j) = 1$ if attribute i is set to level j and $X(i,j) = 0$ otherwise. Let $Y(k)$ be the variable describing the preference of respondent k. We have $Y(k) = 1$ if respondent k prefers the new designed product to all products proposed by the competitors and $Y(k) = 0$ otherwise. The part-worth $u(k,i,j)$ represents the utility perceived by respondent k if attribute i is set to level j. Let $X_c(i,j)$ be the product configuration for competitor c.

The SOC model can thus be written as follows.

$$\max_{X,Y} \sum_{k \in K} Y(k). \tag{1}$$

subject to

$$\sum_{i \in I} \sum_{j \in J_i} u(k,i,j) \cdot (X(i,j) - X_c(i,j)) \geq u_m(k) + (Y(k) - 1) \cdot M \qquad \forall k \in K, \forall c \in C, \tag{2}$$

$$\sum_{j \in J_i} X(i,j) = 1 \qquad \forall i \in I, \tag{3}$$

$$X(i,j), Y(k) \quad \text{binary} \qquad \forall k \in K, \forall i \in I, \forall j \in J_i, \tag{4}$$

where M is a big number. The objective is to maximize the share of preference. Equation (2) ensures that a respondent is counted as a new potential client only if the perceived utility of the proposed product is sufficiently greater than the perceived utility of each product proposed by competition. In this equation, $u_m(k)$ represents the minimum increase in utility that will turn respondent k away from competition. This equation is slightly different from the one in [27], where the perceived utility must be greater than a given hurdle utility that does not depend explicitly on products proposed by competition. Equation (3) ensures that one and only one level is chosen for each attribute. This deterministic SOC model is a mixed-integer linear program that can be solved using standard methods or dedicated algorithm [29,30].

3. The Robust Model

In order to take into account the possible error in part-worths estimation, we will follow the work of Wang and Curry [27] and use their notations when possible. Like them, we suppose that the true part-worth $\hat{u}$ lies in the interval $[u - \bar{u}, u + \bar{u}]$ where u represent the part-worth estimation and $\bar{u}$ the maximal error of estimation. However, we enhance their model in order to take into account cases where some attribute have ordinal levels with respect to utility. For instance, the area of an hotel room is such an attribute, as for everybody, the larger the better. For such attributes, the part-worth of a given level cannot be lower than the part-worth of a less desired level. We can collect all these constraints in a compact notation using the incidence matrix S as follows

$$\sum_{i \in I} \sum_{j \in J_i} S(n,i,j) \cdot \hat{u}(k,i,j) \leq 0 \qquad \forall k \in K, \forall n \in N, \tag{5}$$

where N is the set of constraints on part-worths.

Following Bertisimas and Sim [20], we suppose that the number of part-worths prone to uncertainty is given by T and can be chosen by the modeller. The idea behind this robust approach is to take into account the worst case if T part-worths are uncertain. The modeller can vary this number to obtain a compromise between robustness (high T) and less over-conservatism (low T). Note that if T is equal the total number of part-worths, the model is equivalent to conservative model proposed by Soyster [16], and if $T = 0$, the model is equivalent to the deterministic SOC model (1)–(4). In order to take into account this uncertainty, we introduce in Equation (2) a so called protection function that will count for the relative decrease in utility considering this worst case scenario. The robust SOC model writes then

$$\max_{X,Y} \sum_{k \in K} Y(k) \tag{6}$$

subject to

$$\sum_{i \in I} \sum_{j \in J_i} u(k,i,j) \cdot (X(i,j) - X_c(i,j)) - \beta(c,k) \geq u_m(k) + (Y(k) - 1) \cdot M \qquad \forall k \in K, \forall c \in C, \tag{7}$$

$$\sum_{j \in J_i} X(i,j) = 1 \qquad \forall i \in I, \tag{8}$$

$$X(i,j), Y(k) \quad \text{binary} \qquad \forall k \in K, \forall i \in I, \forall j \in J_i, \tag{9}$$

where the protection function $\beta(c,k)$ is given by

$$\beta(c,k) = \max_{Z^+,Z^-} \sum_{i \in I} \sum_{j \in J_i} \bar{u}(k,i,j) \cdot (X(i,j) - X_c(i,j)) \cdot (Z^+(c,k,i,j) - Z^-(c,k,i,j)) \tag{10}$$

subject to

$$\sum_{i \in I} \sum_{j \in J_i} (Z^+(c,k,i,j) + Z^-(c,k,i,j)) \leq T, \tag{11}$$

$$0 \leq Z^+(c,k,i,j) \leq 1 \qquad \forall i \in I, j \in J_i, \tag{12}$$

$$0 \leq Z^-(c,k,i,j) \leq 1 \qquad \forall i \in I, j \in J_i, \tag{13}$$

$$\sum_{i \in I} \sum_{j \in J_i} S(n,i,j) \cdot \bar{u}(k,i,j) \cdot \left(Z^+(c,k,i,j) - Z^-(c,k,i,j)\right) \leq -\sum_{i \in I} \sum_{j \in J_i} S(n,i,j) \cdot u(k,i,j) \qquad \forall n \in N. \tag{14}$$

Equation (7) is identical to Equation (2), except that a protection function β is introduced in order to take into account the relative reduction in utility in the worst case when T part-worths are prone to uncertainty. Z^+ and Z^- can take value between zero and one and can be seen as variables that describe if the part-worth estimation is considered exact or prone to estimation errors. More precisely, Z^+, respectively Z^-, is different from zero if the part-worth is considered as underestimated, respectively overestimated, and zero otherwise. Note that at optimality, these variables are either null or equal to one and they cannot be both different from zero. In reference [27], the single variable Z is used instead. This is possible as in their model the competition's products are not described endogenously and consequently, the worst case scenario is always an overestimation of the part-worths. Equation (14) comes from the constraints on the part-worth given in Equation (5). We have indeed

$$\begin{aligned}
&\sum_{i\in I}\sum_{j\in J_i} S(n,i,j)\cdot \hat{u}(k,i,j) \\
=\ &\sum_{i\in I}\sum_{j\in J_i} S(n,i,j)\cdot \big(u(k,i,j)+\bar{u}(k,i,j)\cdot (Z^+(c,k,i,j)-Z^-(c,k,i,j))\big) \\
=\ &\sum_{i\in I}\sum_{j\in J_i} S(n,i,j)\cdot \bar{u}(k,i,j)\cdot \big(Z^+(c,k,i,j)-Z^-(c,k,i,j)\big)+\sum_{i\in I}\sum_{j\in J_i} S(n,i,j)\cdot u(k,i,j).
\end{aligned}$$

At this point, we can see that the robust SOC model (6)–(14) consists in two optimization models. To merge them into a single model keeping linear properties, we have to use standard techniques based on duality theory. Let us denote with Q, P^+, P^- and R the dual variables associated with the constraints (11)–(14), respectively. Then, the dual of model (10)–(14) writes

$$\begin{aligned}
\beta(c,k)=\min_{Q,P,R}\quad & T\cdot Q(c,k)+\sum_{i\in I}\sum_{j\in J_i}\big(P^+(c,k,i,j)+P^-(c,k,i,j)\big) \\
&-\sum_{n\in N}\sum_{i\in I}\sum_{j\in J_i} S(n,i,j)\cdot u(k,i,j)\cdot R(n,c,k)
\end{aligned}\tag{15}$$

subject to

$$\begin{aligned}
& Q(c,k)+P^+(c,k,i,j)+\sum_{n\in N} S(n,i,j)\cdot \bar{u}(k,i,j)\cdot R(n,c,k) \\
\geq\ & \bar{u}(k,i,j)\cdot (X(i,j)-X_c(i,j)) \quad \forall i\in I,\ j\in J_i,
\end{aligned}\tag{16}$$

$$\begin{aligned}
& Q(c,k)+P^-(c,k,i,j)-\sum_{n\in N} S(n,i,j)\cdot \bar{u}(k,i,j)\cdot R(n,c,k) \\
\geq\ & -\bar{u}(k,i,j)\cdot (X(i,j)-X_c(i,j)) \quad \forall i\in I,\ j\in J_i,
\end{aligned}\tag{17}$$

$$Q(c,k),P^+(c,k,i,j),P^-(c,k,i,j),R(n,c,k)\geq 0 \qquad \forall c\in C,\ \forall k\in K,\ \forall i\in I,\ \forall j\in J_i. \tag{18}$$

By strong duality, the robust SOC problem (6)–(14) can therefore be rewritten as a single mixed-integer linear programming model as follows

$$\max_{X,Y}\sum_{k\in K} Y(k) \tag{19}$$

$$\sum_{i\in I}\sum_{j\in J_i} u(k,i,j)\cdot (X(i,j)-X_c(i,j))-\beta(c,k) \geq u_m(k)+(Y(k)-1)\cdot M \qquad \forall c\in C,\ \forall k\in K, \tag{20}$$

$$\sum_{j\in J_i} X(i,j)=1 \qquad \forall i\in I, \tag{21}$$

$$X(i,j),Y(k)\quad \text{binary}, \tag{22}$$

$$\begin{aligned}
\beta(c,k)\ =\ & T\cdot Q(c,k)+\sum_{i\in I}\sum_{j\in J_i}\big(P^+(c,k,i,j)+P^-(c,k,i,j)\big) \\
&-\sum_{n\in N}\sum_{i\in I}\sum_{j\in J_i} S(n,i,j)\cdot u(k,i,j)\cdot R(n,c,k) \qquad \forall c\in C,\ \forall k\in K,
\end{aligned}\tag{23}$$

$$
\begin{aligned}
& Q(c,k) + P^+(c,k,i,j) + \sum_{n \in N} S(n,i,j) \cdot \bar{u}(k,i,j) \cdot R(n,c,k) \\
\geq \;& \bar{u}(k,i,j) \cdot (X(i,j) - X_c(i,j)) \qquad \forall c \in C, \forall k \in K, \forall i \in I, \forall j \in J_i, \qquad (24)\\
& Q(c,k) + P^-(c,k,i,j) - \sum_{n \in N} S(n,i,j) \cdot \bar{u}(k,i,j) \cdot R(n,c,k) \\
\geq \;& -\bar{u}(k,i,j) \cdot (X(i,j) - X_c(i,j)) \qquad \forall c \in C, \forall k \in K, \forall i \in I, \forall j \in J_i, \qquad (25)
\end{aligned}
$$

$$
Q(c,k), P^+(c,k,i,j), P^-(c,k,i,j), R(n,c,k) \geq 0 \qquad \forall c \in C, \forall k \in K, \forall i \in I, \forall j \in J_i, \qquad (26)
$$

where M is a big number. One can easily verify that it is sufficient to take

$$
M = \max_{k \in K} u_m(k) + \sum_{i \in I} \max_{k \in K, j \in J_i} (u(k,i,j) + \bar{u}(k,i,j)) - \sum_{i \in I} \min_{k \in K, j \in J_i} (u(k,i,j) - \bar{u}(k,i,j)) \qquad (27)
$$

To sum up, the robust SOC model (19)–(26) is a mixed-integer linear program that can be solved using standard techniques. It finds the product configuration that will maximize the share of preference taking into account, in a robust manner, possible errors in part-worths estimation.

4. Numerical Illustration

In this section, we present a brief numerical experiment to illustrate the method. The purpose of this small case study is to show the feasibility of the method as well as to serve as a tutorial for the Supplementary Materials given with the present paper. It is kept as small as possible for didactic reasons.

4.1. The Case Study

The case study aims to design the service of a hotel. To take the model as simple as possible, we suppose that the proposed service must be the same for all clients. The model could easily be extended to take into account the more realistic case where different services can be proposed (i.e., a PLD). We suppose the service has four salient attributes: room area, breakfast type, location and style. The levels considered for the service design are given in Table 1.

Table 1. Salient attributes and their levels.

Attribute	Level
room	standard superior suite
breakfast	none American
location	city sea mountain
style	casual formal

Two competitors are already on the market, the services they propose are given in Table 2.

Table 2. Competition service design.

Attribute	Level	
	Competitor 1	**Competitor 2**
room	superior	suite
breakfast	American	without
location	sea	city
style	formal	formal

We consider ten respondents; their respective part-worths are given in Table 3. At this point, it is worth mentioning few things about the sample. For this case study, the small sample was chosen arbitrarily as its purpose is to show how a model that does not take into account Equation (5) can lead to an unrealistic worst case scenario. For a real case study, the sample size must be obviously much bigger (see [31] for a rule of thumb to choose the sample size). Let $\bar{u}$, the maximal error of estimation, equal 2 for the attribute room and 1 for the others attributes. These values are a little bit greater than what we encounter in real cases, we did so to emphasize the effects of estimation errors.

Table 3. Utility part-worth for each respondent.

Attribute	Level	Respondent									
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
room	standard	0	0	0	0	0	0	0	0	0	0
	superior	3	1	1	1	1	2	1	2	2	1
	suite	4	2	2	2	2	3	2	3	3	2
breakfast	without	0	0	0	0	0	0	0	0	0	0
	American	1	1	1	1	1	1	2	1	2	1
location	city	2	4	1	0	0	5	4	2	0	1
	mountain	1	0	2	3	2	0	4	2	2	4
	sea	1	3	2	3	2	0	0	3	1	2
style	casual	0	0	0	0	0	1	1	1	1	1
	formal	1	1	1	1	0	0	1	0	0	0

Finally, we suppose that the minimum increase in utility that will turn respondent k away from competition, i.e., $u_m(k)$, is equal to 0.5 for all respondents.

4.2. The Model in AMPL

The method has been implemented using the AMPL modelling environment [32]. All files used for the experiment are provided as Supplementary Materials. The model consist in three files. The file robust_SOC.mod contains the algebraic definition of the robust SOC model (19)–(26). This file contains no data and remains the same whatever the case study. It can be therefore considered as a black-box by the final user. The file robust_SOC.dat contains model's data and must be filled by the final user (see Figure 1).

For the majority of parameters the format is obvious. Let us just indicate few points that are less evident. The set ORDINAL_ATTRIBUTE contains the list of attributes that have ordinal levels with respect to utility. For these attributes, it is important to list their level by increasing desirability (see ATTRIBUTE_LEVEL). Indeed, for these attributes, the program will generate automatically the incidence matrix S that describes the constraints on part-worths associated with these preferences. The parameter X_COMP$[c, i, j]$ is the product configuration for competitor c. Its value is zero by default and must be set to one if, for competitor c, attribute i is set to level j (see Table 2). The parameter utility$[k, i, j]$ represents the part-worth $u(k, i, j)$ and is given in a table where each line represents a given attribute level and each double column a given respondent. For each double column, the first number represents k and the second one $u(k, i, j)$ as given in Table 3. The same format is used for utility_uncertainty$[k, i, j]$ which represents $\bar{u}$ the maximal error of estimation. The model must be run using the file robust_SOC.run in the AMPL environment. Note that it can also be handled using the open source environment GNU MathProg [33].

```
param K:=10;#number of respondents
param C:=2; #number of competitors
param T:=1; #number of coefficients prone to uncertainty

set ATTRIBUTE:= room breakfast location style;
set ORDINAL_ATTRIBUTES:= room breakfast;

set ATTRIBUTE_LEVEL[room]:= standard superior suite;
set ATTRIBUTE_LEVEL[breakfast]:= without American;
set ATTRIBUTE_LEVEL[location]:= city mountain sea;
set ATTRIBUTE_LEVEL[style]:= casual formal;

param X_COMP :=
[1,room,superior]           1
[1,breakfast,American]      1
[1,location,sea]            1
[1,style,formal]            1
[2,room,suite]              1
[2,breakfast,without]       1
[2,location,city]           1
[2,style,formal]            1
;#competition service design.

param utility_min_increase default 0.5;

param utility:=
[*,room,standard]           1 0    2 0    3 0    4 0    5 0    6 0    7 0    8 0    9 0    10 0
[*,room,superior]           1 3    2 1    3 1    4 1    5 1    6 2    7 1    8 2    9 2    10 1
[*,room,suite]              1 4    2 2    3 2    4 2    5 2    6 3    7 2    8 3    9 3    10 2
[*,breakfast,without]       1 0    2 0    3 0    4 0    5 0    6 0    7 0    8 0    9 0    10 0
[*,breakfast,American]      1 1    2 1    3 1    4 1    5 1    6 1    7 2    8 1    9 2    10 1
[*,location,city]           1 2    2 4    3 1    4 0    5 0    6 5    7 4    8 2    9 0    10 1
[*,location,mountain]       1 1    2 0    3 2    4 3    5 2    6 0    7 4    8 2    9 2    10 4
[*,location,sea]            1 1    2 3    3 2    4 3    5 2    6 0    7 0    8 3    9 1    10 2
[*,style,casual]            1 0    2 0    3 0    4 0    5 0    6 1    7 1    8 1    9 1    10 1
[*,style,formal]            1 1    2 1    3 1    4 1    5 0    6 0    7 1    8 0    9 0    10 0
;

param utility_uncertainty :=
[*,room,standard]           1 2    2 2    3 2    4 2    5 2    6 2    7 2    8 2    9 2    10 2
[*,room,superior]           1 2    2 2    3 2    4 2    5 2    6 2    7 2    8 2    9 2    10 2
[*,room,suite]              1 2    2 2    3 2    4 2    5 2    6 2    7 2    8 2    9 2    10 2
[*,breakfast,without]       1 1    2 1    3 1    4 1    5 1    6 1    7 1    8 1    9 1    10 1
[*,breakfast,American]      1 1    2 1    3 1    4 1    5 1    6 1    7 1    8 1    9 1    10 1
[*,location,city]           1 1    2 1    3 1    4 1    5 1    6 1    7 1    8 1    9 1    10 1
[*,location,mountain]       1 1    2 1    3 1    4 1    5 1    6 1    7 1    8 1    9 1    10 1
[*,location,sea]            1 1    2 1    3 1    4 1    5 1    6 1    7 1    8 1    9 1    10 1
[*,style,casual]            1 1    2 1    3 1    4 1    5 1    6 1    7 1    8 1    9 1    10 1
[*,style,formal]            1 1    2 1    3 1    4 1    5 1    6 1    7 1    8 1    9 1    10 1
;
```

Figure 1. The data file robust_SOC.dat.

4.3. Numerical Results

We run the model for different value of T, the number of coefficient prone to uncertainty. For our small model, we have four attributes and therefore the maximal value for T is four. The results for the different value of T are given in Table 4.

Table 4. Market share and optimal service design for different values of T.

	$T = 0$	$T = 1$	$T = 2$	$T \geq 3$
Market Share	**60%**	**30%**	**10%**	**0%**
breakfast	American	American	American	
location	sea	mountain	mountain	
room	suite	suite	superior	
style	formal	casual	casual	

For the case $T = 0$, i.e., the deterministic model, the market share is 60%. As expected, we see that the greater the number of coefficient prone to uncertainty the lower the market share for the worst case scenario. In our small example, we see that if more than two parameters (out of four) are prone to uncertainty the market share is zero. This shows clearly the limits of the robust method if T is taken too large, i.e., in the case of over-conservatism. We also see that the robust optimal design in case of uncertainty is not necessary the same as the optimal design for the deterministic case.

To show the necessity of including the constraints on part-worth for attributes with ordinal levels with respect to utility (Equation (5)), we also run a model without these constraints. For the case $T = 0$ the result is, by definition, the same as in Table 4. For $T = 1$, the result is the same as in Table 4, but for $T \geq 2$ the market share is zero. This small example shows that, if these constraints are not present, the worst case scenario can be an unrealistic event, which leads to wrong decision and which is obviously not acceptable.

5. Conclusions

In this paper, we propose an approach to take into account, in a robust way, part-worth uncertainty in a SOC model. More precisely, we extend the method proposed by Wang and Curry [27] by including endogenously competition. Indeed, Wang and Curry describe competition through hurdle utilities that are treated as exogenous parameters and part-worth uncertainty for competition's products cannot therefore be taken into account. Our approach permits us to take into account all effects of part-worth uncertainty, even those relative to the competition, and consequently improve substantially Wang and Curry's approach. We also present a small case study to show the feasibility of the method as well as to serve as a tutorial for the Supplementary Materials given with the present paper. With this small case study we also show numerical how a model that does not take into account Equation (5) can lead to an unrealistic worst case scenario and therefore can induce a bad decision.

The method proposed in the present paper is an extension of Wang and Curry's method. Consequently, for the validity of the present method, we let readers refer to Wang and Curry's study, where a Monte-Carlo experiment is conducted to assess the predictive ability of their method. It would be interesting to compare our extension of Wang and Curry's method to the method proposed by Bertsimas and Mišić [28], which is to our knowledge the only other robust method for designing products. However, it is beyond the scope of this paper to conduct such a study and we leave it for further research.

The method has been implemented using the AMPL modelling environment. All files are provided as Supplementary Materials and can easily be used even by persons not familiar with robust optimisation.

The robust SOC model proposed in the present paper is a mixed-integer linear model and can easily be combined with other models to form a meta-model. We think more particularly of two fields of application. Firstly, it can be coupled with a comprehensive production model to offer a sophisticated PLD model that can take into account in a precise manner all production specificities and constraints. The resulting PLD is a versatile model that can be used to design products and services in any field. For instance, it can be used to design sophisticated tailored services. Indeed, we are currently working on a real case to help design the wine tourism experience offered by a company. Secondly, it can be coupled with an energy model in order to describe the real consumers choices. Indeed, consumer behaviour is very complex and even sometimes not economically rational. Unfortunately, standard techno-economic energy planning models assumed the economic rationality hypothesis and, consequently, represented consumers' behaviour incorrectly. Coupling the robust SOC model with an energy model is a way to describe in a correct manner the real behaviour of consumers, taking into account the inaccuracy of part-worths estimation.

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Abbreviations

The following abbreviations are used in this manuscript:

PLD	product line design
SOC	share-of-choice

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