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Total Roman {3}-Domination: The Complexity and Linear-Time Algorithm for Trees

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Abstract: For a simple graph $G=(V,E)$ with no isolated vertices, a total Roman {3}-dominating function(TR3DF) on G is a function $f:V(G)\rightarrow\{0,1,2,3\}$ having the property that (i) $\sum_{w\in N(v)} f(w)\geq 3$ if $f(v)=0$; (ii) $\sum_{w\in N(v)} f(w)\geq 2$ if $f(v)=1$; and (iii) every vertex v with $f(v)\neq 0$ has a neighbor u with $f(u)\neq 0$ for every vertex $v\in V(G)$. The weight of a TR3DF f is the sum $f(V)=\sum_{v\in V(G)} f(v)$ and the minimum weight of a total Roman {3}-dominating function on G is called the total Roman {3}-domination number denoted by $\gamma_{t\{R3\}}(G)$. In this paper, we show that the total Roman {3}-domination problem is NP-complete for planar graphs and chordal bipartite graphs. Finally, we present a linear-time algorithm to compute the value of $\gamma_{t\{R3\}}$ for trees.

Keywords: dominating set; total roman {3}-domination; NP-complete; linear-time algorithm

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1. Introduction

Let $G=(V,E)$ be a graph with vertex set $V=V(G)$ and edge set $E=E(G)$. For every vertex $v\in V$, the open neighborhood $N_G(v)=N(v)=\{u\in V(G):uv\in E(G)\}$ and the closed neighborhood $N_G[v]=N[v]=N(v)\cup\{v\}$. We denote the degree of v by $d_G(v)=d(v)=|N_G(v)|$. A vertex of degree one is called a leaf and its neighbor is a support vertex, and a support vertex is called a strong support if it is adjacent to at least two leaves. Let S_n be a star with order n. A tree T is an acyclic connected graph. $G=(G_1\cup G_2)$ is a union graph G such that $V(G)=V(G_1)\cup V(G_2)$ and $E(G)=E(G_1)\cup E(G_2)$.

Given a graph G and a positive integer k, assume that $f:V(G)\rightarrow\{0,1,2,...,k\}$ is a function, and suppose that $(V_0,V_1,..,V_k)$ is the ordered partition of V introduced by f, where $V_i=\{v\in V(G):f(v)=i\}$ for $i\in\{0,1,...,k\}$. Then we can write $f=(V_0,V_1,..,V_k)$ and $\omega_f(V(G))=\sum_{v\in V(G)} f(v)$ is the weight of a function f of G.

A subset S of a vertex set $V(G)$ is a dominating set of G if for every vertex $v\in V(G)\setminus S$, there exists a vertex $w\in S$ such that wv is an edge of G. The domination number of G denoted by $\gamma(G)$ is the smallest cardinality of a dominating set S of G [1]. A function $f:V(G)\rightarrow\{0,1\}$ is called a dominating function(DF) on G if every vertex u with $f(u)=0$ has a vertex $v\in N(u)$ such that $f(v)=1$ [2]. The dominating set problem(DSP) is to find the domination number of G, which has been deeply and widely studied in recent years [3–7].

A subset S of a vertex set $V(G)$ is a total dominating set of G if $\bigcup_{v\in S} N(v)=V(G)$. The total domination number of G denoted by $\gamma_t(G)$ is the smallest cardinality of a total dominating set S of G [8]. The literature on the subject of total domination in graphs has been surveyed and provided in detail in a recent book [9]. Moreover, Michael A. Henning et al. presented a survey of selected recent results on total domination in graphs [10].

The mathematical concept of Roman domination is originally defined and discussed by Stewart et al. [11] and ReVelle et al. [12]. A Roman dominating function(RDF) on graph G is a function $f:V(G)\rightarrow\{0,1,2\}$ such that every vertex $v\in V(G)$ for which $f(u)=0$ is adjacent to at least one vertex u with $f(u)=2$ [13]. The Roman domination number of

G is the minimum weight overall RDFs, denoted by $\gamma_R(G)$ [14]. On the basis of Roman domination, signed Roman domination [15], double Roman domination [16] and total Roman domination [17] have been proposed recently.

The total Roman dominating function(TRDF) on G is an RDF f on G with an additional property that every vertex $v \in V(G)$ with $f(v) \neq 0$ has a neighbor u with $f(u) \neq 0$. Let $\gamma_{tR}(G)$ denote the minimum weight of all TRDFs on G. A TRDF on G with weight $\gamma_{tR}(G)$ is called a $\gamma_{tR}(G)$-function. The conception of TRDF was first defined by Hossein Ahangar et al. [18]. In addition, Nicolás Campanelli et al. studied the total Roman domination number of the lexicographic product of graphs [17] and Chloe Lampman et al. presented some basic results of Edge-Critical Graphs [19].

The Roman $\{2\}$-dominating function (also named Italian domination) f [20] introduced by Chellali et al. which is defined as follows: $f : V(G) \to \{0, 1, 2\}$ has the property that $\sum_{u \in N(v)} f(u) \geq 2$ for $f(v) = 0$ [21]. Chellali et al. proved that the Roman $\{2\}$-domination problem is NP-complete for bipartite graphs [21]. Hangdi Chen showed that the Roman $\{2\}$-domination problem is NP-complete for split graphs, and gave a linear-time algorithm for finding the minimum weight of Roman $\{2\}$-dominating function in block graphs [22]. As a generalization of Roman domination, Michael A. Henning et al. studied the relationship between Roman $\{2\}$-domination and dominating set parameters in trees [20].

A Roman $\{3\}$-dominating function(R{3}DF) f defined by Mojdeh et al. [23], which is defined as follows: $f : V(G) \to \{0, 1, 2, 3\}$ has the property that for every vertex $v \in V(G)$ with $f(v) \in \{0, 1\}$ and $\sum_{u \in N(v)} f(u) \geq 3$. Mojdeh et al. presented an upper bound on the Roman $\{3\}$-domination number of a connected graph G, characterized the graphs attaining upper bound and showed that the Roman $\{3\}$-domination problem is NP-complete, even restricted to bipartite graphs [23] .

The total Roman $\{3\}$-domination [24] was studied recently . The total Roman $\{3\}$-dominating function(TR3DF) on a graph G is an R{3}DF on G with the additional property that every vertex $v \in V(G)$ with $f(v) \neq 0$ has a neighbor w with $f(w) \neq 0$. The minimum weight of a total Roman $\{3\}$-dominating function on G denoted by $\gamma_{t\{R3\}}(G)$ is named the total Roman $\{3\}$-domination number of G. A $\gamma_{t\{R3\}}(G)$-function is a total Roman $\{3\}$-dominating function on G with weight $\gamma_{t\{R3\}}(G)$. Doost Ali Mojdeh et al. showed the relationship among total Roman $\{3\}$-domination, total domination, and total Roman$\{2\}$-domination parameters. They also presented an upper bound on the total Roman $\{3\}$-domination number of a connected graph G and characterized the graphs arriving this bound. Finally, they investigated that total Roman $\{3\}$-domination problem is NP-complete for bipartite graphs [24].

In this paper, we further investigate the complexity of total Roman $\{3\}$-domination in planar graphs and chordal bipartite graphs. Moreover, we give a linear-time algorithm to compute the $\gamma_{t\{R3\}}$ for trees which answer the problem that it is possible to construct a polynomial algorithm for computing the number of total Roman $\{3\}$-domination for trees [24].

2. Complexity

In this section, we study the complexity of total Roman {3}-domination of graph. We show that the total Roman {3}-domination problem is NP-complete for planar graphs and chordal bipartite graphs. Consider the following decision problem.

Total Roman {3}-Domination Problem TR3DP.
Instance: Graph $G = (V, E)$, and a positive integer m.
Question: Does G have a total Roman {3}-function with weight at most m?

Please note that the dominating set problem is NP-complete for planar graphs [25] and chordal bipartite graphs [26]. We show the NP-completeness results by reducing the well-known NP-complete problem, dominating set, to TR3D.

Let G be a graph on n vertices. Let T_v be the tree with $V(T_v) = \{v, v_a, v_b, v_c, v_d, v_e, v_f, v_p, v_q\}$, $E(T_v) = \{vv_a, v_av_c, v_cv_e, v_cv_f, vv_b, v_bv_d, v_dv_p, v_dv_q\}$, as depicted in Figure 1.

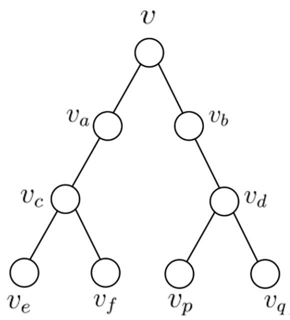

Figure 1. The tree T_v.

Let G' be the graph obtained by adding edges between $v' \in T_{v'}$ and $v'' \in T_{v''}$ if $v'v'' \in E(G)$ from the union of the trees T_v for $v \in V(G)$. Please note that $|V(G')| = n \times |V(T_v)| = 9n$ and $|E(G')| = |E(G)| + n \times |E(T_v)| = |E(G)| + 8n$.

Lemma 1. *If G is a planar graph or chordal bipartite graph , so is G'.*

Lemma 2. *([24]) Let S_n be a star with $n \geq 3$, then $\gamma_{t\{R3\}}(S_n) = 4$.*

Lemma 3. *Let g be a TR3DF of G. If v is a strong support vertex of G, then $\omega_g(N[v]) \geq 4$.*

Proof of Lemma 3. Let $v_1, v_2, .., v_k$ be leaves of v with $k \geq 2$. Since $g(N[v_i]) \geq 3$ for $i \in \{1, 2, .., k\}$, we have $g(v_i) \geq 3 - g(v)$ for $i \in \{1, 2, .., k\}$. Then $\omega_g(N[v]) = g(v) + \sum_{i \in \{1,2,...,k\}} g(v_i) \geq g(v) + g(v_1) + g(v_2) \geq 6 - g(v)$. If $g(v) \leq 2$, it is clear that $\omega_g(N[v]) \geq 4$. If $g(v) = 3$, there exists a vertex $u \in N(v)$ with $g(u) \neq 0$. Then $\omega_g(N[v]) \geq 4$. □

Lemma 4. *If f is a DF of G with $\omega_f(G) \leq \ell$, then there exists a TR3DF g of G' with $\omega_g(G') \leq \ell + 8n$.*

Proof of Lemma 4. For each $v \in V(G)$, we define g as follows: $V(T_v) \to \{0, 1, 2, 3\}$, $g(v_a) = g(v_b) = 1$, $g(v_c) = g(v_d) = 3$, $g(v) = f(v)$, $g(x) = 0$ otherwise. It is clear that g is a $TR3DF$ of G'. Therefore we have that $\omega_g(G') = \omega_f(G) + 8n \leq \ell + 8n$. □

Claim 1. *Let g be a TR3DF of G', then $\omega_g(T'_v) \geq 8$.*

Proof of Claim 1. By Lemmas 2, 3 and definition, we have that $\omega_g(N[v_c]) \geq 4$ and $\omega_g(N[v_d]) \geq 4$. Since $N(v_c) \bigcap N(v_d) = \emptyset$, then we can reduce $\omega_g(T'_v) = \omega_g(N[v_c]) + \omega_g(N[v_d]) \geq 8$. □

Claim 2. *If there exists a TR3DF h of G' with $h(v_a) + h(v_b) \geq 3$ for $v_a, v_b \in V(T_v)$, then there exists a TR3DF g of G' such that $\omega_g(G') \leq \omega_h(G')$ and $g(v_a) + g(v_b) \leq 2$.*

Proof of Claim 2. By the definition of $TR3DF$, we have $\omega_h(N[v_e]) \geq 3$ and $\omega_h(N[v_p]) \geq 3$, then we have $\omega_h(T'_v) \geq 9$.

If $h(v) = 0$, then we define $g : V(G') \to \{0, 1, 2, 3\}$ such that $g(v_e) = g(v_f) = g(v_p) = g(v_q) = 0$, $g(v) = g(v_a) = g(v_b) = 1$, $g(v_c) = g(v_d) = 3$, $g(x) = h(x)$ otherwise, seeing Figure 2. Therefore g is a $TR3DF$ of G' such that $g(v_a) + g(v_b) \leq 2$ and $\omega_g(G') = \omega_h(G')$.

If $h(v) \geq 1$, then we define $g : V(G') \to \{0, 1, 2, 3\}$ such that $g(v_e) = g(v_f) = g(v_p) = g(v_q) = 0$, $g(v_a) = g(v_b) = 1$, $g(v_c) = g(v_d) = 3$, $g(x) = h(x)$ otherwise. Therefore g is a $TR3DF$ of G' such that $g(v_a) + g(v_b) \leq 2$ and $\omega_g(G') \leq \omega_h(G')$. □

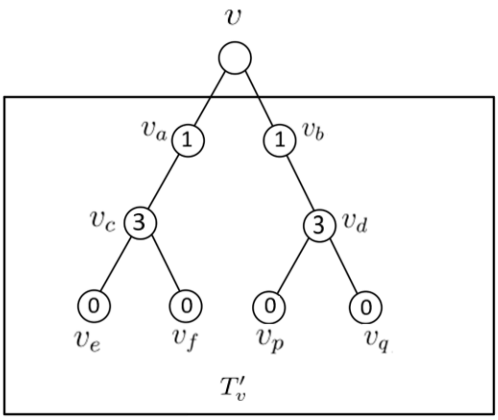

Figure 2. Pre-labeling of g.

Lemma 5. *If g is a TR3DF of G with $\omega_g(G') \le \ell + 8n$, then there exists a DF f of G with $\omega_f(G) \le \ell$.*

Proof of Lemma 5. By Claim 2, w.l.o.g, let g be a $TR3DF$ of G' with $g(v_a) + g(v_b) \le 2$ for $v_a, v_b \in V(T_v)$, $v \in V(G)$. Define $f : V(G) \to \{0,1\}$ such that $f(v) = g(v)$ if $g(v) \le 1$, and $f(v) = 1$ if $g(v) \ge 2$. For each vertex $v \in V(G)$, since $g(v_a) + g(v_b) \le 2$, we have $g(v) \ge 1$ or there exists a vertex $u \in N(v) \cap V(G)$ such that $g(u) \ge 1$. Therefore f is DSF of G and $\omega_f(G) \le \omega_g(G) - 8n \le \ell$ by Claim 1. □

Theorem 1. *By Lemmas 1, 4, 5, the total Roman {3}-domination problem is NP-complete for planar graphs and chordal bipartite graphs.*

3. A Linear-Time Algorithm for Total Roman {3}-Domination in Trees

In this section, we present a linear-time algorithm to compute the minimum weight of total Roman $\{3\}$-dominating function for trees. First, we define the following concepts:

Definition 1. *Let u be a vertex of G, and let $F^{(i,j)}_{u,G}$ on G be a function $f : V(G) \to \{0,1,2,3\}$ having the property that (i) $f(u) = i$, $\sum_{w \in N(u)} f(w) \ge j$; (ii) $\forall v \in V(G) \setminus \{u\}$, $\sum_{p \in N[v]} f(p) \ge 3$ if $f(v) \le 2$ and $\sum_{p \in N(v)} f(p) \ge 1$ if $f(v) = 3$.*

Definition 2. *The minimum weight overall $F^{(i,j)}_{u,G}$ functions on G denoted by $\gamma^{(i,j)}_{tR3}(u,G)$ is the $F^{(i,j)}_{u,G}$ number of G, and a $\gamma^{(i,j)}_{tR3}(u,G)$-function is an $F^{(i,j)}_{u,G}$ function on G with weight $\gamma^{(i,j)}_{tR3}(u,G)$.*

Definition 3. *Let $coil(x)$ be a function defined as follows:* $coil(x) = \begin{cases} x, x \ge 0; \\ 0, x < 0. \end{cases}$

Lemma 6. *For any graph G with specific vertex u, we have*

$$\gamma_{t\{R3\}}(G) = min\{\gamma^{(0,3)}_{tR3}(u,G), \gamma^{(1,2)}_{tR3}(u,G), \gamma^{(2,1)}_{tR3}(u,G), \gamma^{(3,1)}_{tR3}(u,G)\}.$$

Lemma 7. *Suppose T_1 and T_2 are trees with specific vertices v and u, respectively. Let T_3 be the tree with the specific vertex u, which is obtained by joining a new edge uv from the union of T_1 and T_2, as depicted in Figure 3.*

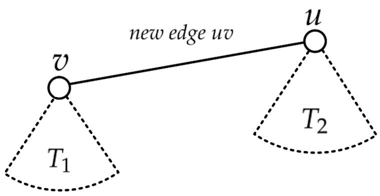

Figure 3. T_3.

Then the following statements hold for $\gamma_{tR3}^{(i,j)}(u, T_k)$.

(a) *For* $i = 0$, $j \in \{0,1,2,3\}$, *we have :*

$$\gamma_{tR3}^{(0,j)}(u, T_3) = min\{\gamma_{tR3}^{(3,1)}(v, T_1) + \gamma_{tR3}^{(0,0)}(u, T_2),\ \min\{\gamma_{tR3}^{(s,3-s)}(v, T_1) + \gamma_{tR3}^{(0,coli(j-s))}(u, T_2) | s = 0,1,2\}\}$$

(b) *For* $i \in \{1,2,3\}$, $j \in \{0,1,2,3\}$, *we have :*

$$\gamma_{tR3}^{(i,j)}(u, T_3) = min\{\gamma_{tR3}^{(s,coil(3-i-s))}(v, T_1) + \gamma_{tR3}^{(i,coil(j-s))}(u, T_2) | s = 0,1,2,3\}$$

Proof of Lemma 7. Let $V(T_1') = V(T_1) \cup \{u\}$, $E(T_1') = E(T_1) \cup \{vu\}$, f be a $\gamma_{tR3}^{(i,j)}(u, G)$-function of T_3, f' be the restriction of f on T_1' and f'' be the restriction of f on T_2.

(a) If f is a $\gamma_{tR3}^{(0,j)}(u, T_3)$-function on T_3 , for $j \in \{0,1,2,3\}$. By the definition of $\gamma_{tR3}^{(i,j)}(u, G)$-function, we have that if $f(v) = 3$, then $\sum_{w \in N_{T_3 \setminus \{u\}}} f(w) \geq 1$. It follows from the fact that f is a $\gamma_{tR3}^{(0,j)}(u, G)$-function of T_3 if and only if $f = f'' \cup f'$, where at least one of followings holds: (i) f'' is a $\gamma_{tR3}^{(0,0)}(u, G)$-function of T_2 , f' is a $\gamma_{tR3}^{(3,1)}(v, T_1)$-function of T_1 ; (ii) f'' is a $\gamma_{tR3}^{(0,coil(j-s))}(u, G)$-function of T_2 , f' is a $\gamma_{tR3}^{(s,3-s)}(v, T_1)$-function of T_1,for $s \in \{0,1,2\}$.

(b) It follows from the fact that f is a $\gamma_{tR3}^{(i,j)}(u, T_3)$-function of T_3, for $i \in \{1,2,3\}$, $j \in \{0,1,2,3\}$ if and only if f=$f'' \cup f'$, where f'' is a $\gamma_{tR3}^{(i,coli(j-s))}(u, T_2)$-function of T_2 and f' is a $\gamma_{tR3}^{(t,coil(3-i-s))}(v, T_1)$-function of T_1, for $s \in \{0,1,2,3\}$. $\square$

Lemmas 6 and 7 give the following dynamic programming algorithm 1 for the total Roman $\{3\}$-domination problem in trees.

Algorithm 1 Counting $\gamma_{t\{R3\}}$ in trees.

Input: A tree T with a tree ordering $[v_1, v_2, .., v_n]$.
Output: the TR3D number $\gamma_{t\{R3\}}(T)$ of T.

for $p = 1$ *to* n **do**
  for $i = 0$ *to* 3, $j = 0$ *to*3 **do**
    if *j=0* **then**
      $\gamma^{(i,j)}(v_p) \leftarrow i$;
    else
      $\gamma^{(i,j)}(v_p) \leftarrow \infty$;
for $p = 1$ *to* $n - 1$ **do**
  let v_q be the parent of v_p
  for $i = 0$ *to* 3 *and* $j = 0$ *to* 3 **do**
    if *i=0* **then**
      $\gamma^{(i,j)}(v_q)$=min{min$\{\gamma^{(s,3-s)}(v_p) + \gamma^{(i,coil(j-s))}(v_q) | s = 0,1,2\}; \gamma^{(3,1)}(v_p) + \gamma^{(i,0)}(v_q)\}$;
    else
      $\gamma^{(i,j)}(v_q)$=min$\{\gamma^{(s,coil(3-i-s))}(v_p) + \gamma^{(i,coil(j-s))}(v_q) | s = 0,1,2,3\}$;
return $min\{\gamma^{(0,3)}(v_n), \gamma^{(1,2)}(v_n), \gamma^{(2,1)}(v_n), \gamma^{(3,1)}(v_n)\}$

4. Conclusions

The total Roman {3}-domination problem was introduced and studied in [24] , and it was proven to be NP-complete for bipartite graphs. In this paper , we prove that the total Roman {3}-domination problem is NP-complete for planar graphs or chordal bipartite graphs , and showed a linear-time algorithm for total Roman {3}-domination problem on trees. For the algorithmic aspects of the total Roman {3}-domination problem , designing exact algorithms or approximation algorithms on general graphs , or polynomial algorithms for total Roman {3}-domination problem on some special classes graphs deserve further research.

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Abbreviations

The following abbreviations are used in this manuscript:

DF	Dominating function
DSP	Dominating set problem
TRDF	Total Roman dominating function
R3DF	Roman $\{3\}$-domination
TR3DF	Total Roman $\{3\}$-domination

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