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FREE VIBRATION ANALYSIS OF CARBON NANOTUBES BASED ON SHEAR DEFORMABLE BEAM THEORY BY DISCRETE SINGULAR CONVOLUTION TECHNIQUE

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Abstract- In this study, free vibration analysis of carbon nanotubes is investigated based on Timoshenko beam theory. Discrete singular convolution (DSC) method is used for free vibration problem of numerical solution of carbon nanotubes. Numerical results are presented and compared with that available in the literature. It is shown that reasonable accurate results are obtained.

Keywords-Carbon Nanotubes, Timoshenko Beam, Free Vibration, Discrete Singular Convolution.

1. INTRODUCTION

Carbon nanotubes were discovered in 1991 by Sumio Iijima [1]. Carbon nanotubes (CNT) are molecular-scale tubes of graphitic carbon with outstanding properties (Fig. 1). It is accepted that CNT are unique nanostructures with remarkable electronic and mechanical properties. Since the CNT were discovered extensive theoretical and experimental studies on mechanical properties of CNT has been performed [2-10]. Vibration, bending and buckling behavior of CNT has been a subject of interest in the past five years. Molecular dynamics or atomistic model has been used in order to look into the mechanics of nanotubes. Moreover, many authors have employed a continuum or structural mechanics approach for more practical and efficient modeling. For this purpose, rod, beam and shell theories have been used by researchers [11-16].

Recently, much attention has been devoted to the mechanical behavior of micro/nano structures such as nanobeam, nanorods, nanotubes and microtubules. Beam theories have been always used for modeling of this kind of nanodevices. The main objective of this study is to give a numerical solution of free vibration analysis of carbon nanotubes based on the theory of Timoshenko beam. To the author knowledge, it is the first time the DSC method has been successfully applied to carbon nanotubes based on Timoshenko beam for the numerical analysis of vibration.

2. DISCRETE SINGULAR CONVOLUTION (DSC)

Discrete singular convolution (DSC) method is a relatively new numerical technique in applied mechanics. The method of discrete singular convolution (DSC) was proposed to solve linear and nonlinear differential equations by Wei [17], and later it was introduced to solid and fluid mechanics by Wei [18], Wei et al. [19], Zhao et al.[20,21], and Civalek [22-29].

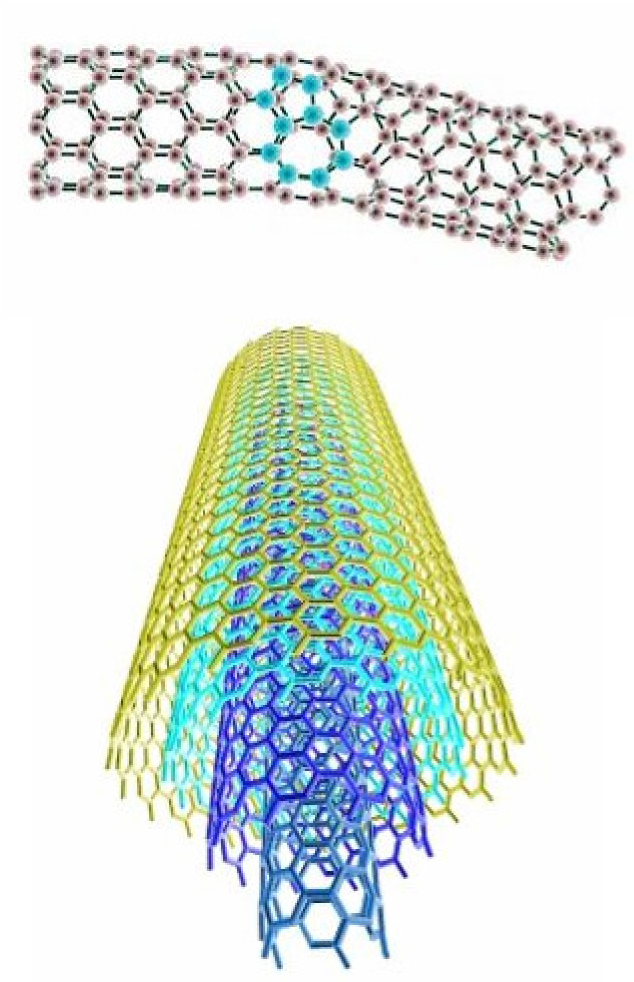

Figure 1. Typical single and multi-walled carbon nanotubes

For more details of the mathematical background and application of the DSC method in solving problems in engineering, the readers may refer to some recently published reference [21-30]. In the context of distribution theory, a singular convolution can be defined by [17]

$$F(t) = (T * \eta)(t) = \int_{-\infty}^{\infty} T(t-x)\eta(x)dx \tag{1}$$

Where T is a kind of singular kernel such as Hilbert, Abel and delta type, and $\eta(t)$ is an element of the space of the given test functions. In the present approach, only singular kernels of delta type are chosen. This type of kernel is defined by [18]

$$T(x) = \delta^{(r)}(x);\ (r = 0,1,2,...,). \tag{2}$$

where subscript r denotes the rth-order derivative of distribution with respect to parameter x. In order to illustrate the DSC approximation, consider a function $F(x)$. In the method of DSC, numerical approximations of a function and its derivatives can be treated as convolutions with some kernels. According to DSC method, the r*th* derivative of a function $F(x)$ can be approximated as [19]

$$F^{(r)}(x) \approx \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(r)}(x_i - x_k) f(x_k); \quad (r{=}0,1,2,...,). \tag{3}$$

where Δ is the grid spacing, x_k are the set of discrete grid points which are centered around x, and $2M+1$ is the effective kernel, or computational bandwidth. It is also known, the regularized Shannon kernel (RSK) delivers very small truncation errors when it use the above convolution algorithm. The regularized Shannon kernel (RSK) is given by [20]

$$\delta_{\Delta,\sigma}(x - x_k) = \frac{\sin[(\pi/\Delta)(x - x_k)]}{(\pi/\Delta)(x - x_k)} \exp\left[-\frac{(x - x_k)^2}{2\sigma^2} \right]; \ \sigma{>}0 \tag{4}$$

The researchers is generally used the regularized delta Shannon kernel by this time. The required derivatives of the DSC kernels can be easily obtained using the below formulation

$$\delta_{\Delta,\sigma}^{(r)}(x - x_j) = \frac{d^r}{dx^r}\left[\delta_{\Delta,\sigma}(x - x_j)\right]\Big|_{x = x_i}, \tag{5}$$

3. SOLUTION OF GOVERNING EQUATIONS

A typical single walled carbon nanotubes (SWCNTs) based on beam theory is depicted in Fig. 2. In this figure, the letter d is the diameter of beam, L length of the beam.

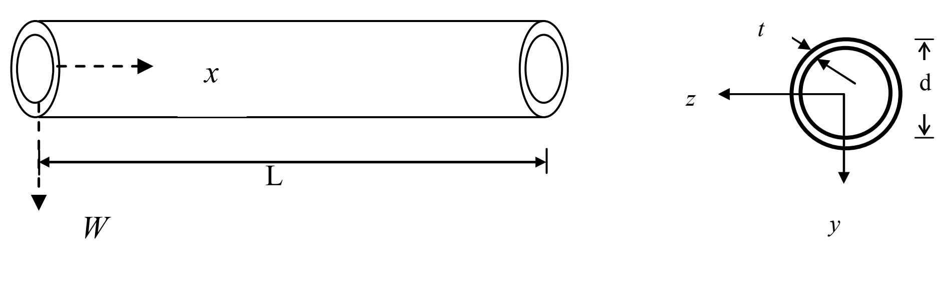

Figure 2. The illustration of carbon nanotubes as Timoshenko beam

The governing equations for free vibration of carbon nanotubes based on Timoshenko beam can be written as

$$kGA\frac{d^2W}{dx^2} - kGA\frac{d\theta}{dx} + \rho A\omega^2 W = 0, \tag{6}$$

$$EI\frac{d^2\theta}{dx^2} + kGA\frac{dW}{dx} - kGA\theta + \rho I\omega^2\theta = 0, \tag{7}$$

By using DSC discretization the Eqs. (6-7) take the form

$$kGA\sum_{j=1}^{N}\delta_{\pi/\Delta,\sigma}^{(2)}(\Delta x)W(x_i) - kGA\sum_{j=1}^{N}\delta_{\pi/\Delta,\sigma}^{(1)}(\Delta x)\theta(x_i) = -\rho A\omega^2 W_i, \tag{8}$$

$$EI\sum_{j=1}^{N}\delta_{\pi/\Delta,\sigma}^{(2)}(\Delta x)\theta(x_i) + kGA\sum_{j=1}^{N}\delta_{\pi/\Delta,\sigma}^{(1)}(\Delta x)W(x_i) = -\rho I\omega^2\theta_i, \tag{9}$$

Two-types of boundary conditions are considered. These are:

Clamped (C)

$$\theta = 0 \text{ and } W = 0 \tag{10}$$

Simply supported (S)

$$M = 0 \text{ and } W = 0 \tag{11}$$

In these equations V and M are the shear and moment resultants and given by

$$V = kGh\left(\frac{\partial W}{\partial x} - \theta\right),\ M = EI\frac{\partial\theta}{\partial x} \tag{12,13}$$

After implementation of the given boundary conditions, Eqs. (8) and (9) can be expressed by

$$[\mathbf{R}]\{\mathbf{U}\} = \omega^2\{\mathbf{U}\}, \tag{14}$$

where $\mathbf{U}$ is the displacements vector, $\mathbf{R}$ is the stiffness matrix. The frequency values for Timoshenko beam are given by the following non-dimensional form

$$\Omega^2 = \omega L^2 \sqrt{\frac{\rho A}{EI}} \tag{15}$$

where ρ is the mass density, A the cross-sectional area, I the second moment of area of cross-section, E the Young's modulus, L is the length of the carbon nanotubes, ω is the circular frequency. For tapered nanotubes Eqs. (8-9) take the form

$$kGA\sum_{j=1}^{N}\delta^{(2)}_{\pi/\Delta,\sigma}(\Delta x)W(x_i) - kGA\sum_{j=1}^{N}\delta^{(1)}_{\pi/\Delta,\sigma}(\Delta x)\theta(x_i) = -\rho A\omega^2 W_i, \tag{16}$$

$$EI(x)\sum_{j=1}^{N}\delta^{(2)}_{\pi/\Delta,\sigma}(\Delta x)\theta(x_i) + kGA\sum_{j=1}^{N}\delta^{(1)}_{\pi/\Delta,\sigma}(\Delta x)W(x_i) = -\rho I(x)\omega^2\theta_i, \tag{17}$$

4. RESULTS

In the manuscript, following material and geometric parameters have been used: ρ =2300 kg/m^3, E=10^{12} N/m^2, L=10^{-8} m, d=33×10^{-9} m, t=0.34×10^{-9} m. The results given in this section are aimed to illustrate the numerical accuracy of the proposed DSC method. The obtained results are listed in Table 1. In this table υ is the Poisson's ratio and k is shear correction factor. First four non-dimensional frequency parameters of simply supported carbon nanotubes are given in Table 1 for d/L=0.1. It is observed that a good agreement between the present calculated results and the results of literature [20,21] has been obtained. Frequency values with mode number of CNTs are depicted in Fig.3 for three different boundary conditions. Effect of diameter-to-length ration on frequency of CNTs is given in Fig. 4 for first two mode numbers. It is shown from these figures that frequency values are increased with mode number and diameter. Variations of mode shapes with the taper ratios (Fig. 5) are presented in Figure 6 for S-S carbon nanotubes with linearly tapered. It is shown that the increasing value of taper ratio, always increases the frequency parameter. The taper ratio is given as $\alpha = d_1 / d_0$.

Table 1. Comparison of non-dimensional frequency parameters ($\Omega^2 = \omega L^2 \sqrt{\rho A / EI}$) of S-S nanotubes ($d/L$=0.1; k=5/6; $\upsilon = 0.3$)

Mode	Reddy Exact [30]	Heireche et al. [31]	DSC N=11	DSC N=13	DSC N=15	DSC N=18
1	3.1217	3.0929	3.1405	3.0962	3.1405	3.1405
2	-	5.4658	6.2747	6.2747	6.2747	6.2747
3	-	8.444	9.3965	9.3964	9.3963	9.3963
4	-	10.6260	10.7219	10.7219	10.7218	10.7218

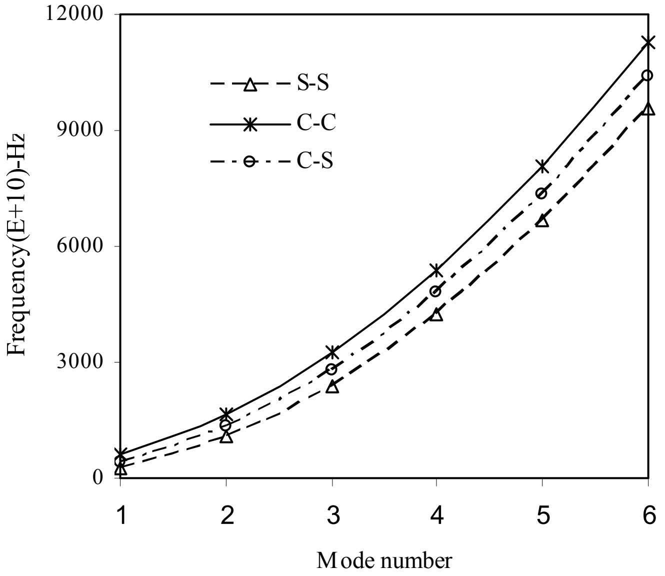

Figure 3. Frequency values of CNTs for different boundary conditions

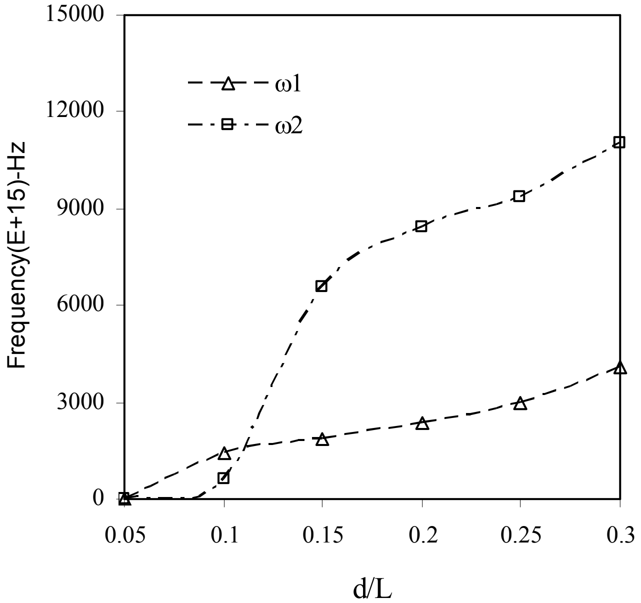

Figure 4. Effect of diameter-to-length ration on frequency of CNTs

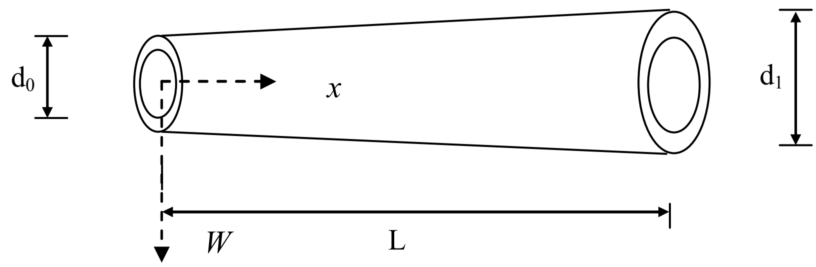

Figure 5. Tapered carbon nanotubes

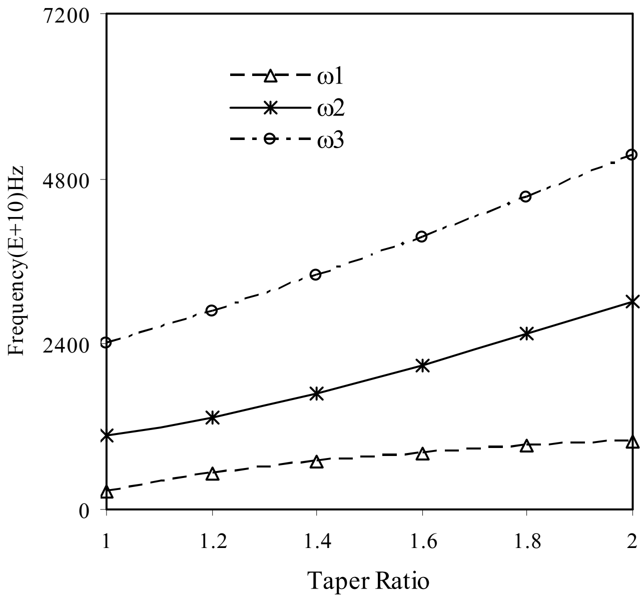

Figure 6. Frequency values for S-S tapered carbon nanotubes

5. CONCLUSIONS

A numerical approach for the free vibration analysis of carbon nanotubes based on Timoshenko beam theory is presented. Several examples were worked to demonstrate the convergence of the method. Excellent convergence behavior and accuracy in comparison with exact results or results obtained by other numerical methods were obtained. Nonlinear beam-mass model [32] is another efficient approach for modeling of CNTs. Although not provided here, the method is also useful in providing vibration solutions of multi-walled carbon nanotubes [33,34]. The present study is being further developed to overcome the convergence problems encountered in the nonlinear vibration analysis of carbon nanotubes.

Acknowledgements

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