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CALCULATION OF GENERALIZED LEVEL DENSITIES FOR NUCLEI IN MASS REGION $20 < A < 50$

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Abstract- In this study, a relation between generalized level density and standard level density is derived. Using this relation and Bethe formula of Fermi gas model for standart level density we obtained a generalized nuclear level density formula for nucleus. Generalized level densities were calculated for some nuclei in mass region between 20 and 50 for different q values close to 1. Our results explain experimental data better than those of Gilbert-Cameron (GC) and Rohr, which are two of the leading compilations in evaluating nuclear level density.

Key Words- Level Density, Nonextensive Statistical Mechanics, Fermi Gas Model

1.INTRODUCTION

Nuclear structure physics is devoted to the study of the properties of nuclei at low excitation energies, where individual energy levels can be solved. This means that typically quantum effects are predominant. In contrast, at higher energies and especially for heavy-ion reactions, quantum mechanics becomes less important and preeminant place is instead given to the methods of statistical mechanics. In all statistical theories, the nuclear level density is the most characteristic quantity and plays an essential role in predicting the energy levels of nucleus which provides a severe challange to our theoretical understanding. A number of related areas of physics and technology are also dependent on the studies of the level density. These include nucleosynthesis studies in astrophysics, and fission and fusion reactor design.

Theoretical investigations of level density started with the pionering work of Bethe [1,2], in which he has obtained a simple level density formula for a gas of non-interacting fermions with equally spaced non-degenerate single particles. Various corrections to the model have been added since the work of Bethe. One correction was the addition of an energy shift to the energy to include the pairing correlations and shell effects[3-9]. It is well known that level density of magic and near magic nuclei cannot be well reproduced over a large energy interval by using Bethe formula with constant level density parameter a. This problem has been investigated by inclusion of collective excitations associated mainly with the statical or dynamical deformation of mean field to the level density parameter [10-14]. Furthermore, combinatorial calculations have also been performed to solve this problem and, in particular, allowed better reproduction of total level densities data coming from counting the neutron resonance spacings and from the analysis of evaporation spectra using the independent particle model level density [5, 12, 15, 16]. Recently, Oslo Group [17, 18], developed a new tecnique to extract level density data which is based on γ- decay energy distributions from a number of initial excitation energies, and showed that although, for example, level densities of ^{26}Al exhibits Fermi-Gas behaviour up to 8 MeV of excitation energy [14, 19], those of ^{56,57}Fe measured with Oslo method [20] have more complicated behaviour which cannot be described by simple Fermi-Gas formula. The influence of pairing correlations leading step structure in vicinity of proton and neutron pairing energies and above might be reason of this complicated structure [21]. Hovewer, nonextensive statistical mechanics, based on the q-generalized entropy proposed by Tsallis [22] and developed by many researhers [23-33], has become powerful tool to deal with some systems which (a) have long range interactions, (b) have long range memory effects and (c) evolve in a multi-fractal space-time. In particular, it has been succesfully used to study the properties of the generalized Bose system and a large number of significant results have been obtained [26, 29]. Obviously, it is very meaningful to investigate the properties of a generalized Fermi system by using nonextensive statistical mechanics. Since nucleus is a Fermi system, it might also be interesting to consider nuclear level density in the framework of nonextensive statistical mechanics. In this direction, Lenzi et al. [34] established a relation between the clasical q-partition function and the level density using q-Laplace transform; for classical ideal gas, they obtained a level density formula from inverse q- Laplace transform of partition function which is the same with that of derived from Laplace transform of canonical partition function within Boltzmann-Gibbs extensive statistics.

In this work, instead of inverse q-Laplace transform of partition function we use a different approach to calculate the level density within statistical mechanics. In this approach, we use a relationship between the generalized nuclear level density and the standard level density which is obtained by following Curilef's prescription [35] for the derivation of the relation between generalized statistical quantity and its standart quantity $q \to 1$. Details of this derivation are given in next section. Advantage of this relation is that the generalized level density can be calculated directly without using inverse integral transform which is not available now. Using this relation and traditional Bethe theory of nuclear level density calculations for standart nuclear level density, we obtained a new formula for nuclear level density which depends on the entropic index q. This formula contains 3 parameter; two from previous theory, i.e. level density and energy shift, and one from q-generalized statistics characterized by the parameter q which is based on the so-called Tsallis' entropy. In section 3, the results obtained from generalized nuclear level density formula are presented for 12 nuclei in the mass region $20 < A < 50$ and compared with the experimental data and two models of Bethe formula.

2.NUCLEAR LEVEL DENSITY

In this section, we obtain a useful relationship between the generalized level density and the standart level density which allows to calculate the generalized level density without using q-generalization of inverse Laplace transformation. To this aim, we calculate the generalized partition function Z_q in terms of a parametric integral over the usual grand canonical partition function $Z_1(\beta(q-1)\xi,\mu)$. The grand-canonical partition function for $q > 1$ is obtained by using the Hilhorst integral representation of Gamma function, [35]

$$\eta_+^{-\nu} = \frac{1}{\Gamma(\nu)} \int_0^\infty d\xi \xi^{\nu-1} e^{-\eta_+ \xi}, \tag{1}$$

as

$$Z_q = \frac{1}{\Gamma(\nu)} \int_0^\infty d\xi \xi^{\nu-1} e^{-\xi} Z_1(\beta(q-1)\xi, \mu). \tag{2}$$

where $\nu = 1/(q-1)$ and $\eta_+ = 1 + \beta(q-1)(E - \mu A)$. The corresponding partition function for $q = 1$ can be written in terms of level density regarded as the function of energy E and the number of particle A

$$Z_1\left(\beta', \mu\right) = \int_0^\infty \int_0^\infty dN\, dE\, \rho_1(E, A, \xi) e^{-\beta' (E - \mu A)} \tag{3}$$

where $\beta' = \beta(q-1)\xi$. Substituting $Z_1\left(\beta', \mu\right)$ in Eq.(3) into Eq.(2), the generalized partition function for $q > 1$ is associated with the standart level density in the following form:

$$Z_q = \frac{1}{\Gamma(\nu)} \int d\xi\, \xi^{\nu-1} e^{-\xi} \int_0^\infty \int_0^\infty dN\, dE \rho_1(E, A, \xi)\, e^{-\beta' (E - \mu A)}. \tag{4}$$

The level density obtained from the inverse Laplace transform of $Z_1(\beta(q-1)\xi, \mu)$ in Eq.(3) for especially ideal fermi gas depends on β', and therefore q and ξ, but for classical ideal gas it is independent of these parameters. ξ dependence of ρ_1 also appears in Eq (10). The generalized partition function related to the physical system can also be defined with respect to level density as

$$Z_q = \sum_{A,E} \left[1 - \beta(1-q)(E - \mu A)\right]^{-\nu} = \int_0^\infty \int_0^\infty dN\, dA\, \rho_q(E, A) \left[1 - (1-q)\beta(E - \mu A)\right]^{-\nu} \tag{5}$$

Comparing Eq.(4) with Eq.(5), for $q > 1$ we obtain a relationship between the generalized level density and the standard level density

$$\rho_q(E, A) = \frac{\eta_+^{\nu}}{\Gamma(\nu)} \int_0^\infty d\xi \xi^{\nu-1} \rho_1(E, A, \xi) \exp(-\eta_+ \xi), \tag{6}$$

The extension of the partition function for $q < 1$ shown by Prato [36] is

$$Z_q = \Gamma(1+\alpha) \frac{i}{2\pi} \oint d\xi \left(-\xi\right)^{-\alpha-1} e^{-\xi} Z_1(-\beta(1-q)\xi, \mu), \tag{7}$$

which can be derived from another integral representation of Gamma function

$$\eta_-^{\alpha} = \Gamma(1+\alpha) \frac{i}{2\pi} \oint d\xi \xi^{-\alpha-1} e^{-\eta_- \xi}, \tag{8}$$

where $\alpha = 1/(1-q)$ and $\eta_- = 1 - (1-q)\beta(E - \mu A)$. Following the lines from Eq. (1) to Eq. (6) for $q > 1$ case, one can obtain the generalized level density for $q < 1$

$$\rho_q(E, A) = \frac{\Gamma(1+\alpha)}{\eta_-^{\alpha}} \frac{i}{2\pi} \oint d\xi \left(-\xi\right)^{-\alpha-1} \rho_1(E, A, \xi)\, e^{-\eta_- \xi}. \tag{9}$$

At this stage, we need to adopt the relationship appeared in Eq.(9) to perform the calculations for nuclear level density, because nucleus is composed of two kinds of particles, neutrons and protons. The nuclear level density must now depend on neutron number N and proton number Z. For years, the simple models such as Fermi gas model, have been still used to calculate nuclear level density at high energies (or low temperatures). The nuclear level density for Bethe theory, ρ_1, is given by

$$\rho_1(E,N,Z,\xi)=\frac{\sqrt{3}\beta'^{\,5/2}\,e^{\pi^2 g(\varepsilon_F)/3\beta'}}{2\pi^3[g(\varepsilon_F)]^2} \tag{10}$$

where $g(\varepsilon_F)$ is total single-particle level spacing of nucleons at Fermi energy and $g_p(\varepsilon_F)=g_n(\varepsilon_F)=g(\varepsilon_F)/2$. For the case $q<1$ (and $q>1$), β' in Eq.(10) is equal to $-\beta(1-q)\xi$ (and $\beta(q-1)\xi$). The details of the derivation of Bethe formula for nuclear level density can be found in Refs.[1-3, 37].

For $q>1$ and sufficiently large N, the integral in Eq.(6) diverges when $q\geq 1+1/N$. For the case $q>1$, previous works indicate that in the thermodynamic limit ($N\to\infty$) employing nonextensive statistical mechanics is not suitable to the classical ideal gas [34], the classical systems with N harmonic oscilators [38] and Fermi systems in a general power-law external potential [39]. Therefore, in this work, we consider only the case $q<1$. Replacing the nuclear level density $\rho_1(E,N,Z,\xi)$ in the integrand of the Eq.(9), the generalized level density for $q<1$ is obtained as

$$\rho_q(U,N,Z)=\frac{\sqrt{2\pi}a\Gamma(1+\alpha)}{3x^{(\alpha+5/2)/2}}I_{\alpha-5/2}(2\sqrt{x}), \tag{11}$$

with $x=\frac{2\sqrt{aU}}{1-q}-2a(U-U_0)$. Above formula is valid through in the interval $1-\sqrt{\frac{U}{a}}\frac{1}{U-U_0}<q<1$, which comes from the cut-off condition. Level density parameter is $a=\frac{\pi^2 g(\varepsilon_F)}{6}$ and U is the excitation energy above the energy of fully degenerate states U_0, i.e. $U=E-U_0$. One might think that U_0 was simply the ground state, so that U could be simply the excitation energy. Malyshev [40] showed that there was a systematic difference in the values of a for neighboring even-even, odd-A, and odd-odd nuclei. In another study, Newton [7] also showed that to obtain U these discrepancies could be removed by substracting the pairing energy from the excitation energy. Thus, $U_0=\delta=\Delta_p+\Delta_n$, Δ_p (Δ_n) is proton (neutron) pairing energy, and $U=E-\Delta_p-\Delta_n$.

3.RESULTS AND DISCUSSION

Our calculations of level density have been performed with using formula in Eq.(11) with energy shift δ that was simply due to pairing. This formula contains 3 parameters; i.e. one more parameter q in addition to the parameters of level density parameter a and energy shift δ (a and δ appears in Bethe formula). In general, the compilation of the parameters of Bethe formula is based on the fit of two parameters. The fits of Braga-Marcazzan and Milazzo-Colli (BMMC) [41] and that of Rohr [42] were starting point. In BMMC compilation, a values deduced individually for a number of nuclei whereas pairing energies of Gilbert-Cameron (GC) [3] determined from semi emprical mass formula were used as the energy shift δ. Rohr compilation is the same with BMMC in that of energy shift wheras a values were fitted to function $a = \alpha A + C$, where A is the mass number, and α and C fitting constants. In GC model, a values are connected to shell correction S as $a/A = \alpha S + C$. Further compilations include the shell and collective effects into level density parameter a [13, 14, 43]. For δ, better results were obtained with the parameters of Myers and Swiatecki [44], and Grimes [6]. For energy shift δ, we use the pairing energy values of GC. The remaining parameters of generalized level density formula are the level density parameter a and the order parameter q which is less than 1 and has a lower limit which depends on level density parameter a. In our calculations, we fit only the parameter a for a fixed value of q to the experimental level density data [45] because the dependence of one parameter on another parameter makes difficult to fit both parameters simultaneously.

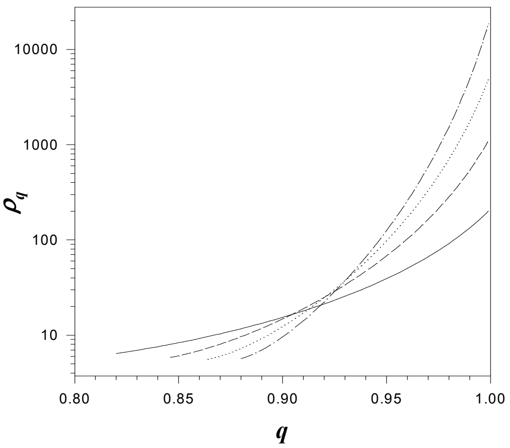

Figure 1. q-dependence of level density for $^{24}_{11}$Na at excitation energies $E = 5$ MeV (solid), $E = 7$ MeV (dashed), $E = 9$ MeV (dotted) and $E = 11$ MeV (dot-dashed).

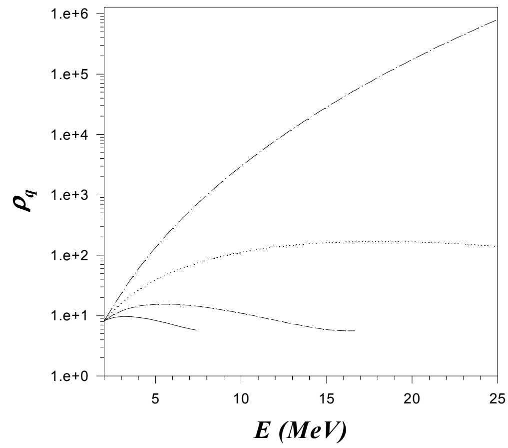

Figure 2. The generalized level density of $^{24}_{11}$Na as a function of excitation energy for a fixed value of $a = 6$. The solid, dashed, dotted, and dot-dashed lines correspond to $q = 0.85$, $q = 0.9$, $q = 0.95$ and $q = 0.99$ respectively.

The fitting values of a for a fixed value of q are given in Table 1. As q values are getting closer to 1, we obtained better fitting values for a. Our values of level density parameter lie between $a \approx A/15$ and $a \approx A/8$, which are the predictions of Fermi Gas and the emprical values extracted from average spacings at neutron binding energies [2, 3, 5, 37, 46], respectively. q dependence of nuclear level densities at various excitation energies for $^{24}_{11}$Na is shown in Fig. 1. Level density increases rapidly at higher energies, but the contribution of this parameter shows a different behaviour as away from and close to 1. For example, as the excitation energy increases, the level density also increases through region where q takes the values between 0.92 and 1 whereas it decreases for

$q < 0.92$. For the entropix index q, there is a lower limit arising from cut-off condition which also depends on excitation energy. These limits therefore change with excitation energy; higher excitation energy higher the limit value. However, when q is fixed, the cut-off condition produces an upper limit for excitation energy. For various q values and $a = 6$, the level density of $^{24}_{11}$Na is plotted as a function of excitation energy and those limits are shown in Fig. 2. The variation of level density with energy is different from the q dependence of level density. While the former one increases with increasing q value, the latter one has the same value for different excitation energies at one value of q.

Table.1: Fitted values of level density parameter a which are obtained by fixing the q values for nuclei in mass region $20 < A < 50$.

Nuclei	*q*	*a*	*Nuclei*	*q*	*a*
^{24}Na	0.98	4.503	^{33}S	0.98	4.927
	0.99	3.621		0.99	3.733
	0.9999	2.98		0.9999	2.944
^{25}Mg	0.982	4.923	^{34}Cl	0.98	3.223
	0.99	3.955		0.99	2.882
	0.9999	3.109		0.9999	2.582
^{27}Al	0.985	4.407		0.98	3.736
	0.99	3.955		0.99	3.26
	0.9999	3.109		0.9999	2.861
^{28}Al	0.98	4.299	^{38}Ar	0.993	4.861
	0.99	3.36		0.997	4.299
	0.9999	2.079		0.9999	3.947
^{31}P	0.98	3.343	^{40}K	0.98	5.182
	0.99	2.917		0.99	4.308
	0.9999	2.56		0.9999	3.635
^{32}P	0.98	2.944	^{41}Ca	0.995	4.909
	0.99	2.661		0.997	4.603
	0.9999	2.409		0.9999	4.2

In Figs. 3-5, we compare our results obtained from formula in Eq.(11) with the results of GC and Rohr models and also with the experimental data [45] for $^{24}_{11}$Na, $^{25}_{12}$Mg, $^{27}_{13}$Al, $^{28}_{13}$Al, $^{31}_{15}$P, $^{32}_{15}$P, $^{33}_{16}$S, $^{34}_{17}$Cl, $^{36}_{18}$Ar, $^{38}_{16}$Ar, $^{40}_{19}$K and $^{41}_{20}$Ca. Although the spin effects are not taken into account in our model, the generalized level density results are in better agreement than those of GC and Rohr models with experimental data. The contribution of q (i.e. nonextensivity) to the behaviour of the level density is significant especially at higher excitation energies. The reason for it is that the entropic index q is a-dependent and level densities are sensitive to a at high excitation energies. The determination of the best fitted values of q close to 1 indicates that level density could not exhibit nonextensive character.

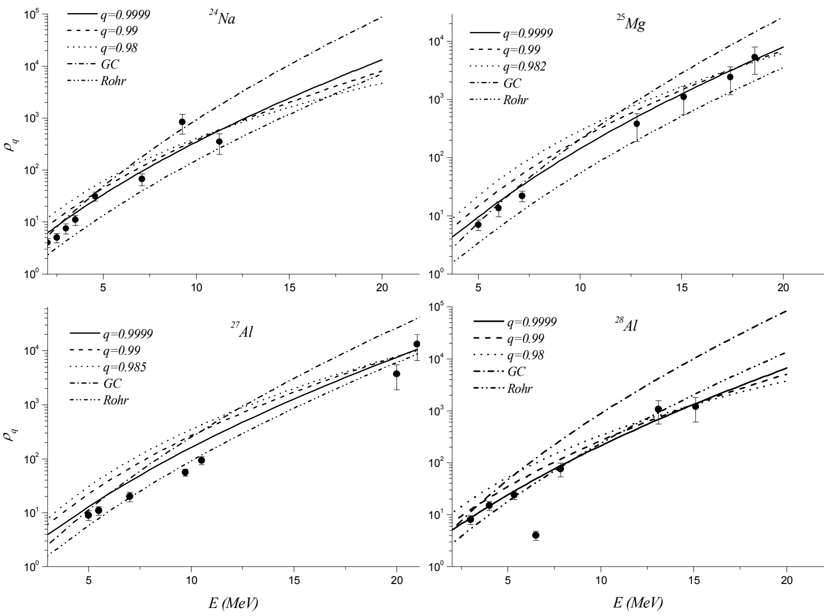

Figure 3. The level densities of ^{24}Na, ^{25}Mg, ^{27}Al and ^{28}Al for some q values.

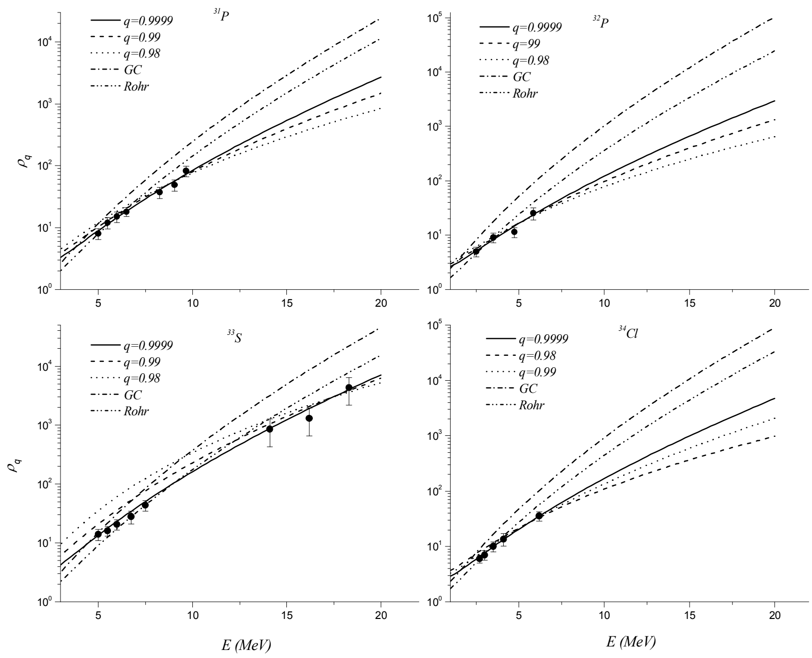

Figure 4. The level densities of ^{31}P, ^{32}P, ^{33}S and ^{34}Cl for some q values.

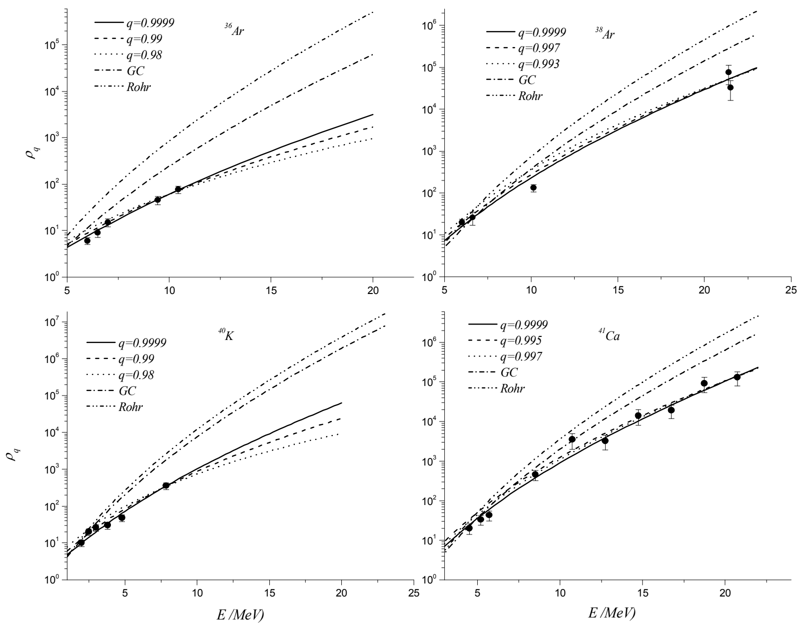

Figure 5. The level densities of ^{36}Ar, ^{38}Ar, ^{40}K and ^{41}Ca for some q values.

Nevertheless we need to consider the spin effects in our model and also need more experimental data to decide whether the physical system might exhibit a nonextensive character or not.

4. CONCLUSION

By using the relation between nonextensive and standart (Boltzmann-Gibbs) partition functions for grand canonical ensemble, we derived a similar relation between nonextensive and standart level density formula $q < 1$ and $q > 1$. From this relation we obtained a generalized nuclear level density formula in the framework of Fermi gas model for only $q < 1$. This is consistent with previous works in which they indicate that in the thermodynamic limit nonextensive statistical mechanics is not suitable.

In the light of discussions in previous section, the generalized level density formula seems to be appropriate to perform calculations for nuclei with $20 \leq A \leq 50$.

Especially, at higher energies the level density is sensitive to q, but the effect of this parameter decreases at lower energies. However, since it has a lower limit which depends on the level density parameter, we should consider the total effects of these parameters in the calculations.

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