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# APPLICATION OF $(G'/G)$-EXPANSION METHOD TO THE COMPOUND KDV–BURGERS-TYPE EQUATIONS

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**Abstract-** In this Letter, the $(G'/G)$-expansion method is proposed to seek exact solutions of nonlinear evolution equations. For illustrative examples, we choose the compound KdV-Burgers equation, the compound KdV equation, the KdV-Burgers equation, the mKdV equation. The power of the employed method is confirmed.

**Key Words-** $(G'/G)$-expansion method, the compound KdV-Burgers equation, Travelling wave solutions

## 1. INTRODUCTION

Nonlinear evolution equations (NLEEs) have been the subject of study in various branches of mathematical–physical sciences such as physics, biology, chemistry, etc. The analytical solutions of such equations are of fundamental importance since a lot of mathematical–physical models are described by NLEEs.

In recent years, searching for explicit solutions of NLEEs by using various methods has become the main goal for many authors. Many powerful methods to construct exact solutions of NLEEs have been established and developed $[1-10]$. But up to now a unified method that can be used to deal with all types of NLEEs has not been discovered.

Recently, Wang et al. $[11]$introduced an expansion technique called the $(G'/G)$-expansion method and they demonstrated that it is powerful technique for seeking analytic solutions of nonlinear partial differential equations. It has been shown that the proposed method is direct, concise, basic and effective. Applications of the method can be found in $[12-22]$.

Our aim in this paper is to present an application of the $(G'/G)$-expansion method to the compound KdV–Burgers-type equations.

## 2. DESCRIPTION OF THE $(G'/G)$-EXPANSION METHOD

We suppose that a nonlinear equation, say in two independent variables $x$ and $t$, is given by

$$P(u,u_x,u_t,u_{xx},u_{xt},u_{tt},...)=0 \tag{1}$$

where $u=u(x,t)$ is an unknown function, $P$ is a polynomial in $u=u(x,t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. In the following we give the main steps of the $(G'/G)$-expansion method.

**Step 1.** Seek traveling wave solutions of Eq.$(1)$ by taking $u(x,t)=U(\xi)$ , $\xi=x-Vt$, where $V$ is the wave speed, and transform Eq.$(1)$ to the ordinary differential equation

$$Q(U,U',-VU',U'',V^2U'',...)=0 \tag{2}$$

where prime denotes the derivative with respect to $\xi$.

**Step 2.** If possible, integrate Eq.$(2)$ term by term one or more times. This yields constant(s) of integration. For simplicity, the integration constant(s) can be set to zero.

**Step 3.** Introduce the solution $U(\xi)$ of Eq.$(2)$ in the finite series form

$$U(\xi)=\sum_{m=0}^{N}a_m\left(G'(\xi)/G(\xi)\right)^m \tag{3}$$

where $a_m$ are real constants with $a_N\neq 0$ to be determined. The function $G(\xi)$ is the solution of the auxiliary linear ordinary differential equation

$$G''(\xi)+\lambda G'(\xi)+\mu G(\xi)=0 \tag{4}$$

where $\lambda$ and $\mu$ are real constants to be determined. Eq.$(2)$ can be changed into

$$\frac{d}{d\xi}(G'/G)=-(G'/G)^2-\lambda(G'/G)-\mu \tag{5}$$

**Step 4.** Determine N. This, usually, can be accomplished by considering homogeneous balance between the highest order derivatives and nonlinear terms appearing in Eq.$(2)$.

**Step 5.** Substituting $(3)$ together with $(4)$ into Eq.$(2)$ yields an algebraic equation involving powers of $(G'/G)$. Equating the coefficients of each power of $(G'/G)$ to zero gives a system of algebraic equations for $a_i,\lambda,\mu$ and $V$. Then, we solve the system with the aid of a computer algebra system (CAS), such as Mathematica, to determine these constants. On the other hand, depending on the sign of the discriminant $\Delta=\lambda^2-4\mu$, the solutions of Eq.$(4)$ are well known to us. Then substituting $a_i,\lambda,\mu$ and $V$ and general solution of Eq.$(4)$ into Eq.$(3)$,we have more travelling wave solutions of the nonlinear evolution Eq.$(1)$.

## 3. THE COMPOUND KDV-BURGERS EQUATION

Let us consider the Compound KdV-Burgers equation

$$u_t+puu_x+qu^2u_x+ru_{xx}-su_{xxx}=0 \tag{6}$$

where $p$, $q$, $r$, $s$ are constants. This equation can be thought of as a generalization of the KdV, mKdV and Burgers equations, involving nonlinear dispersion and dissipation effects. The KdV-type Eq. $(6)$ have some application in quantum field theory, plasma physics and solid-state physics $[23-26]$.

As particular cases,

$(i)$ when $r=0$ and $p,q,s\neq 0$ Eq. $(6)$ becomes the compound KdV equation

$$u_t + puu_x + qu^2u_x - su_{xxx} = 0 \tag{7}$$

$(ii)$ when $p=0$ and $q,r,s\neq 0$ Eq. $(6)$ becomes the KdV-Burgers equation

$$u_t + qu^2u_x + ru_{xx} - su_{xxx} = 0 \tag{8}$$

and

$(iii)$ when $p,r=0$ and $q,s\neq 0$ in Eq. $(6)$, then we get the mKdV equation

$$u_t + qu^2u_x - su_{xxx} = 0. \tag{9}$$

is obtained $[23]$. Now, we introduce the variable $\xi = x - Vt$ and make transformation $u(x,t)=U(\xi)$, to reduce Eq. $(6)$ to the ODE

$$-VU' + pUU' + qU^2U' + rU'' - sU''' = 0, \tag{10}$$

integrating it with respect to $\xi$ once yields

$$-VU + \frac{p}{2}U^2 + \frac{q}{3}U^3 + rU' - sU'' + C = 0, \tag{11}$$

where $C$ is integrating constant. Assume that the solution of Eq. $(11)$ can be expressed as an ansatz $(3)$ together with $(4)$. Then, balancing the terms $U^3$ and $U''$ in Eq. $(11)$, we get $3m = m+2$ which yields the leading order $N=1$. Therefore, we can write the solution of Eq. (18) in the form

$$U = a_0 + a_1\left(G'/G\right), \quad a_1 \neq 0. \tag{12}$$

By $(4)$ and $(12)$ we derive that

$$U'' = 2a_1\left(G'/G\right)^3 + 3a_1\lambda\left(G'/G\right)^2 + \left(2a_1\mu + a_1\lambda^2\right)\left(G'/G\right) + a_1\lambda\mu \tag{13}$$

Substituting $(12)-(13)$ into $(11)$ and setting coefficients of $\left(G'/G\right)^m$ $(m=1,2,..,4)$ to zero, we obtain following undetermined system of algebraic equations for $a_0, a_1, C, \lambda$ and $\mu$:

$$\left(G'/G\right)^0 : C - Va_0 + \frac{pa_0^2}{2} + \frac{qa_0^3}{3} - r\mu a_1 - s\lambda\mu a_1$$

$$\left(G'/G\right)^1 : -Va_1 - r\lambda a_1 - s\lambda^2 a_1 - 2s\mu a_1 + pa_0a_1 + qa_0^2a_1$$

$$\left(G'/G\right)^2 : -ra_1 - 3s\lambda a_1 + \frac{pa_1^2}{2} + qa_0a_1^2$$

$$\left(G'/G\right)^3 : -2sa_1 + \frac{qa_1^3}{3}$$

Solving the above system with the aid of Mathematica, we obtain following two results:

Case 1:

$$s,q \neq 0,\ \ a_1 = \mp\sqrt{\frac{6s}{q}}\ ,\lambda = \frac{-2r + pa_1 + 2qa_0a_1}{6s},V = \frac{1}{6}\left(-4r\lambda - 12s\mu + 6pa_0 + 6qa_0^2 - p\lambda a_1 - 2q\lambda a_0 a_1\right)$$

$$C = \frac{1}{6}\left(-4r\lambda a_0 + 3pa_0^2 + 4qa_0^3 + 4r\mu a_1 - p\lambda a_0 a_1 - 2q\lambda a_0^2 a_1 + p\mu a_1^2\right) \tag{14}$$

where $a_0$ and $\mu$ are arbitrary constants. Substituting $(14)$ together with the solutions of Eq.$(4)$ into $(12)$, we have three types of travelling wave solutions of the Compound KdV-Burgers equation as follows:

When $\lambda^2 - 4\mu \succ 0$, we obtain hyperbolic function solution

$$U_{1,2} = \pm\left[\left(\frac{r\sqrt{6sq} - 3sp}{6sq}\right) \mp \frac{\sqrt{\left(\lambda^2 - 4\mu\right)6sq}}{2q}\left(\frac{C_1 \sinh\frac{\sqrt{\left(\lambda^2 - 4\mu\right)}}{2}\xi + C_2 \cosh\frac{\sqrt{\left(\lambda^2 - 4\mu\right)}}{2}\xi}{C_1 \cosh\frac{\sqrt{\left(\lambda^2 - 4\mu\right)}}{2}\xi + C_2 \sinh\frac{\sqrt{\left(\lambda^2 - 4\mu\right)}}{2}\xi}\right)\right] \tag{15}$$

where $\xi = x - \left(\frac{-4r - 12s\mu + 6pa_0 + 6qa_0^2 - p\lambda a_1 - 2q\lambda a_0 a_1}{6}\right)$t and $C_1, C_2$ are two arbitrary constants.

When $\lambda^2 - 4\mu \prec 0$, we have trigonometric function solution

$$U_{3,4} = \pm\left[\left(\frac{r\sqrt{6sq} - 3sp}{6sq}\right) \mp \frac{\sqrt{\left(4\mu - \lambda^2\right)6sq}}{2q}\left(\frac{C_1 \cos\frac{\sqrt{\left(4\mu - \lambda^2\right)}}{2}\xi - C_2 \sin\frac{\sqrt{\left(4\mu - \lambda^2\right)}}{2}\xi}{C_2 \cos\frac{\sqrt{\left(4\mu - \lambda^2\right)}}{2}\xi + C_1 \sin\frac{\sqrt{\left(4\mu - \lambda^2\right)}}{2}\xi}\right)\right] \tag{16}$$

where $\xi = x - \left(\frac{-4r - 12s\mu + 6pa_0 + 6qa_0^2 - p\lambda a_1 - 2q\lambda a_0 a_1}{6}\right)$t and $C_1, C_2$ are two arbitrary constants.

When $\lambda^2 - 4\mu = 0$, we get rational solution

$$U_{5,6} = \pm\left[\left(\frac{r\sqrt{6sq} \mp 3sp}{6sq}\right) + \frac{\sqrt{6sq}}{q}\left(\frac{C_2}{C_1 + C_2\xi}\right)\right] \tag{17}$$

where $\xi = x - \left(\frac{-4r - 12s\mu + 6pa_0 + 6qa_0^2 - p\lambda a_1 - 2q\lambda a_0 a_1}{6}\right)$t and $C_1, C_2$ are two arbitrary constants.

Case 2:

$$s,q = 0,\ p,r \neq 0,\ a_1 = \frac{2r}{p}\ ,\ V = -r\lambda + pa_0\ ,\ \mathrm{C} = \frac{1}{2}\left(-2r\lambda a_0 + pa_0^2 + 2r\mu a_1\right) \tag{18}$$

where $a_0, \lambda$ and $\mu$ are arbitrary constants. Substituting $(18)$ together with the solutions of Eq.$(4)$ into $(12)$, we have three types of travelling wave solutions of the Compound KdV-Burgers equation as follows:

When $\lambda^2 - 4\mu \succ 0$,

$$U_7=\frac{V}{p}+\frac{r\sqrt{\left(\lambda^2-4\mu\right)}}{p}\left(\frac{C_1\sinh\frac{\sqrt{\left(\lambda^2-4\mu\right)}}{2}\xi+C_2\cosh\frac{\sqrt{\left(\lambda^2-4\mu\right)}}{2}\xi}{C_1\cosh\frac{\sqrt{\left(\lambda^2-4\mu\right)}}{2}\xi+C_2\sinh\frac{\sqrt{\left(\lambda^2-4\mu\right)}}{2}\xi}\right) \tag{19}$$

where $\xi=x-\left(pa_0-r\lambda\right)t$ and $C_1,C_2$ are two arbitrary constants.

When $\lambda^2-4\mu\prec 0$,

$$U_8=\frac{V}{p}+\frac{r\sqrt{\left(4\mu-\lambda^2\right)}}{p}\left(\frac{C_1\cos\frac{\sqrt{\left(4\mu-\lambda^2\right)}}{2}\xi-C_2\sin\frac{\sqrt{\left(4\mu-\lambda^2\right)}}{2}\xi}{C_2\cos\frac{\sqrt{\left(4\mu-\lambda^2\right)}}{2}\xi+C_1\sin\frac{\sqrt{\left(4\mu-\lambda^2\right)}}{2}\xi}\right) \tag{20}$$

where $\xi=x-\left(pa_0-r\lambda\right)t$ and $C_1,C_2$ are two arbitrary constants.

When $\lambda^2-4\mu=0$,

$$U_9=\frac{V}{p}+\frac{2r}{p}\left(\frac{C_2}{C_1+C_2\xi}\right) \tag{21}$$

where $\xi=x-\left(pa_0-r\lambda\right)t$ and $C_1,C_2$ are two arbitrary constants.

## 4.THE COMPOUND KDV EQUATION

Let us consider the Compound KdV equation

$$u_t+puu_x+qu^2u_x-su_{xxx}=0 \tag{22}$$

where $p,q$ and $s$ arbitrary real constants with $p,q,s\neq 0$ .Now, letting $u\left(x,t\right)=U\left(\xi\right)$ , $\xi=x-Vt$ in $\left(22\right)$, to reduce Eq.$\left(22\right)$ to the ODE

$$-VU'+pUU'+qU^2U'-sU'''=0, \tag{23}$$

integrating it with respect to $\xi$ once yields

$$-VU+\frac{p}{2}U^2+\frac{q}{3}U^3-sU''+C=0, \tag{24}$$

where $C$ is integrating constant. Assume that the solution of Eq.$\left(24\right)$ can be expressed as an ansatz $\left(3\right)$ together with $\left(4\right)$. Balancing the terms $U^3$ and $U''$ in Eq. $\left(24\right)$ , $3m=m+2$ , yields the leading order N = 1. Therefore, we can assume the solution of Eq. $\left(24\right)$ in the form

$$U=a_0+a_1\left(G'/G\right),\quad a_1\neq 0. \tag{25}$$

By $\left(4\right)$ and $\left(25\right)$ we derive that

$$U''=2a_1\left(G'/G\right)^3+3a_1\lambda\left(G'/G\right)^2+\left(2a_1\mu+a_1\lambda^2\right)\left(G'/G\right)+a_1\lambda\mu \tag{26}$$

Substituting $\left(25\right)-\left(26\right)$ into $\left(24\right)$, setting coefficients of $\left(G'/G\right)^m$ $\left(m=1,2,..,4\right)$ to zero, we obtain following undetermined system of algebraic equations for $a_0, a_1, C, \lambda, \mu$:

$$\left(G'/G\right)^0 : C - Va_0 + \frac{pa_0^2}{2} + \frac{qa_0^3}{3} - s\lambda\mu a_1$$

$$\left(G'/G\right)^1 : -Va_1 - s\lambda^2 a_1 - 2s\mu a_1 + pa_0 a_1 + qa_0^2 a_1$$

$$\left(G'/G\right)^2 : -3s\lambda a_1 + \frac{pa_1^2}{2} + qa_0 a_1^2$$

$$\left(G'/G\right)^3 : -2sa_1 + \frac{qa_1^3}{3}$$

Solving the above system with the aid of Mathematica, we obtain following results:

$$q \neq 0,\ a_1 = \mp\sqrt{\frac{6s}{q}}\ ,\ \lambda = \frac{p + 2qa_0}{qa_1}\ ,\quad V = \frac{1}{6}\left(-12s\mu + 4pa_0 + 2qa_0^2 - p\lambda a_1\right),$$

$$\mathrm{C} = \frac{1}{6}\left(pa_0^2 - p\lambda a_0 a_1 + p\mu a_1^2\right) \tag{27}$$

where $a_0$ and $\mu$ are arbitrary constants.

Substituting $(27)$ together with the solutions of Eq. $(4)$ into $(25)$, we get three types of travelling wave solutions of the Compound KdV equation as follows:

When $\lambda^2 - 4\mu \succ 0$,

$$U_{1,2} = \left(\frac{-p}{2q}\right) \mp \frac{\sqrt{\left(\lambda^2 - 4\mu\right)6sq}}{2q} \left( \frac{C_1 \sinh \frac{\sqrt{\left(\lambda^2 - 4\mu\right)}}{2}\xi + C_2 \cosh \frac{\sqrt{\left(\lambda^2 - 4\mu\right)}}{2}\xi}{C_1 \cosh \frac{\sqrt{\left(\lambda^2 - 4\mu\right)}}{2}\xi + C_2 \sinh \frac{\sqrt{\left(\lambda^2 - 4\mu\right)}}{2}\xi} \right) \tag{28}$$

where $\xi = x - \left(\frac{-12s\mu + 4pa_0 + 2qa_0^2 - p\lambda a_1}{6}\right)t$ and $C_1, C_2$ are two arbitrary constants.

When $\lambda^2 - 4\mu \prec 0$,

$$U_{3,4} = \left(\frac{-p}{2q}\right) \mp \frac{\sqrt{\left(4\mu - \lambda^2\right)6sq}}{2q} \left( \frac{C_1 \cos \frac{\sqrt{\left(4\mu - \lambda^2\right)}}{2}\xi - C_2 \cosh \frac{\sqrt{\left(4\mu - \lambda^2\right)}}{2}\xi}{C_2 \cos \frac{\sqrt{\left(4\mu - \lambda^2\right)}}{2}\xi + C_1 \sin \frac{\sqrt{\left(4\mu - \lambda^2\right)}}{2}\xi} \right) \tag{29}$$

where $\xi = x - \left(\frac{-12s\mu + 4pa_0 + 2qa_0^2 - p\lambda a_1}{6}\right)t$ and $C_1, C_2$ are two arbitrary constants.

When $\lambda^2 - 4\mu = 0$,

$$U_{5,6} = \frac{-p}{2q} \mp \frac{\sqrt{6sq}}{q}\left(\frac{C_2}{C_1 + C_2\xi}\right) \tag{30}$$

where $\xi = x - \left(\frac{-12s\mu + 4pa_0 + 2qa_0^2 - p\lambda a_1}{6}\right)t$ and $C_1, C_2$ are two arbitrary constants.

## 5. THE KDV-BURGERS EQUATION

Let us consider the KdV-Burgers equation

$$u_t + qu^2u_x + ru_{xx} - su_{xxx} = 0 \qquad (31)$$

where $p, r$ and $s$ arbitrary real constants with $q, r, s \neq 0$. Now, letting $u(x,t) = U(\xi)$, $\xi = x - Vt$ in $(31)$, to reduce Eq. $(31)$ to the ODE

$$-VU' + qU^2U' + rU'' - sU''' = 0, \qquad (32)$$

integrating it with respect to $\xi$ once yields

$$-VU + \frac{q}{3}U^3 + rU' - sU'' + C = 0, \qquad (33)$$

where $C$ is integrating constant. Assume that the solution of Eq. $(33)$ can be expressed as an ansatz $(3)$ together with $(4)$. Then, balancing the terms $U^3$ and $U''$ in Eq. $(11)$, we get $3m = m + 2$ which yields the leading order $N = 1$. Therefore, we can assume the solution of Eq. $(33)$ in the form

$$U = a_0 + a_1\left(G'/G\right), \quad a_1 \neq 0. \qquad (34)$$

By $(4)$ and $(34)$ we derive that

$$U'' = 2a_1\left(G'/G\right)^3 + 3a_1\lambda\left(G'/G\right)^2 + \left(2a_1\mu + a_1\lambda^2\right)\left(G'/G\right) + a_1\lambda\mu \qquad (35)$$

Substituting $(34)-(35)$ into $(33)$, setting coefficients of $\left(G'/G\right)^m$ $(m = 1, 2, .., 4)$ to zero, we obtain an undetermined system of algebraic equations for $a_0, a_1, C, \lambda, \mu$.
Solving this system with the aid of Mathematica, we obtain following results.

$$s, q \neq 0, \ a_1 = \mp\sqrt{\frac{6s}{q}}, \ \lambda = \frac{-r + qa_0a_1}{3s}, \quad V = \frac{1}{3}\left(-2r\lambda - 6s\mu + 3qa_0^2 - q\lambda a_0a_1\right),$$

$$C = \frac{1}{3}\left(-2r\lambda a_0 + 2qa_0^3 + 2r\mu a_1 - q\lambda a_0^2a_1\right) \qquad (36)$$

where $a_0$ and $\mu$ are arbitrary constants. Substituting $(36)$ together with the solutions of Eq. $(4)$ into $(34)$, we get three types of travelling wave solutions of the KdV-Burgers equation as follows:
When $\lambda^2 - 4\mu \succ 0$,

$$U_{1,2} = \mp\left[\frac{r\sqrt{6sq}}{6sq} + \frac{\sqrt{\left(\lambda^2 - 4\mu\right)6sq}}{2q}\left(\frac{C_1\sinh\frac{\sqrt{\left(\lambda^2-4\mu\right)}}{2}\xi + C_2\cosh\frac{\sqrt{\left(\lambda^2-4\mu\right)}}{2}\xi}{C_1\cosh\frac{\sqrt{\left(\lambda^2-4\mu\right)}}{2}\xi + C_2\sinh\frac{\sqrt{\left(\lambda^2-4\mu\right)}}{2}\xi}\right)\right] \qquad (37)$$

where $\xi = x - \left(\frac{-2r\lambda - 6s\mu + 3qa_0^2 - q\lambda a_0a_1}{3}\right)t$ and $C_1, C_2$ are two arbitrary constants.

When $\lambda^2 - 4\mu \prec 0$,

$$U_{3,4} = \mp\left[\frac{r\sqrt{6sq}}{6sq} + \frac{\sqrt{(4\mu-\lambda^2)6sq}}{2q}\left(\frac{C_1\cos\frac{\sqrt{(4\mu-\lambda^2)}}{2}\xi - C_2\sin\frac{\sqrt{(4\mu-\lambda^2)}}{2}\xi}{C_2\cos\frac{\sqrt{(4\mu-\lambda^2)}}{2}\xi + C_1\sin\frac{\sqrt{(4\mu-\lambda^2)}}{2}\xi}\right)\right] \tag{38}$$

where $\xi = x - \left(\frac{-2r\lambda - 6s\mu + 3qa_0^2 - q\lambda a_0 a_1}{3}\right)t$ and $C_1, C_2$ are two arbitrary constants.

When $\lambda^2 - 4\mu = 0$,

$$U_{5,6} = \mp\left[\frac{r\sqrt{6sq}}{6sq} + \frac{\sqrt{6sq}}{q}\left(\frac{C_2}{C_1 + C_2\xi}\right)\right] \tag{39}$$

where $\xi = x - \left(\frac{-2r\lambda - 6s\mu + 3qa_0^2 - q\lambda a_0 a_1}{3}\right)t$ and $C_1, C_2$ are two arbitrary constants.

## 6. THE MKDV EQUATION

Let us consider the mKdV equation

$$u_t + qu^2u_x - su_{xxx} = 0. \tag{40}$$

where $q$ and $s$ arbitrary real constants with $q, s \neq 0$. Now, letting $u(x,t) = U(\xi)$, $\xi = x - Vt$ in $(40)$, to reduce Eq. $(40)$ to the ODE

$$-VU' + qU^2U' - sU''' = 0, \tag{41}$$

integrating it with respect to $\xi$ once yields

$$-VU + \frac{q}{3}U^3 - sU'' + C = 0, \tag{42}$$

where $C$ is integrating constant. Assume that the solution of Eq. $(42)$ can be expressed as an ansatz $(3)$ together with $(4)$. Then, balancing the terms $U^3$ and $U''$ in Eq. $(42)$, we get $3m = m + 2$ which yields the leading order $N = 1$. Therefore, we can assume the solution of Eq. $(42)$ in the form

$$U = a_0 + a_1(G'/G), \quad a_1 \neq 0. \tag{43}$$

By $(4)$ and $(43)$ we derive that

$$U'' = 2a_1(G'/G)^3 + 3a_1\lambda(G'/G)^2 + (2a_1\mu + a_1\lambda^2)(G'/G) + a_1\lambda\mu \tag{44}$$

Substituting $(43)-(44)$ into $(42)$, setting coefficients of $(G'/G)^m$ $(m = 1,2,..,4)$ to zero, we obtain an undetermined system of algebraic equations for $a_0, a_1, C, \lambda, \mu$.
Solving this system with the aid of Mathematica, we obtain following results.

$$q \neq 0, \; a_1 = \mp\sqrt{\frac{6s}{q}}, \; \lambda = \frac{2a_0}{a_1}, \quad V = \frac{1}{3}\left(-6s\mu + qa_0^2\right), \; C = 0 \tag{45}$$

where $a_0$ and $\mu$ are arbitrary constants.

Substituting $(45)$ together with the solutions of Eq. $(4)$ into $(43)$, we obtain three types of travelling wave solutions of the mKdV equation as follows:
When $\lambda^2 - 4\mu \succ 0$,

$$U_{1,2} = \mp \frac{\sqrt{6sq\left(\lambda^2 - 4\mu\right)}}{2q} \left( \frac{C_1 \sinh \frac{\sqrt{\left(\lambda^2 - 4\mu\right)}}{2}\xi + C_2 \cosh \frac{\sqrt{\left(\lambda^2 - 4\mu\right)}}{2}\xi}{C_1 \cosh \frac{\sqrt{\left(\lambda^2 - 4\mu\right)}}{2}\xi + C_2 \sinh \frac{\sqrt{\left(\lambda^2 - 4\mu\right)}}{2}\xi} \right) \qquad (46)$$

where $\xi = x - \left(\frac{-6s\mu + qa_0^2}{3}\right)t$ and $C_1, C_2$ are two arbitrary constants.

When $\lambda^2 - 4\mu \prec 0$,

$$U_{3,4} = \mp \frac{\sqrt{6sq\left(4\mu - \lambda^2\right)}}{2q} \left( \frac{C_1 \cos \frac{\sqrt{\left(4\mu - \lambda^2\right)}}{2}\xi - C_2 \sin \frac{\sqrt{\left(4\mu - \lambda^2\right)}}{2}\xi}{C_2 \cos \frac{\sqrt{\left(4\mu - \lambda^2\right)}}{2}\xi + C_1 \sin \frac{\sqrt{\left(4\mu - \lambda^2\right)}}{2}\xi} \right) \qquad (47)$$

where $\xi = x - \left(\frac{-6s\mu + qa_0^2}{3}\right)t$ and $C_1, C_2$ are two arbitrary constants.

When $\lambda^2 - 4\mu = 0$,

$$U_{5,6} = \mp \frac{\sqrt{6sq}}{q} \left( \frac{C_2}{C_1 + C_2\xi} \right) \qquad (48)$$

where $\xi = x - \left(\frac{-6s\mu + qa_0^2}{3}\right)t$ and $C_1, C_2$ are two arbitrary constants.

## 7. CONCLUSIONS

In this paper we have seen that three types of travelling solutions of the compound KdV-Burgers types equations, namely, the compound KdV-Burgers equation, the compound KdV equation, the KdV-Burgers equation, and the mKdV equation, are successfully found out by using the $\left(G'/G\right)$-expansion method.

Advantages of this method is being direct, concise, more powerful and effective. The performance of this method is reliable and allows us to solve complicated and tedious algebraic calculation. This verifies that the method can be used for many NLEEs in mathematical physics.

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