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BAYESIAN ESTIMATOR OF A CHANGE POINT IN THE HAZARD FUNCTION

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Abstract- This article presents a new approach for obtaining the change point in the hazard function. The proposed approach is developed with the Bayesian estimator. Using a simulation study, mean value and mean square error (MSE) of proposed estimator are obtained and compared with the mean and MSE of traditional estimators. It is showed that the proposed estimator is more efficient than the traditional estimators in many cases. Furthermore, a numerical example is discussed to demonstrate the practice of the proposed estimator.

Key Words- Bayesian Estimator, Change Point, Constant Hazard, Survival Analysis.

1. INTRODUCTION

Let T denote an independent identically disrtributed random variable of survival times. The hazard model of T is given by

$$h(t) = \begin{cases} \alpha & 0 \le t \le \tau \\ \beta & t > \tau \end{cases} \tag{1}$$

where α>0, β>0, τ>0, α and β are hazard rates, and τ is the change point. Hence, the hazard function, $h(t)$, is assumed to have a constant value α until time τ, and a constant value β after time τ. Therefore, obtaining a correct estimate of the change point plays an important role in medical and biological researches. Some recent studies in these fields are given as Gupta *et al.* [1], Tabnak *et al.* [2], Faucett *et al.* [3], Gijbels and Gurler [4], Lin [5], Daniel and Nader [6], Karasoy and Kadilar [7], etc.

It is well known that the probability density function and survival function of a random variable T are given by

$$f(t) = \begin{cases} \alpha\, e^{-\alpha t} & 0 \le t \le \tau \\ \beta\, e^{-(\alpha-\beta)\tau - \beta t} & t > \tau \end{cases} \tag{2}$$

and

$$S(t) = \begin{cases} e^{-\alpha t} & 0 \le t \le \tau \\ e^{-(\alpha-\beta)\tau - \beta t} & t > \tau \end{cases} \tag{3}$$

respectively. Note that $f(t)$ and $S(t)$ have a jump point at τ [8,9,10].

This article is organized as follows. Section 2 introduces the available traditional estimates of τ. In Section 3, my proposed estimator is presented. A simulation study is performed and the results of this simulation are discussed in Section 4. In Section 5, I present a numerical example to demonstrate the application of my proposed estimator.

2. TRADITIONAL ESTIMATORS

Let $T_1, \ldots, T_n$ be a random sample from (1). From this point onwards, without loss of generality, assume that $T_1 \leq \ldots \leq T_n$, in other words, after making suitable rearrangement the order statistics, $T_1, \ldots, T_n$, can always be observed.

Nguyen *et al.* [11] obtained a consistent estimator of τ in the model (1) as follows:

$$X_n(t) = \frac{\sqrt{\upsilon(t)}}{n}\left\{[n - R(t)]\log\frac{n}{n - R(t)} - R(t)\right\} + \frac{R(t)E(t)}{n} - \frac{\bar{T}}{n}\log\left(\frac{n}{n - R(t)}\right),$$

where $\bar{T} = \dfrac{T_1 + T_2 + \ldots + T_n}{n}$ is the mean of the sample; $R(t) = \sum_{i=1}^{n} I_{[T_i \leq t]}$ is the number of left-hand portion of the sample; $E(t) = \dfrac{\sum_{i=R(t)+1}^{n} T_i}{n - R(t)}$ and $\upsilon(t) = \dfrac{\sum_{i=R(t)+1}^{n} T_i^2}{n\text{-}R(t)} - \{E(t)\}^2$ are the mean and variance of the righ-hand portion of the sample, respectively. Here I is an indicator function and the survival times, $T_1, \ldots, T_n$, are ordered as $T_1 \leq \ldots \leq T_n$. A value of t for which $X_n(t)$ is close to 0 is a candidate for an estimate of τ. This method will be abbreviated as NRW in the rest of the article.

Basu *et al.* [12] presented two estimators for the change point, namely

$$\hat{\tau}_{BGJ1} = \inf\{t>0: y_n(t + h_n) - y_n(t) \leq h_n\hat{\beta} + \varepsilon_n\}, \tag{4}$$
$$\hat{\tau}_{BGJ2} = \inf\{t>0: -y_n(t) - \log(1 - p_0) \leq \hat{\beta}\,(\hat{\xi}_{p_0} - t) + \varepsilon_n\},$$

where $\hat{\beta}$ and $\hat{\xi}_{p_0}$ are the estimates of β and ξ_{p_0}, respectively; $p_0 < 1$; $y_n(t) = -\log[S_n(t)]$; $\varepsilon_n = \dfrac{c}{\sqrt{n}}(\log n)$; $h_n = \dfrac{1}{\sqrt[4]{n}}$. Here ξ_{p_0} is the p_0-th population quartile and c is a constant. Note that $\alpha > \beta$ in (1) for this method, which is abbreviated as BGJ in the rest of this article.

The motivation for these estimates are as follows. Consider the hypothesis testing problem:

H_0: $h(t) = \beta$

H_1: $h(t) > \beta$

If $\hat{h}_n(t)$ is a consistent estimator of *h(t)* then one can construct tests by accepting H_0 if $\hat{h}_n(t) \leq \hat{\beta} + \varepsilon_n$. Then the smallest t for which H_0 is accepted would be a candidate for an estimate of τ. For $\hat{\tau}_{BGJ1}$, take $\hat{h}_n(t) = \dfrac{y_n(t+h_n) - y_n(t)}{h_n}$ and for $\hat{\tau}_{BGJ2}$, take $\hat{h}_n(t) = \dfrac{-y_n(t) - \log(1-p_0)}{(\hat{\xi}_{p_0} - t)}$.

In addition, Basu *et al.* [12] found $\hat{\tau}_{BGJ1}$ to be more efficient than $\hat{\tau}_{BGJ2}$ by simulation, so I take only $\hat{\tau}_{BGJ1}$ for the simulation study in this article.

Ghosh and Joshi [13] also investigated the asymptotic distributions of $\hat{\tau}_{\mathrm{BGJ1}}$ and $\hat{\tau}_{\mathrm{BGJ2}}$.

Ghosh *et al.* [14] considered the following likelihood function:

$$L(\alpha, \beta, t \setminus \boldsymbol{D}) = \alpha^{R(t)} e^{-\alpha Q(t)} \beta^{[n-R(t)]} e^{-\beta[T_{tot} - Q(t)]} , \tag{5}$$

where $\boldsymbol{D}$ denotes the data $\{T_1,\ldots,T_n\}$ with $T_0 = 0$ and $T_{n+1} = \infty$; $A(t) = \sum_{i=1}^{n} T_i I_{[T_i \le t]}$ is the sum of survival times for the left-hand portion of the sample; $Q(t) = A(t) + \{n - R(t)\}t$ and $T_{tot} = \sum_{i=1}^{n} T_i$. This method will be abbreviated as GJM in the rest of the article.

In GJM, the non-informative prior distribution is given by

$$\pi(\alpha, \beta, t) = \frac{1}{\alpha\beta} . \tag{6}$$

Multiplying (5) and (6), the posterior distribution is obtained as

$$\pi(\alpha, \beta, t \setminus \boldsymbol{D}) \propto \alpha^{R(t)-1} e^{-\alpha Q(t)} \beta^{[n-R(t)-1]} e^{-\beta[T_{tot} - Q(t)]} \tag{7}$$

and using (7), the marginal posterior distribution for the change point was given by

$$\pi(t \mid \mathbf{D}) \propto \frac{(i-1)!}{[Q(t)]^i} e^{-T_{tot}\beta_0} \sum_{j=0}^{i-1} \left[\left\{ \sum_{k=0}^{n-i+j-1} \frac{(T_{tot}\beta_0)^k}{k!} \right\} \frac{(n-i+j-1)!}{j!} \frac{[Q(t)]^j}{T_{tot}^{n-i+j}} \right]. \tag{8}$$

A value of t which maximizes (8) is a candidate for the estimate of τ. Note that α>β in (1) for this method. Ghosh *et al.* [15] proved that the posterior distribution of α, β and t were asymptotically independent of each other.

Achcar and Loibel [8] defined a uniform prior distribution for α and β in (6) as

$$\pi(\alpha, \beta, t) = \frac{1}{n}\frac{1}{\alpha\beta} , \tag{9}$$

and showed that the choice of informative prior densities gave very accurate inference results on the data set for a medical research study.

Karasoy and Kadilar [16] used the least square estimates of α and β given by Gijbels and Gurler [4] in place of α and β in (6). They obtained the posterior distribution using $L(t_i, \beta, \theta)$ function given by Gijbels and Gurler [4]. Similar to the estimator in GJM, a value of t, which maximizes the posterior distribution, is a candidate for the estimate of τ. This method will be abbreviated as KK in the rest of the article.

3. PROPOSED ESTIMATOR

Considering the definition of the prior distribution, (6), as in GJM, and another prior distribution, (9), as suggested by Achcar and Loibel [8], I decide to use a prior distribution defined in KK with the least square estimates of α and β given by Gijbels and Gurler [4] in place of α and β in (6) as follows:

$$\pi(\hat{\alpha}, \hat{\beta}, t) = \frac{1}{\hat{\alpha}(t_i)\hat{\beta}(t_i)}, \tag{10}$$

where

$$\hat{\alpha}(t_i) = \bar{Y}_n - \frac{1}{n}\hat{\theta}(t_i)\sum_{j=i+1}^{n}(1 - t_{ij}), \tag{11}$$

$$\hat{\beta}(t_i) = \hat{\alpha}(t_i) + \hat{\theta}(t_i). \tag{12}$$

Here

$$\bar{Y}_n = \frac{1}{n}\sum_{j=1}^{n} Y_n(t_j), \text{ and}$$

$$\hat{\theta}(t_i) = \frac{\sum_{j=i+1}^{n} Y_n(t_j)(1 - t_{ij}) - \bar{Y}_n \sum_{j=i+1}^{n}(1 - t_{ij})}{\sum_{j=i+1}^{n}(1 - t_{ij})^2 - \frac{1}{n}\left[\sum_{j=i+1}^{n}(1 - t_{ij})\right]^2}.$$

(for details, see Gijbels and Gurler [4]).

It is clear that $\hat{\alpha}(t_i)$ and $\hat{\beta}(t_i)$ are the least squares estimates of α and β, respectively, for each t_i; $t_{ij} = t_i / t_j$ for $t_j > t_i$, $j = 1,2,\ldots,n$.

I obtain the posterior distribution for the change point in (1) by following the path described in Section 2, as follows:

$$\pi(t \mid \mathbf{D}) \propto \frac{1}{\hat{\alpha}(t_i)\hat{\beta}(t_i)} L(t_i), \tag{13}$$

where $L(t_i)$ is a likelihood function in (5). Note that Karasoy and Kadilar [16] used $L(t_i, \beta, \theta)$ function given by Gijbels and Gurler [4] for the posterior distribution in (13).

Similar to the GJM and KK methods, a value of t, which maximizes (13), is a candidate for the estimate of τ. However, the condition $\alpha > \beta$ in (1) is not required for this method.

4. SIMULATION

In this section, I try to find out which estimator has the smallest mean square error (MSE) under different conditions. In this simulation study, I take 1000 samples of sizes n = 25, 50, 100 and various values for the parameters in (1), as shown in Table 1, by coding a program in Visual Basic 6.0. The computed mean and MSE values of the traditional estimators and the proposed estimator are also given in Table 1. From Table 1, I observe that the proposed estimator generally has a smaller MSE than the other estimators, except for a few cases. Therefore, I can infer that the proposed estimator is generally more efficient than the traditional estimators. For all cases, the proposed estimator is more efficient than the NRW estimator, and gives more accurate estimates considering mean values. For all cases where α is bigger than β, the proposed estimator is more efficient than the KK estimator. Also, the proposed estimator is more efficient than the GJM estimator, except for the case $n = 100$, $\alpha = 2$, $\beta = 0.5$, $\tau = 1$. In addition, when the difference between α and β gets higher for the large sample sizes, the proposed estimator is more efficient than the BGJ estimator. I can claim that the proposed estimator is more efficient than the BGJ estimator when α gets bigger for the invariants of β and τ. Note that the BGJ estimator is unable to find a suitable t satisfying the condition (4) in some cases. Also, the BGJ and the GJM estimators can not be applied for $\alpha<\beta$ in the hazard function. These cases are shown by a dash in Table 1.

Table 1. Estimates of the Change Point and Their MSE Values.

α	β	τ	n	NRW	BGJ	GJM	KK	Proposed
1	0.5	2.5	25	0.0413 (6.0465)	3.3709 (2.8313)	1.1317 (1.9628)	1.4755 (1.8473)	2.0179 (**0.8969**)
			50	0.0202 (6.1497)	3.1541 (1.1561)	1.1737 (1.8051)	1.6625 (1.6493)	2.5326 (**0.5939**)
			100	0.0097 (6.2018)	2.7894 (**0.1989**)	1.3710 (1.3042)	1.7044 (1.6102)	2.9615 (0.6809)
1	0.5	3	25	0.0414 (8.7551)	3.7629 (3.4582)	1.1081 (3.6649)	1.5559 (2.8597)	2.1018 (**1.5142**)
			50	0.0202 (8.8795)	3.8526 (2.4368)	1.1737 (3.3814)	1.8298 (2.3346)	2.7247 (**0.7169**)
			100	0.0097 (8.9421)	3.5310 (0.8929)	1.3710 (2.6832)	1.9403 (2.2485)	3.2226 (**0.5622**)
1	2	1	25	0.0414 (0.9207)	– (–)	– (–)	1.6759 (**0.7833**)	1.7234 (0.9186)
			50	0.0202 (0.9604)	– (–)	– (–)	2.068 (1.4207)	0.8224 (**0.9566**)
			100	0.0097 (0.9808)	– (–)	– (–)	2.4454 (2.3687)	0.6332 (**0.9790**)
2	0.5	1	25	0.0207 (0.9595)	1.7257 (1.4249)	0.5428 (0.2258)	0.5046 (0.3579)	0.8831 (**0.1422**)
			50	0.0107 (0.9792)	1.3679 (0.3371)	0.5868 (0.1822)	0.5011 (0.3149)	1.0719 (**0.1199**)
			100	0.0055 (0.9895)	1.1690 (0.0607)	0.7966 (**0.0529**)	0.4908 (0.2826)	1.2333 (0.1437)
2	0.5	1.5	25	0,0207 (2.1888)	2.3709 (3.2920)	0.5665 (0.8952)	0.6912 (0.8122)	1.0506 (**0.3715**)
			50	0.0116 (2.2166)	2.3758 (2.4097)	0.5868 (0.8454)	0.7206 (0.7569)	1.3217 (**0.1746**)
			100	0.0089 (2.2276)	2.0317 (0.8922)	0.6855 (0.6708)	0.6977 (0.7412)	1.5492 (**0.1172**)
2	1	1	25	0.0207 (0.9595)	1.3581 (0.3589)	0.5798 (0.1989)	0.6898 (0.3032)	0.9354 (**0.1583**)
			50	0.0101 (0.9799)	1.1839 (**0.0843**)	0.5868 (0.1822)	0.7681 (0.3084)	1.1551 (0.1602)
			100	0.0048 (0.9904)	1.0845 (**0.0152**)	0.7896 (0.0534)	0.7564 (0.2710)	1.3206 (0.1865)

Table 1 (continued)

α	β	τ	n	**NRW**	**BGJ**	**GJM**	**KK**	**Proposed**
2	1	1.5	25	0.0207 (2.1888)	1.8815 (0.8646)	0.5540 (0.9162)	0.7779 (0.7149)	1.0509 (**0.3786**)
			50	0.0101 (2.2199)	1.9263 (0.6092)	0.5868 (0.8454)	0.9149 (0.5836)	1.3624 (**0.1792**)
			100	0.0048 (2.236)	1.7655 (0.2232)	0.7826 (0.5253)	0.9702 (0.5621)	1.6113 (**0.1406**)
2	3	1.5	25	0.0207 (2.1888)	– (–)	– (–)	0.9471 (**0.5564**)	1.0891 (2.1882)
			50	0.0101 (2.2199)	– (–)	– (–)	1.2592 (**0.3361**)	1.0398 (2.2190)
			100	0.0048 (0.236)	– (–)	– (–)	1.6072 (0.2531)	1.0095 (**0.2350**)
3	0.5	1	25	0.0146 (0.9716)	1.9069 (3.2612)	0.3811 (0.3941)	0.4299 (0.3885)	0.6962 (**0.1670**)
			50	0.0124 (0.9787)	1.8836 (2.4047)	0.3912 (0.3757)	0.4292 (0.3746)	0.8655 (**0.0785**)
			100	0.0199 (0.9726)	1.5319 (0.8921)	0.5278 (0.2276)	0.4048 (0.3776)	0.9994 (**0.0460**)
3	0.5	1.5	25	0.0146 (2.2070)	1.5867 (1.8467)	0.3633 (1.3007)	0.5660 (0.9747)	1.5459 (**0.6616**)
			50	0.0101 (2.2216)	2.0294 (2.7213)	0.3912 (1.2344)	0.6896 (0.7829)	1.9827 (**0.3516**)
			100	0.0183 (2.2074)	2.4475 (3.6855)	0.5152 (0.9742)	0.7320 (0.7234)	1.2160 (**0.1456**)
3	1	1	25	0.0138 (0.9728)	1.4175 (0.8338)	0.3739 (0.4020)	0.4862 (0.3427)	0.7026 (**0.1645**)
			50	0.0072 (0.9859)	1.4341 (0.6042)	0.3912 (0.3757)	0.5252 (0.3059)	0.8952 (**0.0783**)
			100	0.0037 (0.9928)	1.2657 (0.2231)	0.4570 (0.2981)	0.5152 (0.2969)	1.0518 (**0.0559**)
3	1	1.5	25	0.0138 (2.2089)	1.3728 (**0.6115**)	0.3617 (1.3042)	0.5773 (0.9566)	0.7449 (0.6622)
			50	0.0067 (2.2299)	1.6690 (0.7453)	0.3912 (1.2345)	0.7159 (0.7398)	1.1803 (**0.3531**)
			100	0.0032 (2.240)	1.9346 (0.9419)	0.5144 (0.9757)	0.8017 (0.6302)	1,2179 (**0.1452**)

Table 1 (continued)

α	β	τ	n	**NRW**	**BGJ**	**GJM**	**KK**	**Proposed**
3	2	1	25	0.0138 (0.9728)	1.1727 (0.2269)	0.3667 (0.4103)	0.5558 (0.2968)	1.1974 (**0.1744**)
			50	0.0067 (0.9866)	1.2093 (0.1541)	0.3912 (0.3757)	0.6813 (0.2302)	0.9110 (**0.0855**)
			100	0.0032 (0.994)	1.1326 (0.0559)	0.5196 (0.2355)	0.7656 (0.2129)	1.0942 (**0.0545**)
3.5	1	1.5	25	0.0118 (2.2148)	1.1381 (**0.5058**)	0.3094 (1.4238)	0.5055 (1.0686)	0.6419 (0.8052)
			50	0.0061 (2.2318)	1.3839 (0.5087)	0.3353 (1.3602)	0.6422 (0.8345)	0.8490 (**0.4880**)
			100	0.0028 (2.2417)	1.6456 (0.6774)	0.3917 (1.2307)	0.7654 (0.6603)	1.0644 (**0.2435**)
4	0.5	1	25	0.0112 (0.9781)	1.4556 (2.4423)	0.2764 (0.5291)	0.3985 (0.4149)	0.8535 (**0.2491**)
			50	0.0137 (0.9770)	1.8126 (3.1172)	0.2934 (0.5021)	0.4571 (0.3544)	0.7209 (**0.1217**)
			100	0.0382 (0.9453)	2.0062 (3.2135)	0.3885 (0.3765)	0.4396 (0.3634)	0.8622 (**0.0482**)
4	0.5	1.5	25	0.0109 (2.2177)	1.0305 (**0.9101**)	0.2709 (1.5153)	0.4437 (1.1769)	0.5627 (0.9315)
			50	0.0067 (2.2307)	1.2916 (1.1129)	0.2934 (1.4587)	0.5663 (0.9482)	1.3458 (**0.6191**)
			100	0.0134 (2.2167)	1.6114 (1.7733)	0.3858 (1.2439)	0.6956 0.7478	0.9359 (**0.3608**)
4	0.5	3	25	0.0103 (8.9381)	– (–)	0.2705 (7.4547)	0.4463 (6.5820)	2.5638 (**5.9883**)
			50	0.0051 (8.969)	– (–)	0.2934 (7.3285)	0.5727 (5.9659)	1.7472 (**5.1260**)
			100	0.0024 (8.986)	– (–)	0.3857 (6.8368)	0.7115 (5.3287)	1.9560 (**4.2142**)
4	1	1	25	0.0103 (0.9795)	1.1393 (0.6597)	0.2732 (0.5333)	0.4110 (0.4010)	0.5538 (**0.2489**)
			50	0.0058 (0.9888)	1.3648 (0.7972)	0.2934 (0.5021)	0.4867 (0.3264)	0.7237 (**0.1196**)
			100	0.0047 (0.9919)	1.4905 (0.8074)	0.3866 (0.3787)	0.4955 (0.3109)	0.8766 (**0.0471**)

* Estimated value for the change point (mean of the estimates for the change point).
The value in parenthesis is the MSE value of the estimation.
The bold number represents the smallest MSE value.

5. APPLICATION

In this section I apply the proposed estimator to data set about the survival times for 124 breast-cancer patients (44 of which are censored) obtained from the Oncology Department in Hacettepe University Hospital [17]. The data are reported in Karasoy and Kadilar [16].

Applying the proposed method to this data set, I obtain the following estimator of the hazard function

$$h(t) = \begin{cases} 0.00012 & t \le 53 \\ 0.00637 & t > 53 \end{cases}$$

where, of course, 53 is an estimate of the change point. Note that Karasoy and Kadilar [16] estimated the change point as 48 for this data set.

6. CONCLUSION

In this article, I have developed a new Bayesian estimator for the change point in the hazard function. This estimator has been compared with the existing estimators. Simulation results show that the proposed estimator can be used to obtain the most accurate estimate of the change point in the hazard function.

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