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THE INVESTIGATION OF ^{130,132}Te BY IBM-2

Sait İnan[1], Nureddin Türkan[2] and İsmail Maraş[3]

[1]Celal Bayar University, Department of Science Teaching, Demirci, Manisa, TURKEY
[2]Bozok University, Faculty of Arts and Science, Yozgat, TURKEY
[3]Celal Bayar University, Faculty of Arts and Science, Manisa, TURKEY
[1]saitinan@yahoo.com, [2]nturkan@hotmail.com, [3]maras.ismail@yahoo.com

Abstract- In this study, we determined the most appropriate Hamiltonian that is needed for present calculations of nuclei in the A$\cong$130 region by interacting boson model. The second version of interacting boson model (IBM-2) has been widely used for describing the quadrupole collective states of the medium heavy nuclei. The proton and neutron variables are distinguished when this version of the model is applied. Because it is important to describe the proton and neutron variables explicitely. Using the best-fitted values of parameters in the Hamiltonian of the IBM-2, we have calculated energy levels and B(E2) values for ^{130,132}Te. The results were compared with the previous experimental and theoretical data and it is observed that they are in good agreement. Some B(E2) values that are still not known so far are stated and the set of parameters used in these calculations is the best approximation that has been carried out so far. It has turned out that the interacting boson approximation (IBA) is fairly reliable for the calculation of spectra in the entire set of such Te isotopes.

Key Words- Interacting Boson Model, Energy Levels, Electromagnetic Transitions, Even-Even Te.

1. INTRODUCTION

The even stable tellurium isotopes with only two protons outside the magic shell of 50 proton and a number of neutrons ranging from 78 to 80 could be expected to be nearly spherical and their level structure should be reasonably well explained by the vibrational model. Very little is known about the multipolarities of interband transitions in tellurium nuclei. In earlier studies on measurements of the reorientation effect in ^{122}Te, ^{126}Te and ^{128}Te showed that the quadrupole moment of the 2^+ levels of these nuclei was appreciably different from zero [1]. In order to explain these large Q_{2+} values, a considerable amount of theoretical work has been developed. N. K. B. Shu et al. [2] stated that the even Te nuclei exhibit general collective nuclei properties and theoretical calculations on these nuclei have been based on the vibrational model, the quasiparticle model, and the interacting boson approximation model(IBA). The spectra of these nuclei have been investigated by β- and γ-spectroscopic measurements and by neutron capture and inelastic scattering experiments [3]. The low-lying states of this region show a rich collective structure and they were investigated extensively in terms of various models, such as the interacting boson model (IBM) [4-13], the fermion dynamical symmetry model (FDSM) [14-16], the pair-truncated shell model (PTSM) [17-22] and the nucleon-pair shell model [23-26].

In the present work, the structures of ^{130,132}Te was undertaken to provide more detail on the neutron-rich isotopes. So, the aim is to carry out B(E2) transition probabilities of such Te nuclei which are around the mass region A$\cong$130 by employing the most appropriate Hamiltonian of IBM-2 and to give a clear description about their structure in the dynamic symmetry limits. In Section 2, we give a review of the theoretical background of the study. In Section 3, the previous experimental and theoretical [27-30] data are compared with calculated values and the general features of Te (Z=52, N=78,80) isotopes are reviewed. The last section contains some concluding remarks.

2. THEORY

The IBM-2 formalism is a general model specifying the parameters of the Hamiltonian and it is considered as the neutrons' and protons' degrees of freedom are taken into account explicitly. It has the advantage of being closer to a microscopic theory, however the matrices that have to be diagonalized are much larger. Also, one can regard the IBM-1 model space , in which neutron and proton degrees of freedom are distinguished as a subspace of the IBM-2 model space, namely that of the fully symmetric states. From the IBA-2 Hamitonian one can thus project out its IBA-1 piece [31]. In the projection form IBA-2 onto IBA-1 the number of neutron (N_ν) and proton (N_π) bosons play an important role. The program code PHINT [32] is used within the option of specifying the parameters in the neutron-proton formalism where it takes care of projecting on maximum symmetry states. If the lowest states are indeed fully symmetric, the calculation with PHINT will give exactly the same excitation energies. Thus the Hamiltonian can be written as

$$H=\varepsilon_\nu n_{d\nu}+\varepsilon_\pi n_{d\pi}+\kappa Q_\pi . Q_\nu+V_{\pi\pi}+V_{\nu\nu}+M_{\pi\nu} \tag{1}$$

where $n_{d\rho}$ is the neutron (proton) d-boson number operator.

$$n_{d\rho}=d^+\tilde{d},\rho=\pi,\nu$$
$$\tilde{d}_{\rho m}=(-1)^m d_{\rho,-m} \tag{2}$$

where s^+_ρ, $d^+_{\rho m}$ and s_ρ, $d_{\rho m}$ represent the s-boson and d-boson creation and annihilation operators. The parameters ε, κ, χ_ρ and $C_{L\rho}$ are the free parameters that have been determined so as to reproduce as closely as possible the excitation-energy of all positive parity levels for which a clear indication of the spin value exists, following the same procedure described in [33]. The value of χ_π has been kept fixed along the isotopic chain as suggested by microscopic considerations which predict that this parameter depends only on the proton number. Due to admixtures of non-fully symmetric states in the IBA-2 the projection gives different results and parameters, mostly ε (ED) and κ (RKAP), have to be normalized. The rest of the operators in the Eq. 1 are defined as,

$$Q_\rho = (s^+_\rho \tilde{d}_\rho + d^+_\rho s_\rho)^{(2)} + \chi_\rho (d^+_\rho \tilde{d}_\rho)^{(2)}$$

$$V_{\rho\rho} = \sum_{L=0,2,4} C_{L\rho} ((d^+_\rho d^+_\rho)^{(L)} (d^+_\rho \tilde{d}_\rho)^{(L)})^{(0)} \qquad ;\rho = \pi, \nu \tag{3}$$

and $M_{\nu\pi}$ is referred to as the Majorana force. In particular, in this work we focus on the E2 transitions that are one of the important factors within the collective nuclear structure. So the electromagnetic transitions can also be analyzed in the framework of the IBM-2 and the most general E2 transition operator can be written as [4,34] ,

$$\begin{aligned} T(E2) &= e_\rho (s^+_\rho \times \tilde{d}_\rho + d^+_\rho \times \tilde{s}_\rho)^{(2)} + \chi_\rho (d^+_\rho \times \tilde{d}_\rho)^{(2)} \\ &= e_\nu Q_\nu + e_\pi Q_\pi \end{aligned} \tag{4}$$

where χ_ρ is a dimensionless coefficient and e_ρ is the effective quadrupole charges.

3. CALCULATIONS AND RESULTS

The $^{120-132}$Te isotopes have N_π = 1, and N_ν varies from 2 to 1, while the parameters κ, χ_ρ and ε, as well as $C_{L\rho}$, with L=0,2,4 were treated as free parameters and their values were estimated by fitting to the measured level energies (Table 1). This procedure was made by selecting the "traditional" values of parameters and then allowing one parameter to vary while keeping the others constant until a best fit was obtained. This was carried out iteratively until an overall fit was achieved. Having obtained wave functions for the states in $^{120-132}$Te after fitting the experimental energy levels in IBM-2, we can calculate the electromagnetic transition rates between states using the program PHINT [32]. We take χ_π =-1.2 in the fit for Te isotopes. In particular, the spectrum of the SU(5) nuclei is dominated by value of ε, large in comparison with the other parameters, whereas O(6) nuclei are characterized by value of κ, large compared to ε [35].

Table 1 . IBM-2 Parameters Used in The Present Study.

^A_ZX	N_π	N_ν	N	ε	κ	χ_ν	χ_π	**$C_{L\nu}$** (L=0,2,4)	**$C_{L\pi}$** (L=0,2,4)
$^{130}_{52}$Te$_{78}$	1	2	3	0.820	0.00	0.9	-1.2	0.0 , 0.0 , 0.0	0.0 , 0.0 , 0.0
$^{132}_{52}$Te$_{80}$	1	1	2	0.860	0.00	0.9	-1.2	0.0 , 0.0 , 0.0	0.0 , 0.0 , 0.0

Table 2 shows that the calculated results of energies are in good agreement with the experimental ones.

Table 2. Comparison of Calculated IBM-2 Energies with Experimental Results for ^{130,132}Te.

Isotope	Spin Parity (I^{π})	This Work(MeV)	Experiment (MeV) [26]
$^{130}_{52}Te_{78}$	2^+_1	0.820	0.840
	4^+_1	1.640	1.633
	6^+_1	2.460	1.815
$^{132}_{52}Te_{80}$	2^+_1	0.860	0.974
	4^+_1	1.720	1.671

The following table (Table 3) is related to B(E2) values of some transitions for ^{130,132}Te isotopes. As it is seen from the table, except $B(E2; 2^+_1 \rightarrow 0^+_1)$, there is no any experimental and theoretical transition probability values of such nuclei. They are compared with the calculated results of present study and there is a very nice agreement between them.

Table 3. Comparison of Calculated B(E2) Values with Some Previous Theoretical and Experimental Results for ^{130,132}Te.

Isotope	$J^+_i \rightarrow J^+_s$	B(E2) (e^2b^2)		
		This Work	Theory	Experimental
$^{130}_{52}Te_{78}$	$2^+_1 \rightarrow 0^+_1$	0.296	0.290(b) 0.295(d) 0.058(a)	0.302(b) 0.300(c)
	$2^+_1 \rightarrow 0^+_2$	0.079	-	-
	$3^+_1 \rightarrow 2^+_1$	0.0	-	-
	$3^+_1 \rightarrow 4^+_1$	0.085	-	-
	$4^+_1 \rightarrow 2^+_1$	0.394	-	-
	$0^+_2 \rightarrow 2^+_1$	0.394	-	-
	$2^+_2 \rightarrow 0^+_1$	0.0	-	-
	$4^+_2 \rightarrow 4^+_1$	0.141	-	-
	$4^+_2 \rightarrow 2^+_2$	0.155	-	-
	$2^+_2 \rightarrow 2^+_1$	0.394	-	-

Table 3. (Continued)

Isotope	$J_i^+ \rightarrow J_s^+$	B(E2) (e^2b^2) This Work	Theory	Experimental
$^{132}_{52}Te_{80}$	$2_1^+ \rightarrow 0_1^+$	0.189	0.190$^{(d)}$	-
	$0_2^+ \rightarrow 2_1^+$	0.188	-	-
	$2_1^+ \rightarrow 0_2^+$	0.038	-	-
	$2_2^+ \rightarrow 0_1^+$	0.0	-	-

(a) ref.[28], (b)ref.[29], (c) ref.[30], (d) ref.[1]

Some B(E2) transition ratios of ^{130,132}Te isotopes are discussed as R_1= $B(E2;4_1^+\rightarrow2_1^+)/B(E2;2_1^+\rightarrow0_1^+)$,$R_2$=$B(E2;2_2^+\rightarrow2_1^+)/B(E2;2_1^+\rightarrow0_1^+)$,$R_3$= $B(E2;0_2^+\rightarrow2_1^+)/B(E2;2_1^+\rightarrow0_1^+)$,$R_4$=$B(E2;2_2^+\rightarrow0_1^+)/B(E2;2_2^+\rightarrow2_1^+)$, R_5= $B(E2;3_1^+\rightarrow2_1^+)/B(E2;3_1^+\rightarrow4_1^+)$, R_6=$B(E2;4_2^+\rightarrow4_1^+)/B(E2;4_2^+\rightarrow2_2^+)$ and R_7= $B(E2;4_1^+\rightarrow2_1^+)/B(E2;2_2^+\rightarrow2_1^+)$ and the calculated ratios are compared with that of SU(5),O(6),SU(3) ratio limits [35] in Table 4. The similar calculations for the quantities in this table are also made in some previous works like [36] (for $^{70-82}$Se), [37] (for $^{122-134}$Xe) and [38] (for $^{128-140}$Nd).

Table 4. Comparison of R_1, R_2, R_3, R_4, R_5, R_6 And R_7 Ratios with Those of SU(5),O(6),SU(3) Ratio Limits for ^{130,132}Te

Isotopes		R_1	R_2	R_3	R_4	R_5	R_6	R_7
SU(5)		2.00	2.00	2.00	0.0110	0.060	0.72	1.00
SU(3)		1.60	0.02	0.00	0.7000	2.500	0.03	6.93
O(6)		1.60	0.79	0.00	0.0700	0.120	0.75	1.84
$^{130}_{52}Te_{78}$	Present	1.33	-	-	0	0	0.91	-
	Experiment	-	-	-	-	-	-	-
$^{132}_{52}Te_{80}$	Present	-	-	0.99	-	-	-	-
	Experiment	-	-	-	-	-	-	-

4. CONCLUSION

In this paper we have described the results of calculations for ^{130,132}Te isotopes in terms of the neutron-proton interacting boson model IBM-2 model, in which protonic and neutronic distinction made between the bosons and it has found that, in many cases, there is a good agreement between our calculations and experiments. Using the best-fitted values of parameters in the Hamiltonian of the IBM-2, we have calculated energy levels and B(E2) values for ^{130,132}Te. While the results are compared with the previous experimental and theoretical data it is observed that they are in good agreement. Some B(E2) values that are still not known so far are stated and the set of parameters used in these calculations is the best approximation that has been carried out so far. It has turned out that the interacting boson approximation (IBA) is fairly reliable for the calculation of spectra in the entire set of such Te isotopes. The energy values are better reproduced by the calculation for such Te isotopes along the N=78,80. On the contrary, electromagnetic properties seem to be better reproduced in the process. The systematics of the recent studies, obtained by different counting schemes for effective boson number, are tested by the tables of energies and B(E2) values. Finally, it should be remarked that the reported value in the literature agrees well with the presented results.

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