Mathematical and Computational Applications, Vol. 19, No. 2, pp. 161-171, 2014

APPROXIMATE SOLUTIONS FOR COUPLED SYSTEMS OF NONLINEAR PDES USING THE REDUCED DIFFERENTIAL TRANSFORM METHOD

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Abstract- In this paper, we are concerned with finding approximate solutions to systems of nonlinear PDEs using the Reduced Differential Transform Method (RDTM). We examine this method to obtain approximate numerical solutions for two different types of systems of nonlinear partial differential equations, such as the two-component KdV evolutionary system of order two and the Broer-Kaup (BK) system of equations. The theoretical analysis of the RDTM is investigated for these systems of equations and is calculated in the form of power series with easily computable terms. Illustrative examples will be presented to support the proposed analysis.

Keywords and phrases- Reduced Differential Transform Method (RDTM), KdV Evolutionary System, Broer-Kaup (BK) equations

1. INTRODUCTION

The Reduced Differential Transform Method [1, 2, 3], was first introduced by Keskin to solve linear and nonlinear PDEs that appears in many Mathematical Physics and engineering applications. The method provides solutions in an infinite series form and the obtained series may converge to a closed form solution if the exact solution exists. For concrete problems where exact solution does not exist, the truncated series may be used for numerical purposes. For nonlinear models, the RDTM [4, 5, 6], has shown dependable results and gives analytical approximation that converges very rapidly and in some cases gives exact solutions. Many numerical methods were used in the past to solve systems of nonlinear partial differential equations, such as, Adomian Decomposition Method (ADM) [7, 8], Differential Transform Method (DTM) [9], the Tanh-Coth Method [10], and the Variational Iteration Method (VIM) [11] and others. In this paper, we find approximate solutions to the following systems of NLPDEs:
First, the two-component KdV evolutionary system of order two:

$$\begin{aligned} u_t + 3v_{xx} &= 0 \\ v_t - u_{xx} - 4u^2 &= 0 \end{aligned}, \tag{1.1}$$

subject to the initial conditions

$$u(x,0) = \frac{-3\mu^2}{4+4cos(\mu x)}\, ; \, v(x,0) = \frac{\sqrt{3}\mu^2}{4} tan\left(\frac{\mu x}{2}\right). \tag{1.2}$$

Second, the Broer-Kaup (BK) system of equations:

$$\begin{aligned} u_t + uu_x + v_x &= 0 \\ v_t + u_x + (uv)_x + u_{xxx} &= 0 \end{aligned}, \tag{1.3}$$

subject to the initial conditions

$$u(x,0)=1+2\tanh(x);\; v(x,0)=1-2\tanh^2(x) . \tag{1.4}$$

The goal of our study is to use the RDTM to find approximate solutions to two different types of systems of nonlinear partial differential equations and to show how accurate and efficient is the method in finding approximate solutions to other complicated systems of nonlinear partial differential equations.

Keskin, in his PhD thesis [3], introduced the reduced form of the differential transform method (DTM) as a reduced differential transform method (RDTM). Keskin used the RDTM to solve the Gas Dynamics Equation, linear and nonlinear Klein-Gordon Equations and more. Also, Keskin and Oturanc [1] used the RDTM to solve linear and nonlinear wave equations and they showed the effectiveness, and the accuracy of the method. Moreover, Alquran, Al-Khaled and Ananbeh [12], gave new Soliton solutions for the Broer-Kaup (BK) system of equations using the rational sine-cosine method. Finally, M. Abdou and A. Soliman [13] used the RDTM to give numerical simulations of nonlinear evolution equations and M. Abdou [14] finds numerical solutions to the coupled MKdV system of equations and the coupled Schrodinger-KdV system of equations.

2. ANALYSIS OF THE RDTM

In this section, we start with a function of two variables $u(x,t)$ which is analytic and $k-$times continuously differentiable with respect to time t and space x in the domain of our interest. Assume we can represent this function as a product of two single-variable functions, namely, $u(x,t) = f(x).g(t)$. From the definitions of the DTM, the function can be represented as follows:

$$u(x,t)=\left(\sum_{i=0}^{\infty} F(i)x^i\right)\left(\sum_{j=0}^{\infty} G(j)t^j\right)=\sum_{k=0}^{\infty} U_k(x).\,t^k , \tag{2.1}$$

where $U_k(x)$ is the transformed function of $u(x,t)$ which can be defined as:

$$U_k(x) = \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k}u(x,t)\right]_{t=0} . \tag{2.2}$$

From equations (2.1) and (2.2) we can deduce

$$u(x,t) = \sum_{k=0}^{\infty} U_k t^k . \tag{2.3}$$

Some basic operations of the reduced differential transformation obtained from equations (2.1) and (2.2) are given in the table below:

Table 1. Basic operations of the RDTM [3]

Functional Form	**Transformed form**
$u(x,t)$	$\frac{1}{k!}\left[\frac{\partial^k}{\partial t^k}u(x,t)\right]_{t=0}$
$\alpha u(x,t) \pm \beta v(x,t)$	$\alpha U_k(x) \pm \beta V_k(x)$, α and β are constant.
$u(x,t).v(x,t)$	$\sum_{i=0}^{k} U_i(x) V_{k-i}(x)$
$u(x,t).v(x,t).w(x,t)$	$\sum_{i=0}^{k}\sum_{j=0}^{i} U_j(x) V_{i-j}(x) W_{k-i}(x)$
$\frac{\partial^n}{\partial t^n}u(x,t)$	$\frac{(k+n)!}{K!}U_{k+n}(x)$
$\frac{\partial^n}{\partial x^n}u(x,t)$	$\frac{\partial^n}{\partial x^n}U_k(x)$
$x^m t^n u(x,t)$	$x^m U_{k-n}(x)$
$x^m t^n$	$F_k(x) = x^m \delta(k-n)$, where $\delta(k-n) = \begin{cases}1, & k=n\\ 0, & k\neq n\end{cases}$
$\frac{\partial^{n+m}}{\partial x^n \partial t^m}u(x,t)$	$\frac{\partial^n}{\partial x^n}\left[\frac{(k+m)!}{k!}U_{k+m}(x)\right]$

To illustrate the RDTM, we write the two-component KdV evolutionary system of order two in standard form

$$\begin{aligned} &L_t\left(u(x,t)\right) + 3L_{xx}\left(v(x,t)\right) = 0 \\ &L_t\left(v(x,t)\right) - L_{xx}\left(u(x,t)\right) + 4N\left(u(x,t)\right) = 0 , \end{aligned} \tag{2.4}$$

subject to initial conditions

$$u(x,0) = f(x) , \; v(x,0) = g(x) , \tag{2.5}$$

where $L_t = \frac{\partial}{\partial t}$, $L_{xx} = \frac{\partial^2}{\partial x^2}$, and $N(u(x,t))$ is the nonlinear term.

Now from equation (2.4) and (2.5), we can derive the recursive formulas (using the formulas in Table 1) as follows:

$$(k+1)U_{k+1}(x) = -3\frac{\partial^2}{\partial x^2}\left(V_k(x)\right)$$
$$(k+1)V_{k+1}(x) = \frac{\partial^2}{\partial x^2}\left(U_k(x)\right) + 4N\left(u(x,t)\right) \quad , \tag{2.6}$$

and

$$U_0(x)=f(x)\,;\, V_0(x)=g(x)\,. \tag{2.7}$$

To find the rest of the iterations, we first substitute equation (2.7) into equation (2.6) and then we find the values of $U_k(x)$'s and $V_k(x)$'s. Finally, we apply the inverse transformation to all the values of $\{U_k(x)\}_{k=0}^n$ and $\{V_k(x)\}_{k=0}^n$ to obtain the approximate solutions:

$$\hat{u}(x,t)=\sum_{k=0}^{n} U_k(x)t^k\;;\;\hat{v}(x,t)=\sum_{k=0}^{n} V_k(x)t^k\;, \tag{2.8}$$

where n is the number of iterations we have used to find the approximate solution. Hence, the exact solutions of our problem is given by

$$u(x,t)=\lim_{n\to\infty}\hat{u}(x,t)\;;\;v(x,t)=\lim_{n\to\infty}\hat{v}(x,t)\;. \tag{2.9}$$

3. APPLICATIONS

In this section, we test the RDTM on two numerical examples and then compare our approximate solutions to the exact solutions.

3.1. Examples

In this section, we present two examples to show the efficiency of the RDTM.

Example. 3.1.1

We consider the two-component KdV Evolutionary System of order two:

$$\begin{aligned} u_t + 3v_{xx} &= 0 \\ v_t - u_{xx} - 4u^2 &= 0 \end{aligned}\quad , \tag{3.1}$$

subject to the initial conditions

$$u(x,0) = \frac{-3\mu^2}{4+4cos(\mu x)}\;;\; v(x,0) = \frac{\sqrt{3}\mu^2}{4}\,tan\left(\frac{\mu x}{2}\right). \tag{3.2}$$

where the exact solutions are, [6]

$$u(x,t) = \frac{-3\mu^2}{4+4cos\left(\mu\left(x+\sqrt{3}\mu t\right)\right)}\;;\; v(x,t)=\frac{\sqrt{3}\mu^2}{4}tan\left(\frac{\mu\left(x+\sqrt{3}\mu t\right)}{2}\right). \tag{3.3}$$

Applying the RDTM to (3.1) and (3.2) with $\mu=1$, we obtain the following recursive relations:

$$U_{k+1}(x) = \left(\frac{-3}{(k+1)}\right)\frac{\partial^2}{\partial x^2}V_k(x)$$
$$V_{k+1}(x) = \left(\frac{1}{(k+1)}\right)\left(\frac{\partial^2}{\partial x^2}U_k(x)+4\sum_{i=0}^{k}U_i(x)U_{k-i}(x)\right), \tag{3.4}$$

and

$$U_0(x) = \frac{-3}{4+4cos(x)}\ ; \quad V_0(x)=\frac{\sqrt{3}}{4}tan\left(\frac{x}{2}\right) \tag{3.5}$$

where, the $U_k(x)$, $V_k(x)$ are the transform function of the $t-$dimensional spectrum.

Now, substitute Eq. (3.5) into Eq. (3.6) to obtain the following:

$$U_1(x) = -3\sqrt{3}\,csc^2(x)\,sin^4\left(\frac{x}{2}\right)\ ;\ V_1(x)=\frac{3}{4+4cos(x)}\ ;\ U_2(x)=\frac{9}{32}\left(cos(x)-2\right)\sec^4\left(\frac{x}{2}\right)\ ;$$
$$V_2(x)=\frac{3\sqrt{3}tan\left(\frac{x}{2}\right)}{8+8cos(x)} \tag{3.6}$$

We continue in this manner and after the sixth iteration, the differential inverse transform of $\{U_k(x)\}_{k=0}^{6}$, $\{V_k(x)\}_{k=0}^{6}$ will give the following approximate solutions:

$$\hat{u}(x,t)=\sum_{k=0}^{6}U_k(x)t^k = U_0(x)+U_1(x)t+U_2(x)t^2+U_3(x)t^3+...$$
$$u(x,t)=-\frac{3}{4+4cos(x)}-3\sqrt{3}\,csc^2(x)sin^4\left(\frac{x}{2}\right)t+\frac{9}{32}\left(cos(x)-2\right)\sec^4\left(\frac{x}{2}\right)t^2...$$
$$\hat{v}(x,t)=\sum_{k=0}^{4}V_k(x)t^k = V_0(x)+V_1(x)t+V_2(x)t^2+V_3(x)t^3+...$$
$$v(x,t)=\frac{\sqrt{3}}{4}tan\left(\frac{x}{2}\right)+\frac{3}{4+4cos(x)}t+\frac{3\sqrt{3}tan\left(\frac{x}{2}\right)}{8+8cos(x)}t^2+...$$

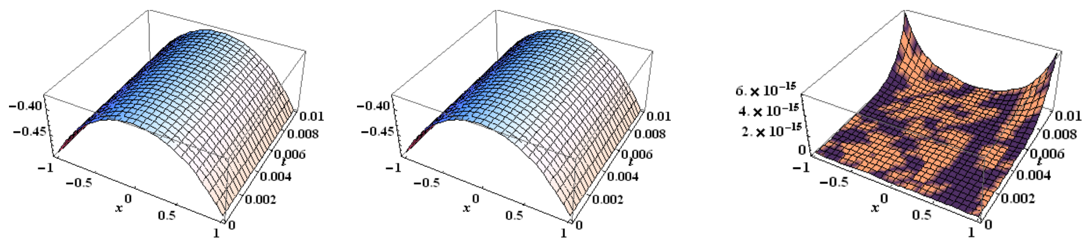

Figure 1. The approximate, exact solutions and absolute error of *u(x, t)* respectively for example 3.1.1 when -1< x <1 and 0< t <0.01.

Note that figure 1 shows the exact solution, approximate solution and the absolute error for $u(x,t)$, respectively.

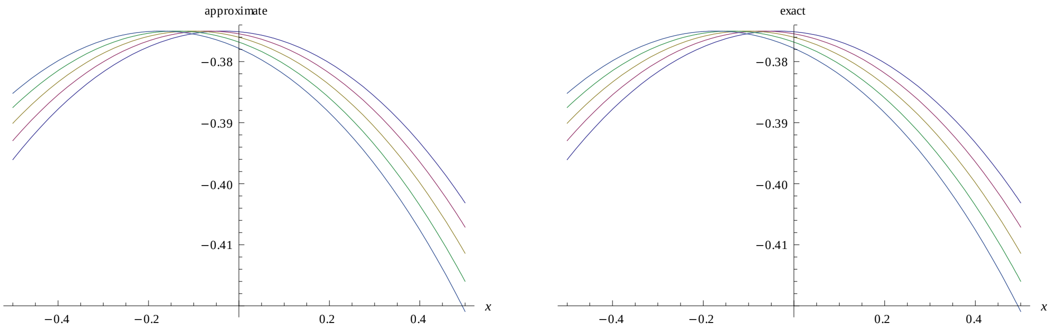

Figure 2. The approximate and exact solutions of *u(x, t)* for example 3.1.1 when -0.5< *x* <0.5 and *t* =0.02, 0.04, 0.06, 0.08, 0.1.

Note that figure 2 shows the exact solution, approximate solution of $u(x,t)$ for the values of $x = \{-1, -0.5, 0.5, 1\}$ and $t = \{0.002, 0.004, 0.006, 0.01\}$.

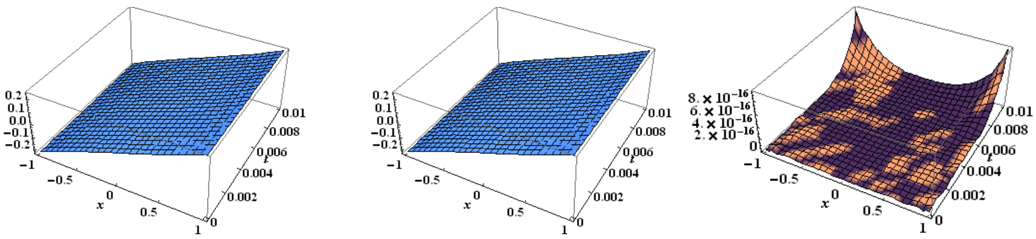

Figure 3.The approximate, exact solutions and absolute error of *v(x, t)*, respectively for example 3.1.1 *v(x, t)* when -1< *x* <1 and 0< *t* <0.01.

Note that figure 3 shows the exact solution, approximate solution and the absolute error for $v(x,t)$, respectively.

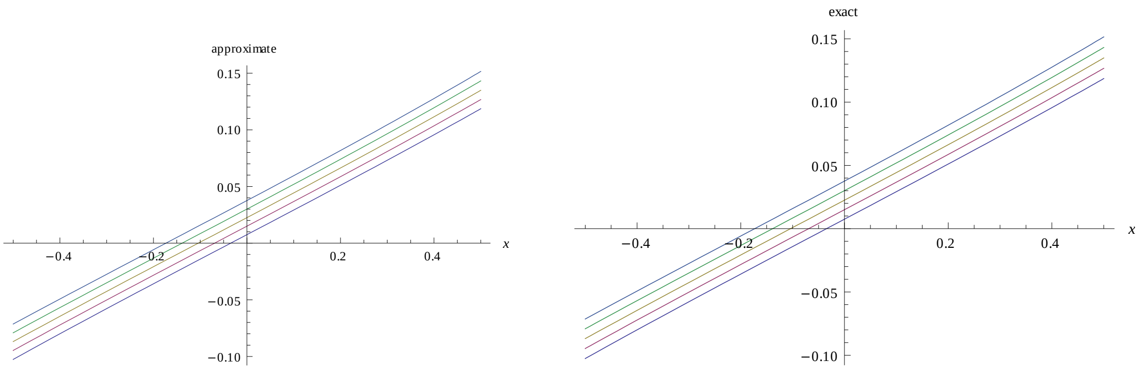

Figure 4. The approximate and exact solutions of *v(x, t)* for example 3.1.1 when -0.5< *x* <0.5 and *t* =0.02, 0.04, 0.06, 0.08, 0.1.

Note that figure 4 shows the exact solution, approximate solution of $v(x,t)$ for the values of $x = \{-1, -0.5, 0.5, 1\}$ and $t = \{0.002, 0.004, 0.006, 0.01\}$.

Example 3.1.2

We consider the Broer-Kaup (BK) system of equations:

$$\begin{aligned} &u_t + uu_x + v_x = 0 \\ &v_t + u_x + (uv)_x + u_{xxx} = 0 \end{aligned}, \tag{3.7}$$

subject to the initial conditions

$$u(x,0)=1+2\tanh(x),\ v(x,0)=1-2\tanh^2(x). \tag{3.8}$$

The exact solutions are, [6]

$$u(x,t)=1-2\tanh(t-x),\ v(x,t)=1-2\tanh^2(t-x). \tag{3.9}$$

Applying the RDTM to (3.8) and (3.7), we obtain the recursive relation

$$\begin{aligned} U_{k+1}(x) &= \left(\frac{-1}{k+1}\right)\left(\frac{\partial}{\partial x}V_k(x)+\sum_{i=0}^{k}U_i(x)\frac{\partial}{\partial x}U_{k-i}(x)\right) \\ V_{k+1}(x) &= \left(\frac{-1}{k+1}\right)\left(\frac{\partial}{\partial x}U_k(x)+\frac{\partial}{\partial x}\left(\sum_{i=0}^{k}U_i(x)V_{k-i}(x)\right)+\frac{\partial^3}{\partial x^3}U_k(x)\right) \end{aligned}, \tag{3.10}$$

and

$$U_0(x)=1+2\tanh(x),\qquad V_0(x)=1-2\tanh^2(x). \tag{3.11}$$

Now, substitute Eq. (3.11) into Eq. (3.10) to obtain the following:

$$\begin{aligned} &U_1(x) = -2sech^2(x), U_2(x) = -2sech^2(x)\tanh(x), U_3(x) = -\frac{2}{3}(cosh(2x)-2)\,sech^4(x), \dots \\ &V_1(x) = 4sech^2(x)\tanh(x), V_2(x) = 2(cosh(2x)-2)\,sech^4(x), V_3(x) = \frac{4}{3}(cosh(2x)-5)\,sech^4(x)\tanh(x), \dots \end{aligned} \tag{3.12}$$

We continue in this manner and after the sixth iteration, the differential inverse transform of $\{U_k(x)\}_{k=0}^{6}$, $\{V_k(x)\}_{k=0}^{6}$ will give the following approximate solutions:

$$\tilde{u}(x,t)=\sum_{k=0}^{6}U_k(x)t^k = U_0(x)+U_1(x)t+U_2(x)t^2+U_3(x)t^3+\dots$$

$$u(x,t)=1+2\tanh(x)-2sech^2(x)t-2sech^2(x)\tanh(x)t^2-\frac{2}{3}(cosh(2x)-2)sech^4(x)t^3+\dots\ ,$$

$$\tilde{v}(x,t)=\sum_{k=0}^{6}V_k(x)t^k = V_0(x)+V_1(x)t+V_2(x)t^2+V_3(x)t^3+\dots$$

$$v(x,t)=1-2\tanh^2(x)+4sech^2(x)\tanh(x)t+2(cosh(2x)-2)sech^4(x)t^2+\frac{4}{3}(cosh(2x)-5)sech^4(x)\tanh(x)t^3+\dots$$

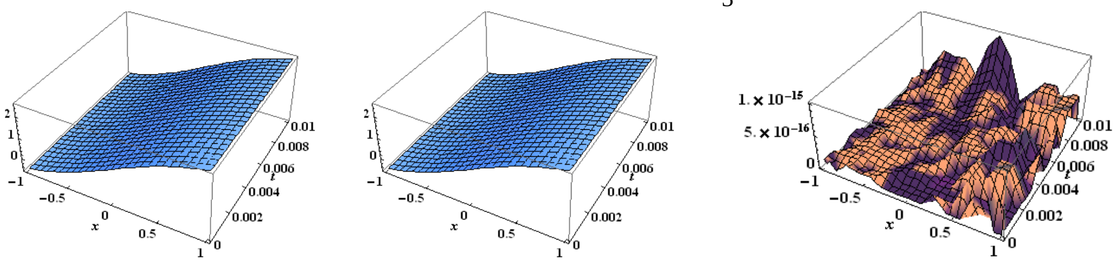

Figure 5. The approximate, exact solutions and absolute error of *u(x, t)* respectively for example 3.1.2 when -1< *x* <1 and 0< *t* <0.01.

Note that figure 5 shows the exact solution, approximate solution and the absolute error for $u(x,t)$, respectively.

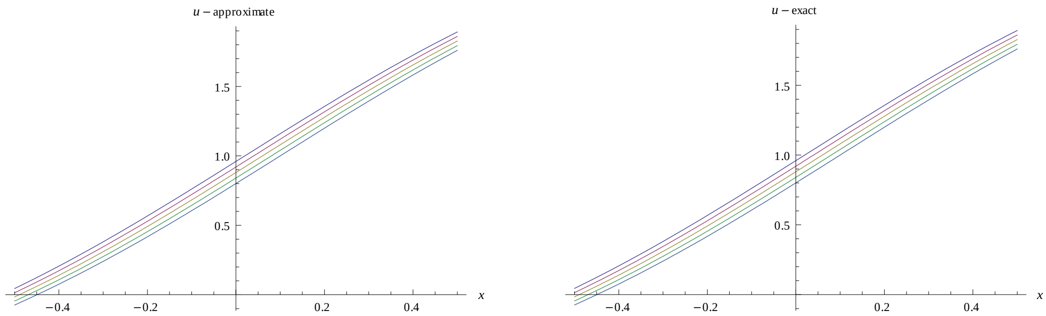

Figure 6. The approximate and exact solutions of *u(x, t)* for example 3.1.2 when -0.5< *x* <0.5 and *t* =0.02, 0.04, 0.06, 0.08, 0.1.

Note that figure 6 shows the exact solution, approximate solution of $u(x,t)$ for the values of $x = \{-1, -0.5, 0.5, 1\}$ and $t = \{0.002, 0.004, 0.006, 0.01\}$.

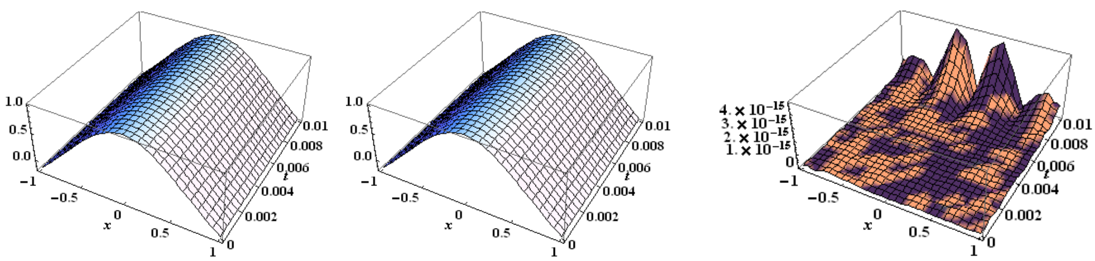

Figure 7. The approximate, exact solutions and absolute error of *v(x, t)* respectively for example 3.1.2 when -1< *x* <1 and 0< *t* <0.01.

Note that figure 7 shows the exact solution, approximate solution and the absolute error for $v(x,t)$, respectively.

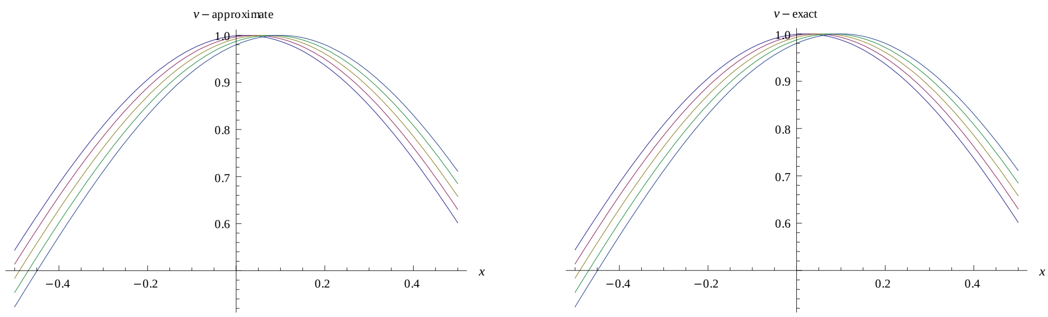

Figure 8. The approximate and exact solutions of *v(x, t)* for example 3.1.2 when -0.5< *x* <0.5 and *t* =0.02, 0.04, 0.06, 0.08, 0.1

Note that figure 8 shows the exact solution, approximate solution of $v(x,t)$ for the values of $x = \{-1, -0.5, 0.5, 1\}$ and $t = \{0.002, 0.004, 0.006, 0.01\}$.

3.2. Tables

In this section, we shall illustrate the accuracy and efficiency of the RDTM by comparing the approximate and exact solutions. In Table 2 and 3, we consider the same value for x, and t, specifically, $x = \{-1, -0.5, 0.5, 1\}$ and $t = \{0.002, 0.004, 0.006, 0.01\}$.

Table 2. Comparison of absolute errors of *u(x, t)* and *v(x, t)* for the two-component KdV evolutionary system of order 2, by RDTM for different values of *x, t.*

x	t	$u(x,t)$ Exact	$u(x,t)$ Approx.	$v(x,t)$ Exact	$v(x,t)$ Approx.	Error(RDTM)(n=6) $v(x,t)$	Error(RDTM)(n=6) $u(x,t)$
-1	.002	-0.4859987017	-0.4859987017	-0.2355830020	-0.2355830020	0	$1.11022303E^{-16}$
	.004	-0.4850855048	-0.4850855048	-0.2346119187	-0.2346119187	0	$5.55111512E^{-17}$
	.006	-0.4841777817	-0.4841777817	-0.2336426563	-0.2336426563	0	$2.22044605E^{-16}$
	.01	-0.4823786318	-0.4823786318	-0.2317095507	-0.2317095507	$8.88178420E^{-16}$	$5.82867088E^{-15}$
-0.5	.002	-0.3990979170	-0.3990979170	-0.1097677480	-0.1097677480	$1.38777878E^{-17}$	$9.59351708E^{-18}$
	.004	-0.3987488789	-0.3987488789	-0.1089699016	-0.1089699016	$1.38777878E^{-17}$	$4.37629829E^{-17}$
	.006	-0.3984026879	-0.3984026879	-0.1081727506	-0.1081727506	0	$1.83433918E^{-17}$
	.01	-0.3977188108	-0.3977188108	-0.1065805113	-0.1065805113	$1.80411242E^{-16}$	$9.35662399E^{-16}$
0.5	.002	-0.3998045711	-0.3998045711	0.1113655491	0.1113655491	0	$1.17459666E^{-17}$
	.004	-0.4001622057	-0.4001622057	0.1121655154	0.1121655154	$2.77555756E^{-17}$	$6.96101917E^{-17}$
	.006	-0.4005227246	-0.4005227246	0.1129661997	0.1129661997	0	$9.57615746E^{-18}$
	0.01	-0.4012524536	-0.4012524536	0.1145697464	0.1145697464	$1.94289029E^{-16}$	$8.50952883E^{-16}$
1	.002	-0.4878416434	-0.4878416434	0.2375306753	0.2375306753	$2.77555756E^{-17}$	0
	.004	-0.4887714524	-0.4887714524	0.2385072875	0.2385072875	$5.55111512E^{-17}$	$5.55111512E^{-17}$
	.006	-0.4897068631	-0.4897068631	0.2394857648	0.2394857648	$5.55111512E^{-17}$	$1.11022303E^{-16}$
	0.01	-0.4915946213	-0.4915946213	0.2414483603	0.2414483603	$9.71445147E^{-16}$	$6.05071548E^{-15}$

Table 3. Comparison of absolute errors of *u(x, t)* and *v(x, t)* for the Broer-Kaup (BK) system equations, by RDTM for different values of *x, t.*

x	t	$u(x,t)$ Exact	$u(x,t)$ Appr.	$v(x,t)$ Exact	$v(x,t)$ Approx.	Error(RDTM)(n=6) $v(x,t)$	Error (RDTM)(n=6) $u(x,t)$
-1	.002	−0.5248656521	−0.5248656521	−0.1626076285	−0.1626076285	$4.44089210E^{-16}$	$4.44089210E^{-16}$
	.004	−0.5265378847	−0.5265378847	−0.1651589567	−0.1651589567	0	$2.22044605E^{-16}$
	.006	−0.5282050196	−0.5282050196	−0.1677052910	−0.1677052910	$2.22044605E^{-16}$	$2.22044605E^{-16}$
	.01	−0.5315240365	−0.5315240365	−0.1727829372	−0.1727829372	$8.88178420E^{-16}$	0
-0.5	.002	0.0726228035	0.0726228035	0.5699857677	0.5699857677	$5.17342545E^{-17}$	$3.16248385E^{-17}$
	.004	0.0694857454	0.0694857454	0.5670716110	0.5670716110	$1.79166394E^{-17}$	$4.32816210E^{-18}$
	.006	0.0663545110	0.0663545110	0.5641530588	0.5641530588	$5.96930588E^{-18}$	$6.53956485E^{-17}$
	.01	0.0601096021	0.0601096021	0.5583030199	0.5583030199	$1.14439064E^{-15}$	$6.86646623E^{-16}$
0.5	.002	1.9210856177	1.9210856177	0.5758006425	0.5758006425	$1.01678014E^{-16}$	$9.18898932E^{-17}$
	.004	1.9179311150	1.9179311150	0.5787012341	0.5787012341	$6.94107766E^{-17}$	$8.09162501E^{-17}$
	.006	1.9147708158	1.9147708158	0.5815971773	0.5815971773	$3.04420465E^{-17}$	$3.27189770E^{-17}$
	0.01	1.9084328654	1.9084328654	0.5873748646	0.5873748646	$1.40966802E^{-15}$	$4.85812888E^{-16}$
1	.002	2.5215058541	2.5215058541	−0.1574900320	−0.1574900320	0	0
	.004	2.5198182687	2.5198182687	−0.1549237850	−0.1549237850	$2.22044605E^{-16}$	0
	.006	2.5181255459	2.5181255459	−0.1523525866	−0.1523525866	$2.22044605E^{-16}$	0
	0.01	2.5147246484	2.5147246484	−0.1471953803	−0.1471953803	$6.66133815E^{-16}$	0

4. CONCLUSION

In this paper, the Reduced Differential Transform Method (RDTM) was implemented for solving the two-component KdV Evolutionary System of order two and the Broer-Kaup (BK) system of equations. We successfully found approximate solutions for both systems of nonlinear PDEs by first applying the RDTM to both physical models. The results we obtained were in excellent agreement with the exact solutions. The RDTM introduces a significant improvement in the field over existing techniques. Our goal in the future is to apply the RDTM to other systems of nonlinear PDEs that arises in other fields of science.

Acknowledgement- This work was supported by research grant from Jordan University of Science and Technology No. (87/2013). Also, the author would like to thank the Editor and the anonymous referees for their comments and suggestions on this paper. Also, I would like to thank the Deanship of Research (DR) at Jordan University of Science and Technology (JUST) for their support in this project.

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