Mathematical and Computational Applications

MDPI

Article

3-Homogeneous Groups and Block-Transitive 7–(v, k, 3) Designs

Xiaolian Liao [1,*], Guohua Chen [1] and Shangzhao Li [2]

1 Department of Mathematics Hunan, University of Humanities Science and Technology, Loudi 417000, China; hnldcgh@163.com
2 School of Mathematics and Statistics, Changshu Institute of Technology, Changshu 215500, China; lszfd2004@163.com
* Correspondence: hnldlxl2005@126.com; Tel.: +86-134-6738-9235

Academic Editor: Fazal M. Mahomed
Received: 25 November 2016; Accepted: 29 May 2017; Published: 15 June 2017

Abstract: The classification of a block-transitive designs is an important subject on algebraic combinatorics. With the aid of MATLAB software, using the classification theorem of 3-homogeneous permutation groups, we look at the classification problem of block-transitive 7–(v, k, 3) design and prove our main theorem: If the automorphism group of a 7–(v, k, 3) design is block-transitive, then it is neither isomorphic to Affine Type Groups nor Almost Simple Type Groups.

Keywords: block-transitive; 3-homogeneous groups; permutation group; Affine Type Groups; Almost Simple Type Groups

---

1. Introduction

A t–design is a finite incidence structure $D = (X, \mathrm{B})$, where X is a set of points and B a set of blocks, such that (1) each block is incident with k points and (2) each t-subset of X is incident with λ blocks. If $|X| = v$, then we call D a $t - (v,\ k,\ \lambda)$ design [1].

Usually we denote $|\mathrm{B}| = b$. A t-design D is trivial, if each t-subset of X is a block and all t-subsets of X are contained in B. If any two blocks of D are different, then D is called simple. If a permutation of X preserves a block of D to a block of itself, the permutation is called an automorphism of D, and all the automorphisms of D form a group G, called the automorphism group of D, and we denote it $G \leq Aut(D)$. A group of automorphism of D is block-transitive if G acts transitively on the blocks of D. This is equivalent to that group G is transitive on the blocks and a block stable subgroup of G is also transitive on the blocks. Here we are interested in a design that is nontrivial and simple.

How to construct a block design with given parameters is one of the most important topics of combinatorial mathematics. It is especially difficult to construct a t–design with larger parameters. By [2], we have the following result: if D is a nontrivial $t - (v,\ k,\ \lambda)$ design with a block-transitive automorphism group, then $t \leq 7$. Thus, it is necessary to discuss block-transitive designs with $t \leq 7$.

Praeger and Cameron proved a very meaningful theorem in 1993 [2]: when $t > 8$, there is no non-trivial block-transitive t-design, and when $t > 7$, there is no non-trivial flag-transitive t-design. At present, the research results of flag-transitive design are relatively perfect. However, the research on block-transitive t-designs is scarce. In 2010, Huber completed the proof that there is no block-transitive Steiner 6-design (unless $G = P\Gamma L(2, p^e)$, $p = 2, 3$, and e is odd prime) [3]. However, when $\lambda \geq 2$, the current research progress is slow and has yielded few results. In this paper, we are going to study the existence of block-transitive $7 - (v,\ k,\ 3)$ designs. The main results are as follows.

Theorem. *Let D be a non-trivial $7 - (v,\ k,\ 3)$ design and G be an automorphism group of D. If G acts block-transitively on D, then G is neither isomorphic to Affine Type Groups nor Almost Simple Type Groups.*

2. Preliminary Results

Lemmas 1–5 can be found in the results of Beth et al. [4].

Lemma 1. *Let D be a $t-(v,\ k,\ \lambda)$ design, where $t\ \geq\ 2$ and $G\ \leq\ Aut(D)$ [4]. Therefore, the following holds:*

1. *If G acts block-transitively on D , then G acts point $[t/2]$ -homogeneously on D.*
2. *If G acts flag-transitively on D , then G acts point $[(t+1)/2]$-homogeneously on D.*

In particular, when $t\ =\ 7,\ \lambda\ =\ 3$ and G acts block-transitively on D, then G acts point 3-homogeneously on D. We can use the classification of finite 3-homogeneous permutation groups to discuss a block-transitive $7-(v,\ k,\ 3)$ design.

Lemma 2. *Let D be a $t-(v,\ k,\ \lambda)$ design. Therefore, the following holds:*

1. $bk = vr$
2. $\begin{pmatrix} v \\ t \end{pmatrix}\lambda = b\begin{pmatrix} k \\ t \end{pmatrix}$
3. *For any $1 \leq\ s\ <\ t$, a $t-(v,\ k,\ \lambda)$ design is also a $s-(v,\ k,\ \lambda_s)$ design, where:*

$$\lambda_s = \lambda\frac{(v-s)(v-s-1)\dots(v-t+1)}{(k-s)(k-s-1)\dots(k-t+1)}$$

4. *If $t = 7$, then*

$$r(k-1)(k-2)(k-3)(k-4)(k-5)(k-6) = \lambda(v-1)(v-2)(v-3)(v-4)(v-5)(v-6)$$

Lemma 3. *If D is a non-trivial $t-(v,\ k,\ \lambda)$ design, then*

$$\lambda\begin{pmatrix} v-s \\ t-s \end{pmatrix} \equiv 0\left(\mathrm{mod}\begin{pmatrix} k-s \\ t-s \end{pmatrix}\right),\ 0 \leq s \leq t$$

Lemma 4. *If D is a non-trivial $t-(v,\ k,\ \lambda)$ design, then $v > k\ +\ t$.*

Lemma 5. *If D is a $t-(v,\ k,\ \lambda)$ design, then*

$$\lambda(v-t+1) \geq (k-t+1)(k-t+2), t\ >\ 2.$$

In this case, when $t = 7$, we deduce from Lemma 5 the following upper bound for the positive integer k.

Lemma 6. *Let D be a non-trivial $7-(v,\ k,\ 3)$ design, then*

$$k \leq \left[\sqrt{3v-\frac{71}{4}}+\frac{11}{2}\right]$$

Proof. By Lemma 5, when $t = 7, \lambda = 3$, we have $3(v-7+1) \geq (k-7+1)(k-7+2)$, then

$$k \leq \left[\sqrt{3v-\frac{71}{4}}+\frac{11}{2}\right]$$

□

Lemma 7. *Let D be a non-trivial $t-(v,\ k,\ \lambda)$ design and G be an automorphism group of D [2]. If G acts block-transitively on D, then* $b=\frac{|G|}{|G_B|}$.

Lemma 8. *Let G be a finite 3-homogeneous permutation group on a set X with $|X| \ge 4$, then G is either of Affine Type or Almost Simple Type [5].*

Lemma 9. *In finite 3-homogeneous permutation groups, the classification of Affine Type Group is one of the following [5]:*

1. $G \cong AGL(1,8),\ A\Gamma L(1,8)$ *or* $A\Gamma L(1,32)$
2. $G \cong SL(d,2),\ d \ge 2$
3. $G \cong A_7,\ v=2^4$

Lemma 10. *In finite 3-homogeneous permutation groups on a set X with $|X| \ge 4$, let G be of Almost Simple Type [4]. Therefore, G contains a simple normal subgroup N, where N is the socle of group G and $N \le G \le Aut(N)$. Thus, N and $v=|X|$ are given as follows:*

1. $A_v,\ v \ge 5$
2. $PLS(2,q),\ v=q+1,\ q>3$
3. $M_v,\ v=11,12,22,23,24$
4. $M_{11},\ v=12$

Lemma 11. *Let ε be a 3-dimensional subspace in vector space $V=V(d,2)$, then $GL(d,2)=SL(d,2)$ acts point-transitively on $V\backslash\varepsilon$ ([6]).*

3. Proof of the Main Theorem

Let D be a non-trivial $7-(v,\ k,\ 3)$ design, the automorphism group G of D acts block-transitively on D, then G is a finite 3-homogeneous permutation group. Using Lemma 8, we know that a permutation group of a $7-(v,\ k,\ 3)$ design is either Affine Type or Almost Simple Type. For D is non-trivial, then we can suppose $k>7$. Now we are going to discuss the problem in two cases.

3.1. G Is an Affine Type Group

Corollary 1. *G is not isomorphic to* $AGL(1,8),\ A\Gamma L(1,8)$ *or* $A\Gamma L(1,32)$.

If $v=8$, then $k<v-t=8-7=1$, which is a contradiction to the condition of $k>7$.

If $v=32$ and $|G|=|A\Gamma L(1,32)|=4960$, by Lemma 6, we have $k \le \left[\sqrt{3v-\frac{71}{4}}+\frac{11}{2}\right] \le 14$, and combining the condition of $k>7$, the parameter k satisfies condition $8 \le k \le 14$. It is clear here that these are not possible if $10 \le k \le 14$. Because of Lemma 2 (2), we have $29 \mid b$, but $29 \perp |G|$ (where $29 \perp |G|$ means that $|G|$ cannot be divisible by 29), which leads to a contradiction. Corollary 1 is impossible.

Corollary 2. *G is not isomorphic to* $SL(d,2),\ d \ge 2$.

Here $|V|=|V(d,2)|=2^d$. If $d=2,\ 3$, then $v=|V|=4,\ 8$, and by the proof of the above Corollary 1, none of the designs satisfying the above conditions exist. Now we assume $d>3$. Let $e_1,e_2,\cdots,e_d$ be a set of bases of vector space $V(d,2)$. Therefore, the dimension of the subspace generated by any 7 points in vector space $V(d,2)$ is at least 3. Let $\varepsilon=<e_1,e_2,e_3>$ be a vector subspace generated by the base vectors e_1,e_2,e_3 and $S=\{0,e_1,e_2,e_3,e_1+e_2,e_1+e_3,e_2+e_3\}$ be any 7-subset of ε. By the definition of t-design, for a 7-design, each 7-subset is contained in exactly 3 blocks and we record these three blocks as B_1,B_2,B_3. Thus, $S \subseteq B_1 \cap B_2 \cap B_3$. If B_1 contains a vector α and $\alpha \in V(d,2)\backslash\varepsilon$, then by the transitivity of $SL(d,2)_\varepsilon$ on the vector space $V(d,2)$, we have $\alpha^{SL(d,2)_\varepsilon} = V(d,2)\backslash S$ and $\varepsilon \cup V(d,2)\backslash S \subseteq B_1^{SL(d,2)_\varepsilon} \subseteq B_1 \cup B_2 \cup B_3$.

Therefore, we have $2^d - 8 \leq 3(k-7)$ or $v = 2^d \leq 3k - 13$, on the other hand, by Lemma 5, we have $3(v-6) \geq (k-6)(k-5)$, so $k < 14$. As the same proof method of Corollary 1, we can rule out this case.

Corollary 3. *G is not isomorphic to* A_7, $v = 2^4$.

Here $v = 2^4 = 16$, and for $k \leq \left[\sqrt{3v - \frac{71}{4}} + \frac{11}{2}\right] \leq 11$ and $k \geq 8$, the possible value of k is 8, 9, 10, or 11 by calculation. The corresponding r are not positive integers. This is impossible.

Thus, Corollary 3 is not possible.

3.2. G Is an Almost Simple Type

Corollary 4. *G is not isomorphic to* A_v, $v \geq 5$.

Since D is a non-trivial 7-design, then, by Lemma 4, $k < v - 7$. Of course, we have $k \leq v - 2$, so $A_v (v \geq 5)$ acts v-2-transitively on D, and then G is k-transitive on D. It means that D contains all k-subsets and D is a trivial design, a contradiction.

Corollary 5. *G is not isomorphic to* $N = PSL(2,q), v = q+1, q = p^e \geq 3$.

Here, $N \leq G \leq Aut(N)$ and $Aut(N) = P\Gamma L(2,q)$. For $|N| = |PSL(2,q)| = \frac{(q+1)q(q-1)}{n}$, $n = (2, q-1)$.

$$|PGL(2,q)| = (q+1)q(q-1),\ |P\Gamma L(2,q)| = |PGL(2,q)| \cdot e = e(q+1)q(q-1)$$

Then

$$|G| = \frac{a(q+1)q(q-1)}{n},\ q = p^e,\ a|ne$$

For $v = q+1 > k \geq 8$, then $q \geq 8$. By Lemmas 2 (1) and 7, we have

$$b = \frac{|G|}{|G_B|} = \lambda \frac{\binom{v}{t}}{\binom{k}{t}}$$

and

$$\frac{a(q+1)(q-1)q}{|G_B|n} = \frac{3(v-1)(v-2)(v-3)(v-4)(v-5)(v-6)}{k(k-1)(k-2)(k-3)(k-4)(k-5)(k-6)} \tag{1}$$

For $v = q + 1$, we have

$$\frac{a(q+1)(q-1)q}{|G_B|n} = \frac{3(q-1)(q-2)(q-3)(q-4)(q-5)(q+1)q}{k(k-1)(k-2)(k-3)(k-4)(k-5)(k-6)} \tag{2}$$

After simplification, we obtain the following:

$$3(q-2)(q-3)(q-4)(q-5)|G_B|n = ak(k-1)(k-2)(k-3)(k-4)(k-5)(k-6) \tag{3}$$

Using Lemma 5 again, then

$$3(q-5) \geq (k-6)(k-5) \tag{4}$$

Add Equation (4) into Equation (3), we have

$$(q-2)(q-3)(q-4)|G_B|n \le ak(k-1)(k-2)(k-3)(k-4) \tag{5}$$

Additionally, by Equation (4), we can obtain an Inequality (6) under the conditions of $k \ge 27$,

$$k(k-1)(k-2)(k-3) \le 2\,[(k-6)(k-5)\,]^2 \tag{6}$$

By Equations (5) and (6) and Lemma 6, we have

$$|\mathrm{G}_B|n < \frac{18a(q-5)^2(\sqrt{3q-\frac{59}{4}}+\frac{3}{2})}{(q-2)(q-3)(q-4)} \tag{7}$$

Now we are going to discuss it in three steps.

1. $p = 2,\ q = 2^{\mathrm{e}}$.

Here $n = (2, q-1) = 1$. By $a|ne$, we know $a \le e$, so

$$|\mathrm{G}_B| < \frac{18e(2^e-5)^2(\sqrt{3\cdot 2^e-\frac{59}{4}}+\frac{3}{2})}{(2^e-2)(2^e-3)(2^e-4)} \tag{8}$$

Now, we construct an auxiliary function as follows:

$$y = f(e) = \frac{18e(2^e-5)^2(\sqrt{3\cdot 2^e-\frac{59}{4}}+\frac{3}{2})}{(2^e-2)(2^e-3)(2^e-4)} \tag{9}$$

Equation (9) is a decreasing function of e ($e \ge 3$). Using a computer to calculate, we find that $f(18) \approx 1.0979006$, $f(19) \approx 0.8190221 < 1$. Using MATLAB software, we map Figures 1 and 2 as follows (see Appendices A and B):

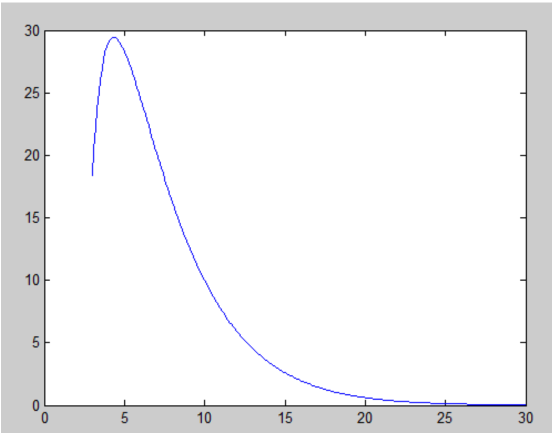

Figure 1. Partial graph of $f(e)$ ($3 \le e \le 30$).

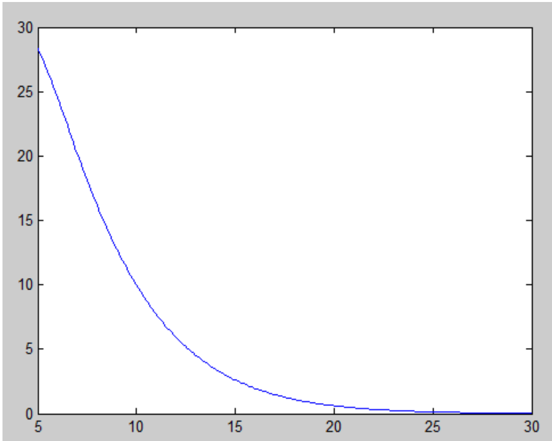

Figure 2. Partial graph of $f(e)$ $(5 \le e \le 30)$.

For $|G_B| \ge 1$, by properties of a decreasing function, we obtain $e \le 18$. As a result, the corresponding values of q and v are as shown in Table 1.

Table 1. Values of q and v.

e	q	v
3	8	9
4	16	17
5	32	33
6	64	65
7	128	129
8	256	257
9	512	513
10	1024	1025
11	2048	2049
12	4096	4097
13	8192	8193
14	16,384	16,385
15	32,768	32,769
16	65,536	65,537
17	131,072	131,073
18	262,144	262,145

Since $8 \le k \le \left[\sqrt{3v - \frac{71}{4}} + \frac{11}{2}\right]$ and $v = q + 1 \ge 15$. By Equation (3), the possible values of v and k are only one of the following three kinds $(v, k) = (65537, 8)$, (131073,8), and (262145,8). They are all in contradiction with the hypothesis $k \ge 26$.

Now, we are going to discuss the case that k is less than 27. We can introduce Inequality (10) with Equation (3) and $|G_B| \ge 1$:

$$\frac{3(2^e-2)(2^e-3)(2^e-4)(2^e-5)}{e} \le k(k-1)(k-2)(k-3)(k-4)(k-5)(k-6). \tag{10}$$

Let

$$f(e) = \frac{3(2^e-2)(2^e-3)(2^e-4)(2^e-5)}{e}$$

and

$$h(k) = k(k-1)(k-2)(k-3)(k-4)(k-5)(k-6)$$

Then, $f(e)$ is an increasing function of e, and the maximum value of $h(k)$ is $h(k)_{max} = h(26) = 552,552,000$. It can be calculated that the maximum value of e is $e_{max} = 9$. Due to $k \geq 8$ and $v = 2^e + 1 > k + t \geq 15$, the minimum value of e is $e_{\min} = 4$.

Again, with Equation (3), we have

$$(q-2)(q-3)(q-4)(q-5) \mid k(k-1)(k-2)(k-3)(k-4)(k-5)(k-6)\,e, \tag{11}$$

and

$$(q-2)(q-3)(q-4)(q-5) \leq k(k-1)(k-2)(k-3)(k-4)(k-5)(k-6)e \tag{12}$$

The admissible parameter sets (with Inequality (12) and Lemmas 3–6) are shown in Table 2.

Table 2. Values of q, v and k.

e	q	v	k
4	16	17	8, 9
5	32	33	8, 9, 10, 11, 12, 13, 14
6	64	65	12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24
7	128	129	16, 17, 18, 19, 20, 21, 22, 23, 24
8	256	257	21, 22, 23, 24, 25, 26
9	512	513	k is at least equal to 29, contradiction.

At last, the admissible parameter sets (v, k) in Table 2 do not satisfy Equation (11). Therefore, we show that it does not occur.

2. $p = 3,\ q = 3^e$.

The proof is similar to the above. However, here $n = (2,\ q-1) = 2$, and by $a|ne$, we have $a \leq 2e$. Correspondingly, we can construct the following auxiliary function:

$$y = f(e) = \frac{36e(3^e-5)^2(\sqrt{3 \cdot 3^e - \frac{59}{4}} + \frac{3}{2})}{(3^e-2)(3^e-3)(3^e-4)} \tag{13}$$

Similar to the proof method of the above, we can rule out it.

3. $p \geq 5,\ q = p^e$ and p is a prime.

Since $n = (2, q-1) = 2$ and $a|ne$, then $a \leq 2e$. If $p \geq 5,\ q = p^e \geq 5^e$, then $q = p^e \geq 5^e \geq 5^{\frac{a}{2}} > 2^a$, and $a < \log_2 q$. By Inequality (7), we have

$$2 \leq |G_B|n < \frac{18e(q-5)^2(\sqrt{3q - \frac{59}{4}} + \frac{3}{2})\log_2 q}{(q-2)(q-3)(q-4)} \tag{14}$$

Let

$$f(q) = \frac{18(q-5)^2(\sqrt{3q - \frac{59}{4}} + \frac{3}{2})\log_2 q}{(q-2)(q-3)(q-4)} \tag{15}$$

since $f(q)$ is a reduced function when q is greater than 28 (Figures 3 and 4) (see Appendices C and D).

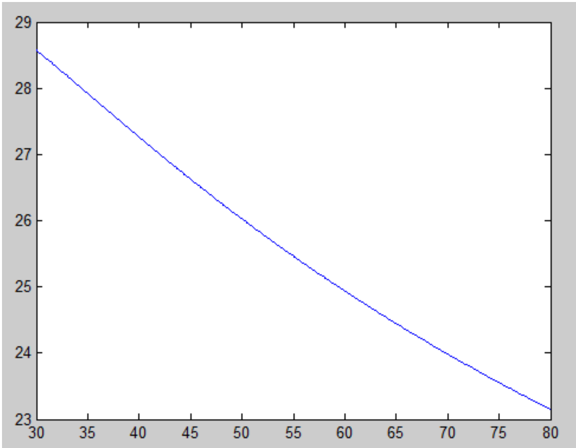

Figure 3. Partial graph of $f(q)$.

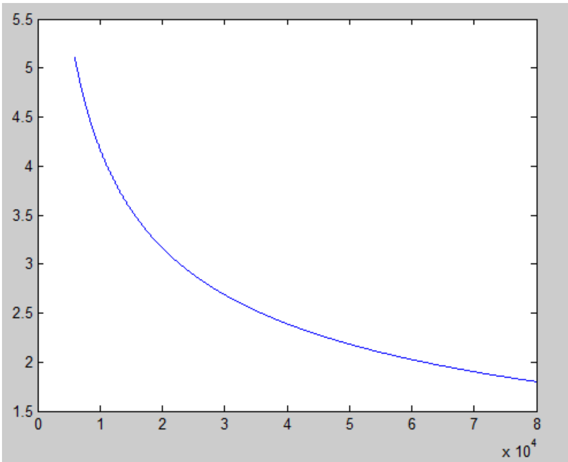

Figure 4. Partial graph of $f(q)$.

For $f(65000) = 1.9617,\ f(60000) = 2.0273$, binding Inequality (11), the range of possible values of q is $28 < q < 60000$. Accordingly, the values of k satisfy the condition $13 < k < 430$. For each pair (q, k), there is no corresponding $7-(q+1,\ k,\ 3)$ design by Lemma 2.

In a word, if $N = PLS(2,\ q),\ v = q+1,\ q = p^e \geq 3$, and $k \geq 26$, G cannot act block-transitively on any $7-(v,\ k,\ 3)$ designs.

Now, we discuss the case of $8 \leq k < 26$. By Equation (3),

$$\frac{3(q-2)(q-3)(q-4)(q-5)}{\log_2 q} \leq k(k-1)(k-2)(k-3)(k-4)(k-5)(k-6)$$

After calculating, the prime number that is suitable for the above inequality does not exist.

In summary, Corollary 2 is not possible.

Corollary 6. *G is not isomorphic to* $N = M_v,\ v = 11, 12, 22, 23, 24$.

By inequality $8 \leq k \leq \left[\sqrt{3v - \frac{71}{4}} + \frac{11}{2}\right]$ and Lemma 2 (4), we know that admissible parameter sets are 7–(11, 8, 3), 7–(12, 8, 3), 7–(22, 8, 3), 7–(23, 8, 3), 7–(24, 8, 3), and 7–(22, 9, 3), 7–(22, 10, 3).

By Lemma 3, if $t = 7,\ \lambda = 3$, we have

$$3\begin{pmatrix} v-s \\ 7-s \end{pmatrix} \equiv 0\left(\bmod\begin{pmatrix} k-s \\ 7-s \end{pmatrix}\right),\ 0 \le s \le 7 \tag{16}$$

- For parameter set 7–(11, 8, 3), Equation (16) does not hold if we take , and 7–(11, 8, 3) design does not exist.
- For parameter sets 7–(12, 8, 3), 7–(22, 8, 3), 7–(22, 9, 3), and 7–(22, 10, 3), Equation (16) does not hold if we take $s = 3$, thus neither of these four designs exist.
- For parameter sets 7–(23, 8, 3), Equation (16) does not hold when $s = 2$, and this design does not exist.

At last, using Lemma 2, we have

$$bk(k-1)(k-2)(k-3)(k-4)(k-5)(k-6) = 3v(v-1)(v-2)(v-3)(v-4)(v-5)(v-6) \tag{17}$$

For parameter sets 7–(24, 8, 3), the right side of Equation (17) can be divisible by 19, so b can be divisible by 19. By Lemma 7, $|G|$ can be divisible by 19, but if $|G| = 244823040a$ (where a = 1 or 2), this is impossible. Therefore, the 7–(24, 8, 3) design does not exist.

Corollary 7. *G is not isomorphic to* $N = M_{11}$, $v = 12$.

By the proof of **Corollary 3,** we know that this case will not occur. In this way, we have completed the proof of the main theorem.

4. Conclusions

With the aid of the MATLAB software, using the classification theorem of 3-homogeneous permutation groups, we have proved that a block-transitive automorphism group of a 7–(v, k, 3) design is neither isomorphic to Affine Type Groups nor Almost Simple Type Groups.

Acknowledgments: The authors thank the referees for their valuable comments and suggestions on this paper. In particular, the first author thanks Weijun Liu and Huiling Li for their guidance and help. This research was supported by Science Research Project of Hunan Provincial Department of Education (Xiang Jiao Tong(2016)No365).

Author Contributions: This article was completed under the joint efforts of all three authors. The main writing work of the article was completed by L.X., C.G. mainly used the MATLAB software to produce the auxiliary graph, and L.S. used the MATLAB software for the larger parameters.

Conflicts of Interest: There is no conflict of interest.

Appendix A

```
e = 3:1/10:30;

y = (18.*e.*(2.^e-5).^2.*((3.*2.^e-59./4).^(1./2) + (3./2)))./((2.^e-2).*(2.^e-3).*(2.^e-4));

plot(e,y);
```

Appendix B

```
e = 5:1/10:30;

y = (18.*e.*(2.^e-5).^2.*((3.*2.^e-59./4).^(1./2) + (3./2)))./((2.^e-2).*(2.^e-3).*(2.^e-4));

plot(e,y);
```

Appendix C

```
q = 30:1/10:80;
```

```
y = (18.* (q-5).^2.*((3.*q-59./4).^(1./2) + (3./2))).*log2(q)./((q2).*(q-3).*(q-4));
plot(q,y);
```

Appendix D

```
q = 6000:1/100:80000;
y = (18.*(q-5).^2.*((3.*q-59./4).^(1./2) + (3./2))).*log2(q)./((q2).*(q-3).*(q-4));
plot(q,y);
```

References

1. Biggs, N.L.; White, A.T. *Permutation Groups and Combinatorial Structures*; Peking University Press: Beijing, China, 1979.
2. Cameron, P.J.; Praeger, C.E. Block-transitive t-designs II. In *Finite Geometry and Combinatorics*; de Clerck, F., Ed.; Cambridge University Press: Cambridge, UK, 1993; pp. 103–119.
3. Huber, M. On the Cameron Praeger conjecture. *J. Comb. Theory Ser. A* **2010**, *117*, 196–203. [CrossRef]
4. Beth, T.; Jungnickel, D.; Lenz, H. *Design Theory*, 2nd ed.; Cambridge University Press: Cambridge, UK, 1999.
5. Kantor, W.M. K-homogeneous groups. *Math. Z.* **1972**, *124*, 261–265. [CrossRef]
6. Xu, X.; Zhang, J. 3-Homogeneous Groups and 5-(v,k,3) Designs. *Math. Theory Appl.* **2012**, *32*, 5–11.