



Editorial

Computational Methods in Interdisciplinary Applications of Nonlinear Dynamics

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Nonlinear dynamics takes its origins from physics and applied mathematics. In the last few decades, this interdisciplinary field of interest for many engineers, physicists, and mathematicians has spawned useful applications in almost all branches of science and technology. Looking for the closest example, well-developed asymptotic methods are among the principal methods of nonlinear analyses. Nevertheless, many theoretical and also real-world physical systems comply with interdisciplinary mathematical and numerical methods that have to be taken into account in the modeling, analysis, identification, and control of nonlinear dynamical systems, representing challenges in mathematical and computational applications.

In this Special Issue, the aim is to offer state-of-the-art current computational methods and their interdisciplinary applications oriented to solving problems in nonlinear dynamics. Potential topics include (but not limited to):

- mathematical modeling of physical systems,
- dedicated numerical methods,
- asymptotic methods in computations,
- interdisciplinary nonlinear nature of engineering problems,
- discontinuity driven nonlinear behavior,
- complex nonlinear dynamics,
- identification of nonlinear systems,
- optimization principles of nonlinear behavior,
- control schemes in nonlinear engineering systems,
- nonlinearity caused engineering problems,
- numerical methods in analysis of periodic and chaotic nonlinear systems.

In this Special Issue the authors of selected works have contributed to the topics listed above, since contents of their works can be synthesized as follows.

The work entitled “Nonlinear Elimination of Drugs in One-Compartment Pharmacokinetic Models: Nonstandard Finite Difference Approach for Various Routes of Administration” [1] by Egbelowo presents a study on nonstandard finite difference (NSFD) schemes, capable of solving one-compartment pharmacokinetic models. These models are represented by both linear and nonlinear ordinary differential equations. “Exact” finite difference schemes, which are a special NSFD, are provided for the linear models while the NSFD rules are applied, based on Mickens’ idea of transferring nonlinear models into discrete schemes. The method used was compared with other established methods to verify its efficiency and accuracy. One-compartment pharmacokinetic models are considered for different routes of administration: I.V. bolus injection, I.V. bolus infusion and extravascular administration.

A systematic spreadsheet method for modeling and optimizing general partial differential algebraic equations has been considered by Ghaddar in the paper entitled “Rapid Modeling and Parameter

Estimation of Partial Differential Algebraic Equations by a Functional Spreadsheet Paradigm” [2]. The method exploits a pure spreadsheet PDAE solver function design that encapsulates the Method of Lines and permits seamless integration with an Excel spreadsheet nonlinear programming solver. Two alternative least-square dynamical minimization schemes are devised and demonstrated on a complex parameterized PDAE system with discontinuous properties and coupled time derivatives. Applying the method involves no more than defining a few formulas that closely parallel the original mathematical equations, without any programming skills. It offers a simpler alternative to more complex environments which require nontrivial programming skill and effort.

The work entitled “Lyapunov Exponents of Early Stage Dynamics of Parametric Mutations of a Rigid Pendulum with Harmonic Excitation” [3] by Śmiechowicz, Loup and Olejnik takes into consideration three dynamic systems composed of a mathematical pendulum suspended on a sliding body subjected to harmonic excitation. A dynamic analysis, having a comparative form of the studied parametric mutations of the rigid pendulum with inertial suspension point and damping was performed. The examined system with parametric mutations is solved numerically, where phase planes and Poincaré maps were used to observe the system response. Lyapunov exponents were computed in two ways to classify the dynamic behavior at relatively early stage of forced responses using two proven methods. The results show that with some parameters three systems exhibit a very similar dynamic behavior, i.e., quasi-periodic and even chaotic motions.

In the contribution entitled “On the Approximation of a Nonlinear Biological Population Model Using Localized Radial Basis Function Method” [4] by Uddin, Ali and Taufiq a localized radial basis function meshless method applied to approximate a nonlinear biological population model is presented. The method approximates the derivatives at every point corresponding to their local support domain. The method is well suited for arbitrary domains. Compared to the finite element and element free Galerkin methods, no integration tool is required. Four examples are demonstrated to check the efficiency and accuracy of the method. The results are compared with an exact solution and other methods available in literature.

Finally, “A Tutorial for the Analysis of the Piecewise-Smooth Dynamics of a Constrained Multibody Model of Vertical Hopping” [5] by Zana, Bodor, Bencsik and Zelei is presented. The authors consider the dynamic modeling and analysis of legged locomotion: on the one hand, the high degrees-of-freedom (DoF) descriptive models are geometrically accurate, but the analysis of self-stability and motion pattern generation is extremely challenging; on the other hand, low DoF models of locomotion are thoroughly analyzed in the literature; however, these models do not describe the geometry accurately. They contribute by narrowing the gap between the two modeling approaches, developing a dynamic analysis methodology for the study of self-stable controlled multibody models of legged locomotion. An efficient way of modeling multibody systems is to use geometric constraints among the rigid bodies. There is shown that the methodology is especially effective when closed kinematic loops are present, such as in the case of walking models, when both legs are in contact with the ground. The mathematical representation of such constrained systems is the differential algebraic equation (DAE). Mathematical analysis methods of piecewise-smooth dynamic systems and their application for constrained multibody models of self-stable locomotion represented by DAE is presented. The numerical approach is demonstrated on a linear model of hopping and compared with analytically obtained reference results.

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It was a great pleasure to cooperate in such conditions. I look forward to collaborating with MCA in the future.

Conflicts of Interest: The author declares no conflict of interest.

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