

## A METHOD FOR SOLVING 0-1 MULTIPLE OBJECTIVE PROGRAMMING PROBLEM, USING DATA ENVELOPMENT ANALYSIS TECHNIQUE

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**Abstract-** In this paper a method for finding efficient solutions of a 0-1 Multiple Objective Linear (Nonlinear) programming Problem using Data Envelopment Analysis (DEA) technique is proposed. In this method, for each feasible solution of 0-1 Multiple Objective Programming (0-1 MOP) problem, a Decision Making Unit (DMU) is introduced. Using the additive model, the relative efficiency of these DMUs is evaluated. Each feasible solution corresponding to an efficient DMU is the efficient solution of 0-1 MOP problem.

**Keywords-** 0-1 Multiple Objective Programming, Data Envelopment Analysis, Efficient Solution

### 1. INTRODUCTION

Data Envelopment Analysis (DEA) is a mathematical programming technique, which is used for the evaluating relative efficiency of Decision Making Units (DMUs) and has been proposed by Charnes et al. [4]. This technique has been extended by Banker et al. (BCC model) [2]. Also the additive model, which is used in this paper, has been proposed by Charnes et al. [5]. There is a close relation between DEA and Multiple Objective Programming (MOP) [7,8]. In this paper, we have used from this relation for solving 0-1 MOP by additive model. The methods by Liu et al. [8], Bitran [3] and Deckro et al. [6] have been proposed for solving 0-1 Multiple Objective Linear Programming Problem (0-1 MOLP). These methods are not able to solve the 0-1 MOP problems with nonlinear structure.

In the next section, MOP is considered. DEA is introduced in section 3. In section 4 a method for finding efficient solutions of 0-1 MOP by using DEA is proposed. Section 5 illustrates the procedure with some numerical examples and in the last section conclusion and some remark are put forward.

### 2. MULTIPLE OBJECTIVE PROGRAMMING

A multiple objective programming problem is defined in the following form:

$$\begin{aligned} & \text{Max} \quad (f_1(W), f_2(W), \dots, f_k(W)) \\ & \text{Min} \quad (g_1(W), g_2(W), \dots, g_l(W)) \\ & \text{s.t.} \quad W \in \Omega \end{aligned} \tag{1}$$

where  $f_1, f_2, \dots, f_k$  and  $g_1, g_2, \dots, g_l$  are objective functions and  $\Omega$  is feasible region. If all objective functions are linear and  $\Omega$  is a convex polyhedral, then the problem (1) is called a multiple objective linear programming problem.

**Definition 2.1.**  $\bar{W} \in \Omega$  is said to be an efficient solution of the problem (1) if and only if there does not exist  $W^o \in \Omega$ , such that

$$(f_1(W^o), \dots, f_k(W^o), -g_1(W^o), \dots, -g_l(W^o)) \geq (f_1(\bar{W}), \dots, f_k(\bar{W}), -g_1(\bar{W}), \dots, -g_l(\bar{W}))$$

and inequality holds strictly for at least one index.

If in the problem (1) all variables are restricted to be zero – one, then the problem (1) is called 0-1 MOP problem and is defined as follows in which  $W = (w_1, w_2, \dots, w_n)$ ,

$$\begin{aligned} \text{Max} \quad & (f_1(W), f_2(W), \dots, f_k(W)) \\ \text{Min} \quad & (g_1(W), g_2(W), \dots, g_l(W)) \\ \text{s.t.} \quad & W \in \Omega \\ & w_j \in \{0, 1\} \quad j = 1, 2, \dots, n. \end{aligned} \quad (2)$$

### 3. DATA ENVELOPMENT ANALYSIS

Consider  $n$  decision making units  $DMU_j$  ( $j = 1, 2, \dots, n$ ), which each DMU consumes a  $m$ -vector input to produce a  $s$ -vector output. Suppose that  $X_j = (x_{1j}, x_{2j}, \dots, x_{mj})$  and  $Y_j = (y_{1j}, y_{2j}, \dots, y_{sj})^T$  are the vectors of inputs and outputs values, respectively for  $DMU_j$ , in which it has been assumed that  $X_j \geq 0$  &  $X_j \neq 0$  and  $Y_j \geq 0$  &  $Y_j \neq 0$ . Consider the set  $S$  and its convex hull as  $S = \{(X_j, Y_j) \mid j = 1, 2, \dots, n\}$

$$C(S) = \left\{ (X, Y) \mid (X, Y) = \sum_{j=1}^n \lambda_j (X_j, Y_j), \sum_{j=1}^n \lambda_j = 1, \lambda_j \geq 0, j = 1, 2, \dots, n \right\}$$

Let  $(X_p, Y_p)$  corresponds to  $DMU_p$ . If a vector  $(X, Y) \in C(S)$  can be found such that  $(-X, Y) \geq (-X_p, Y_p)$  &  $(-X, Y) \neq (-X_p, Y_p)$ ,

then  $DMU_p$  is called inefficient; otherwise, it is called efficient. To evaluate relative efficiency of DMUs, the following model is used which is known as additive model:

$$\begin{aligned} h_p^* = \text{Min} \quad & -\sum_{i=1}^m s_i^- - \sum_{r=1}^s s_r^+ \\ \text{s.t.} \quad & \sum_{j=1}^n \lambda_j y_{rj} - s_r^+ = y_{rp}, \quad r = 1, \dots, s \\ & -\sum_{j=1}^n \lambda_j x_{ij} - s_i^- = -x_{ip}, \quad i = 1, \dots, m \\ & \sum_{j=1}^n \lambda_j = 1 \\ & \lambda_j \geq 0, s_i^- \geq 0, s_r^+ \geq 0, j = 1, \dots, n, i = 1, \dots, m, r = 1, \dots, s. \end{aligned} \quad (3)$$

We know that  $DMU_p$  is efficient in the additive model if and only if  $h_p^* = 0$ .

**LEMMA 3.1.** *The additive model is translation invariant (see [1]).*

Since the additive model is translation invariant, it can be used for evaluating the relative efficiency of DMUs with zero or negative components in the input or output vectors. Note that the additive model can be used for evaluating the relative efficiency of DMUs without input or output which are defined as follows:

The envelopment side of the additive model without input

$$\begin{aligned}
 Q_p^* = \text{Min} \quad & - \sum_{r=1}^s s_r^+ \\
 \text{s.t.} \quad & \sum_{j=1}^n \lambda_j y_{rj} - s_r^+ = y_{rp}, \quad r = 1, \dots, s \\
 & \sum_{j=1}^n \lambda_j = 1 \\
 & \lambda_j \geq 0, s_r^+ \geq 0, \quad j = 1, \dots, n, \quad r = 1, \dots, s.
 \end{aligned} \tag{4}$$

and

The envelopment side of the additive model without output

$$\begin{aligned}
 Q_p^* = \text{Min} \quad & - \sum_{i=1}^m s_i^- \\
 \text{s.t.} \quad & - \sum_{j=1}^n \lambda_j x_{ij} - s_i^- = -x_{ip}, \quad i = 1, \dots, m \\
 & \sum_{j=1}^n \lambda_j = 1 \\
 & \lambda_j \geq 0, s_i^- \geq 0, \quad j = 1, \dots, n, \quad i = 1, \dots, m.
 \end{aligned} \tag{5}$$

**THEOREM 3.1.** *The envelopment side of the additive model without input is feasible and bounded.*

**Proof:** It can be easily verified that  $\lambda = (\lambda_1, \dots, \lambda_p, \dots, \lambda_n) = e_p = (0, 0, \dots, 0, 1, 0, \dots, 0, 0)$  and  $S^+ = (s_1^+, \dots, s_s^+) = (0, 0, \dots, 0)$  is a feasible solution for this model.

To prove that the envelopment side of the additive model without input is bounded, consider its dual, which is as follows:

$$\begin{aligned}
& \text{Max} \quad \sum_{r=1}^s u_r y_{rp} + u_o \\
& \text{s.t.} \quad \sum_{r=1}^s u_r y_{rj} + u_o \leq 0, \quad j = 1, \dots, n \\
& \quad \quad u_r \geq 1, \quad r = 1, \dots, s.
\end{aligned}$$

Since  $(u_1, u_2, \dots, u_s, u_o) = (1, 1, \dots, 1, u_o)$  with  $u_o = \min_{1 \leq j \leq n} \{-\sum_{r=1}^s y_{rj}\}$  is a feasible solution of the above problem, the dual is feasible. Therefore, the envelopment side of the additive model without input is bounded.

**LEMMA 3.2.** *The additive model without input is translation invariant.*

The proof is straightforward.

**LEMMA 3.3.** *In the additive model without input,  $DMU_p$  is efficient if and only if  $Q_p^* = 0$ .*

**Proof :** Let  $Q_p^* = 0$  and by contradiction, suppose that  $DMU_p$  is inefficient. So, there exists a  $\bar{\lambda}$  such that  $\sum_{j=1}^n \bar{\lambda}_j y_{rj} \geq y_{rp}$ ,  $r = 1, \dots, s$  in which at least one inequality holds strictly. That is, there exists an  $l$  which  $\sum_{j=1}^n \bar{\lambda}_j y_{lj} \geq y_{lp}$  and this means that  $s_l^+ > 0$ . So, there exists a feasible solution, say  $(\bar{\lambda}, S^+)$ , such that  $Q_p^* < 0$  and this is a contradiction.

Conversely, suppose that  $DMU_p$  is efficient in the additive model without input. We will prove that  $Q_p^* = 0$ . If  $Q_p^* < 0$ , then there is an  $l$  which  $s_l^+ > 0$ . Hence,  $\sum_{j=1}^n \lambda_j y_{lj} > y_{lp}$  which is a contradiction.

#### 4. SOLVING 0-1 MOP USING DEA TECHNIQUE

Consider the following problem:

$$\begin{aligned}
& \text{Max} \quad (f_1(W), f_2(W), \dots, f_k(W)) \\
& \text{s.t.} \quad g_i(W) \leq b_i, \quad i = 1, 2, \dots, m \\
& \quad \quad w_j \in \{0, 1\}, \quad j = 1, 2, \dots, n
\end{aligned} \tag{6}$$

where  $W = (w_1, w_2, \dots, w_n)$  and functions  $f_1, f_2, \dots, f_s$  and  $g_1, g_2, \dots, g_m$  are not necessarily linear. Let  $\Omega = \{W \mid g_i(W) \leq b_i, w_j \in \{0, 1\}, i = 1, 2, \dots, m, j = 1, 2, \dots, n\}$  which is called the set of the feasible solutions of the problem (6).

Two different cases are considered for the problem (6) in the following forms:

##### 4.1. Case 1: Efficient solution with less sources

In this case, suppose that consuming less sources is important. In other words, it is assumed that in addition to the objective functions are satisfied, less sources are consumed. Hence, instead of problem (6), the following problem is proposed.

$$\begin{aligned}
& \text{Max} \quad (f_1(W), f_2(W), \dots, f_k(W)) \\
& \text{Min} \quad (g_1(W), g_2(W), \dots, g_m(W)) \\
& \text{s.t.} \quad g_i(W) \leq b_i, \quad i = 1, 2, \dots, m \\
& \quad \quad w_j \in \{0, 1\}, \quad j = 1, 2, \dots, n
\end{aligned} \tag{7}$$

where  $f_r(W)$  is the value of  $r^{\text{th}}$  ( $r=1, 2, \dots, s$ ) objective function, and  $g_i(W)$  is the value of consuming  $i^{\text{th}}$  ( $i=1, 2, \dots, m$ ) source in which  $W \in \Omega$ . Corresponding to each feasible solution  $W_d$  of the problem (7), the vectors  $X_d$  and  $Y_d$  are defined as follows:

$$X_d = (x_{1d}, x_{2d}, \dots, x_{md}) \quad \& \quad Y_d = (y_{1d}, y_{2d}, \dots, y_{sd})$$

where,

$$y_{rp} = f_r(W_d), \quad r = 1, 2, \dots, s \tag{8}$$

$$x_{id} = g_i(W_d), \quad i = 1, 2, \dots, m$$

In order to use DEA technique for finding efficient solutions of the problem (7), each vector  $(X_d, Y_d)$  is considered as a DMU where,  $X_d$  and  $Y_d$  are input and output vectors, respectively. To evaluate the relative efficiency of these DMUs, the additive model is used.

**THEOREM 4.2.** *If DMU<sub>d</sub> is efficient in the model (3) then  $W_d$  is efficient solution for problem (7).*

**Proof:** Let DMU<sub>d</sub> be efficient in model (3) and by contradiction, suppose that  $W_d$  is not efficient solution for the problem (7). So, there exists  $W_\beta$  such that:

$$f_r(W_\beta) \geq f_r(W_d), \quad r = 1, 2, \dots, s \tag{9}$$

$$g_i(W_\beta) \leq g_i(W_d), \quad i = 1, 2, \dots, m$$

and strict inequality holds for at least one index, say index  $l$  or  $k$ , i.e:

$$f_l(W_\beta) > f_l(W_d) \quad \text{or} \quad g_k(W_\beta) < g_k(W_d) \tag{10}$$

In either case, if we consider the vector  $\bar{\lambda}$  in the form  $\bar{\lambda} = e_l = (0, 0, \dots, 0, 1, 0, \dots, 0, 0)$  or  $\bar{\lambda} = e_k = (0, 0, \dots, 0, 1, 0, \dots, 0, 0)$ . From (8) and (10), we have  $x_{k\beta} < x_{kd}$  or  $y_{l\beta} > y_{ld}$ . Hence:

$$\sum_{j=1}^n \bar{\lambda}_j y_{lj} > y_{ld} \quad \text{or} \quad \sum_{j=1}^n \bar{\lambda}_j x_{kj} < x_{kd}.$$

Consequently,  $s_l^+ > 0$  or  $s_k^- > 0$ . This means that there exists a feasible solution, say  $(\bar{\lambda}, S^-, S^+)$ , where the optimal value of objective function in model (3) should be negative, and this is a contradiction.

#### 4.2. Case 2: Efficient solution in general case

Recall that  $W$  is an efficient solution of the problem (6) if and only if there does not exist  $W^o$  such that:

$$(f_1(W^o), f_2(W^o), \dots, f_s(W^o)) \geq (f_1(W), f_2(W), \dots, f_s(W))$$

and strict inequality holds at least for one index.

Corresponding to each feasible solution  $W_d$  of the problem (6), the vector  $Y_d$  is defined as  $Y_d = (y_{1d}, y_{2d}, \dots, y_{sd})$

where,

$$y_{rp} = f_r(W_d), \quad r = 1, 2, \dots, s. \quad (11)$$

Each vector  $Y_d$  is considered as a decision making unit ( $DMU_d$ ) without input vector. In this case, we have assumed that the value of consuming sources does not effect in the appointment of the efficient solutions. Therefore, for finding the efficient solutions of the problem (6), the constructed DMUs are evaluated by the additive model without input (model (4)).

**THEOREM 4.3.** *If  $DMU_d$  is efficient in the model (4), then  $W_d$  is an efficient solution of the problem (6).*

**Proof:** Let  $DMU_d$  be efficient in the model (4) and by contradiction, suppose that  $W_d$  is not efficient solution of the problem (6). So, there should be a  $W_\beta$  such that:

$$f_r(W_\beta) \geq f_r(W_d), \quad r = 1, 2, \dots, s$$

and strict inequality holds for at least one index, say index  $l$ , i.e.  $f_l(W_\beta) > f_l(W_d)$ . If we consider the vector  $\bar{\lambda}$  in the form  $\bar{\lambda} = e_l = (0, 0, \dots, 0, 1, 0, \dots, 0, 0)$ , then from (11) and  $f_l(W_\beta) > f_l(W_d)$  we will have  $y_{l\beta} > y_{ld}$ . Hence,  $\sum_{j=1}^n \bar{\lambda}_j y_{lj} > y_{ld}$ . Consequently,  $s_l^{**} > 0$ .

This means that there exists a feasible solution, say  $(\bar{\lambda}, S^+)$ , where the optimal value of the objective function in the model (4) should be negative, and this is a contradiction.

Therefore, by considering the models, the algorithms for case 1 and case 2 can be summarized as follows:

An algorithm for finding efficient solution with less input (case 1)

Step 0: Start,

Step 1: Find all feasible solution of the problem (7),

Step 2: For each feasible solution, construct a DMU according to (8),

Step 3: Evaluate the relative efficiency of the constructed DMUs using the model (3),

Step 4: End.

An algorithm for finding efficient solution (case 2)

Step 0: Start,

Step 1: Find all feasible solution of problem (6),

Step 2: For each feasible solution, construct a DMU according to (11),

Step 3: Evaluate the relative efficiency of the constructed DMUs using the model (4),

Step 4: End.

## 5. NUMERICAL EXAMPLES

Example 1: Consider the following 0-1 MOP problem.

$$\text{Max } 3w_1 + 6w_2^2 + 5w_3^3 - 2w_4^4 + 3w_5^5$$

$$\text{Max } 6w_1 + 7w_2 + 4w_3 + 3w_4 - 8w_5$$

$$\text{Max } 5w_1 - 3w_2 + 8w_3 - 4w_4 + 3w_5$$

$$\text{s.t. } -2w_1 + 3w_2 + 8w_3 - w_4 + 5w_5 \leq 13$$

$$6w_1 + 2w_2 + 4w_3 + 4w_4 - 3w_5 \leq 15$$

$$4w_1 - 2w_2 + 6w_3 - 2w_4 + w_5 \leq 11$$

$$w_1, w_2, w_3, w_4, w_5 \in \{0,1\}.$$

The problem has 5 variables therefore, the number of feasible solutions is equal or less than  $2^5$ . In the Table 1, the column 2 denotes all feasible solutions of the problem. The corresponding outputs and inputs of the feasible solutions have been presented in columns 3 and 4, respectively.

**Table 1:** Feasible solutions, inputs and outputs

No	$W_i$	Output	Input
1	(0,0,0,0,1)	(3,-8,3)	(5,-3,1)
2	(0,0,0,1,0)	(-2,3,-4)	(-1,4,-2)
3	(0,0,1,0,0)	(5,4,8)	(8,4,6)
4	(0,1,0,0,0)	(6,7,-3)	(3,2,-2)
5	(1,0,0,0,0)	(3,6,5)	(-2,6,4)
6	(0,0,0,1,1)	(1,-5,-1)	(4,1,-1)
7	(0,0,1,0,1)	(8,-4,11)	(13,1,7)
8	(0,1,0,0,1)	(9,-1,0)	(8,-1,-1)
9	(1,0,0,0,1)	(6,-2,8)	(3,3,5)
10	(0,0,1,1,0)	(3,7,4)	(7,8,4)
11	(0,1,0,1,0)	(4,10,-7)	(2,6,-4)
12	(1,0,0,1,0)	(1,9,1)	(-3,10,2)
13	(0,1,1,0,0)	(11,11,5)	(11,6,4)
14	(1,0,1,0,0)	(8,10,13)	(6,10,10)
15	(1,1,0,0,0)	(9,13,2)	(1,8,2)
16	(0,0,1,1,1)	(6,-1,7)	(12,5,5)
17	(0,1,0,1,1)	(7,2,-4)	(7,3,-3)
18	(1,0,0,1,1)	(4,1,4)	(2,7,3)
19	(1,0,1,0,1)	(11,2,16)	(11,7,11)
20	(1,1,0,0,1)	(12,5,5)	(6,5,3)
21	(0,1,1,1,0)	(9,14,1)	(10,10,2)

22	(1,0,1,1,0)	(6,13,9)	(5,14,8)
23	(1,1,0,1,0)	(7,16,-2)	(0,12,0)
24	(1,1,1,0,0)	(14,17,10)	(9,12,8)
25	(1,1,0,1,1)	(10,8,1)	(5,9,1)
26	(1,0,1,1,1)	(9,5,12)	(10,11,9)
27	(1,1,1,1,1)	(15,12,9)	(13,13,7)
28	(0,0,0,0,0)	(0,0,0)	(0,0,0)

The results of the evaluation of the constructed DMUs by models (4) and (3) has been presented in the columns 3 and 5 of the Table 2, respectively.

**Table 2:** The obtained results from models of (3) and (4)

No	$W_i$	Opt. Value of (4)	$W_i^*$	Opt. value of (3)	$W_i^*$
1	(0,0,0,0,1)	43	-	0	Efficient
2	(0,0,0,1,0)	44	-	0	Efficient
3	(0,0,1,0,0)	24	-	0	Efficient
4	(0,1,0,0,0)	31	-	0	Efficient
5	(1,0,0,0,0)	27	-	0	Efficient
6	(0,0,0,1,1)	46	-	12.76	-
7	(0,0,1,0,1)	24	-	0	Efficient
8	(0,1,0,0,1)	33	-	0	Efficient
9	(1,0,0,0,1)	29	-	0	Efficient
10	(0,0,1,1,0)	27	-	16.72	-
11	(0,1,0,1,0)	34	-	0	Efficient
12	(1,0,0,1,0)	30	-	0	Efficient
13	(0,1,1,0,0)	14	-	0	Efficient
14	(1,0,1,0,0)	0	Efficient	0	Efficient
15	(1,1,0,0,0)	17	-	0	Efficient
16	(0,0,1,1,1)	29	-	15.86	-
17	(0,1,0,1,1)	36	-	0	Efficient
18	(1,0,0,1,1)	32	-	12.30	-
19	(1,0,1,0,1)	0	Efficient	0	Efficient
20	(1,1,0,0,1)	19	-	0	Efficient
21	(0,1,1,1,0)	17	-	9.31	-
22	(1,0,1,1,0)	13	-	0	Efficient
23	(1,1,0,1,0)	20	-	0	Efficient
24	(1,1,1,0,0)	0	Efficient	0	Efficient
25	(1,1,0,1,1)	22	-	0	Efficient
26	(1,0,1,1,1)	11	-	12	-
27	(1,1,1,1,1)	0	Efficient	0	Efficient
28	(0,0,0,0,0)	41	-	0	Efficient



The last column of table 2 shows that DMU<sub>6</sub>, DMU<sub>10</sub>, DMU<sub>16</sub>, DMU<sub>18</sub>, DMU<sub>21</sub> and DMU<sub>26</sub> are inefficient and other DMUs are efficient. Therefore,  $\Omega' = \Omega - \{W_6, W_{10}, W_{16}, W_{18}, W_{21}, W_{26}\}$  is the set of the efficient solutions of the Example 1 in the case 1. Column 4 of table 2 shows that DMU<sub>14</sub>, DMU<sub>19</sub>, DMU<sub>24</sub> and DMU<sub>27</sub> are efficient and other DMUs are inefficient. Therefore,  $X' = \{W_{14}, W_{19}, W_{24}, W_{27}\}$  is the set of the efficient solutions of the Example 1 in the case 2.

Example 2: Consider the following 0-1 MOP problem.

$$\begin{aligned} \text{Max } & 4w_1 - 3w_2 + 5w_3 \\ \text{Max } & 2w_1 + 7w_2 - w_3 \\ \text{s.t. } & w_1 + 2w_2 + w_3 \leq 7 \\ & 3w_1 + w_2 + 2w_3 \leq 6 \\ & w_1, w_2, w_3 \in \{0,1\}. \end{aligned}$$

The feasible solutions, inputs, outputs and the obtained results from models (3) and (4) are shown in the Tables (3) and (4).

**Table 3:** Feasible solutions, inputs and outputs of Example 2

No	$W_i$	Output	Input
1	(0,0,0)	(0,0)	(0,0)
2	(1,0,0)	(4,2)	(1,3)
3	(0,1,0)	(-3,7)	(2,1)
4	(0,0,1)	(5,-1)	(1,2)
5	(1,1,0)	(1,9)	(3,4)
6	(1,0,1)	(9,1)	(2,5)
7	(0,1,1)	(2,6)	(3,3)
8	(1,1,1)	(6,8)	(4,6)

**Table 4:** The obtained results from models (3) and (4)

No	$W_i$	Opt. value of (4)	$W_i^*$	Opt. value of (3)	$W_i^*$
1	(0,0,0)	14	-	0	Efficient
2	(1,0,0)	8	-	0	Efficient
3	(0,1,0)	10	-	0	Efficient
4	(0,0,1)	10	-	0	Efficient
5	(1,1,0)	0	Efficient	0	Efficient
6	(1,0,1)	0	Efficient	0	Efficient
7	(0,1,1)	6	-	0	Efficient
8	(1,1,1)	0	Efficient	0	Efficient

## 6. CONCLUSION

In general, solving 0-1 MOP in which objective functions and constraints are not linear is a hard task. It seems that the suggested algorithms in this paper are the only ones, which solve the nonlinear problems and find the efficient solutions. If the problem has a rather large size, then to solving the problem needs more computational effort. By considering convexity constraint of the additive model and the additive model without input, some efficient solutions of the problem may be lost.

## 7. REFERENCES

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