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EFFECTS OF PERIODIC MAGNETIC FIELD TO THE DYNAMICS OF VIBRATING BEAM

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Abstract- Dynamics of a magnetoelastic beam in a periodic magnetic field is investigated. For this aim, a new measurement tool for the observation of vibrations of the beam is introduced in place of using common strain-gauge technique. Several analyses including power spectra, maximal Lyapunov exponent, correlation dimension and time series clustering are carried out to determine vibrational aspects of the system. As a result of these analyses, it is found that the system extremely depends on the frequency of external field, even, the nonlinear character of the motion exhibits more complexity for the odd excitation frequencies.

Keywords- magnetoelastic beam, nonlinearity, Lyapunov exponent, dendogram.

1. INTRODUCTION

Deterministic systems with linear and nonlinear behaviours have been investigated frequently in many phenomena, for instance elastic structures, chemical reactions and electrical circuits for their wide usage in different areas [1,2,3]. In this manner, there has been growing interest in investigating complex behaviors of such systems. One of the investigation areas, elastic structure has been studied by many scientists for some decades because of the common - usage in many technical devices, for instance, generators, motors, transformers and fusion reactors [4].

The first comprehensive studies on vibrations of buckled and curved plates were carried out by Cummings and Eisley [5,6]. Tseng and Dugundji studied the nonlinear vibrations of a buckled beam with fixed ends and observed both periodic and nonperiodic motions in their experimental and theoretical works [7,8]. They also explored the snap-through motion for their dynamic system. In addition, Moon and Holmes examined the vibrations of a forced magnetoelastic beam which was buckled by magnetic forces and it was found out that the harmonic excitation of this system exhibited chaotic snap-through behavior [4]. Another study was realized by Saymonds and Yu with an elastic-plastic beam and they gave numerical results for the transient response of the system using nine different finite element codes [9]. Then, their work was extended to a chaotic vibration problem [10].In the present work, the dynamic behavior of the magnetoelastic beam in a periodic magnetic field is investigated. The apparatus which is used to obtain the dynamics of magnetoelastic beam and the whole experimental process are introduced in Section 2. Following section gives brief explanation about methods used to identify the dynamics and both experimental findings and analyses are also evaluated in this section. Finally, Section 4 mentions main conclusions of the present work.

2. EXPERIMENTAL

The block diagram of apparatus used to explore the dynamic behavior of the magnetoelastic beam in a periodic magnetic field is given *Fig. 1.* The magnetoelastic beam is positioned as

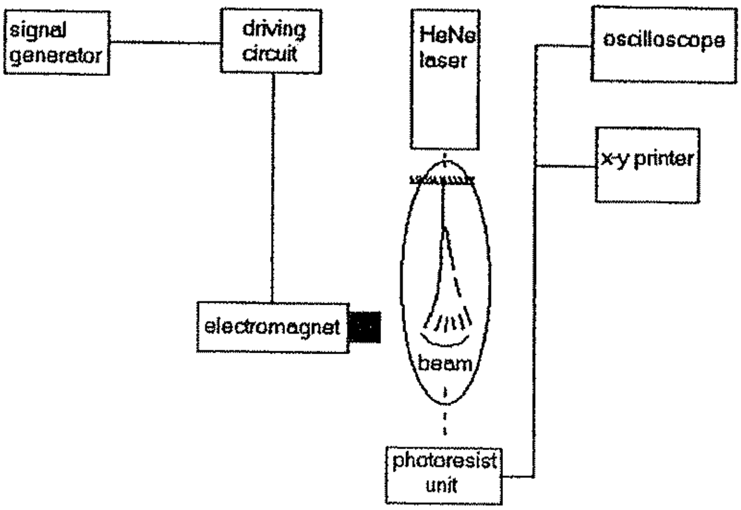

Figure 1. Block diagram of apparatus used to obtain the vibrations of magnetoelastic beam.

perpendicular to experiment table as indicated by eliptical shape in this figure. In the first part of the apparatus, we obtain a driving current to provide a preferable working condition for the electromagnet, for this aim, a *Leader Lag-27* signal generator with square wave is used. As seen in *Fig. 2a*, only positive pulses of square wave are passed through the system with the help of *1N4001* type diode, so the electromagnet is only driven during the half of whole period. In this sense, the electromagnet produces a magnetic field as follows:

$$B = 0 \quad \rightarrow \quad 0 < t \leq \frac{1}{2f}$$

$$B = B_0 \quad \rightarrow \quad \frac{1}{2f} < t \leq \frac{1}{f}$$

Here, B and f represent the magnetic field strength and its frequency, respectively. In the experiment, a coil as electromagnet with *250* loops has been used and the magnetic field, $B_0 = 0.028T$ is measured constantly for every half period. This field magnitude is not changed during the experimental process.

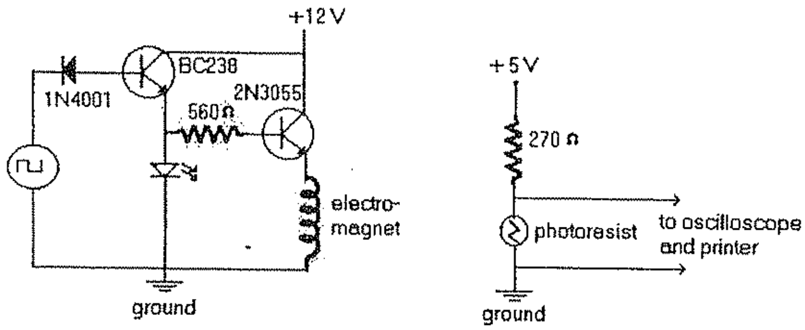

Figure 2. (a) The circuit diagram of driving circuit. (b) The circuit diagram of photoresist.

The second part of the experimental system is constructed for the detection of the vibrations of beam. In this part, an *He-Ne* laser beam is used to convey the vibrations to oscilloscope and printer in place of well-known strain-gauge technique. This kind of recording technique has been known from dripping faucet experiments relevant to recording the time intervals between successive drops [*see in 11*]. However, this technique is applied to the vibrations from an elastic structure for the first time. This kind of measurement idea for vibrating structures can also be thought of an application of Poincare section determination with respect to physical position of the beam, because the vibrations which are at the equilibrium point are only recorded.

The photoresist unit is given in *Fig. 2b*. According to this circuitry, when the laser beam has illuminated the photoresist, the current passing through the resistor creates a potential between the coupling cable. Meanwhile, it can be considered that measuring the motion at the equilibrium point is more useful owing to recording vibrations with small amplitude. One can then arrange the system to take the potential maximum at the equilibrium point of the beam. As a result of this arrangement, the illuminating light decreases the potential difference between the cables and when the laser beam which is illuminating the photoresist to be interrupted by the magnetoelastic beam, then the potential difference on the photoresist increases. This is provided with coupling the photoresist transversely. This potential is conveyed as an electrical pulse to measurement devices.

The magnetoelastic beam is always ensured to start its motion from stable condition and the laser beam is cut by magnetoelastic beam at its equilibrium position during the whole experimental process. The distance to coil from the equilibrium point of beam has been arranged as *5 cm* to enable the observation of large amplitude vibrations and the vibration amplitude is generally measured as nearly *1.2 cm*. We have used a *35.5 cm* length ferromagnet as magnetoelastic beam with *7 mm* width, *0.5 mm* thickness and *10.78 g* weight.

The vibrations have been observed by a *Gould DSO 4068* type oscilloscope to ensure that all motion of system to be recorded by *Pasco CDL 8010* printer, simultaneously. The amplitudes of vibrations are obtained from printer as *mV*, measured voltage on the photoresist unit. These obtained amplitudes are not only the reasonable results of displacement but also the results of beam velocity.

Since beam vibrations in higher magnitudes, for instance, do not interrupt the beam in a long time interval, as a result of this behavior, the decrease in potential becomes smaller. Thus, these amplitudes are acceptable as the function of displacement and velocity values of the beam.

3. ANALYSES OF EXPERIMENTAL DATA

In this section, the time series are initially introduced. During the experimental procedure, the frequencies of the magnetic field are changed from *1 Hz* to *10 Hz* with *1 Hz* frequency interval and the behavior of the beam with respect to these frequencies is observed. In *Fig. 3(a-j)*, the amplitudes with respect to time are seen. The time scale is defined as *0.366 s* for the first seven plots and the scale is keept at *0.330 s* for the rest of plots.

It is clear from the time histories that vibrations exhibit considerably different behaviors with respect to the field frequencies. The field frequencies with odd numbers generally yield to more complex structure i.e. $f = 1Hz$, $f = 3Hz$ and $f = 9Hz$ for the field frequencies give more complexity to magnetoelastic beam. (That will be proven later with the help of correlation dimension and maximal Lyapunov exponent measurements.) For the frequency values of $f = 2Hz$, $f = 4Hz$ and $f = 8Hz$, the system indicates less complex behavior. However, determining these differences may be more clear by utilizing power spectrum analysis. As seen in *Fig. 4(a-j)*, the motion contain different frequencies with their subharmonics.

The frequency values from power spectrum indicate that fundamental frequencies are the same as the field frequencies for only $f = 4Hz$, $f = 5Hz$ and $f = 6Hz$.The behavior of system is autonomous from the excitation frequency for the other field frequencies. The spectra also indicate a complexity except the field frequencies of $f = 2Hz$, $f = 4Hz$, $f = 8Hz$ and $f = 10Hz$ as mentioned above. The complex structures are especially seen for odd frequencies such as, $f = 1Hz$, $f = 3Hz$, $f = 5Hz$, $f = 7Hz$ and $f = 9Hz$. In addition,

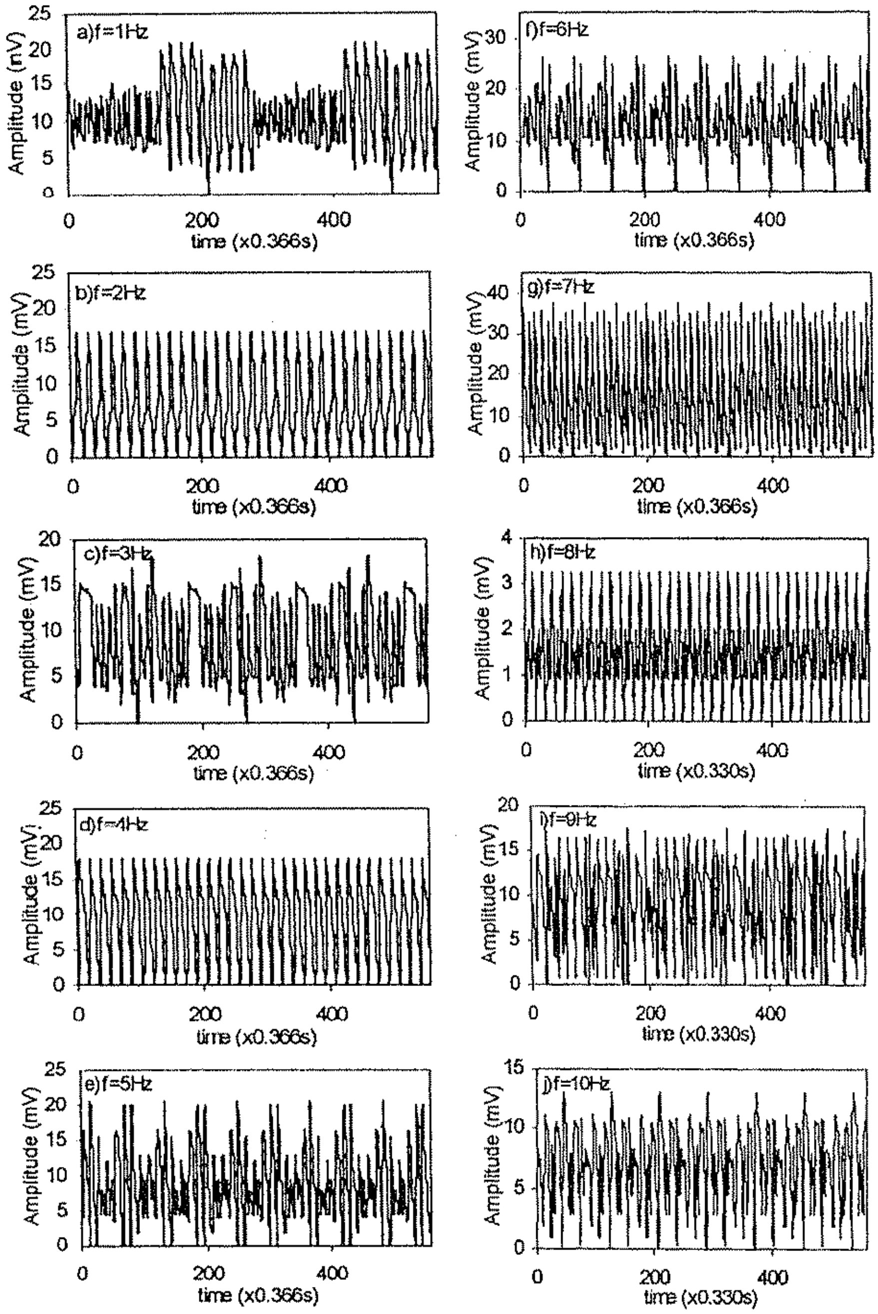

Figure 3. (a-j) Vibrations of the magnetoelastic beam for various magnetic field frequencies.

$f = 6Hz$ causes a nonlinear motion as an even field frequency. Another remarkable thing about spectra is that $f = 6Hz$ and $f = 7Hz$ frequencies for magnetic field indicate a wide range of frequency distribution as seen in *Fig. 4(f, g)*.

Other technique to identify the complexities of the system – correlation dimension gives more quantitative result to identify the phase space. In this manner, correlation dimension requires a quantity named embedding dimension, m. Takens stated that if any phase space has N dimension then, in general, one must reconstruct an embedding value of $2N+1$ dimensions [2]. Using the values $m = 5$ and $N = 2$ for the present case, the aim is to identify the phase space points in a neighborhood of e.

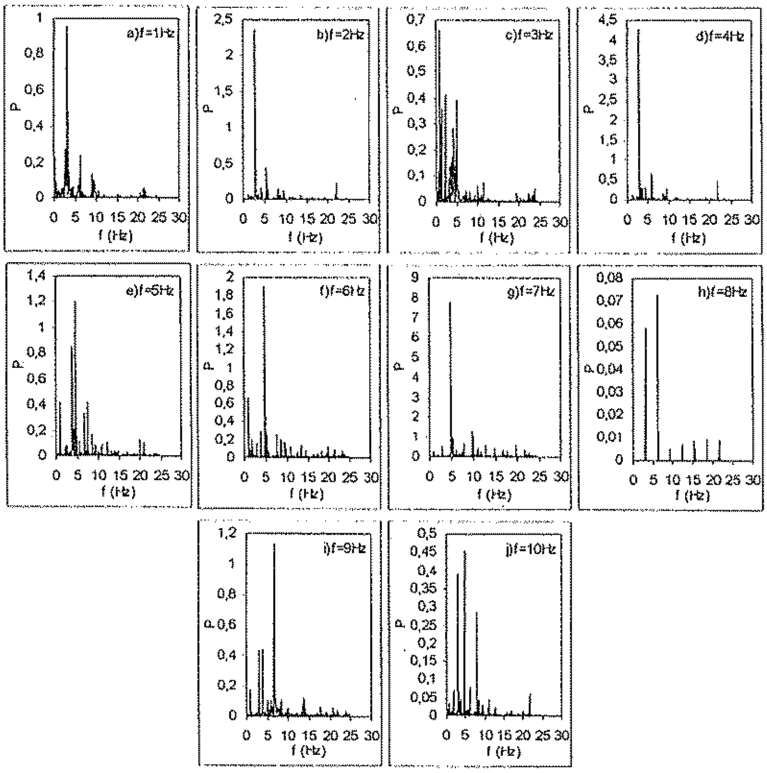

Figure 4. (a-j) Power spectrum analyses of the vibrations.

For this process, the distances between the pairs of points, $s_{ij} = |x_i - x_j| + |y_i - y_j|$ are calculated using Euclidean measure. Then, the correlation function is defined as,

$$C(e) = \lim_{N \to \infty} \frac{1}{N^2} \; [\, number\ of\ pairs\ (i,j)\ with\ distance\ s_{ij} < e \,]$$

In addition to the above relation, the correlation dimension can be determined by finding out the slope of the $\ln C(e)$ versus $\ln e$ curve such as,

$$D_G = \lim_{e \to 0} \frac{\ln C(e)}{\ln e}$$

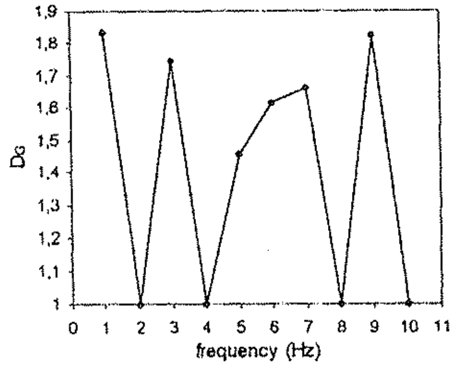

Figure 5. Variation of the correlation dimension values for various field frequencies.

As seen in *Fig. 5*, dimension values are changeable and they support the power spectrum analysis. For any two dimensional system, it is clear for the expected behaviour of the dimension that $D_G = 1$ is provided for ordered regions, while $1<D_G<2$ is provided for nonlinear regions. In this sense, dimension values are found above *1.45* for odd frequencies and below *1.61* for even frequencies. According to dimension measurements, even frequencies except $f = 6Hz$ have linear behavior as in power spectrum analysis.

Positive Lyapunov exponents are indicators of a nonlinear, especially chaotic motion in any system and describe that the system exhibits a sensitive dependence on the initial conditions in the phase space [2]. Meanwhile, maximal Lyapunov exponent characterizes the fastest rate of exponential divergence of two nearby trajectories. This maximal exponent is given by

$$MLE = \frac{1}{(t_N - t_0)} \sum_{k=1}^{N} log_2 \frac{d(t_{k+1})}{d(t_k)}$$

where $d(t_k)$ denotes the distance between two neighboring trajectories. For the computational process, we use a well-known algorithm given by Wolf, *et al.* [12]. As seen in *Fig. 6*, the exponents are positive except $f = 2Hz$, $f = 4Hz$, $f = 8Hz$, $f = 10Hz$ and such a result determines a chaotic structure for other frequencies. This result also proves the complexity in the power spectra relating to the drive

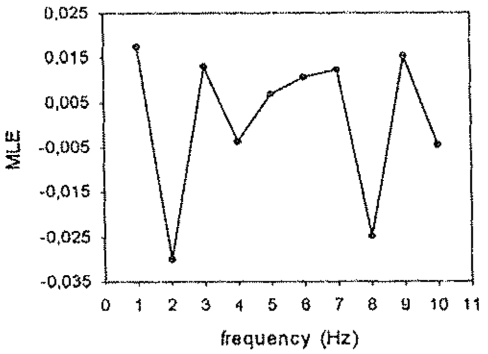

Figure 6. Variation of the maximal Lyapunov exponents for various field frequencies.

parameter. Meanwhile, field frequencies with odd numbers have generally higher exponents than frequency values with even numbers have. In addition, motion at $f = 6Hz$ indicates a nonlinear character.

The last analysis is the hierarchical clustering which defines time series in a successive merging or divisioning manner. There are several ways to identify time series with getting them grouped depending on researches. Initially, it should be stated that we need to realize this part of study because of having many time series data each of which belongs to various frequencies. Structural behaviors of vibrations should not be investigated sufficiently by individual analyses with respect to the frequency because of different response of the magnetoelastic beam. This last comment is also important to reach a generalizated idea on the system for the applied magnetic field. As a hierarchical clustering method, average linkage method is used for grouping. Relating to this method, groups are fused according to the average distance between pairs of members in the respective sets in the average linkage. In this manner, the problem is to search the distance to find the nearest objects, for instance, P and Q. These objects are merged to form the cluster. Distances between P, Q and any other cluster R are determined by

$$d_{(PQ)R} = \sum_{i}^{N}\sum_{j}^{N}\frac{d_{ij}}{N_{(PQ)}N_R}$$

where d_{ij} is the distance between object i in the cluster (PQ) and object j in the cluster R. Meanwhile, $N_{(PQ)}$ and N_R are the number of items in clusters (PQ) and R, respectively [13]. In the light of this information, cluster analysis is given by a two - dimensional diagram commonly known as dendogram.
According to *Fig. 7*, the result of hierarchical cluster analysis is given as three different clusters. It is clear that while the fluctuations relating to $f = 1Hz$, $f = 3Hz$, $f = 5Hz$, $f = 7Hz$ and $f = 8Hz$ are in the first group, the second group includes the vibrations related to $f=2Hz$, $f = 4Hz$ and $f = 9Hz$, respectively. The last group is formed by $f = 6Hz$ and $f = 10Hz$ vibrations. As a result of this clustering, it can be concluded

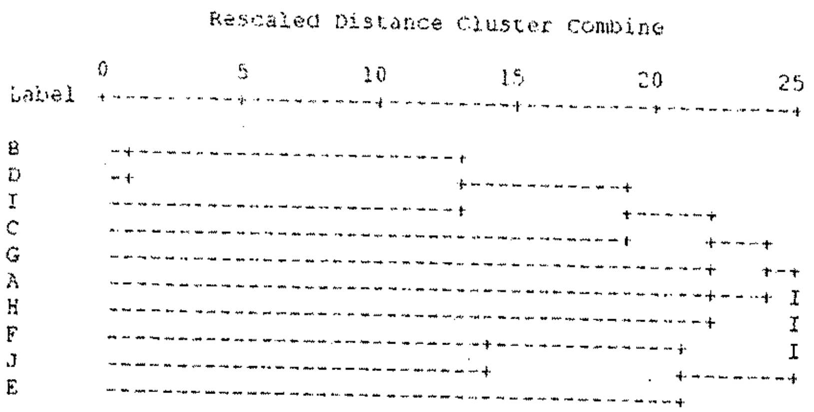

Figure 7. The dendogram graphic of vibrations: External field frequencies are in alphabetical order.

that the fluctuations of frequency values with odd numbers are in the same group except that of $f = 9Hz$. Even values for magnetic field frequencies are in two groups except the frequency value for $f = 8Hz$. This result states that characteristics of these vibrations are similar to other members being in their own group, structurally. Thus, the complex structures stemmed from the field frequency may be determined using dendogram.

4. CONCLUSIONS

The dynamics of magnetoelastic beam being in a periodic magnetic field is studied.For this aim, we have utilized a laser beam which provides to identify the vibrations as a function of displacement and velocity of the beam. As a result of analyses, power spectrum give complex structures for odd excitation frequencies. During the analysis procedure, this comment on odd frequencies has been also proved by other analyses. While the correlation dimension values are changeable with the drive parameter, it is proven that even field frequencies except $f = 6Hz$ have a linear character. Maximal Lyapunov exponents confirm the nonlinearity, especially chaoticity for the vibrations with all odd frequencies and $f = 6Hz$. It has also been clarified that fundamental frequencies of vibrations are different from some excitation frequencies in the investigated frequency range. Related to the last analysis, dendogram indicates that these vibrations may be divided into three groups with respect to the average distance between pairs of fluctuations. This result can be considered as an indication of structural similarity of time series relating to excitation frequencies. Analyses on magnetoelastic beam generally state the same result that odd frequencies have similar structures and degrees of complexity.

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