philosophies

MDPI

Article

Gödel, Turing and the Iconic/Performative Axis

Juliette Cara Kennedy

Department of Mathematics and Statistics, University of Helsinki, 00100 Helsinki, Finland; juliette.kennedy@helsinki.fi

Abstract: 1936 was a watershed year for computability. Debates among Gödel, Church and others over the correct analysis of the intuitive concept "human effectively computable", an analysis at the heart of the Incompleteness Theorems, the Entscheidungsproblem, the question of what a finite computation is, and most urgently—for Gödel—the generality of the Incompleteness Theorems, were definitively set to rest with the appearance, in that year, of the Turing Machine. The question I explore here is, do the mathematical facts exhaust what is to be said about the thinking behind the "confluence of ideas in 1936"? I will argue for a cultural role in Gödel's, and, by extension, the larger logical community's absorption of Turing's 1936 model. As scaffolding I employ a conceptual framework due to the critic Leo Marx of the technological sublime; I also make use of the distinction within the technological sublime due to Caroline Jones, between its iconic and performative modes—a distinction operating within the conceptual art of the 1960s, but serving the history of computability equally well.

Keywords: Gödel; Turing; Greenberg; modernism; technological sublime; technological optimism; computability

check for updates

Citation: Kennedy, J.C. Gödel, Turing and the Iconic/Performative Axis. *Philosophies* **2022**, *7*, 141. https://doi.org/10.3390/philosophies7060141

Academic Editor: Diane Proudfoot

Received: 6 November 2022
Accepted: 25 November 2022
Published: 12 December 2022

1. Introduction

In 1937 Church coined the phrase "Turing Machine"[1] to describe Turing's *mathematical* model, and in so doing he enabled one of the oldest and most powerful metaphors ever to enter the sciences—the *machine metaphor*.

1936 was a watershed year for computability. Debates among Gödel, Church and others over the correct analysis of the intuitive concept "human effectively computable", an analysis at the heart of the Incompleteness Theorems, the *Entscheidungsproblem*, the question of what a finite computation is, and most urgently, for Gödel, the *generality* of the Incompleteness Theorems—which depends in turn on a precise understanding of the notion of "formal system"—were *definitively* set to rest with the appearance, in that year, of the Turing Machine.

Gödel especially was emphatically convinced by the adequacy of Turing's model, where he had been previously unconvinced by the adequacy of all the other models that had been proposed in the period prior to 1936, including his own, the class of Herbrand-Gödel recursive functions; but Church and others had also expressed themselves in similar terms at the time, about their own systems, also in their later writings.[2]

The mathematical facts, which I will describe below, are well known. There is also a small literature in which the principals themselves explain how this turn of events unfolded.[3]

The question I wish to explore in this paper is, do the mathematical facts exhaust what is to be said about the thinking behind the "confluence of ideas in 1936"?[4] Or might there be still more to be said, and from another direction entirely, the direction of cultural influence? The assimilation of Turing's work occurred during a period of high modernism, and within modernism a high, one might say excessive, degree of technological optimism. And indeed, one cannot help noticing parallels between events in the foundations of mathematics of the period and developments central to the modernist culture of the first half of the 20th century—a culture whose icon, if it could be thought of as having one, is surely tied to the *machine*.[5]

I will argue, then, for a cultural role in Gödel's, and, by extension, the larger logical community's absorption of Turing's 1936 model. As scaffolding I will employ a conceptual framework due to the critic Leo Marx from his *The Machine in the Garden: Technology and the Pastoral Ideal in America* [5], of the *technological sublime*; I will also make use of the distinction within the technological sublime that Caroline Jones lays out in her *Machine in the Studio: Constructing the Postwar American Artist* [6], between its iconic and performative modes—a distinction operating within the conceptual art of the 1960s, Jones suggests, but serving the history of computability equally well. As I will argue, the iconic and the performative operate not only within conceptual art; the iconic and the performative are also threaded into *logic*, into the logical mentality, through the Turing machine.

My suggestion raises immediately the problem of influence—the problem of whether it really exists or not, between art and mathematics. Here I am interested in the writings of the modernist critic Clement Greenberg, for whom influence, if not even convergence, was a given:

> ... what their convergence does show, however, is the profound degree to which Modernist art belongs to the same specific cultural tendency as modern science, and this is of the highest significance as a historical fact.[6]

I will return to Greenberg's remark below. It is perhaps worth noting that one often finds *parallelism* asserted in the popular literature, between cubism, for example, and the phenomenon of incompleteness in logic—incompleteness being responsible for inaugurating mathematics' *own* kind of modernism, or so the thought goes, marked by the splitting of (mathematical) truth from (mathematical) proof and by the rise of formalism, nominalism and other anti-realist and relativist ideologies—a modernism that is still with us, arguably, unlike the case of modernism in art, which is thought to have come to an end, perhaps sometime in the early 1960s.

It is difficult to speak of the influence of culture on the sciences. There is no language for this type of ... how might one call it? *criticism*. Mathematics is usually thought of as sealed off from culture; immune to societal influence; answerable to no force of human culture beyond its own internal imperatives. Criticism, in other words, is *unnecessary*.

Or? Might it be the case that mathematics is *porous*? alive and reactive to shifts in the larger culture, if mathematics is not even in a state of symbiosis with culture? This is the thought that mathematics and art can harmonize, at times, being subject to the same forces of human history—or as Greenberg put it, being part of the same specific cultural tendency. And reciprocally: so along with the idea that art and mathematics are subject to the same forces of history, one might also say that they are themselves forces of history: displacing, disrupting, and even constituting history itself, through the constant bringing forth of new *lebenswelte*.

The theory of computability in the 1930s involves several key figures, but I am concerned here with Gödel's part in this history. Gödel was moved by Turing's model; one could almost say that he was *startled* by it—and this startling is what I want to make strange in this essay.[7] For there was nothing inevitable about Gödel's embrace of Turing's model. It was produced, I will argue, within a subjectivity constituted by the modern.

For it is modernism that is at issue here, a modernism indexed to the machine and aiming at *genre purity*; a modernism that transformed both mathematics and art into self-directed, self-critical practices. The aspiration in both cases, was autonomy: for art, autonomy in the Greenbergian sense, taking the form of medium-specificity;[8] and for mathematics philosophical autonomy, taking the form of a (hoped-for) internal consistency proof—itself a form of medium-specificity—and otherwise expressing itself in the belief that in the face of (foundational) crises, mathematics can, and should do for itself. Curtis Franks puts it this way:

> ... Hilbert deliberately intends a deeper foundational investigation than those of his contemporaries, and his chief aim ... is to establish a mathematical autonomy according to which the reliability and correctness of ordinary mathematical

> methods does not rest on *any* epistemological background—neither the failed conceptual framework of nineteenth-century set theory, nor any new philosophically informed framework—since these can only ever provide "ambiguous" foundations—foundations dependent in their conclusiveness on their underlying philosophical principles. Since philosophical principles are, according to Hilbert, eternally contentious, such a defense of mathematics would only be a "half-truth": a truth only in so far as one is willing to subscribe to the relevant philosophical principles.[9]

The outline of this paper is as follows: After some remarks on metaphor, I will sketch the general history of computability in the 1930s, confining myself to a single episode within it, namely Gödel's confrontation with Turing's model by way of the machine metaphor. As an aside—and it seems odd to have to point this out—the Turing Machine is not a machine in the strict sense of being a physical device, it is a mathematical construction, an abstract object. To employ the phrase "Turing Machine" for Turing's model, then, is to employ a metaphor—one that drew its power, I will suggest, from the technological optimism of the time, if the language of optimism does not fall too far short of the sweeping embrace of the machine rampant in all quarters of that period.[10]

Simply put, this is the story of how the vocabulary of the machine entered Gödel's logical vocabulary, and consequently, the vocabulary of logic; the story of how that vocabulary swept away the many obscurities that had clouded the discussion around computability prior to 1936; the story of how that vocabulary resolved, one by one, the problems of: what is a finite computation? how general are the incompleteness theorems? how to define the concept of "formal system"?—that is, the problem of delimiting and precisifying those concepts.

I will then turn to Greenberg's conception of modernism, his account of how the goals of autonomy and genre purity were attained as inevitable developments within late modernist art, contrasting these two concepts with autonomy and genre purity in mathematics. I will end with a few words on pseudomorphism, not in the realm of art but in the realm of ideas.

2. Metaphor

There is much to say about the connection between metaphor and knowledge, about the power a metaphor has to penetrate a science like mathematics and to act forcefully on it. From Richard Rorty:

> Aristotle's metaphorical use of *ousia*, Saint Paul's metaphorical use of *agape*, and Newtons metaphorical use of *gravitas*, were the results of cosmic rays scrambling the fine structure of some crucial neurons in their respective brains. Or, more plausibly, they were the result of some odd episodes in infancy ... It hardly matters how the trick was done. The results were marvelous. There had never been such things before.[11]

Adding some mathematical metaphors to the list: "collapsing cardinals", "continuous function", "interior/exterior" ... every mathematician knows that mathematics is *metaphor-saturated*, and indeed one can hardly imagine mathematics without metaphor.[12]

Rorty's elaborate account of metaphor, of the way metaphor operates in language, is useful here. Metaphors, for Rorty, are "private acts of redescription" originating "outside" of language—"outside", metaphorically, in the sense of unintelligibility; and his account turns on the idea of the *literalized metaphor*, literalization being what happens when a metaphor breaks into sensibility; when a phrase like, for example, "point of view" comes to mean something like an attitude toward something—becomes, in other words, literalized:

> Between ... [between living and dead metaphor] we cross the fuzzy and fluctuating line between natural and non-natural meaning, between stimulus and cognition, between a noise having a place in a pattern of justification of belief. Or, more precisely, we begin to cross this line if and when these unfamiliar noises

> acquire familiarity and lose vitality through being not just mentioned ... but used: used in arguments, cited to justify beliefs, treated as counters within a social practice, employed correctly or incorrectly.[13]

Rorty sees the creation and literalization of metaphors as the "fuel of liberalism", and "a call to change one's language and one's life".[14] As such, metaphors are a sign of the viability of a shared social practice; evidence of the ability of that practice to continually transform itself, to produce new meaning, through the creation of metaphors.[15]

My concern here is with the machine metaphor, which is very old, going back to the classical period, and returning in renewed forms ever since. In the 17th century Descartes famously assimilated the body to a machine in the *Discourse on Method*; in the 20th century the Italian futurists' deplorable machine rhetoric—war to be enjoyed as a grand theater of the machine, etc.—gave rise to a new artistic genre. And reciprocally: so to speak of the machine as a body, or as possessing a body, or as having bodily agency, this is now more than a way of speaking, the metaphor has colonized vast areas of public discourse, not to mention the philosophy of mind, with the human mind being likened, say, to software. There would be much to say about all this, however it is Gödel's use of the machine metaphor, the way he applied the metaphor in logic, that is my interest in this essay.

3. Gödel's Machine Vocabulary

To see how the language of the machine entered Gödel's logical vocabulary, we must revisit his role in the history of computability. Gödel gave a precise definition of the recursive (or primitive recursive) functions in his 1931 paper [16] presenting the Incompleteness Theorems, though the recursive functions were known before. In fact, Gödel was among the first to suggest the problem of isolating the concept, that is, beginning with the pre-theoretic, intuitive concept of computability and ending with a formal mathematical modelling thereof.

As Gödel would realise almost immediately upon proving the Incompleteness Theorems, the question of their generality, that is the question to which formal systems those theorems apply, was left unresolved in the 1931 paper. This is because the generality issue is tied to the availability of a precise and adequate notion of effective computability, and/or, if you like, finite procedure. This is because the formal systems at issue in the Incompleteness Theorems, are to be given effectively.[16]

Gödel stopped short of claiming generality (except in a limited sense) for the Incompleteness Theorems in the paper, writing that it was conceivable that there are finitary systems which are not covered by those theorems. In correspondence with Herbrand later, Gödel would even go so far as to say that the concept of finite computation was "undefinable", a view that he held through 1934 (and beyond), when he wrote that "the notion of finite computation is not defined, but serves as a heuristic principle".[17]

The period saw, of course, other developments: Church developed the λ-calculus together with Kleene, a type-free and, in R. Gandy's words, logic free model of effective computability,[18] based on the primitives "function" and "iteration". In fact, the bifurcation in in the mid-1930s between the logical as opposed to the algorithmic conception of intuitive computability was an important development and is dwelled on at length in [20]. Put briefly, the line drawn was between viewing computation as a form of deduction, vs the algorithmic conception, that is, viewing computation as in some sense logic free—a condition implied by the algorithmic conception, at least in Gandy's view.

In spite of the emergence of these systems, or perhaps one might say because of them, the adequacy question, that is, the question of finding, not just any modeling, but a faithful modeling of the concepts of "humanly computable", and "finite procedure", remained open. The suggestion in early 1934, for example, to equate human computability with λ-definability (Church's system), Gödel found "thoroughly unsatisfactory".[19]

The language of the machine appears in Gödel's writings already in the first sentence of his 1931 paper with the phrase "mechanical rules".[20] The phrase also occurs in the notes of his 1934 Princeton Lectures [22], where again the word "mechanical" modifies "rules".

This falls well short of his view of the *whole formal system* as a machine outright in 1936—so not just that the rules are somehow to be thought of as machine-like, Gödel likens the entire formalism, after Turing, that is, *the entire deductive apparatus*, to a machine.

It is worth mentioning that a machine or protocol-like concept of "formal system" would have been known at the time. As Hodges writes in his [23], on the evidence of terminology, the Polish school seemed to have conceived of "deductive theory" for example, in a protocol-like manner, i.e., as "something to be performed". As Hodges put it, the view was "that a deductive theory is a kind of activity".

By 1935, Gödel began to believe in the possibility of arriving at a definitive notion of formal system. As Gödel wrote to Kreisel in 1965:

> That my [incompleteness] results were valid for all possible formal systems began to be plausible for me (that is since 1935) only because of the Remark printed on p. 83 of 'The Undecidable' ... But I was completely convinced only by Turing's paper.[21]

In his 1936 [25] Turing gave a self-standing analysis of informal, human, effective computability and used it to solve the *Entscheidungsproblem*, as Church in his [26] had solved it just prior, though with a conceptually different proof.

Turing's analysis was exact but informal, defining the concept of "human effectively computable" by means of an "apparatus" consisting of a tape scanned by a reader, with a set of simple instructions adjoined. Precisely, the analysis consisted of, first: a conceptual analysis of human effective computation, and second: a mathematical precisification of the concept "human effectively computable" consisting of rules given by a set of quintuples: "erase", "print a 1", "move left", and "move right".

Turing's construction was homemade, homespun and eminently workable, and the reaction to it among the Princeton logicians was explosively positive. As Kleene would write in 1981,

> Turing's computability is intrinsically persuasive but λ-definability is not intrinsically persuasive and general recursiveness scarcely so (its author Gödel being at the time not at all persuaded).[22]

I quoted Gödel in his 1934 lectures to the effect that "the notion of finite computation is not defined ... " In the published version of these notes in Gödel appended the following postscriptum at this point:

> In consequence of later advances, in particular of the fact that, due to A. M. Turing's work, a precise and unquestionably adequate definition of the general concept of formal system can now be given, the existence of undecidable arithmetical propositions and the non-demonstrability of the consistency of a system in the same system can now be proved rigorously for *every* consistent formal system containing a certain amount of finitary number theory.
>
> Turing's work gives an analysis of the concept of "mechanical procedure" (alias "algorithm" or "computation procedure" or "finite combinatorial procedure"). This concept is shown to be equivalent with that of a "Turing machine". A formal system can simply be deemed to be any mechanical procedure for producing formulas, called provable formulas. For any formal system in this sense there exists one in the sense of page 346 above that has the same provable formulas (and likewise vice versa), provided the term "finite procedure" occurring on page 346 is understood to mean "mechanical procedure". This meaning, however, is required by the concept of formal system, whose essence it is that reasoning is completely replaced by mechanical operations on formulas.[23]

A similar postscript appears as note added to the republication in [18] of his 1931 paper:

> *Note added 28 August 1963.* In consequence of later advances, in particular of the fact that due to A. M. Turing's work a precise and unquestionably adequate definition of the general notion of formal system70 can now be given, a completely

> general version of Theorems VI and XI is now possible. That is, it can be proved rigorously that in every consistent formal system that contains a certain amount of finitary number theory there exist undecidable arithmetic propositions and that, moreover, the consistency of any such system cannot be proved in the system.

The above footnote 70 reads: "In my opinion the term "formal system" or "formalism" should never be used for anything but this notion".

As Gödel would later explain to Hao Wang, Turing's model of human effective calculability is, in some sense, perfect:

> The resulting definition of the concept of mechanical by the sharp concept of "performable by a Turing machine" is both correct and unique ... Moreover it is absolutely impossible that anybody who understands the question and knows Turing's definition should decide for a different concept".[24]

For Gödel, then, the Turing Machine was not just another in the list of acceptable notions of computability—it was the grounding of all of them.

It is at this point that histories of the period can sometimes conclude, taking Gödel's move as completely natural—as just what one would expect. Perhaps though, there is more to be said. After all, considering the arc of Gödel's thought as a whole, his abrupt change of mind on so many issues fundamental to logic, not to mention to his own work in logic, was drastically out of character. So what happened?

One cannot help but notice the frequent and, on reflection, somewhat strange use of the words "machine" or "mechanical" in the texts. Strange, because after all, *there are no machines in the picture*. The view taken here is that Gödel's conversion to Turing's model was enabled by a metaphor, the machine metaphor; one embedded in the cultural context of high modernism and high technological optimism; a metaphor, if taken in the sense of *Turing Machine*, tied to step-by-stepness and homespun protocols; to images of exercise books and children at their desks doing their sums—to a kind of pastorality, of the mathematical kind.

4. Autonomy, Self-Critique and the Technological Sublime

Modernism is a multidimensional *Weltanschauung* sweeping across the creative domains of the late 19th and 20th centuries—a Weltanschauung indexed, as we have said, to the *machine*. As a general phenomenon, modernism can be thought of as tied, briefly put, to industrialization. In art, modernism is often identified with the collapse of realism and the rise of abstraction in the early part of the 20th century; in modernist literature entrenched forms of expression were also rejected, for different reasons in different quarters. There was *experiential veridicality*, as Gertrude Stein put what she, James Joyce and others were thought to be after.[25] So not: "my love, as sweet as a rose" but rather: "A rose is a rose is a rose".

The canon of modernist critique is immense. Here I am interested in the trajectory within modernism involving the technological sublime, the impact of this specific cultural tendency on the theory of computability in the 1930s. Keeping Jones' warning in mind, that "modernism does not exist outside its articulation in culture, and in individual thinking minds. Not a "zetgeist" that suffuses us like a fog, not automatic, not without *work* ..."[26] I want to operate on as *granular* a level as possible.

Before turning to the technological sublime, if we think merely about modernism writ large, Greenberg writes that:

> The essence of Modernism lies, as I see it, in the use of characteristic methods of a discipline to criticize the discipline itself, not in order to subvert it but in order to entrench it more firmly in its area of competence. Kant used logic to establish the limits of logic, and while he withdrew much from its old jurisdiction, logic was left all the more secure in what there remained to it.[27]

Greenberg's is as precise a description as one can find, of the self-critical methodological strategies of the foundational programs of the early part of the 20th century in Vienna, Göttingen, Cambridge and elsewhere. The Hilbert Program, for example, developed exact methods whereby the notion of proof itself became the object of mathematical study. What Hilbert did was no less than to turn the entire machinery of mathematics onto *itself*, in order to set mathematics more firmly on its foundations—or as Greenberg put it, to entrench it more firmly in its area of competence.

Autonomy, an essential achievement of late modernist painting, comes, for Greenberg, from the sciences:

> ... Scientific method alone asks, or might ask, that a situation be resolved in exactly the same terms as that in which it is presented. But this kind of consistency promises nothing in the way of aesthetic quality ...

Autonomy in painting, according to Greenberg, emerges in the late modernist era, and aims to reach the viewer through "eyesight alone"; to transform the viewer into a purely *optical* subject. Flatness and frontality are key words here; and where Mondrian is criticized for creating "form scenarios", or mere depictions of forms, Pollock, later on, would "[achieve] a dissolution of the picture into into sheer texture, sheer sensation",[28] entering the body through eyesight alone.

What does autonomy in mathematics look like? This emerges as a foundational principle in a fairly completed form in Hilbert's work of the 1920s, going by the name "finitary standpoint". As the Hilbert scholar Richard Zach describes it:

> This methodological standpoint consists in a restriction of mathematical thought to those objects which are "intuitively present as immediate experience prior to all thought", and to those operations on and methods of reasoning about such objects which do not require the introduction of abstract concepts, in particular, without appeal to completed infinite totalities.[29]

And as Hilbert himself puts it:

> If logical inference is to be reliable, it must be possible to survey these objects completely in all their parts, and the fact that they occur, that they differ from one another, and that they follow each other, or are concatenated, is immediately given intuitively, together with the objects, as something that can neither be reduced to anything else nor requires reduction. This is the basic philosophical position that I consider requisite for mathematics and, in general, for all scientific thinking, understanding, and communication.[30]

As Zach remarks, the objects in question are concrete spacio-temporal tokens, namely the *signs*. The point of view here is that a reduction of the entire deductive corpus of mathematics to an entirely surveyable, finitary system, in which deduction takes the form of combinatorial manipulation of signs, will set mathematics on firm ground—assuaging worries about consistency; delivering syntactic completeness.

As an aside, in both the case of late modernist art and in the case of foundational formalism we have a trajectory leading toward collapse, or so it was argued in [32]. As is known, the Incompleteness Theorems, the infinities of set theory, auto-referentiality and so forth, were in the air for conceptual artists of the 1960s, albeit lightly. The suggestion in that paper was that just as in the case of the foundations of mathematics of the 1920s and 30s, also in late Modernist art self-critique imposed the aspiration to produce a grounded, necessary, and finite set of laws aiming towards a completion of the subject. The aspiration was ultimately self-undermining, or so we argued, for modernism;[31] in mathematics this self-undermining is even provable, if one wants to read Gödel's Incompleteness Thoerems that way.

We suggested in [32] that this happened on the heels of (what one might think of as) art's own linguistic turn, namely the "eruption of language" into the aesthetic field, as Craig Owens [33] described it (in the form of the art of Bochner, for example).

As it turns out, the eruption of language into the foundations of mathematics in the period in question, namely the emergence of an exact syntax in the context of foundational formalism, brought modernism *into* mathematics. This set the stage for the Incompleteness Theorems; and whether the importation of syntax sparked a collapse in the form of those Theorems, Greenberg's assimilation of art and science under the same cultural tendency—the search for autonomy—clearly reaches beyond the domains of application he envisaged in "Modernism in Painting".

With that Greenbergian convergence in mind, let us finally turn to the technological sublime, a term coined by the mid-century critic Leo Marx in his book *The Machine in the Garden* [5]. Put briefly, the technological sublime holds the romantic (sublime) conception of the American landscape of the late 19th century, seeing that terrain as a kind of virginal paradise, up against the language of industrial progress. *The machine in the garden*, then, is meant to mark the conflict between technology and pastorialism, the conflict being, that

> … this ideal [of the pastoral idyll] has been used?in the service of a reactionary or false ideology, thereby helping to mask the real problems of industrial civilization.[32]

Marx terms this "complex hybrid of technological progressivism and the pastoral ideal"[33] as "the rhetoric of the "technological sublime". And while for Marx the technological sublime is a mid-nineteenth century American phenomenon, Jones suggests that the technological sublime reemerges in the 1960s, when the "appearance of moon shots, superhighways, and the penetration of the technological into human nature" existed side by side with a growing ecology movement.[34]

Jones sees this later incarnation of the technological sublime operating along two axes, the *iconic* and the *performative*; and the merging of the two in the mind of the artist leads to a kind of an endgame built into the logic of late modernist art. The machine in the studio, then, marks the demise of the heroic conception of the (usually male) artist and the outsourcing of artistic production—and here the word "production" is very revealing.

Returning to the case of computability, here production is also offloaded, from the "computor", which was Turing's (and other's) way of referring to people who calculate, to his "machine". The payoff is substantial: computation is now error-free, being carried out by a machine; reliability is now total.

One may ask, reliable in what way? Wittgenstein's qualms around rule-following will come to mind here.[35] As it turns out though, such qualms did not impinge, at least not at the time. That these "machines for proving theorems", in Gödel's terms, were seen as adequately or faithfully modeling what the human "computor" does; that the activity of the machine had something, had everything! to do with *us*, with what we do—this was Gödel's realization. For, thinking now of finite procedures in terms of Turing Machines, Gödel's idea here is that mathematical production itself, *when it is finitary in a precise sense*, can be off-loaded altogether onto a "machine for proving theorems", i.e the formalism. The machine, in other words, *is* us—in our finite deductive mode.

The machine metaphor is drawn on here, obviously; and its iconic and performative modes are right in front of us: the Turing Machine is on the one hand bathed in the iconography of the *mathematical-pastoral*—in a word, all the "exercise book", and "pencil and paper" of it; while on the other hand there is the Machine's performative character, encoding perfectly the protocol-like conception of deduction.

It is a perfect merging of the *raw* and the *cooked*; and rather than marking a conflict, a conflict indicated by the phrase "the machine on the garden", the Turing Machine, in entering the *mathematical garden*, lays conflicts to rest: the model resolved, for Gödel, the adequacy question; and it settled, at least for a time, the philosophical difficulties surrounding the notion "finite procedure".

5. Pseudomorphism

The main claim here is that Gödel's uncharacteristic turn of mind on issues fundamental to logic and to his own work in logic was enabled by the technological optimism of the time—not a general optimism but one brought to earth by a specific view of technology. I used the twofold rhetoric of the technological sublime as conceptual scaffolding, in its iconic/performative modes. Here the *mathematical pastoral* stood in for the virginal landscape; and the protocol-like conception of deduction stood in for industrial progressivism. My point is that Gödel's whole board acceptance of Turing's model was conditioned by this merging of *pastoral* and *protocol*.

I adapted Marx's discourse of the *machine in the garden* to my own purpose, along with Jones's conception of the *machine in the studio*. The broad point I wished to make is that while the machine in the studio may mark the demise of the heroic conception (of the artist) and the outsourcing of artistic production, here the machine in the *mathematical* garden was the spark that inaugurated the modern era in metamathematics.

The parallels here are striking—but are they confirmatory? Greenberg's view of modernism—the ambient manifold within which all of this takes place—is that it is essentially oriented toward autonomy. It is very natural to read an orientation toward autonomy into the Hilbert program, which was a formative episode within mathematics' *own* modernism. But are these autonomies the same? For that matter, are these two modernisms the same?

For the worker in foundations of mathematics, Greenberg's remark that "the essence of Modernism lies ... in the use of characteristic methods of a discipline to criticize the discipline itself, not in order to subvert it but in order to entrench it more firmly in its area of competence" will induce a shock of recognition. Or is the strong sense of familiarity the historian of mathematics experiences when reading this passage merely, to borrow a phrase from the art historian Yves-Alain Bois, a *pseudomorphic frisson*?[36] Erwin Panofsky defines pseudomorphism as:

> The emergence of a form A, morphologically analogous to, or even identical with, a form B, yet entirely unrelated to it from a genetic point of view.[37]

In his [35] Bois cites examples of artworks that are pseudomorphically related, but from a genetic point of view completely unrelated—where a genetic similarity relation implies causality, presumably; something like transmission at the point of origin.

In the area of ideas, pseudomorphism takes the form of the (mis)reading of shared tendencies, resemblances of ideas, similar moves within a framework and the like, as indicative of an underlying connection, perhaps causal in nature; however the resemblance is only a superficial one. Causal links are tenuous, at best; the apparent shared tendencies could be, in fact, antithetical to eachother.

Bois illustrates the distinction between genetic and pseudomorphic relatedness with a comparison of two works by the artists Sol LeWitt and François Morellet, works that are visually (morphically) similar, but which convey, nevertheless, "entirely opposite messages".[38] The key point here is that for the works to have been related genetically, LeWitt, in this case, would have had to recognize the Morellet work as produced within a certain systematic framework—to *see* it as such; to take it fully in.

In the context of this paper, to take seriously Greenberg's phrase "belonging to the same specific cultural tendency" is to acknowledge the existence of a diffusionist mechanism. The proposal is dubious. In the case here the artists of the period could have been aware of these specific events in the foundations of mathematics; but they would have also needed to see those programs as oriented toward autonomy, grasp their meaning as such. As Bois writes, "in order for a form to be received in a context different from that of its origin, it must first of all be receivable".[39]

It is not enough, in other words, that two fields engage in self-examination, what matters is the specific nature of that reflection. In computability in the period in question, i.e before 1936, one sees the development of the field away from the logical conception of computability, as was noted. One way of describing what Turing achieved was to bring this development to a completion by smuggling bodily experience into the picture—*experiential viridicality*, if you like.

As for the conceptual art of the 1960s, however one accounts for the outsourcing of production and the demise of the heroic conception of the artist, it strikes this nonspecialist as involving more of a repudiation of the body than an embrace of it—if the body figures into the story at all.

6. Conclusions

The job of the critic, Greenberg writes, is to "ascertain the master-current",[40] to bring it out. The problem of cultural influence is very difficult. Pseudomorphosis is thick on the ground; and the drawing of facile connections is all too common in the literature. But we must try to see the outlines of cultural influence where we can, using what is in front of us. As Bois reminds us, pseudomorphism is not always pseudo;[41] and even Gödel engaged in criticism, preoccupied as he was with the left- and rightward motions of the *zeitgeist*.[42]

There is of course the problem of technological optimism, which has in one sense fallen away, and in another sense become entrenched now to a degree well beyond anything imagined during the industrial revolution. The contours of the master-current operating in that domain are barely visible, if they are not out of view altogether—but everything depends on our being able to ascertain them.

Funding: Research partially supported by grant 40734 of the Academy of Finland. This paper was finished during my visit to the Institute for Advanced Study in fall of 2022, for whose support I am also grateful.

Acknowledgments: For helpful discussions and correspondence during the writing of this paper I am very grateful to Yve-Alain Bois, David Gray, Caroline Jones and Juliet Mitchell.

Conflicts of Interest: The author declares no conflict of interest.

Notes

1 [1].

2 See [2].

3 See e.g., Kleene [2,3].

4 This is the title of Robin's Gandy's landmark 1988 paper [4], "The Confluence of ideas in 1936".

5 As David Gray put it in a personal communication: "The machine, mechanical or electronic, is surely the icon of modernism. It was a moment of mad optimism in Dessau ... "

6 "Modernist Painting", in [7], p. 85.

7 I adapt here Jones's use of the phrase "to make strange" on p. 7 of her [6], which she adapts in turn from Shklovsky [8].

8 Greenberg explains medium-specificity in above-cited essay "Modernist Painting":

> It quickly emerged that the unique and proper area of competence of each art coincided with all that was unique in the nature of its medium. The task of self-criticism became to eliminate from the specific effects of each art any and every effect that might conceivably be borrowed from or by the medium of any other art. Thus would each art be rendered "pure", and in its "purity" find the guarantee of its standards of quality as well as of its independence. "Purity" meant self-definition, and the enterprise of self-criticism in the arts became one of self-definition with a vengeance.

9 [9], p. 35.

10 The embrace of technology is of course indicated already decades before. See, for example, the line "Singing the strong light works of engineers", from Walt Whitman's poem of 1869 written after the opening of the Suez Canal.

11 [10], p. 17.

12 It was argued in [11] that Wittgenstein's critique of extensionalism in mathematics turns in part on the fact that it involves an unintelligible use of metaphor. See also [12].

13 [13], p. 171.

14 [14], pp. 12–13.

15 The poet Robert Frost, for whom metaphor was "all of thinking", is an interesting fellow-traveler. For Frost what matters is the fragility of human society sans metaphor:

> What I am pointing out is that unless you are at home in the metaphor, unless you have had your proper poetical education in the metaphor, you are not safe anywhere. Because you are not at ease with figurative values: you don't know the metaphor in its strength and its weakness. You are not safe with science; you are not safe in history. ("Education by Poetry", Amherst College address, *Amherst Graduates' Quarterly*, February 1931).

Another interesting fellow traveler is Quine, who recognized the importance of metaphor for science and philosophy:

> Pleasure precedes business. The child at play is practicing for life's responsibilities. Young impalas play at fencing with one another, thrusting and parrying. Art for art's sake was the main avenue, says Cyril Smith, to ancient technological

breakthroughs. Such also is the way of metaphor: it flourishes in playful prose and high poetic art, but it is vital also at the growing edges of science and philosophy.

The molecular theory of gases emerged as an ingenious metaphor: a gas was likened to a vast swarm of absurdly small bodies. So pat was the metaphor that it was declared literally true and thus became straightway a dead metaphor; the fancied miniature bodies were declared real, and the term "body" was extended to cover them all [15].

16 As Shapiro put the point in his [17]:

It is natural to conjecture that Gödel's methods [in the Incompleteness Theorems JK] can be applied to any deductive system acceptable for the Hilbert program. If it is assumed that any legitimate deductive system must be effective (i.e., its axioms and rules of inference must be computable), the conjecture would follow from a thesis that no effective deductive system is complete, provided only that it is ω-consistent and sufficient for arithmetic. But this is a statement about all computable functions, and requires a general notion of computability to be resolved.

17 [18], p. 348. As Gödel wrote to Herbrand in 1931:

Clearly, I do not claim either that it is certain that some finitist proofs are not formalizable in Principia Mathematica, even though intuitively I tend toward this assumption. In any case, a finitist proof not formalizable in Principia Mathematica would have to be quite extraordinarily complicated, and on this purely practical ground there is very little prospect of finding one; but that, in my opinion, does not alter anything about the possibility in principle [19], p. 23.

18 [4], section 14.8.

19 Church, letter to Kleene of 29 November 1935. Quoted in Sieg, op. cit., and in Davis [21].

20 See [16]. That first sentence reads:

The development of mathematics toward greater precision has led, as is well known, to the formalization of large tracts of it, so that one can prove any theorem using nothing but a few mechanical rules.

21 Quoted in Sieg [24], in turn quoting from an unpublished manuscript of Odifreddi.

22 See [3].

23 [18], p. 369.

24 Remark to Hao Wang [27], p. 203.

25 See [28].

26 [29], p. xviii.

27 Greenberg, op cit.

28 Greenberg, quotes in [6], p. 293.

29 See [30], section 2.

30 See [31].

31 From that paper:

While we accept the premise laid out by Kwon, Briony Fer and others that the collapse of modernism in the visual arts developed through a logic delineated within modernism itself, we here argue that this collapse was symptomatic of a much broader unraveling of the intellectual fabric of modernism writ large. Making this case requires a shift in our understanding of what comprises the defining feature(s) of modernism. Rather than the internal features to which Clement Greenberg insisted painting should aspire in order to entrench itself "more firmly in its areas of competence", we posit that the development of modernist painting can be understood as an example of an attempt to produce a mutually exclusive and collectively exhaustive set of parameters, an encyclopedic system of types.

32 [5], p. 7.

33 [6], p. 154

34 [6], op cit.

35 See Floyd [34].

36 [35], p. 134.

37 [36], p. 26.

38 [35], p. 146.

39 [35], p. 147.

40 [37], p. 189

41 Bois' essay "On the uses and abuses of look-alikes" was inspired by a visit to Musées de la Ville de Rouen, where he encountered the work *Young Woman in Her Death Bed*. He concludes the essay thus:

Following my visit to the museum in Rouen, I was itching to find out about what made *Young Woman in Her Death Bed* possible, and whether there was something in the society of seventeenth-century Flanders in common with that of mid-nineteenth-century France or America that had produced such disturbing photographs of dead children with eyes wide open. But something in common, as well, in the cultures of these two societies soon after the occurrences of such works, something that would have motivated their repression and thus destruction and would explain why they seem so exceptional today. I should also note again that the open eyes are by no means the only thing that triggered in me this immediate conviction, when looking at the seventeenth-century painting in Rouen, that I was in front of a nineteenth-century painting (I would have felt just the same if the young woman's eyes were closed)—they were just the tip of the iceberg. I am still itching to, of course, and probably forever will be, being a specialist of neither period. The pseudomorphosis in that case might indeed be pseudo, a total fluke, but if it is not, the flash that floored me could be the occasion of a redistribution of the art-historical cards—which is, as far as I am concerned, the only really interesting part of the game in which we are all so passionately participating [35], p. 149.

42 For example, Gödel draws this moral from his Incompleteness Theorems, " ... [one must either] either give up the old rightward aspects of mathematics or attempt to uphold them in contradiction to the spirit of the time". Quoted in Wang [27], p. 156.

References

1. Church, A. Review of Turing 1936. *J. Symb. Logi* **1937**, *2*, 42–43.
2. Kleene, S.C. The theory of recursive functions, approaching its centennial. *Bull. Am. Math. Soc. (N. S.)* **1981**, *5*, 43–61. [CrossRef]
3. Kleene, S.C. Origins of Recursive Function Theory. *Ann. Hist. Comput.* **1981**, *3*, 52–67. [CrossRef]
4. Gandy, R. The Confluence of Ideas in 1936. In *The Universal Turing Machine: A Half-Century Survey*; Oxford Science Publications; Oxford University Press: New York, NY, USA, 1988; pp. 55–111.
5. Marx, L. *The Machine in the Garden: Technology and the Pastoral Ideal in America*; Oxford University Press: Oxford, UK, 1964.
6. Jones, C. *Machine in the Studio: Constructing the Postwar American Artist*; University of Chicago Press: Chicago, IL, USA, 1996.
7. Greenberg, C. *Clement Greenberg: The Collected Essays and Criticism*; O'Brian, J., Ed.; University of Chicago Press: Chicago, IL, USA, 1986; Volume 4.
8. Shklovsky, V. *Theory of Prose*; Dalkey Archive Press: Funks Grove, IL, USA, 1925.
9. Franks, C. *The Autonomy of Mathematical Knowledge*; Hilbert's Program Revisited; Cambridge University Press: Cambridge, UK, 2009; p. xiv+213. [CrossRef]
10. Rorty, R. *Contingency, Irony and Solidarity*; Cambridge University Press: Cambridge, UK, 1989.
11. Floyd, J.; Mühlhölzer, F. *Wittgenstein's Annotations to Hardy's Course of Pure Mathematics—An Investigation of Wittgenstein's Non-Extensionalist Understanding of the Real Numbers*; Nordic Wittgenstein Studies; Springer: Cham, Switzerland, 2020; Volume 7, p. xx+322. [CrossRef]
12. Kennedy, J. Wittgenstein's annotations to Hardy's course of pure mathematics: An investigation of Wittgenstein's non-extensionalist understanding of the real numbers [book review of 4230095]. *Philos. Math.* **2022**, *30*, 256–272. [CrossRef]
13. Rorty, R. *Objectivity, Relativism, and Truth*; Philosophical Papers; Cambridge University Press: Cambridge, MA, USA, 1991; Volume 1.
14. Rorty, R. *Essays on Heidegger and Others*; Philosophical Papers; Cambridge University Press: Cambridge, MA, USA, 1991; Volume 2.
15. Quine, W.V. A Postscript on Metaphor. *Crit. Inq.* **1978**, *5*, 161–162. [CrossRef]
16. Gödel, K. Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I. *Monatsh. Math. Phys.* **1931**, *38*, 173–198. [CrossRef]
17. Shapiro, S. The open texture of computability. In *Computability—Turing, Gödel, Church, and Beyond*; MIT Press: Cambridge, MA, USA, 2013; pp. 153–181.
18. Gödel, K. *Collected Works*; The Clarendon Press; Oxford University Press: New York, NY, USA, 1986; Volume I, p. xvi+474.
19. Gödel, K. *Collected Works*; Feferman, S., Dawson, J.W., Jr., Goldfarb, W., Parsons, C., Sieg, W., Eds.; The Clarendon Press; Oxford University Press: Oxford, UK, 2003; Volume V, p. xxvi+664.
20. Kennedy, J. Turing, Gödel and the "Bright Abyss". In *Philosophical Explorations of the Legacy of Alan Turing*; Boston Studies in Philosophy; Springer: Berlin/Heidelberg, Germany, 2017; Volume 324.
21. Davis, M. Why Gödel didn't have Church's Thesis. *Inform. Control* **1982**, *54*, 3–24. [CrossRef]
22. Davis, M. (Ed.) *The Undecidable*; Dover Publications Inc.: Mineola, NY, USA, 2004; p. ii+413.
23. Hodges, W. Tarski's Theory of Definition. In *New essays on Tarski and Philosophy*; Oxford University Press: Oxford, UK, 2008; pp. 94–132. [CrossRef]
24. Sieg, W. Gödel on computability. *Philos. Math.* **2006**, *14*, 189–207. [CrossRef]
25. Turing, A.M. On Computable Numbers, with an Application to the Entscheidungsproblem. *Proc. Lond. Math. Soc.* **1937**, *2-42*, 230–265. [CrossRef]
26. Church, A. A Note on the Entscheidungsproblem. *J. Symb. Log.* **1936**, *1*, 40–41; Correction *1*, 101–102. [CrossRef]
27. Wang, H. *A Logical Journey. Representation and Mind*; MIT Press: Cambridge, MA, USA, 1996.
28. Stein, G. *The Autobiography of Alice B. Toklas*; Harcourt, Brace and Company: San Diego, CA, USA, 1933.
29. Jones, C. *Eyesight Alone: Clement Greenberg's Modernism and the Bureaucratization of the Senses*; University of Chicago Press: Chicago, IL, USA, 2005.
30. Zach, R. Hilbert's Program. In *The Stanford Encyclopedia of Philosophy*, Fall 2019 ed.; Zalta, E.N., Ed.; Metaphysics Research Lab, Stanford University: Stanford, CA, USA, 2019.

31. Hilbert, D. On the Infinite. In *From Frege to Gödel: A Source Book in Mathematical Logic*; van Heijenoort, J., Ed.; Harvard University Press: Cambridge, MA, USA 1965; pp. 367–392.
32. Kennedy, J.; Maizels, M. The System that Destroys Itself, or Greenberg's Modernism and the Liar's Paradox. *Crisis Crit.* **2018**, *5*, 211–234.
33. Owens, C. Earthwords. *October* **1979**, *10*, 121–130. [CrossRef]
34. Floyd, J. Wittgenstein's diagonal argument: a variation on Cantor and Turing. In *Epistemology Versus Ontology*; Logic, Epistemology, and the Unity of Science; Springer: Dordrecht, The Netherlands, 2012; Volume 27, pp. 25–44. [CrossRef]
35. Bois, Y.A. On the Uses and Abuses of Look-alikes. *October* **2015**, *154*, 127–149. [CrossRef]
36. Panofsky, E. *Tomb Sculpture: Four Lectures on Its Changing Aspects from Ancient Egypt to Bernini*; Abrams: New York, NY, USA, 1992; p. 319.
37. Greenberg, C. *Clement Greenberg: The Collected Essays and Criticism*; O'Brian, J., Ed.; University of Chicago Press: Chicago, IL, USA, 1986; Volume 3.