Supplementary Materials: Topologically Protected Wormholes in Type-III Weyl Semimetal $Co_3In_2X_2$ (X = S, Se)

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1. Landau Levels in the Weyl Regime

First, the type-III Weyl phase must be confirmed to be physically possible. The simple Hamiltonian that describes a 3D Weyl point can be described as:

$$H(k) = \pm vk \cdot \sigma \tag{1}$$

From this formula, a Dirac point can be constructed with two 3D Weyl points of opposite chirality, thus forming the Dirac cone.

$$H = \begin{bmatrix} H_K & 0 \\ 0 & H_{K'} \end{bmatrix} \tag{2}$$

From this formulation, the prototypical type-I Weyl semimetals can be easily constructed and formed. However, magnetism must be introduced in order to tilt the Hamiltonian (and therefore the Weyl cone) so that a critically tilted weyl cone can be formed which leads to a type-III Weyl phase. (where magnetism is added in the$\hat{z}$ direction in order to allow for the Weyl cone to tilt).

$$H(k) = Ck_z \pm k \cdot \sigma \tag{3}$$

This realizes a Weyl point with +1 or -1 Chern number. The type of Weyl cone depends on the value of the parameter C where $C > |1|$ is a type-II Weyl semimetal , $C < |1|$ is a type-I Weyl semimetal, and $C = -1$ is a type-III Weyl Semimetal. In the case where magnetic field is applied in the $\hat{z}$ direction, the Hamiltonian can be rewritten as [1]:

$$E_n = Ck_z \pm \sqrt{k_z^2 + \frac{2n}{l^2}} \tag{4}$$

Where $l = \frac{eB}{c}^{-\frac{1}{2}}$ is the magnetic perameter. For all figures $\frac{1}{l} = 0.1$.

References

1. Soluyanov, A.A.; Gresch, D.; Wang, Z.; Wu, Q.; Troyer, M.; Dai, X.; Bernevig, B.A. Type-II Weyl semimetals. *Nature* **2015**, *527*, 495–498. doi:10.1038/nature15768.