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Review

Superconductors with a Topological Gap

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Abstract: I review a new superconductivity mechanism in which the gap is opened through a topological mechanism and not through the Landau mechanism of spontaneous symmetry breaking. As a consequence, the low-energy effective theory which describes these new superconductors is not the Landau–Ginzburg theory, formulated in terms of a local-order parameter, but a topological-field theory formulated in terms of emerging gauge fields. This new mechanism is realized as global superconductivty in Josephson junction arrays and in thin superconducting films with thicknesses comparable to the superconducting coherence length, which exhibits emergent granularity.

Keywords: topological mass; superconductivity; topological field theory

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1. Introduction

The Ginzburg–Landau (GL) approach [1] is generally used to describe superconductivity, one of most interesting phenomena in quantum physics. It unravels the fundamental reasons for the very existence of superconductivity decoupled from specific material characteristics. In the field-theory formulation of the GL theory, superconductivity arises because the relevant gauge fields acquire a mass, representing the superconducting gap. The mass arises via the so-called Anderson–Higgs mechanism. In this framework, the Coulomb interaction is normally neglected, since it is considered a sub-dominant effect with respect to the dominant magnetic forces. In two spatial dimensions (2D), however, fluctuations are stronger [2]. Strictly speaking, the Mermin–Wagner theorem forbids long-range order at finite temperatures. This effect, however, can be completely neglected, in practice, for all reasonable sample sizes used in experiments [3].

Here, I review a Higgsless model of superconductivity first introduced in [4,5], in which the gap is generated by a topological mechanism, and not through the Anderson–Higgs mechanism. This topological mechanism is P- and T-invariant and can be formulated in any space-time dimensions, contrary to anyon superconductivity. Its low- energy effective field theory in (2 + 1) dimensions is formulated in terms of two emergent compact gauge fields. In $(d + 1)$ dimensions, the dominant term in effective low-energy action is the topological BF [6] term. It reduces to a mixed Chern–Simons (CS) [7] term in two spatial dimensions. In these superconductors, the destruction of superconductivity is caused by a proliferation of vortices and not by the breaking of Cooper pairs, as in traditional ones [8,9]. In 2D, this is the famed BKT transition [10]. Cooper pairs, in fact, have been detected above the BKT thermal transition [11].

Materials in which Higssless superconductivity is realized are materials which exhibit the superconductor-to-insulator transition (SIT) [12] and have, as relevant degrees of freedom, Cooper pairs and vortices [13]. They are characterised by a large-enough normal-state dielectric constant ε, so that the Coulomb interaction is the 2D logarithmic interaction up to a screening length εd, which typically exceeds the sample size [8,9] and cannot be neglected. As a consequence, we have to couple the GL model to 2D electromagnetism, obtaining, thus, scalar electrodynamics. The gauge fields, which couple to the charge fluctuations in the condensate, become massive through the Anderson–Higgs mechanism [1] and remain coupled to the charge fluctuations in the condensate. However, in thin films, the screening length, given by the inverse of the Anderson–Higgs mass, is the Pearl length $\lambda_\perp = \lambda_L^2/d$. The Pearl length is comparable or larger than the typical sample sizes in thin superconducting films with thicknesses $d \to \xi$ (the Pearl length of TiN in the 2D limit is ≈ 0.1 mm, the typical size of the platelets used in experiments). This means that, for all purposes, gauge fields remain massless within the film. The limit $d \to \xi$ defines what I call planar superconductors.

Inside the sample, the system is described by non-compact QED in (2 + 1) dimensions. This theory has a coupling constant $\propto \log R/a$ with a, an ultraviolet cutoff (UV), and R, the infrared (IR) cutoff and is, thus, infrared divergent [14,15]. In superconducting films, the UV cutoff is given by the coherence length ξ, and the IR cutoff by the size of the system. For system with sizes $\xi \ll R \leq \lambda_\perp$, the theory that describes them is non-perturbative and the Anderson–Higgs mechanism cannot screen the gauge fields.

How can planar superconductors cure their infrared divergences? There are two possibilities: the first one is via instantons, which represent tunneling events between differnt vacua, and which generate a mass for gauge fields when they are in a plasma phase [15]. The second one is via the Chern–Simons (CS) topological mass generation [14,16]. When only one gauge fields is present, this second mechanism breaks the discrete $\mathcal{P}$ (parity) and $\mathcal{T}$ (time-reversal) symmetries. However, when two gauge fields are present in the theory, as in the case of planar supercpnductors, the mixed topological term that enters the action preserves these symmetries [7]. Depending on the value of d (and the bulk London penetration depth λ_L), the system will choose one of these two possibilities. The first leads to superinsulators [17], the second to superconductors, which are the focus of the present review (actually, there is a third possibility, that of completely eliminating any condensate, leading to Bose metals [7,18,19]). As I will show, the thickness of film d does not enter the topological CS mass, which is given only by the product of the two characteristic magnetic and electric length scales that charactrize the system. For planar superconductors, the screening length is, thus, the bulk London penetration depth and not the Pearl length:

$$\lambda_{\text{top}} = O(\lambda_L)\ . \tag{1}$$

The topological gauge theory of superconductivity replaces the GL model for planar superconductors [4,5]. In this model, there is no Higgs field and and no Abrikosov vortices, which are replaced by Josephson vortices [8,9].

Section 2, will show how the BF theory describes Higgsless superconductivity in any dimensions. The BF term is the wedge product of a (d-p)-form b and the curvature d of a p-form a [6]. For the applications of BF theory to superconductivity, I will consider the special case where a_1 is a 1 form and, correspondingly, b_{d-1} is a $(d-1)$ form. In the special case of (3 + 1) dimensions, b is the well-known Kalb–Ramond tensor field $b_{\mu\nu}$ [20]. In (2 + 1) dimensions, both a and b are 1 forms. Section 3 will show how the BF term, or mixed CS term, arises in (2 + 1) dimensions and how global superconductivity emerges. I will also show that this is the only possible superconductivity mechanism in (2 + 1) dimensions and that, to regularize the infrared divergences, the system decomposes into superconducting droplets with the typical size of the order of the superconducting coherence length. The superconducting films have, thus, a granular structure [8].

2. BF Theory

In $(d+1)$ dimensions, the BF term I will consider is [4]:

$$S_{BF} = \frac{k}{2\pi} \int_{M_{d+1}} b_{d-1} \wedge da_1 = \frac{k}{2\pi} \int_{M_{d+1}} a_1 \wedge db_{d-1}\ , \tag{2}$$

where $k = \frac{k_1}{k_2}$, with k_1 and k_2 integers, is a dimensionless coupling constant and

$M_{d+1} = M_d \times R_1$ with R_1 as the time direction. The action in Equation (2) has a generalized Abelian gauge symmetry under the transformations:

$$
\begin{aligned}
b &\to b + \eta \,, \\
a &\to a + \lambda \,,
\end{aligned}
$$

where η and λ are closed $(d-1)$- and d-form, respectively. The gauge transformations for the a form change the action by a surface term. I will, however, ignore this term, since I will consider only compact spatial manifolds without boundaries and require that the fields go to pure gauge configuration at infinity in the time direction.

In order to describe superconductivity, the Bf term must be P- and T-invariant. I will, thus, consider the b_{d-1} form as a pseudo-tensor, with the conserved current $j_1 = *db_{d-1}$ describing charge fluctuations [4,7]. The form a_1 will be considered as a vector and the conserved current $j_{d-1} = *da_1$ as describing the conserved fluctuations in (d-2)-dimensional vortex lines [4,7].

Following [7], the the low-energy effective theory that describes these type of superconductors can be expressed in terms of a_1 and b_{d-1} only. The BF term is the term that dominates at large distances since it contains only one derivative. Introducing the two coupling constants e_q^2 and e_v^2 with dimension m^{-d+3} and m^{d-1}, respectively, and including the two kinetic terms for the generalized gauge fields, the low-energy effective theory is given by:

$$
\begin{aligned}
S_{TM} &= \int_{M_{d+1}} \tfrac{-1}{2e_q^2} da_1 \wedge *da_1 + \tfrac{k}{2\pi} a_1 \wedge db_{d-1} \\
&\quad + \tfrac{(-1)^{d-1}}{2e_v^2} db_{d-1} \wedge *db_{d-1} \,,
\end{aligned}
\tag{3}
$$

where, for simplicity, I used relativistic notation.

Equation (3) describes two massive gauge fields. The mass is generated trough the BF term, which generalizes to any number of dimensions the Chern–Simons mechanism for the topological mass [16]. This topological mass plays the role of the gap that characterise the ground state of superconductors. Starting from the equation of motion for a and b:

$$
\frac{1}{e_q^2} d * db_{d-1} = \frac{k}{2\pi} da_1 \,, \tag{4}
$$

and

$$
\frac{1}{e_v^2} d * da_1 = \frac{k}{2\pi} db_{d-1} \,, \tag{5}
$$

and, applying $d*$ on both sides of Equations (4) and (5), I obtain:

$$
\begin{aligned}
d\delta da_1 - \tfrac{ke^2}{2\pi} d * db_{d-1} &= 0 \,, \\
d * \delta db_{d-1} - \tfrac{kg^2}{2\pi} d * da_1 &= 0 \,,
\end{aligned}
\tag{6}
$$

where $\delta = *d*$ is the adjoint of the exterior derivative [21]. Introducing $\Delta = d\delta$ (when acting on an exact form) and substituting $d * db_{d-1}$ and $d * da_1$ in Equation (6) with the expression obtained from Equations (4) and (5), I can rewrite the equation of motions as:

$$
\begin{aligned}
(\Delta + m^2) da_1 &= 0 \,, \\
(\Delta + m^2) db_{d-1} &= 0 \,,
\end{aligned}
\tag{7}
$$

The quantity m in Equation (7) is the topological mass $m = \frac{k e_v e_q}{2\pi}$.

As can be derived from Equations (4) and (5), the coupling between charges and vortices, due to the BF term, makes the charges act as sources for the vortex line currents encircling them and viceversa. This mechanism is the origin of the gap, not a local-order parameter which acquires a vacuum expectation value.

Since both the gauge fields are compact, the model described by Equation (3) admits topological defects, both electric and magnetic. In the effective field theory, they represent localized charges and vortices, while the currents j_1 and j_{d-1} represent fluctuations in charge and vortex density. Electric topological defects are closed string-like objects, described by a singular closed 1-form Q_1, which couple to the form a_1. They represent the singular parts of the field strength da_1, allowed by the compactness of the gauge symmetries [22]. Magnetic topological defects are closed (d−1) branes described by a singular (d−1)-dimensional form Ω_{d-1}. They couple to the form b_{d-1} and the singular parts of the field strength db_{d-1}, again allowed by the compactness of the gauge symmetries [22].

The condensation (or lack of) of the topological defects determines the phase diagram of the models. To compact gauge theory, an ultraviolet regularization [23] is necessary. In what follows, I will present a formal derivation implying the ultraviolet regularization and show that, in any dimensions, the phase of electric condensation describes a superconductor, without discussing the conditions for the condensation of topological defects. I will consider the phase in which magnetic topological defects are dilute, while the electric ones form a condensate and introduce a non-local-order parameter, the 't Hooft operator, $\langle L_H \rangle$, which represents the amplitude for creating and separating a pair of vortices with fluxes $\pm\phi$. The computation of $\langle L_H \rangle$ in this phase will show that all flux strengths $\phi \neq 2n\pi/(k_1/k_2)$ with n an integer, are confined: this is nothing more than the Meissner effect.

The formal sum over the electric topological defects Q_1 in the partition function, with magnetic topological defects dilute, gives:

$$\begin{aligned} Z &= \int \mathcal{D}a\mathcal{D}b\mathcal{D}Q \\ &\exp\left[i\tfrac{k}{2\pi}\int_{M_{d+1}}(a_1 \wedge db_{d-1} + a_1 \wedge *Q_1)\right] . \end{aligned} \tag{8}$$

Using the partition function Equation (8), I can compute the expectation value of the 't Hooft operator:

$$\begin{aligned} \langle L_H \rangle &= \tfrac{1}{Z}\int \mathcal{D}a\mathcal{D}b\mathcal{D}Q \\ &\exp\Big[i\tfrac{k}{2\pi}\int_{M_{d+1}}(a_1 \wedge db_{d-1} + a_1 \wedge *Q_1) \\ &\quad + i\tfrac{k}{2\pi}\phi\int_{S_{d-1}} b_{d-1}\Big] . \end{aligned} \tag{9}$$

Using Stokes' theorem and integrating over a_1, I can rewrite $\langle L_H \rangle$ as:

$$\begin{aligned} \langle L_H \rangle \quad &\propto \int \mathcal{D}b\mathcal{D}Q\, \delta(db_{d-1} + *Q_1) \\ &\exp\left[i\tfrac{k}{2\pi}\phi\int_{S_d} db_{d-1}\right] , \end{aligned} \tag{10}$$

where the surface S_d represents a compact orientable surface on M_d and it is such that $\partial S_d \equiv S_{d-1}$.

Integrating over b I, thus, obtain for the 't Hooft operator the expression:

$$\langle L_H \rangle = \propto \int \mathcal{D}Q\ \exp\left[-i\frac{k}{2\pi}\phi\int_{S_d} *Q_1\right] . \tag{11}$$

Using the Poisson summation formula, it is easy to see that all fluxes that are different from

$$\frac{\phi}{k_2} = \frac{2\pi}{k_1}\, n \qquad\qquad n \in N\,, \tag{12}$$

are confined, since $\langle L_H \rangle$ vanishes, giving the the Meissner effect, which characterizes the superconducting phase. Note that $2\pi/(k_1/k_2)$ is the fundamental fluxon since the electric condensate carries k_1 fundamental charges of unit $1/k_2$, as is evident from Equation (8). In this purely topological long-distance theory the confining force is infinite, which is why $\langle L_H \rangle$ vanishes; the area law will be recovered including the higher order kinetic terms Equation (3) and the UV cutoff.

The London equation in the purely topological long-distance theory (zero penetration depth limit) can be obtained by coupling the system to an external electromagnetic field A and computing the induced current:

$$\int_{M_{d+1}} A \wedge (*j_1 + *Q_1) \propto \int_{M_{d+1}} A \wedge (db_{d-1} + *Q_1) \, . \tag{13}$$

In this limit, the induced current is identically zero:

$$j_{\text{ind}} = 0 \, . \tag{14}$$

This can be easily seen by noticing that A can be entirely reabsorbed in a redefinition of the gauge field a_1. As for the Meissner effect, the standard form of the London equation would be obtained considering the higher order kinetic terms for the gauge fields and the UV cutoff.

3. (2 + 1)-Dimensional Case

In the superconducting phase, in (2 + 1) dimensions, the system cures its infrared divergencies by breaking up the condensate into "perturbative" droplets of the size $\mathcal{O}(\xi)$ and characterized by independent phases. These independent phases can form configurations in which their circulation over neighboring droplets is a multiple of 2π, thereby forming Josephson vortices [1]. Josephson vortices have a nontrivial gauge structure but no dissipative core; they are ballistic [24,25] and can Bose condensate near enough to the SIT where the parameter window in which their motion is ballistic becomes wide. On the contrary, Abrikosov vortices have a normal core, dissipative motion and cannot Bose condensate. The granular structure formed by the droplets has been observed in thin superconducting films [26].

A fundamental point in thin superconducting films is that the relevant degrees of freedom are charges and vortices subject to topological Aharonov–Bohm [27] and Aharonov–Casher [28] interactions. Being infrared-dominant, these interactions are the most relevant in the low-energy effective action for the films. Describing the films only in terms of the phases via the XY model, neglecting charges degrees of freedom and, thus, the topological interactions, is not correct.

Following [7,18,19], I will use a continuum space-time notation with coordinates $x = (x_0, \mathbf{x})$, with the droplet size taken as the scale that identifies the necessary ultraviolet cutoff, and introduce the fields:

$$\begin{aligned} Q^\mu &= Q_I \int ds \, \frac{dx_I^\mu}{ds} \, \delta^3(x - x_I(s)) \, , \\ M^\mu &= M_J \int dt \, \frac{dx_J^\mu}{dt} \, \delta^3(x - x_J(t)) \, , \end{aligned} \tag{15}$$

which describe point-like condensate droplets of Cooper pairs and the vortices between them. In Equation (15), $x_I(s)$ and $x_J(t)$ parametrize the closed or infinitely long world line of Cooper pairs and vortices. The fields Q^μ and M^μ satisfiy, thus, the conditions: $\partial_\mu Q^\mu = 0$, $\partial_\mu M^\mu = 0$, and describe the integer charges and vortices that constitute the main dynamical degrees of freedom of the system. The topological Aharonov–Bohm–Casher interaction in the Euclidean partition function is given by the term (in natural units $c = 1$, $\hbar = 1$, $\varepsilon_0 = 1$):

$$S_{\text{top}} = i \, 2\pi \int d^3x \; Q_\mu \epsilon_{\mu\alpha\nu} \frac{\partial_\alpha}{\nabla^2} M_\nu \, , \tag{16}$$

where $\epsilon^{\mu\alpha\nu}$ is the totally antisymmetric tensor. Using the relation:

$$\frac{1}{-\nabla^2}\delta^3(x) = \frac{1}{4\pi}\frac{1}{|x|} \,, \tag{17}$$

in three Euclidean dimensions, Equation (16) can be rewritten as:

$$\begin{aligned} S_{\rm top}(Q_I, M_J) &= i2\pi Q_I M_J \Phi(C_I, C_J) \,, \\ \Phi(C_I, C_J) &= \frac{1}{4\pi}\int_0^1 ds \int_0^1 dt \, \frac{dx_I^\mu}{ds}\epsilon_{\mu\alpha\nu}\frac{(x_I - x_J)^\alpha}{|x_I - x_J|^3}\frac{dx_J^\nu}{dt} \,. \end{aligned} \tag{18}$$

When C_I and C_J are closed Euclidean trajectories, $\Phi(C_I, C_J)$ is the integer Gauss linking number [29] between the curves, and it becomes trivial when the quantization condition $Q_I M_J =$ integer for all I, J, is satisfied. An example of this trivial case is closed trajectories describing a Minkowski space-time fluctuation creating a charge and a hole which annihilate after having encircled a vortex. This is not, however, the general case and the Aharonov–Bohm–Casher (ABC) phases in the Euclidean partition function cannot, in general, be neglected, since they give rise to non-trivial quantum interference effects.

Wilczek [30] showed that the non-local topological interactions, Equation (16), admit a local formulation obtained at the price of introducing to emergent gauge fields a_μ and b_μ, which couple to the charge and vortex trajectories and interact via a mixed CS interaction [7]:

$$\begin{aligned} Z &= \sum_{\{Q^\mu, M^\mu\}} \int \mathcal{D}a_\mu \mathcal{D}b_\mu \mathrm{e}^{-S} \,, \\ S &= \int d^3x \, \frac{i}{2\pi} a_\mu \epsilon^{\mu\alpha\nu}\partial_\alpha b_\nu + i a_\mu Q^\mu + i b_\mu M^\mu \,. \end{aligned} \tag{19}$$

This is the reason why the effective field theory of planar superconductors must be formulated in terms of gauge fields. Also in Equation (19), the emergent gauge fields are compact so the continuous notation is a short-hand for a model which is formally defined on a lattice.

At the classical level, the equation of motion for the dual gauge field strengths:

$$\begin{aligned} f^\mu &= \frac{1}{2}\epsilon^{\mu\alpha\nu} f_{\alpha\nu} = \epsilon^{\mu\alpha\nu}\partial_\alpha b_\nu = 2\pi Q_\mu \,, \\ g^\mu &= \frac{1}{2}\epsilon^{\mu\alpha\nu} g_{\alpha\nu} = \epsilon^{\mu\alpha\nu}\partial_\alpha a_\nu = 2\pi M_\mu \,, \end{aligned} \tag{20}$$

show that they can be seen as the conserved charge and vortex currents, respectively. In what follows, I will consider charges and vortices quantized in integer multiples of $2e$ and $2\pi/2e$ and I will ignore single-electron fluctuations, assuming that their gap is sufficiently large to be neglected.

To construct the long-distance effective field theory describing the system, I start by noticing that it possesses $U(1) \otimes U(1)$ gauge symmetry. I have, thus, to add all power-counting relevant and marginal terms consistent with this symmetry. The possible next-order terms are the Maxwell terms for the two gauge fields, which involve two derivatives and are $U(1)$ gauge invariant:

$$S = \int d^3x \, \frac{i}{2\pi} a_\mu \epsilon^{\mu\alpha\nu}\partial_\alpha b_\nu + \frac{1}{2e_v^2} f_\mu f_\mu + \frac{1}{2e_q^2} g_\mu g_\mu + i a_\mu Q_\mu + i b_\mu M_\mu \,. \tag{21}$$

The two equations, (19) and (21), are nothing more that the (2 + 1)-dimensional limit of Equations (2) and (3), respectively, with the topological excitations due to the compactness of the gauge symmetry included and the parameter $k = 1$.

The two coupling constants $e_{\rm q}^2$ and $e_{\rm v}^2$ represent the two typical energy scales in the problem, giving the orders of magnitude of the electric energy of a charge $2e$, having the spatial scale d and magnetic energy of an elementary flux quantum $\Phi_0 = \pi/e$, and possessing the spatial scale $\lambda_\perp$,

$$e_\mathrm{q}^2 = \mathcal{O}\left(\frac{4e^2}{d}\right) ,$$
$$e_\mathrm{v}^2 = \mathcal{O}\left(\frac{\Phi_0^2}{\lambda_\perp}\right) = \mathcal{O}\left(\frac{\pi^2}{e^2\lambda_\perp}\right) = \mathcal{O}\left(\frac{\pi^2 d}{e^2\lambda L^2}\right) . \tag{22}$$

Their ratio $g = e_\mathrm{v}/e_\mathrm{q} = O(d/(\alpha\lambda_L))$ ($\alpha = e^2/4\pi$ is the fine structure constant) represents the relative strength of magnetic and electric forces and it is the quantum parameter that determines which of the two infrared-catastrophe-avoiding models [7,18] is realized by the system. It plays the role of a dimensionless conductivity.

In terms of e_q^2 and e_v^2, the topological CS mass [14,16] can be written as: $m = e_\mathrm{q}e_\mathrm{v}/2\pi v$, and it is the mass that appears in the dispersion relation of both charges and vortices,

$$E = \sqrt{m^2v^4 + v^2p^2} , \tag{23}$$

with v as the velocity of the propagation in the medium once non-relativistic effects are taken into account.

The two kinetic terms in Equation (21) are naively infrared-irrelevant since the corresponding coupling constant e_q^2 and e_v^2 have the canonical dimension [1/length]. However, considering only the infrared-dominant mixed CS term will lead to wrong results because the limit $m \to \infty$, which gives the pure topological theory, does not commute with quantization because it involves a phase-space reduction [31]. To study physical systems, described by normalizable states, one must consider the $m \to \infty$ limit of the topologically massive gauge theory [16]. This is not true for the purely mathematical applications that lead to the knot theory. When two gauge fields are present, in order to correctly define the topological limit $e_\mathrm{q}^2 \to \infty$ and $e_\mathrm{v}^2 \to \infty$, one has to specify the value of g in this limit. The parameter g is the one that determines the behavior of charges and vortices and, by varying it, one obtains very different ground states and phases for the system [7,18].

To investigate the nature of the different phases obtained by varying g, I will add to the action a term which couples the charge current j^μ to the real electromagnetic field A_μ:

$$S \to S + \frac{i}{2\pi}\int d^3x\, A_\mu f^\mu , \tag{24}$$

and I will compute the effective action $S_\mathrm{eff}(A_\mu, Q_\mu, M_\mu)$, obtained by integrating over the fictitious gauge fields a_μ and b_μ:

$$\begin{aligned} S_\mathrm{eff} &= \int d^3x \left[\frac{e_\mathrm{q}^2}{2}Q_\mu \frac{\delta_{\mu\nu}}{-\Delta + m^2v^2}Q_\nu \right. \\ &+ i2\pi m^2v^2 Q_\mu \frac{\epsilon_{\mu\alpha\nu}\partial_\alpha}{\Delta(-\Delta+m^2v^2)}\left(M_\nu + \frac{1}{2\pi}F_\nu\right) + \\ &+ \left. \frac{e_\mathrm{v}^2}{2}\left(M_\mu + \frac{1}{2\pi}F_\mu\right)\frac{\delta_{\mu\nu}}{-\Delta+m^2v^2}\left(M_\nu + \frac{1}{2\pi}F_\nu\right)\right] , \end{aligned} \tag{25}$$

where $F_\mu = \epsilon_{\mu\nu\alpha}\partial_\nu A_\alpha$ is the dual electromagnetic field strength. For the kernel in Equation (25), as standard in lattice gauge theories [32], I will retain only the self-interaction term:

$$G(x-y) = \frac{1}{m^2v^2 - \Delta}\delta^3(x-y) \to \ell^2 G(mv\ell)\,\delta^3(x-y) , \tag{26}$$

where ℓ is the UV cutoff given, for the films, by the droplet size. At first order in the derivative expansions, the screened potential becomes a delta-function of a strength depending on the ratio of the two length scales ℓ and $1/mv$. In this approximation, Equation (25) becomes:

$$\begin{aligned} S_{\mathrm{eff}} &= \int d^3x \left[\frac{1}{2} \frac{2\pi\ell\eta}{g} Q_\mu^2 + \frac{1}{2} 2\pi\ell\eta g \left(M_\mu + \frac{1}{2\pi} F_\mu \right)^2 + \right. \\ & \left. + i2\pi(mv\ell)\eta Q_\mu \frac{\epsilon_{\mu\nu\alpha}\partial_\nu}{\Delta} \left(M_\alpha + \frac{1}{2\pi} F_\alpha \right) \right] . \end{aligned} \tag{27}$$

The numerical parameter $\eta = (mv\ell)G(mv\ell)$, which appears in Equation (27), is of order $\mathcal{O}(1)$ and depends on the dimensionless quantity $mv\ell$. With g, η determines the quantum-phase structure of the model [7,18].

The superconducting phase is a phase in which vortices are suppressed, $M_\mu = 0$ in (27), and global phase coherence is established. In this phase, there are no more integer-valued charges, droplet charges form a global condensate, and they are connected by quantum tunnelling percolation. Tunnelling percolation on the droplets and the formation of a global condensate has been experimentally observed in high-temperature superconductors [33]. An experimental realization of an artificial superlattice of quantum wells with tunnelling between superconducting units with a separation of the order of coherence length is discussed in [34].

Since there are no more integer-valued charges, the sum over the integer-valued field Q_μ in the partition function becomes an integral over a real-valued field H_μ, which satisfies the constraint $\partial_\mu H_\mu = 0$. I, thus, represent the field H_μ in terms of a new gauge field n_μ as $H_\mu = \epsilon_{\mu\alpha\nu}\partial_\alpha n_\nu$. In terms of the field n_μ, the effective action for the electromagnetic field A_μ can be written as:

$$S_{\mathrm{eff}} = \int d^3x \, \frac{1}{2} \frac{2\pi\ell\eta}{g} H_\mu^2 + \frac{1}{2} \frac{\ell\eta g}{2\pi} F_\mu^2 - i(mv\ell)\eta \; n_\mu \epsilon_{\mu\alpha\nu}\partial_\alpha A_\nu \, , \tag{28}$$

In Equation (28), the electromagnetic field is coupled to the n_μ field through a mixed CS term and it acquires a mass via the topological mechanism [14,16]. The topological mass is given by the expression:

$$m = e_q e_v / 2\pi \, , \tag{29}$$

and the screening length is, thus, $\lambda_{\mathrm{top}} = 1/mv$. This screening length corresponds to the bulk London penetration depth, Equation (1). In fact, by integrating also over the gauge field n_μ representing the global condensate fluctuations, I obtain an effective action for A_μ alone,

$$S_{\mathrm{eff}}(A_\mu) = \int d^3x \, \frac{1}{2\lambda_{\mathrm{top}}} A_\mu \left(\delta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\Delta} \right) A_\nu + \dots \, , \tag{30}$$

with the dots denoting higher order terms in the derivative expansion and λ_{top} given by Equation (1). This is the quantity entering the London equations and representing, thus, the effective London penetration depth of planar superconductivity.

From Equation (28), using Equation (22), we see that the effective coupling constant of the Maxwell term for the electromagnetic field is $O(d\ell/e^2\lambda_{\mathrm{top}})$. The Coulomb coupling constant is, thus, renormalized to $e^2\lambda_{\mathrm{top}}/\ell$. Varying ℓ, one can see that for $\ell \approx \lambda_{\mathrm{top}}$, the coupling constant tends towards the bare value given by the electron charge while, when ℓ becomes small, the effective Coulomb interaction increases.

The dual phase is a phase in which the vortices form a condensate. I, thus, integrate the vortex degrees of freedom M_μ and set $Q_\mu = 0$ to compute the effective electromagnetic action $S_{\mathrm{eff}}(M_\mu, A_\mu)$. Since I am considering materials with a very high dielectric constants, in the effective action I can neglect the magnetic components with respect to the electric ones, since $v \ll 1$. I, thus, obtain:

$$S_{\text{eff}}(M_\mu, A_\mu) = \int d^3x \, \frac{e_v^2 \ell^2 G}{v^2 8\pi^2} (F_i + 2\pi M_i)^2 \,, \tag{31}$$

with latin indices "i" denoting purely spatial coordinates. This is the non-relativistic version [17,35,36] of Polyakov's compact QED [22], in which magnetic monopole instantons create the confining linear potential between the probe charges and generate the photon mass. This dual phase is a superinsulator [7,17].

4. Conclusions

In this paper, I reviewed a superconductivity mechanism which is not based on the Anderson–Higgs mechanism but on a topological mechanism to generate the mass for the gauge fields. The simplest example $(k = 1)$ of this type of Higgsless superconductivity is concretely realized as the global superconductivity mechanism in planar Josephson junction arrays and granular superconducting films [7,8]. This Higgsless, topological superconductivty is the only possibility in 2D [8], but may also be realized in 3D bulk materials [35–37]. Indeed, 3D bulk materials with the emergent granularity typical of planar superconductors have been recently found [38].

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