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Enhancing Calculus Learning through Interactive VR and AR Technologies: A Study on Immersive Educational Tools

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Abstract: In the realm of collegiate education, calculus can be quite challenging for students. Many students struggle to visualize abstract concepts, as mathematics often moves into strict arithmetic rather than geometric understanding. Our study presents an innovative solution to this problem: an immersive, interactive VR graphing tool capable of standard 2D graphs, solids of revolution, and a series of visualizations deemed potentially useful to struggling students. This tool was developed within the Unity 3D engine, and while interaction and expression parsing rely on existing libraries, core functionalities were developed independently. As a pilot study, it includes qualitative information from a survey of students currently or previously enrolled in Calculus II/III courses, revealing its potential effectiveness. This survey primarily aims to determine the tool's viability in future endeavors. The positive response suggests the tool's immediate usefulness and its promising future in educational settings, prompting further exploration and consideration for adaptation into an Augmented Reality (AR) environment.

Keywords: virtual reality; VR; solids of revolution; education; interactive learning; spatial reasoning; immersive environment

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1. Introduction

The challenges in the instructor–student relationship have become more evident, particularly during the shift to online teaching amid the 2020 COVID-19 pandemic. This period highlighted the limitations of traditional instructivist methods, especially in capturing students' interest and imagination in STEM fields. The issue, although accentuated by the pandemic, has been a longstanding concern. Research suggests that subjects like calculus demand spatial reasoning and creative thinking for effective comprehension [1]. Despite the growing recognition of constructivist pedagogies [2] supporting these learning aspects, there seems to be a deficiency in tools designed to foster these essential skills and qualities in students.

Using strictly 2D visualizations in standard teaching environments is increasingly seen as inadequate for developing skills required in subjects that demand spatial intuition. As a constructivist would argue [2], these traditional visualizations often hinder collaborative and interactive learning, failing to address individual students' diverse needs and learning styles. Simply put, this one-size-fits-all approach can be ineffective, as students grasp concepts differently. While many projects attempt to address this problem, many of these projects are seen as lacking in one way or another by the authors: Often, these projects are limited in their range of visualizations or lack the desired level of interactivity, resulting in students passively observing rather than actively engaging with the material by harnessing the potential of AR/VR applications. In such projects, students may not "play" with the concepts placed before them. These projects, in fact, tend to rely heavily on instructor guidance rather than empowering students to explore and build their understanding.

The potential benefits of AR/VR in education are increasingly evident across various fields. In 2019, there were 436 AR/VR education studies submitted to the Web of Science [3], while a notable portion of AR studies within the Social Science Citation Index from 2006 to 2016 were focused on Science or Mathematics Education [4]. These studies have consistently reported several key advantages for students, which include, but are far from limited to:

- Improved Information Retention: While a study of high school Philosophy students using a VR environment showed significant improvements in information retention, studies have shown a far more impressive, far-reaching field in which such environments have improved memory: Treating dementia and traumatic brain injuries [5]. Such retention improvements appear more prevalent in VR environments [6], yet, due to the clear difficulties of using VR in a standard classroom environment, AR may be used in less individualized cases. Additional studies have found similar benefits, particularly in the field of learning languages [7].
- Enhanced Visualization Abilities: As previously mentioned, AR/VR significantly improves students' ability to visualize complex concepts. Beyond the aforementioned retention advantages, one study found that strictly spatial and temporal relationships are remembered 40% more effectively in AR/VR environments [5]. These benefits are likely due to the feeling of physically being in the environment, also known as spatial presence [6]. However, in some cases, these advantages are granted by allowing users to see what they simply cannot; AR's many applications in surgery, particularly overlaying intraoperative imaging techniques over the patient, have given surgeons the ability to see into the patient without having to constantly reference a separate monitor. These techniques have been suggested for training surgeons, so patients may be put at lower risk of complications [8,9]. While that is, of course, a radically different field, the notion of simply allowing students to see what they otherwise can't applies. Other studies have found similar results [2].
- Increased Student Attention and Motivation: One study reported an average score of 4/5 regarding student attention within engineering courses, which is exceptionally high given the common pedagogical struggles within said field [1]. Due to their often-game-like nature, AR/VR environments seem to universally improve student motivation, likely due to simply being more enjoyable [6,7]. In other examples, significant improvements were found among students on the autism spectrum [5]. Additional studies have found similar results for various populations [6,9–11].
- Improved Student Outcomes and Success Rates: Direct and notable improvements in student performance and success rates across various fields have been observed as a result of implementing these technologies [2,7,10,12]. In some cases, this is tied directly to the self-driven learning valued by constructivist pedagogy [5].
- Facilitated Access to Psychological 'Flow' State: Easier access to the psychological "flow" state, a mental state of deep immersion and focused concentration, paired with reduced cognitive load, despite the improvements as mentioned above [11].
- Collaborative Benefits: Given its ability to connect distant individuals, AR/VR technologies are thought to enable social collaboration in unusual circumstances, particularly among students who struggle with such interaction [5]. As early as 2014, this was demonstrated in various projects [13].

While this study represents an initial step and requires further development to become a standard tool in educational settings, it demonstrates the potential to address the limitations of traditional 2D visualizations. The VR-based graphing tool developed in this research, designed to illustrate solids of revolution, their underlying Riemann sums, and other related visual concepts, introduces a new level of interactivity to mathematics. This interactivity is expected to enhance student engagement and understanding of complex mathematical concepts. Preliminary data on student attitudes towards this VR tool and its potential effectiveness have been gathered, making use of the ARCS framework.

ARCS is a thoroughly validated framework for evaluation student motivation [14], which has seen use in several academic contexts [7,15]. The framework's acronym stands for its four evaluation factors: Attention (A), which describes how little the students' focus moves away from the presentation, Relevance (R), which describes how personalized the presentation is to the student, past and future, Confidence (C), which includes a students' own confidence in their capabilities as well as reasonable expectations for what they may accomplish, and Satisfaction (S), which describes a general sense of enjoyment, curiosity, and more broadly, fun [14]. Due to similar work relying on a customized questionnaire for the framework [1], the current study has chosen the same approach.

The overwhelmingly positive feedback suggests promising outcomes and supports the further development of this project for use in a structured course environment. Importantly, as this research involved student responses, ethical considerations were duly noted, and the study received approval from the University's Institutional Review Board (IRB) under approval number #2023.10.006. The inclusion of human participants was conducted by the ethical guidelines set by the IRB, ensuring that the study adhered to the highest standards of research ethics.

Related Works

This study builds upon the authors' previous research [16], which describes an initial version of the graphing tool. The current version retains many of the core functionalities from the earlier work but introduces several new features, as outlined in Section 2.1. These enhancements are a step towards making the tool more applicable in a classroom environment.

As the authors have previously described at length, the goals of the paper include a unification of aspects from the following works:

- Construct3D [17] and a related project, Physics Playground [18]. Both projects were developed circa 2008. While Construct3D was an extremely interactive AR tool aimed at teaching students geometry and included a tool to create simple solids of revolution, Physics Playground was effectively an AR interface for running simple physics simulations. While extremely impressive for the time, it is unlikely the project would be reasonably ported to modern hardware [17].
- An unnamed, recently developed project [19], which cites Construct3D as an inspiration, enabled the creation of arbitrary cross-sections of solids of revolution. However, its functionality had some ambiguities and offered limited student interactivity.
- AVRAM and ARC [20]. Somewhat combining the approaches of Construct3D and the previously mentioned unnamed project, these two projects provide, respectively, a synchronized 3D graphing software in which students can work together to manipulate and understand a particular 3D solid, and an AR implementation of several classes of problems, including polar coordinates and quadric surfaces. However, these two functionalities are split, and, while this study's tool is in early development and requires far more features to match ARC's, ARC seems to take a less procedural approach, making modules for each functionality separately. In the future, the current tool seeks to avoid this by generalizing each functionality as much as possible.
- Manim, an exceptionally powerful mathematics animation engine created by Grant Sanderson [21]. The visualizations produced by the engine have proven extremely useful, not only for Sanderson's YouTube channel, 3Blue1Brown [22–24], but multiple academic circumstances, including the visualization of error theory [25], general mathematics [26], and machine learning courses at the authors' host university. Although not an AR tool, the clarity and effectiveness of Manim's visualizations set a high standard for AR projects in educational contexts.

While the authors acknowledge the existence of projects such as GeoGebra AR [27], they feel that such projects almost entirely lack interactivity, leading to their exclusion from this study.

2. Materials and Methods

2.1. Implementation

A demonstration video showcasing the tool, presented in person to subjects, can be viewed https://youtu.be/7fN3W-e8x14; accessed on 26 February 2024.

As described in our previous work [16], the tool was generated using the Unity 3D engine and extensively utilized standard XR libraries [28,29], Shader Graph [30] as well as a library that provides a simplified Sprache wrapper for Unity [31]. Sprache, a widely-used C# parser library [32], is notable, though it has limited documentation. The tool features various visualization capabilities, including solids of revolution, customized UI elements, cross-section visualization, and an "unfolding" visualization used to demonstrate the basic function of a solid of revolution. For clarity and convenience, key aspects of the implementation are outlined below:

2.1.1. Extended UI Implementation

In response to challenges encountered with Unity's default UI elements, the authors developed a set of custom UI elements placed within the panel shown in Figure 1. These elements are designed for intuitive use, featuring tooltips that appear when users hover over sliders and buttons. These tooltips explain each element's function clearly, with additional details found in the corresponding sections below. The UI management is streamlined through a script associated with the UI panel, ensuring seamless interaction between the UI and other objects. The slider's current value is displayed as the slider is changed, and sliders may be set to their default value by selecting the sphere with both the "grip" and "trigger" buttons on most XR (Extended Reality) controllers.

Figure 1. Current UI Panel.

The aforementioned tooltips are provided below, describing the elements top to bottom, left to right, with further implementation details provided in following sections:

- Unroll Function—This slider causes the solid of revolution displayed to "unroll", displaying a subsection of the solid from 0 to the selected value in radians
- Unroll Sum—This slider is similar to Unroll Function, but "unrolls" the Riemann sum underlying the solid of revolution
- Zoom—This slider allows you to zoom in and out of the function at the origin
- Minimum Value—This slider determines the minimum value of X at which the function will be evaluated (Must be less than Maximum Value)

- Maximum Value—This slider determines the maximum value of X at which the function will be evaluated (Must be greater than Minimum Value)
- Keyboard—No tooltip provided: Used to input function desired
- X Increment—This slider controls the "steps" along the X axis at which the function will be evaluated. For example, a function with a minimum of 0, maximum of 1, and increment of 0.5, will be evaluated at 0, 0.5, and 1
- Degree Increment—This slider controls the "steps" used to rotate around the given axis when generating a solid of revolution, in degrees. For example, a function with a degree increment of 18 will evaluate the solid at 0°, 18°, 36°, etc.
- Axis Buttons—These buttons will set the solid of rotation to be around the given axis
- Graph or Solid?—If this button is grey, a solid of revolution will be generated. If not, a standard 2D graph will be generated
- Update—When activated, this button generates the function you have input
- Cross Section Mode—This button determines whether the entire graph will be visualized, or only a cross section around the teal ball located near this panel
- Cross Sliders—These sliders control the size of the cross section (when enabled) in the given axis

All sliders colored red and the first two rows of buttons do not update continuously, as they alter the construction of the 3D solid. All other UI elements update the display of the current solid or graph.

2.1.2. Procedural Mesh Generation

In the procedural mesh generation process, a string input is processed by the graph generation object using a Sprache-based library. This library was originally designed for single float values rather than the range required for a function evaluation. This was cumbersome and inefficient for function evaluations. To optimize this, the library's operators were overloaded to handle arrays of float values, thereby reducing the need for repetitive parsing. One notable exception to this is Sigma and Pi notation, such as

$$\Sigma_{k=1}^{20} kx^2 \text{ or } \Pi_{k=1}^{20} kx^2$$

These require repeated calls to the parser to handle the variation in k. In our approach, we utilize Unity's Mesh library to procedurally generate graphical representations. For standard graphs, this involves creating a series of simple "strip" of quadrilaterals based on the values input by the user. In the case of solids of revolution, X values are aligned with their corresponding Y values as defined by the user's function and are rotated around an axis of the user's choice to generate disks of the solid. The mesh is complete after each disk is connected to the previous in similar "strips" of quadrilaterals. A basic UV map testing texture applied to a solid of revolution is shown in Figure 2 (The numbers in the figure are used for showing how the texture is mapped to the object). The UV for simple graphs is relatively simple; movement along the X axis in 3D space is equivalent to movement along the X axis in the UV map, and movement along the Z axis in 3D space is equivalent to movement along the Y axis in the UV map.

Users, as depicted in Figure 1, are capable of altering the procedural mesh generation in the following ways:

- Minimum and Maximum value, Alongside X Increment: Determine the fidelity of the graph along the X axis. These values may be mapped to the following code snippet, used to generate the X values at which the function is sampled:

  ```
  for(float x = Minimum Value;x <= Maximum Value;x += X increment)
  ```

- Degree Increment: Determines the fidelity of the solid of revolution by controlling the angles around a circle at which the solid is sampled, in degrees. This may be mapped to the following code snippet:

  ```
  for(float angle = 0;angle <= 360;angle += Degree Increment)
  ```

- X, Y, and Z Axis Buttons: Determines the axis along which the solid of revolution is generated, as seen in Figure 3. The inclusion of the Z-axis in the tool is mainly for demonstration purposes.

Figure 2. Standard UV Testing Textures Placed on Solid of $y = x$.

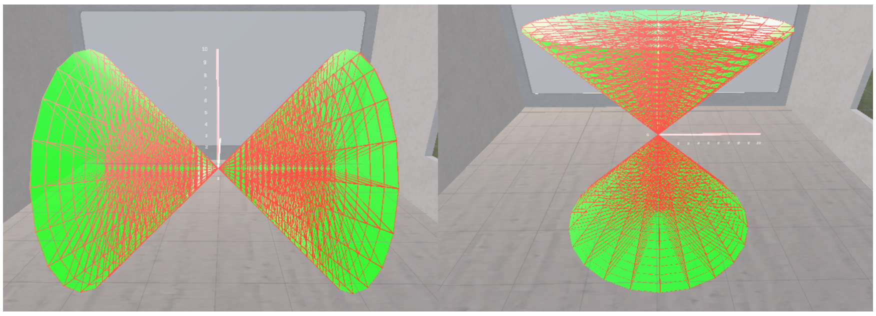

Figure 3. Solid of Revolution Generated Along Both Axes.

2.1.3. Underlying Riemann Sum Visualization

To more effectively demonstrate the mathematical concepts to students, a visualization of the underlying Reimann sum is generated alongside the solids of revolution. The visualization takes a somewhat unorthodox approach; it visualizes the traditional method of infinitesimal disks along the principal axis and infinitesimal arc lengths around the circle. Our aim is to forge a more intuitive link to the general formula for the solid of revolution, represented as

$$\pi \int f(x)^2 dx$$

by illustrating a clearer connection to the area of a circle. A set of points for each vertex is generated (except for the last vertex in each ring, which is co-located with the first to ensure proper UV generation):

1. The corresponding vertex on the previous ring of the solid of revolution
2. The vertex itself
3. The vertex's component along the axis chosen for the solid of revolution
4. The vertex itself, once more
5. The next vertex along the ring in which the vertex is placed, rotating clockwise

This process results in a substantially large set of points for all vertices. These points are then passed to Unity's Line Renderer component and subsequently baked into a mesh.

This baking process ensures the visualization interacts correctly with the shader discussed in Sections 2.1.4 and 2.1.5.

2.1.4. "Unrolling" Visualization

Figure 4 illustrates how users can "unroll" the solid of revolution, a feature that visually represents the process of its creation by rotating around a selected axis, as depicted in Figure 3. This functionality is enabled through a custom shader created using Unity's Shader Graph library, relying on relatively simple trigonometry. The Riemann sum (shown in red) and the solid itself (shown in green) may be "unrolled" independently using an angle from 0 to 2π, as demonstrated in Figure 4.

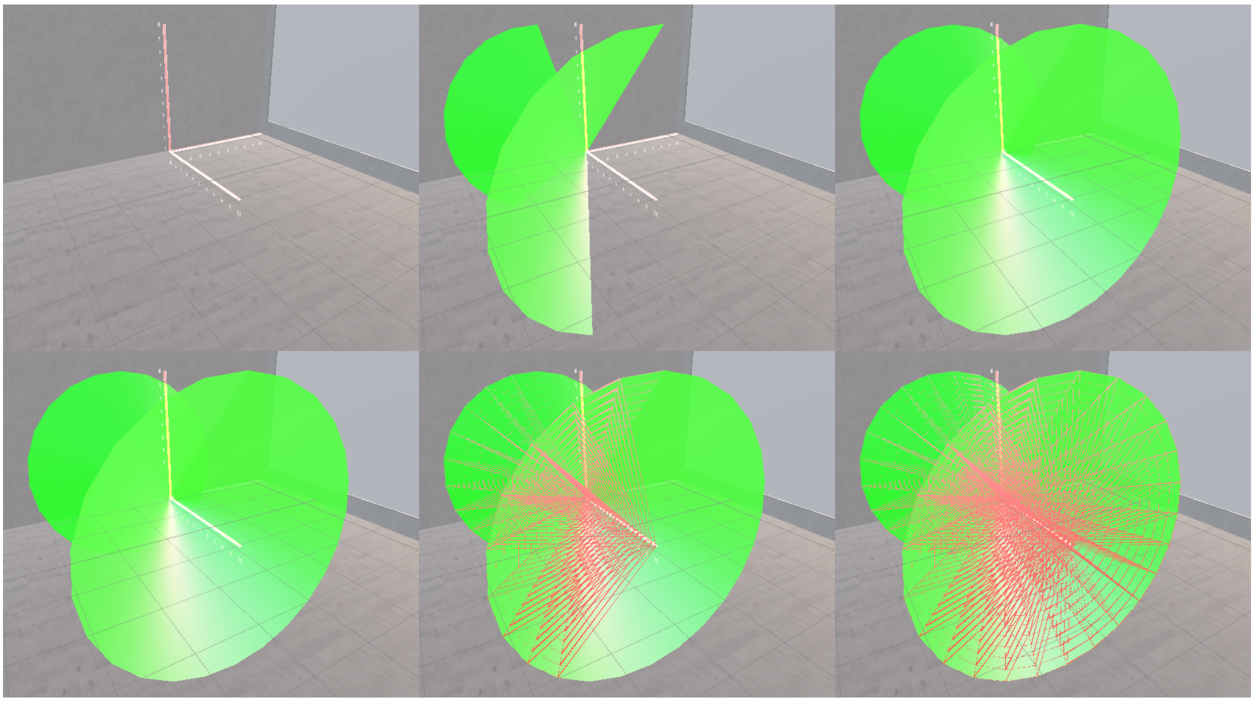

Figure 4. "Unrolling" Visualization for Both Solid and Sum.

2.1.5. Cross Section Visualization

Alongside the "unrolling" visualization discussed in Section 2.1.4, the previously-mentioned custom shader is also used to selectively render a cuboid of the resulting object. Within the tool, after selecting the option via a button shown in Figure 1, UI sliders may be used to define the cuboid centered at the cyan sphere shown in Figure 5. This functionality allows users to create and explore arbitrary cross-sections of the solid.

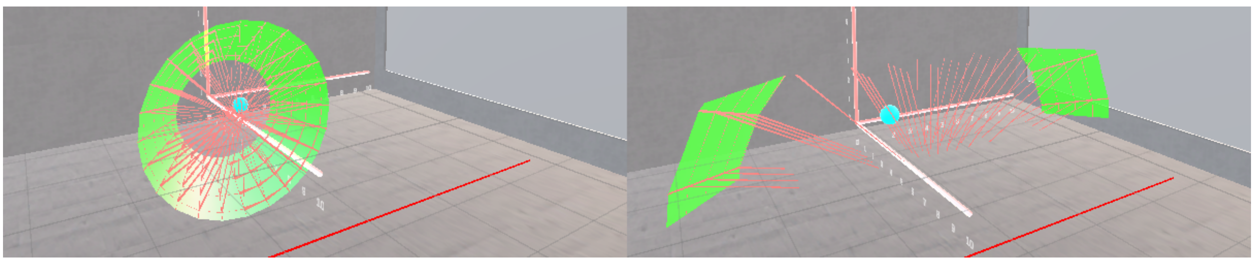

Figure 5. A Demonstration of Generated Cross Sections.

2.1.6. Additional Features

The following features are included in the tool due to their simplicity, current completion status, or plans for future modification:

- Notebook Feature: A simple "Notebook" object is provided within the tool, providing users with essential information such as basic controls and function syntax.
- VR Classroom Environment: The tool features a virtual reality classroom setting, which is somewhat oversized for easy navigation. The authors created the models within the scene, and the textures were taken from freely available online repositories.
- Cross-Platform Functionality: Basic cross-platform functionality is provided within the project, including the ability to deploy directly to the Oculus Quest using an Android build target. Compatibility with additional XR head-mounted displays (HMDs) can be achieved using standard methods within the Unity XR Interaction Toolkit library.
- AR Scene for HoloLens: Alongside the primary VR implementation, a basic AR scene designed for the Microsoft HoloLens is included. However, due to consistent hardware delivery delays, this scene remains incomplete. Given the decreased likelihood of motion sickness due to movement in the real world and the relative ease of use in a classroom environment, the future versions of the tool are expected to shift towards an AR environment.

2.2. Subject Engagement

2.2.1. Preliminary Survey

In conjunction with subjects' engagement with the VR application described in Section 2.1, a distinct subset of students, a portion of whom subsequently participated in the application as mentioned earlier, were administered an initial survey. This survey aimed to gauge their perspectives on AR and 3D applications. The survey comprised questions structured on a Likert scale ranging from 1 to 5, tailored to each query. Additionally, it included open-ended questions to facilitate more detailed responses. These participants were selected exclusively from Calculus II and III students. The primary objective of this survey was to ascertain the students' overarching attitudes toward AR and VR applications by their own reporting. This acts as a relatively simple analysis of students' Confidence and Satisfaction, as defined by ARCS, surrounding both existing teaching materials and AR/VR applications. These factors were chosen as, due to the survey's nature, measuring Attention and Relevance would information more closely related to a particular curriculum, rather than the students' current capabilities. Specifically, for the students in Calculus III, the survey questions were structured as shown in Table 1:

The questions provided to Calculus II students were identical, with the exception of question 1 (a), which was altered to:

1. (a) Do you find it challenging to imagine a solid in 3D space by revolving a region around an axis?

This alteration accounted for the relative lack of 3D concepts within Calculus II. A total of 70 students enrolled in Calculus II and Calculus III courses at West Texas A&M University (WTAMU) participated in the survey.

A number of questions, marked with †, were not used to calculate the overall ARCS scores: They are indicative of their respective factors as they pertain to future AR/VR tools, as well as the practicality of AR over VR in a classroom environment from students' perspectives. Specifically, question 9 gauges the degree to which students believe more interactivity would hold their attention, or simply improve their enjoyment of the tool, while question 10 gauges students' desires for concepts more closely related to concepts they are more familiar with.

Table 1. Preliminary Survey.

No.		Question	Type/Scale	ARCS Factor
1	a	Do you find it challenging to imagine a surface in 3D space?	Not At All Challenging—Extremely Challenging	C
	b	If you find it challenging, what specific aspects do you struggle with?	Short Answer	C
2	a	Are you interested in the use of augmented reality applications to enhance your understanding of 3D visualization in calculus?	Not Interested At All—Extremely Interested	S
	b	What aspects of augmented reality applications do you think would be most beneficial to learning calculus concepts?	Short Answer	C/S [1]
3	a	In your opinion, do you believe that augmented reality techniques could improve your understanding of calculus concepts?	Disagree—Agree	C/S [1]
	b	Please provide any additional comments or suggestions regarding the potential use of augmented reality in your calculus studies.	Short Answer	C/S [1]

[1] Due to their open-ended nature, students provided both Confidence- and Satisfaction-related reasoning to these questions.

2.2.2. VR Application

Students from the WTAMU, who were either enrolled in or had previously taken a Calculus II/III course, were selected as subjects to test the basic functionalities of the project. These participants were recruited through verbal announcements in university courses, student organization events, and similar settings. In line with the consent form, subjects were given three options for their involvement in the study:

1. Participate fully in the study using the tool, which operates on the Meta Quest 2 platform. This option involves standard participation, engaging directly with the VR tool.
2. Opt out of the standard participation but share reasons, such as medical or health concerns. This choice was included to assess if a significant portion of the target group had reservations about using the technology, which could impact the project's overall feasibility.
3. Choose not to participate in the study without providing feedback or data to the research team.

If subjects chose the first option, they signed a consent form before engaging with the VR headset. Upon initiating the experiment, they were instructed to generate simple, familiar functions and explore the provided visualizations. Participants were then allowed to freely interact with the VR tool for up to thirty minutes, with some variation in time when groups were involved. Following their interaction with the VR tool, participants completed a brief questionnaire. This questionnaire included agree/disagree statements rated on a scale from 1 to 5 and short-answer questions focused on evaluating the tool's effectiveness in aiding Calculus II/III instruction. A number of questions were used to measure multiple ARCS factors for various reasons. For example, in question 3, due to the differentiation between a classroom and study environment, students agreeing to this statement likely felt the visualization was more personally relevant, rather than an abstract concept they simply must learn. This would also imply the tool was satisfying enough to use without an instructor. The questionnaire is as shown in Table 2.

Table 2. VR Application Questionnaire.

No.	Question	Type	ARCS Factor
1	After using the visualization, my understanding of the solids of revolution improved.	Disagree—Agree	C
2	The visualization would be beneficial in a classroom setting.	Disagree—Agree	A/S
3	The visualization would serve as a valuable study tool.	Disagree—Agree	R
4	The visualization would be more useful as an Augmented Reality application—super-imposed, for instance, on a classroom table.	Disagree—Agree	A/R †
5	I experienced discomfort or other negative effects while using the visualization.	Disagree—Agree	S
6	The visualization could be enhanced with specific features.	Disagree—Agree	A/R
	If you agree, please provide some recommendations:	Short Answer	
7	Compared to textbooks or other conventional teaching methods, this visualization offers a more intuitive grasp of the solids of revolution.	Disagree—Agree	C
	Do you believe this applies more to new students, current students, or both?	Short Answer	
8	Similar Visualizations would benefit other mathematics, engineering, or computer science subjects.	Disagree—Agree	R
	If you agree, please provide some recommendations:	Short Answer	
9	I would appreciate more direct interactivity in the visualization, with controls similar to the cross-section visualization.	Disagree—Agree	A/S †
10	I would prefer a more intricate visualization capable of representing a broader array of concepts.	Disagree—Agree	R †
11	Did you find any bug that you want to let us know? Please provide the details briefly.	Short Answer	N/A [1]

Question value reversed during analysis; lower scores represented a higher amount of the relevant ARCS factor; † Question value not used in standard analysis; see below: [1] Answers to the final question were used to, when possible, remedy simple mislabelling, visual issues, or other relatively simple errors that the authors felt would not significantly alter user experience or questionnaire answers. This included a UI element mislabelling, the "unrolling" visualization behaving strangely when not placed along the x-axis, etc.

If subjects selected the second option, they would be directed to a shortened questionnaire regarding their reasons for avoiding direct participation. While no students chose this option, it was included in order to evaluate the possibility of VR technology limiting access for an infeasible number of students. The opt-out questionnaire is shown in Table 3.

All information gathered via questionnaires was anonymous and aggregated to avoid privacy breaches and conflicts of interest on the authors' parts. Subjects were informed that the only information that may be used to verify their involvement in the study was their signature on a consent form, which was held within a secure location.

Table 3. Opt-out Questionnaire.

No.	Question	Type
1	I chose not to participate in the VR visualization due to health concerns.	Yes/No
2	Please specify you reasons for opting out of the VR experience:	Short Answer
3	Would you consider using VR visualization if your concerns were addressed or if additional safety features were implemented	Yes/No
	If yes, please specify what changes or features would make you comfortable:	Short Answer

3. Results

3.1. Preliminary Survey

The principal findings from the survey described in Section 2.2.1 are summarized as follows:

1. Difficulty in 3D Visualization:
   - A notable portion of students (average rating: 2.39 ranging from 1, Not at all challenging, to 5, Extremely challenging) acknowledged challenges in imagining solids in 3D space, a fundamental aspect of Calculus III, It is worth noting that any score above 1 indicated some level of difficulty, and Calculus II/III students experiencing any difficulty is indicative of an overall lack of confidence in traditional teaching methods
   - Common difficulties highlighted include the inability to visualize unique shapes in 3D and the struggle to interpret these shapes from 2D diagrams.
2. Interest in AR Applications:
   - Most students expressed high interest (average rating: 3.39 ranging from 1, Not interested at all, to 5, Extremely interested) in using AR applications to enhance their understanding of calculus concepts. Given the term "interest" can only refer to students' subjective outlook of simply enjoying the process, this portrays a degree of satisfaction with even hypothetical VR/AR tools.
   - Students believe that AR could particularly aid in manipulating functions in 3D space, offering a more tangible and interactive learning experience.
3. Impact of AR on Calculus Learning:
   - The respondents showed a strong belief (average rating: 3.89 ranging from 1, Strongly disagree, to 5, Strongly agree) in the potential of AR to improve their understanding of calculus concepts. Students described finding VR more enjoyable while providing a more direct visualization, allowing for greater intuition. These, respectively, represent a degree of satisfaction and confidence in even hypothetical VR/AR tools.
   - Several students noted that AR would offer a more in-depth and engaging way to explore complex calculus topics.

The encouraging results from our preliminary survey significantly underscore the need for innovative educational solutions like our project. Although the survey included elements of AR, we chose to focus on developing a VR-based prototype. This decision was guided not only by VR's capacity to offer a more immersive and interactive learning experience, particularly beneficial for students grappling with the visualization of complex 3D calculus concepts but also due to the non-availability of an AR headset. The enthusiastic feedback from the survey serves as a pivotal motivation for our current work, affirming the value of VR in enhancing understanding and engagement in mathematical learning.

3.2. VR Application Survey

3.2.1. ARCS Criteria

As described in Table 2, the open-ended nature of some questions revealed that some students provided reasoning in terms of multiple ARCS criteria. The questionnaire has been coded to reflect this to the best of the authors' capabilities. Questions coded to more than one criteria are simply used for each one equally, used to generate an average score for each factor.

In addition, a number of questions have their scores inverted, meaning the final score is calculated as $5 - x$, where x is the students' initial response.

The resulting average scores are provided below and are shown in Figure 6:

- Attention (2 Questions): 4.14
- Relevance (3 Questions): 4.21
- Confidence (2 Questions): 4.14
- Satisfaction (2 Questions): 3.73

As mentioned above, some questions were removed from standard ARCS scoring. As shown below, these questions indicate some students desired visualizations of concepts more closely related to their own work, as well as more interactivity, which, given the relatively limited interactivity of traditional teaching tools, further affirms VR's value to teaching.

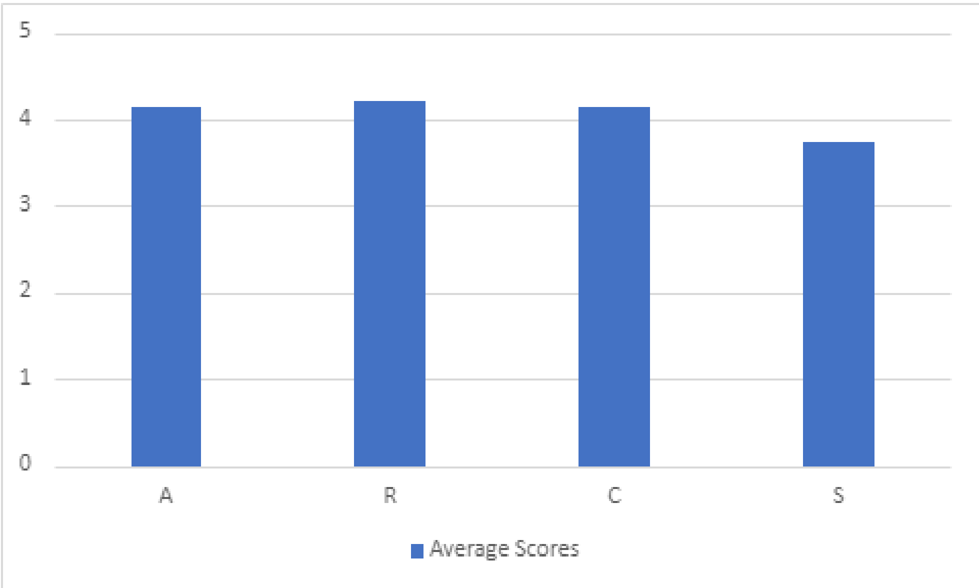

Figure 6. ARCS Criteria Scores of VR Application Questionnaire.

3.2.2. Data Elucidation

This section contains the precise results gathered from each question, rather than their scores on the ARCS framework, alongside demographic information of the participants:

1. Demographic Overview and Conceptual Struggles
   Our survey engaged 21 students with diverse backgrounds. The age range was 19 to 29 years, with a gender distribution of 18 males and 3 females. The racial diversity included 2 Asian, 12 White, 6 Hispanic, and 1 other, providing a broad spectrum of perspectives. Figure 7 illustrates this demographic diversity, which is crucial for understanding the varied impact of the VR Studio. The survey revealed that while Calculus II students often struggle with solids of revolution, Calculus III students

face difficulties with more generalized concepts. This includes challenges with the bounds of triple integrals and visualizing spherical coordinates—areas where VR can offer visual aids. Students' feedback underscores the necessity for tools to aid these complex topics. The accompanying bar graph Figure 8 illustrates the survey results, clearly representing the students' responses and preferences. In our survey, a response score of 1 or 2 indicates low agreement or satisfaction, while a score of 4 or 5 indicates high agreement or satisfaction with the evaluated aspects.

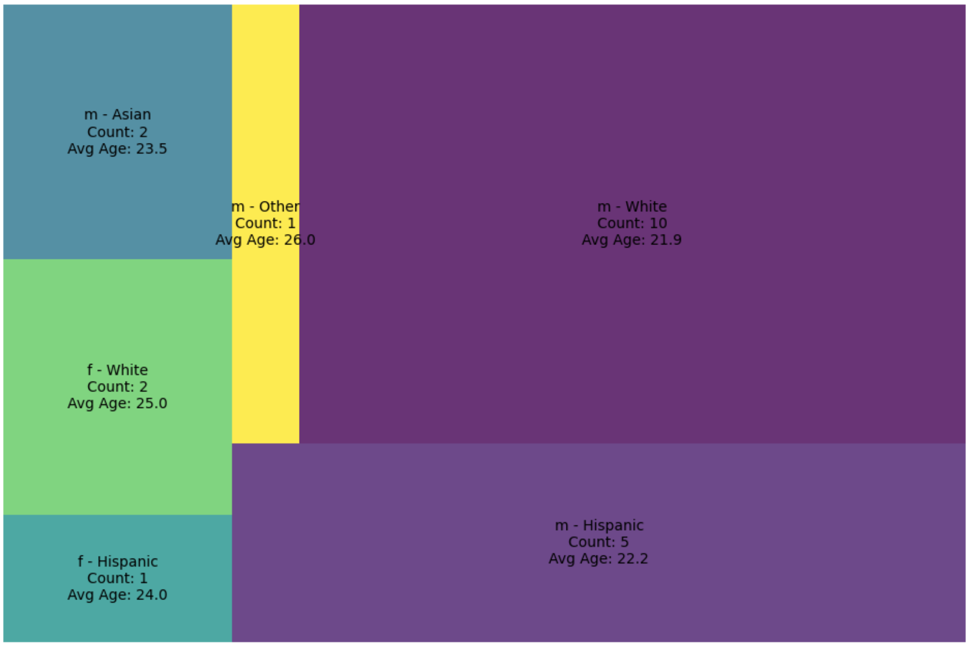

Figure 7. Demographic overview: Treemap of Participants by Gender and Race with Age Gradient.

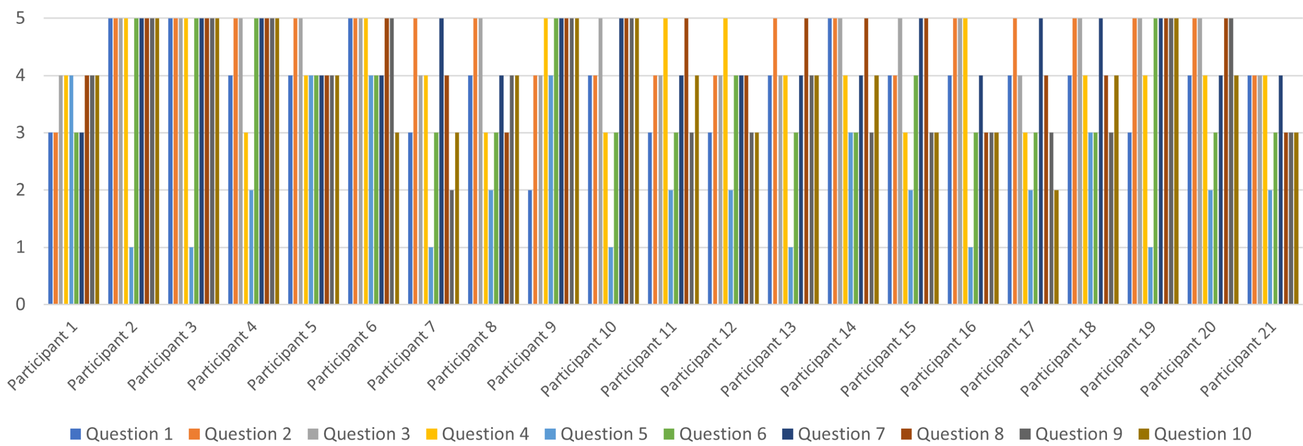

Figure 8. Survey Results: Student Feedback on VR Studio for Advanced Calculus Concepts.

2. General Feedback and Experience
    - Understanding of Solids of Revolution (Q1): 15 of 21 students agreed (score of 4 or 5) that the VR studio helped them understand the solids of revolution. In comparison, only 1 disagreed (score of 1 or 2), indicating a high level of effectiveness in this area.
    - Usefulness as an Augmented Reality Application (Q4): 16 of 21 of students saw the VR studio as beneficial when considering an Augmented Reality application, suggesting an interest in AR technologies for educational purposes.

3. Educational Impact
    - Classroom Utility of VR Studio (Q2): A significant 20 of 21 of students found the VR studio useful in the classroom setting, demonstrating its strong educational value.
    - Value as a Study Tool (Q3): The VR studio was unanimously valued as a study tool, all students agreeing on its usefulness, highlighting its potential as a learning aid.
4. Comparison with Traditional Methods
    - Intuitive Grasp Compared to Conventional Methods (Q7): The VR studio was seen as more intuitive than conventional methods by 20 of 21 students, emphasizing its effectiveness in facilitating understanding.
5. Feedback on Specific Methods, Concepts, and Suggestions
    - Calculus II students highlighted difficulties with the washer method, suggesting a need for visualization tools for plotting multiple graphs. Calculus III students expressed the need for visual aids for triple integrals and spherical coordinates, with suggestions for using distinct colors to differentiate integral bounds.
    - In the survey, students strongly preferred specific features in the VR studio: the ability to manipulate graphs freely, visual enhancements for clearer axis visibility and graph focus, and the functionality to overlay multiple graphs, likely due to usefulness for methods like the washer method in a VR setting. These features align well with planned updates, indicating their relevance and potential impact on learning.
6. Suggestions for Improvement and Reported Issues
    - Potential Enhancements to the VR Studio (Q6): 9 of 21 students suggested potential enhancements to the VR studio, providing valuable feedback for further development.
    - Bugs or Issues Encountered (Q11): Responses to this question were primarily open-ended, providing specific feedback on technical aspects and user experience. As stated in the methods section, any minor issues encountered with the Quest hardware were addressed immediately. This proactive approach ensured that hardware limitations did not significantly hinder the study's results.
7. Enthusiasm for AR/VR Tools
    - Despite varying levels of struggle with visualization, there was strong interest in engaging with AR tools (scoring 3+ on multiple questions)., indicating excitement about the potential of AR in education.
8. Additional Applications and Interactivity Preferences
    - Applicability of VR to Other Subjects (Q8): The survey results revealed a notable interest in extending VR applications beyond mathematics, with 18 of 21 of students recognizing its potential across various subjects. Physics, particularly the 3D visualization of electric and magnetic fields, emerged as a frequent area of interest. Computer Science topics, including neural networks and trees, were also highlighted as areas ripe for VR integration. Furthermore, students' comments underscored the benefits of visualizing spherical coordinates within the VR environment, illustrating its versatility and applicability as a multifaceted educational tool.
    - Desire for More Direct Interactivity (Q9): 12 of 21 students expressed a desire for more direct interactivity in the VR studio, suggesting areas for improvement in user engagement.
    - Preference for Intricate Visualizations (Q10): 14 of 21 students preferred intricate visualizations, highlighting a demand for more complex and detailed graphical representations in educational tools.

9. Health Concerns and Opt-out Reasons
    - Discomfort or Negative Effects Experienced (Q5): Only 4 of 21 students reported experiencing discomfort or negative effects, while a significant 15 of 21 did not face any such issues, indicating good user comfort overall.
10. Revisiting Concepts with VR
    - Students clearly preferred VR tools that allow revisiting mathematical concepts, indicating their potential for reinforcing learning and appealing across different learning styles.
11. General Interest and the Quest for More
    - The feedback reflects a general interest in more complex concepts over basic interactivity, with students eager for tools to help them grasp more challenging subjects.

4. Discussion

The research introduces an immersive and interactive VR graphing tool to improve the comprehension of calculus concepts, especially in 3D visualization. This section highlights the promising outcomes and ethical considerations stemming from introducing our immersive and interactive VR graphing tool designed to enhance the comprehension of calculus concepts, particularly in 3D visualization. We then delve into the study's limitations, addressing areas for potential improvement. Following this, we transition to discussing incorporating student-suggested features as future directions, underlining our commitment to aligning the tool with students' needs and preferences. Finally, we will detail our Contributions to Educational Practice, showcasing the impact of our research on educational methodologies and student learning experiences.

1. Promising Outcomes and Ethical Considerations: Despite a few bugs and potential flaws, the preliminary feedback from students who interacted with the VR tool has been overwhelmingly positive. This positivity encourages further exploration and development of the project, ultimately moving it into a meaningful, course-based environment. Importantly, ethical considerations were diligently addressed, and the study received approval from the University's Institutional Review Board (IRB), ensuring adherence to the highest research ethics standards.
2. Addressing Study Limitations: It is essential to acknowledge certain limitations within the current study:
    - The source of the subjects may have introduced a potential conflict of interest, as some participants may have had pre-existing associations with the authors. This influence could have impacted the subjects' responses.
    - Some participants may have been primarily intrigued by the concept of XR graphing tools rather than critically evaluating their suitability for the classroom environment. The study's small sample size, consisting mainly of students from STEM disciplines, particularly Computer Science, necessitates further investigation to generalize the results to a broader population.
    - Due to the informal nature of the questionnaire, the study should be regarded as an indicator of student engagement and motivation rather than a quantitative analysis of student outcomes.
    - Given the limited number of respondents, this study serves as a strictly exploratory measure, and a much larger study of similar tools must be performed before one may be implemented in a classroom environment.
    - Due to the extremely early status of the visualization tool, these results are more indicative of general excitement and enhanced intuition, rather than a direct measurement of student outcomes. A far more extensive array of features is required for gathering such data.

- The open-ended nature of some questions makes a systemic approach difficult, and may hinder reproducibility; exact feature requests, attitudes on interactivity and complexity, etc. may vary from student to student and organization to organization.
- Given the strictly VR-based implementation, this study cannot provide any information on an AR implementation. This is despite the relative practicality of such a paradigm in a classroom environment.

3. Incorporation of Student-Suggested Features as Future Directions: The feedback and suggestions from students who interacted with the VR graphing tool are invaluable. Prioritized enhancements focus on expanding the tool's capabilities with advanced graphical features, interactive simulations, and comprehensive scientific visualizations. This feedback guides immediate improvements and informs the long-term vision of creating a comprehensive AR platform for advanced calculus education. The positive response from students underscores the tool's potential to revolutionize traditional learning methods, aligning closely with student needs and preferences.
4. Contributions to Educational Practice:
As presented in our study, the innovative application of AR and VR technologies showcases a transformative approach to calculus education, emphasizing the importance of immersive learning experiences. The utilization of these technologies has demonstrated a marked improvement in students' comprehension of complex calculus concepts, particularly through enhanced 3D visualization and interactive learning environments [2,7,10]. This study reveals the immediate benefits, such as increased student engagement and understanding, and lays the groundwork for future educational practices. It underscores the potential for AR and VR technologies to revolutionize traditional teaching methodologies by offering more personalized and engaging learning experiences. Furthermore, students' positive feedback and suggestions highlight the demand for technology-enhanced educational tools, suggesting a shift towards more interactive and visually enriched learning landscapes. Our research contributes to educational practice by providing evidence of the effectiveness of immersive technologies in improving educational outcomes and suggesting pathways for integrating these innovations into mainstream educational settings.

Our research aligns with the extensive evidence supporting AR/VR's benefits in higher education, as documented in the literature. Consistent with studies submitted to the Web of Science in 2019 [3] and findings within the Social Science Citation Index focused on AR in Science or Mathematics Education [4], our findings affirm the positive impact of VR on student engagement, information retention, and comprehension of complex concepts [5,6]. In addition, studies explicitly using the ARCS framework display similar results of high scores on all 4 criteria [1,7], as well as similar preliminary studies on students' perceptions and attitudes[9]. Moreover, our study echoes previous research on increased student motivation and improved outcomes, highlighting VR's role in enhancing educational practices [1,2]. This comparison underscores our contribution to understanding AR/VR's transformative potential in education.

5. Conclusions

The VR graphing tool introduced in this study signifies a pivotal advancement in the teaching and learning of calculus, emphasizing 3D visualization's critical role. By incorporating student feedback, acknowledging limitations, and adhering to ethical research practices, this tool sets the stage for future developments and broader applications. As it progresses towards a more generalized AR platform, the tool promises to enrich educational experiences across STEM fields. It fosters deeper engagement and paves the way for innovative learning pathways to contribute to a more dynamic and effective educational landscape and spur continuous research and development in educational technology. This endeavor represents a significant step towards creating a versatile educational tool that transcends the limitations of isolated visualizations. The goal is to develop it into an interactive, AR-enabled platform that supports a broad spectrum of mathematical and scientific concepts. This vision is rooted in constructivist pedagogy principles, prioritizing collaboration, interaction, and tailored learning experiences. The tool encourages student-led knowledge discovery and exploration by aiming to exceed traditional instructional methods, thereby promoting a more engaging and comprehensive learning environment.

Author Contributions: The contributions to this research by the authors are detailed as follows: Conceptualization was primarily handled by M.F.H.S. and L.P. developed and implemented the methodology. M.F.H.S. conducted the validation of the results. Both L.P. and M.F.H.S. were involved in the formal analysis. M.F.H.S. organized and managed the resources. Data curation was a joint effort by L.P. and M.F.H.S.; L.P. prepared the original draft of the manuscript, while the review and editing were overseen by M.F.H.S. and L.P. led the visualization. Supervision of the entire project was conducted by M.F.H.S. All authors have read and agreed to the published version of the manuscript. These roles are in accordance with the CRediT taxonomy and reflect substantial contributions to the work reported.

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Institutional Review Board Statement: The study was conducted in accordance with the Declaration of Helsinki and approved by the Institutional Review Board of West Texas A&M University. The approval, under protocol code #2023.10.006, was granted on 31 October 2023, and is valid for one calendar year, expiring on 31 October 2024. This approval includes responsibilities for the principal investigators regarding continuing review, completion report, unanticipated problems, potential non-compliance, amendments, consent forms, audit, recruitment, and compliance with FERPA and PPRA regulations.

Informed Consent Statement: Informed consent was obtained from all subjects involved in the study.

Data Availability Statement: The data presented in this study are available on request from the corresponding author.

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Abbreviations

The following abbreviations are used in this manuscript:

VR	Virtual Reality
AR	Augmented Reality
ARCS	Attention, Relevance, Confidence, Satisfaction
STEM	Science, Technology, Engineering, and Mathematics
IRB	Institutional Review Board
XR	Extended Reality
UI	User Interface
HMD	Head Mounted Displays
WTAMU	West Texas A&M University

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