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Jafari Transformation for Solving a System of Ordinary Differential Equations with Medical Application

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Abstract: Integral transformations are essential for solving complex problems in business, engineering, natural sciences, computers, optical science, and modern mathematics. In this paper, we apply a general integral transform, called the Jafari transform, for solving a system of ordinary differential equations. After applying the Jafari transform, ordinary differential equations are converted to a simple system of algebraic equations that can be solved easily. Then, by using the inverse operator of the Jafari transform, we can solve the main system of ordinary differential equations. Jafari transform belongs to the class of Laplace transform and is considered a generalization to integral transforms such as Laplace, Elzaki, Sumudu, G_transforms, Aboodh, Pourreza, etc. Jafari transform does not need a large computational work as the previous integral transforms. For the Jafari transform, we have studied some valuable properties and theories that have not been studied before. Such as the linearity property, scaling property, first and second shift properties, the transformation of periodic functions, Heaviside function, and the transformation of Dirac's delta function, and so on. There is a mathematical model that describes the cell population dynamics in the colonic crypt and colorectal cancer. We have applied the Jafari transform for solving this model.

Keywords: Laplace transform; Jafari transform; inverse Jafari transform; ordinary differential equations

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1. Introduction

Integral transformations have been successfully applied for solving many problems in engineering science, applied mathematics, and mathematical physics for almost two centuries. The history of integral transformations goes back to the monumental work of Joseph Fourier (1768–1830) in 1822 and to the renowned work of P. S. Laplace (1749–1827) on probability theory in the 1780s. Integral transforms introduce powerful methods for solving integral equations and differential equations. The Laplace transform is the most commonly used integral transform in the mathematical literature. Fourier introduced the theory of Fourier series, heat conduction, and Fourier integrals with many applications. The role of the integral transforms is to map a function from its original space into a new space by integration. The properties of the original function in the new space might be more easily manipulated than in the original space.

Integral transforms, as known, solve the differential equations by converting these equations to algebraic equations. As a result, these algebraic equations can be solved easily. Of course, the solution of these algebraic equations is considered a transform of the solution of the original differential equations. To complete the solution, this transform must be inverted [1–6]. In the class of Laplace transform, the senior researchers introduced many integral transforms during the last two decades, such as Natural, Sumudu, Aboodh, Elzaki, Pourreza, G_transform, Mohand, Kamal, and Sawi transform [7–19].

Integral transforms can be used to solve several types of ordinary differential equations (ODEs), integral equations, partial differential equations (PDEs), and fractional-order differential equations (FDEs) [20–28]. These transforms also can be coupled with the Adomian decomposition and the homotopy perturbation methods to solve complicated types of ODEs, PDEs, and FDEs [29–34]. Aggarwal et al. [35–37] solved several problems using the Laplace transformation. In [38], the authors presented the application of Laplace transformation in cryptography. Fatoorehchi et al. proposed a nonlinear differential equations solution based on a novel extension of the Laplace transformation [39]. Higazy et al. [40] solved the HIV-1 infections model by the Shehu transform. The authors of [41] used the Sawi decomposition method for solving the Volterra integral equation. A modified differential transform method has been applied for solving the vibration equations of MDOF systems [42]. Higazy et al. [19] used the Sawi transformation to find the exact solution of ODEs.

This paper aims to find the solution of the system of ordinary differential equations (SODEs) using a new integral transformation [1], we have called it Jafari transformation. SODEs can be used to describe many real-world problems such as the problem of the three-layer beam, electrical circuits, chain of chemical reactions, control of a flying apparatus in cosmic space, mixing growth of species, and mechanical vibration. This motivated us to study and prove some valuable properties and theories of Jafari transformation that have not been checked and confirmed before, such as linearity property, scaling property, first and second shift properties, the transformation of periodic functions, the transformation of Heaviside function, the transformation of Dirac's delta function, and so on in Sections 2–9. In Section 10, we have discussed the solution of SODEs using the Jafari integral transformation. Section 11 has solved a mathematical model describing the cell population dynamics in the colonic crypt and colorectal cancer using the Jafari integral transformation. Finally, in Section 12, the conclusions of this paper are introduced. Now, let us start with the definition of the Jafari transform.

2. Definition of Jafari Transform

The Jafari transform of the function $v(t), t \geq 0, h(\sigma) \neq 0$ and $g(\sigma)$ being positive real functions, is given by [1]

$$\mathcal{J}\{v(t)\} = h(\sigma)\int_0^\infty v(t)e^{-g(\sigma)t}dt = \mathcal{R}(h(\sigma), g(\sigma)), \tag{1}$$

where the integral exists for some $g(\sigma)$. It must be noted that the Jafari transform (1) for those $v(t)$, which are not continuously differentiable, contains terms with negative or fractional powers of $g(\sigma)$.

Suppose that for all $t \geq 0$, the function $v(t)$ is piecewise continuous and satisfies $|v(t)| \leq Me^{\mu t}$, then $\mathcal{R}(h(\sigma), g(\sigma))$ exists for all $g(\sigma) > \mu$.

Since

$$\begin{aligned} ||\mathcal{R}(h(\sigma), g(\sigma))|| = |h(\sigma)\textstyle\int_0^\infty v(t)e^{-g(\sigma)t}dt| &\leq h(\sigma)\textstyle\int_0^\infty |v(t)|e^{-g(\sigma)t}dt \\ &\leq h(\sigma)\textstyle\int_0^\infty Me^{\mu t}e^{-g(\sigma)t}dt \leq \frac{h(\sigma)M}{\mu - g(\sigma)}, \end{aligned}$$

the statement is valid.

3. Some Essential Characteristics of Jafari Transform

In this section, we introduce some useful characteristics of the Jafari transform.

3.1. Linearity of Jafari Transform

Theorem 1. *If* $\mathcal{J}\{v_1(t)\} = \mathcal{R}_1(h(\sigma), g(\sigma))$ *and* $\mathcal{J}\{v_2(t)\} = \mathcal{R}_2(h(\sigma), g(\sigma))$, *then* $\mathcal{J}\{\alpha v_1(t) + \beta v_2(t)\} = \alpha\mathcal{R}_1(h(\sigma), g(\sigma)) + \beta\mathcal{R}_2(h(\sigma), g(\sigma))$, *where* α, β *are arbitrary constants.*

Proof of Theorem 1. From the definition of Jafari transform, we have

$$\begin{aligned}
&\mathcal{J}\{\alpha v_1(t)+\beta v_2(t)\} = h(\sigma)\int_0^\infty [\alpha v_1(t)+\beta v_2(t)]e^{-g(\sigma)t}dt \\
&\Rightarrow \mathcal{J}\{\alpha v_1(t)+\beta v_2(t)\} = \alpha[h(\sigma)\int_0^\infty v_1(t)e^{-g(\sigma)t}dt] + \beta[h(\sigma)\int_0^\infty v_2(t)e^{-g(\sigma)t}dt] \\
&\Rightarrow \mathcal{J}\{\alpha v_1(t)+\beta v_2(t)\} = \alpha\mathcal{J}\{v_1(t)\} + \beta\mathcal{J}\{v_2(t)\} \\
&\Rightarrow \mathcal{J}\{\alpha v_1(t)+\beta v_2(t)\} = \mathcal{R}_1(h(\sigma),g(\sigma)) + \mathcal{R}_2(h(\sigma),g(\sigma)). \ \square
\end{aligned}$$

3.2. Scaling Property of Jafari Transform

Theorem 2. *If* $\mathcal{J}\{v(t)\} = \mathcal{R}(h(\sigma),g(\sigma))$*, then* $\mathcal{J}\{v(\lambda t)\} = \frac{1}{\lambda}\mathcal{R}(h(\sigma),\frac{g(\sigma)}{\lambda})$ *with* $\lambda \neq 0$.

Proof of Theorem 2. From the definition of Jafari transform, we have

$$\mathcal{J}\{v(\lambda t)\} = h(\sigma)\int_0^\infty v(\lambda t)e^{-g(\sigma)t}dt.$$

Let $\lambda t = x \Rightarrow \lambda dt = dx$, then

$$\begin{aligned}
&\mathcal{J}\{v(\lambda t)\} = \tfrac{1}{\lambda}[h(\sigma)\int_0^\infty v(x)e^{-g(\sigma)(\frac{x}{\lambda})}dx] \\
&\Rightarrow \mathcal{J}\{v(\lambda t)\} = \tfrac{1}{\lambda}[h(\sigma)\int_0^\infty v(x)e^{-(\frac{g(\sigma)}{\lambda})x}dx] = \tfrac{1}{\lambda}\mathcal{R}(h(\sigma),\tfrac{g(\sigma)}{\lambda}). \ \square
\end{aligned}$$

3.3. First Shift Property of Jafari Transform

Theorem 3. *If* $\mathcal{J}\{v(t)\} = \mathcal{R}(h(\sigma),g(\sigma))$*, then* $\mathcal{J}\{e^{\lambda t}v(t)\} = \mathcal{R}(h(\sigma),g(\sigma)-\lambda)$.

Proof of Theorem 3. From the definition of Jafari transform, we have

$$\begin{aligned}
&\mathcal{J}\{e^{\lambda t}v(t)\} = h(\sigma)\int_0^\infty e^{\lambda t}v(t)e^{-g(\sigma)t}dt = h(\sigma)\int_0^\infty v(t)e^{-(g(\sigma)-\lambda)t}dt \\
&\Rightarrow \mathcal{J}\{e^{\lambda t}v(t)\} = \mathcal{R}(h(\sigma),g(\sigma)-\lambda). \ \square
\end{aligned}$$

3.4. Second Shift Property of Jafari Transform

Theorem 4. *If* $\mathcal{J}\{v(t)\} = \mathcal{R}(h(\sigma),g(\sigma))$*, then*

$$\mathcal{J}\{v(t-\lambda)H(t-\lambda)\} = e^{-\lambda g(\sigma)}\mathcal{R}(h(\sigma),g(\sigma)).$$

Proof of Theorem 4. From the definition of Jafari transform, we have

$$\mathcal{J}\{v(t-\lambda)H(t-\lambda)\} = h(\sigma)\int_0^\infty v(t-\lambda)H(t-\lambda)e^{-g(\sigma)t}dt = h(\sigma)\int_\lambda^\infty v(t-\lambda)e^{-g(\sigma)t}dt,$$

let $t-\lambda = x \Rightarrow dt = dx$, then

$$\begin{aligned}
&\mathcal{J}\{v(t-\lambda)H(t-\lambda)\} = h(\sigma)\int_0^\infty v(x)e^{-g(\sigma)(x+\lambda)}dx \\
&\Rightarrow \mathcal{J}\{v(t-\lambda)H(t-\lambda)\} = e^{-\lambda g(\sigma)}[h(\sigma)\int_0^\infty v(x)e^{-g(\sigma)x}dx] \\
&\Rightarrow \mathcal{J}\{v(t-\lambda)H(t-\lambda)\} = e^{-\lambda g(\sigma)}\mathcal{R}(h(\sigma),g(\sigma)). \ \square
\end{aligned}$$

4. Jafari Transform of Periodic Functions

A periodic function is sectionally continuous and for some $\lambda > 0$, satisfies

$$v(t) = v(t+\lambda) = v(t+2\lambda) = v(t+3\lambda) = \cdots = v(t+n\lambda).$$

We can write the Jafari transform of $v(t)$ as the series of integrals

$$\mathcal{J}\{v(t)\} = h(\sigma)\int_0^\infty v(t)e^{-g(\sigma)t}dt = h(\sigma)\int_0^\lambda v(t)e^{-g(\sigma)t}dt + h(\sigma)\int_\lambda^{2\lambda} v(t)e^{-g(\sigma)t}dt$$
$$+h(\sigma)\int_{2\lambda}^{3\lambda} v(t)e^{-g(\sigma)t}dt + \cdots$$

For the second integral, put $t = x + \lambda$; for the third integral, put $t = x + 2\lambda$; for the fourth, put $t = x + 3\lambda$; etc.; then the limits on each integral are 0 and λ. Hence,

$$\mathcal{J}\{v(t)\} = h(\sigma)\int_0^\infty v(t)e^{-g(\sigma)t}dt = h(\sigma)[\int_0^\lambda v(t)e^{-g(\sigma)t}dt + \int_0^\lambda v(x+\lambda)e^{-g(\sigma)(x+\lambda)}dx$$
$$+\int_0^\lambda v(x+2\lambda)e^{-g(\sigma)(x+2\lambda)}dx + \int_0^\lambda v(x+3\lambda)e^{-g(\sigma)(x+3\lambda)}dx + \cdots]$$
$$\Rightarrow \mathcal{J}\{v(t)\} = h(\sigma)\int_0^\lambda v(t)e^{-g(\sigma)t}dt + e^{-g(\sigma)\lambda}[h(\sigma)\int_0^\lambda v(x+\lambda)e^{-g(\sigma)x}dx]$$
$$+e^{-2g(\sigma)\lambda}[h(\sigma)\int_0^\lambda v(x+2\lambda)e^{-g(\sigma)x}dx] + \cdots$$

The dummy variable of integration x can be set equal to t, and with the use of

$$v(t) = v(t+\lambda) = v(t+2\lambda) = v(t+3\lambda) = \cdots = v(t+n\lambda).$$
$$\Rightarrow \mathcal{J}\{v(t)\} = h(\sigma)\int_0^\lambda v(t)e^{-g(\sigma)t}dt + e^{-g(\sigma)\lambda}[h(\sigma)\int_0^\lambda v(t+\lambda)e^{-g(\sigma)t}dt]$$
$$+e^{-2g(\sigma)\lambda}[h(\sigma)\int_0^\lambda v(t+2\lambda)e^{-g(\sigma)t}dt] + \cdots$$
$$\Rightarrow \mathcal{J}\{v(t)\} = h(\sigma)\int_0^\lambda v(t)e^{-g(\sigma)t}dt + e^{-g(\sigma)\lambda}[h(\sigma)\int_0^\lambda v(t)e^{-g(\sigma)t}dt]$$
$$+e^{-2g(\sigma)\lambda}[h(\sigma)\int_0^\lambda v(t)e^{-g(\sigma)t}dt] + \cdots$$
$$\Rightarrow \mathcal{J}\{v(t)\} = [1 + e^{-g(\sigma)\lambda} + e^{-2g(\sigma)\lambda} + e^{-3g(\sigma)\lambda} + \cdots][h(\sigma)\int_0^\lambda v(t)e^{-g(\sigma)t}dt].$$

Since

$$\frac{1}{1-e^{-g(\sigma)\lambda}} = 1 + e^{-g(\sigma)\lambda} + e^{-2g(\sigma)\lambda} + e^{-3g(\sigma)\lambda} + \cdots$$
$$\Rightarrow \mathcal{J}\{v(t)\} = \frac{1}{1-e^{-g(\sigma)\lambda}}[h(\sigma)\int_0^\lambda v(t)e^{-g(\sigma)t}dt].$$

5. Jafari Transform of Heaviside Function

$$\mathcal{J}\{H(t-\lambda)\} = h(\sigma)\int_0^\infty H(t-\lambda)e^{-g(\sigma)t}dt = h(\sigma)\int_\lambda^\infty e^{-g(\sigma)t}dt = \frac{h(\sigma)}{g(\sigma)}e^{-g(\sigma)\lambda}.$$

6. Jafari Transform of Dirac's Delta Function

Schwartz space S is the function space of all functions whose derivatives are rapidly decreasing.

The Dirac's delta function δ is defined as follows [43]

$(\delta(t-\lambda), v(t)) = \int_{-\infty}^\infty \delta(t-\lambda)v(t)dt = v(\lambda)$, for any v in the Schwartz space. Then, the Jafari transform for $\delta(t-\lambda)v(t)$ is

$$\mathcal{J}\{\delta(t-\lambda)v(t)\} = h(\sigma)\int_0^\infty \delta(t-\lambda)v(t)e^{-g(\sigma)t}dt = h(\sigma)\int_0^\infty \delta(t-\lambda)v(\lambda)e^{-g(\sigma)\lambda}dt$$
$$= h(\sigma)v(\lambda)e^{-g(\sigma)\lambda}\int_0^\infty \delta(t-\lambda)dt = h(\sigma)v(\lambda)e^{-g(\sigma)\lambda}\int_{-\infty}^\infty \delta(t-\lambda)dt = h(\sigma)v(\lambda)e^{-g(\sigma)\lambda},$$
$$\int_{-\infty}^\infty \delta(t-\lambda)dt = 1.$$

If $v(t) = 1$, then $\mathcal{J}\{\delta(t-\lambda)v(t)\} = \mathcal{J}\{\delta(t-\lambda)\} = h(\sigma)e^{-g(\sigma)\lambda}$.

7. Jafari Transform of Derivatives of the Function

Theorem 5. *Suppose $v(t)$ is differentiable, $h(\sigma)$ and $g(\sigma)$ are positive real functions, then*

$$\mathcal{J}\{v^{(n)}(t)\} = [g(\sigma)]^n\mathcal{J}\{v(t)\} - h(\sigma)\sum_{i=0}^{n-1}[g(\sigma)]^{n-i-1}v^{(i)}(0); n \geq 1.$$

Proof of Theorem 5. From the definition of Jafari transform, we firstly have

$$\begin{aligned}
&\mathcal{J}\{v'(t)\} = h(\sigma)\int_0^\infty v'(t)e^{-g(\sigma)t}dt \\
&\Rightarrow \mathcal{J}\{v'(t)\} = h(\sigma)[v(t)e^{-g(\sigma)t}]_0^\infty + g(\sigma)h(\sigma)\int_0^\infty v(t)e^{-g(\sigma)t}dt \\
&\Rightarrow \mathcal{J}\{v'(t)\} = h(\sigma)[\lim_{t\to\infty} v(t)e^{-g(\sigma)t}] - h(\sigma)v(0) + g(\sigma)h(\sigma)\int_0^\infty v(t)e^{-g(\sigma)t}dt \\
&\Rightarrow \mathcal{J}\{v'(t)\} = g(\sigma)\mathcal{J}\{v(t)\} - h(\sigma)v(0), \text{ where } \lim_{t\to\infty} v(t)e^{-g(\sigma)t} = 0.
\end{aligned}$$

Secondly, since

$$\begin{aligned}
&\mathcal{J}\{v'(t)\} = g(\sigma)\mathcal{J}\{v(t)\} - h(\sigma)v(0) \\
&\Rightarrow \mathcal{J}\{v''(t)\} = g(\sigma)\mathcal{J}\{v'(t)\} - h(\sigma)v'(0) \\
&\Rightarrow \mathcal{J}\{v''(t)\} = g(\sigma)\{g(\sigma)\mathcal{J}\{v(t)\} - h(\sigma)v(0)\} - h(\sigma)v'(0) \\
&\Rightarrow \mathcal{J}\{v''(t)\} = [g(\sigma)]^2\mathcal{J}\{v(t)\} - h(\sigma)[g(\sigma)v(0) + v'(0)] \\
&\Rightarrow \mathcal{J}\{v'''(t)\} = [g(\sigma)]^2\mathcal{J}\{v'(t)\} - h(\sigma)[g(\sigma)v'(0) + v''(0)] \\
&\Rightarrow \mathcal{J}\{v'''(t)\} = [g(\sigma)]^3\mathcal{J}\{v(t)\} - h(\sigma)[[g(\sigma)]^2v(0) + g(\sigma)v'(0) + v''(0)].
\end{aligned}$$

By induction, we can deduce that

$$\mathcal{J}\{v^{(n)}(t)\} = [g(\sigma)]^n\mathcal{J}\{v(t)\} - h(\sigma)\sum_{i=0}^{n-1}[g(\sigma)]^{n-i-1}v^{(i)}(0); n \geq 1. \square$$

Jafari transforms for some important basic functions, which are utilized for finding the solution of important problems in engineering and sciences, are given in Table 1.

Table 1. Jafari and inverse Jafari transforms of some basic functions.

$v(t) = \mathcal{J}^{-1}[\mathcal{R}(h(\sigma), g(\sigma))]$	$\mathcal{J}\{v(t)\} = \mathcal{R}(h(\sigma), g(\sigma))$
1	$h(\sigma)/g(\sigma)$
$e^{\lambda t}$	$\frac{h(\sigma)}{g(\sigma)-\lambda}, g(\sigma) > \lambda$
$t^k, k > 0$	$\frac{\Gamma[k+1]h(\sigma)}{[g(\sigma)]^{k+1}}$
$\sin kt$	$\frac{kh(\sigma)}{[g(\sigma)]^2+k^2}$
$\cos kt$	$\frac{g(\sigma)h(\sigma)}{[g(\sigma)]^2+k^2}$
$\sinh kt$	$\frac{kh(\sigma)}{[g(\sigma)]^2-k^2}$
$\cosh kt$	$\frac{g(\sigma)h(\sigma)}{[g(\sigma)]^2-k^2}$
$e^{\lambda t}\sin kt$	$\frac{kh(\sigma)}{[g(\sigma)-\lambda]^2+k^2}$
$e^{\lambda t}\cos kt$	$\frac{[g(\sigma)-\lambda]h(\sigma)}{[g(\sigma)-\lambda]^2+k^2}$
$e^{\lambda t}\sinh kt$	$\frac{kh(\sigma)}{[g(\sigma)-\lambda]^2-k^2}$
$e^{\lambda t}\cosh kt$	$\frac{[g(\sigma)-\lambda]h(\sigma)}{[g(\sigma)-\lambda]^2-k^2}$
$\delta(t-\lambda)$	$h(\sigma)e^{-\lambda g(\sigma)}$
$H(t-\lambda)$	$\frac{h(\sigma)}{g(\sigma)}e^{-g(\sigma)\lambda}$

Table 1. *Cont.*

$v(t-\lambda)H(t-\lambda)$	$e^{-\lambda g(\sigma)}\mathcal{J}\{v(t)\}$
$v(t)=v(t+n\lambda), n=1,2,3,\ldots$	$\frac{h(\sigma)}{1-e^{-g(\sigma)\lambda}}\int_0^\lambda v(t)e^{-g(\sigma)t}dt$
$v^{(n)}(t),\ n\geq 1$	$[g(\sigma)]^n\mathcal{J}\{v(t)\}-$ $h(\sigma)\sum_{i=0}^{n-1}[g(\sigma)]^{n-i-1}v^{(i)}(0).$

8. The Advantages of Jafari Transform

In this section, we show some advantages of the Jafari transform, as follows:

- All classes of integral transforms are covered by the Jafari transform. Hence, all the transforms in the class of Laplace transform, introduced during the last few decades, are a special case of the Jafari transform.
- Jafari transform can be applied for solving the ODEs with constant and variable coefficients. Further, it can be applied easily for solving the fractional-order differential equations and fractional-order integral equations.
- From the definition of the Jafari transform, several new integral transforms can be defined by choosing new forms for $h(\sigma)$ and $g(\sigma)$.

For the integral transforms, it should be noted that there are no advantages between these transforms unless for special problems. Let us show that in the following example.

Example 1. *Consider the following equation*

$$\alpha t^2 v''(t)+\beta t v'(t)+\gamma v(t)=\delta t^m, m\in\mathbb{N}, v(0)=v'(0)=0, \tag{2}$$

where α,β,γ,and δ are constants.

Applying the Jafari transform on both sides of the above equation gives

$$\begin{aligned}
&\alpha(h(\sigma)/g'(\sigma))\tfrac{d}{d\sigma}[(1/g'(\sigma))(\tfrac{d}{d\sigma}(\tfrac{1}{h(\sigma)}\mathcal{J}\{v''(t)\}))]\\
&-\beta(h(\sigma)/g'(\sigma))\tfrac{d}{d\sigma}[\tfrac{1}{h(\sigma)}\mathcal{J}\{v'(t)\}]+\gamma\mathcal{J}\{v(t)\}=\delta\mathcal{J}\{t^m\},\\
&\alpha(h(\sigma)/g'(\sigma))\tfrac{d}{d\sigma}[(1/g'(\sigma))(\tfrac{d}{d\sigma}(\tfrac{1}{h(\sigma)}g^2(\sigma)\mathcal{R}(\sigma)))]\\
&-\beta(h(\sigma)/g'(\sigma))\tfrac{d}{d\sigma}[\tfrac{1}{h(\sigma)}g(\sigma)\mathcal{R}(\sigma)]+\gamma R(\sigma)=\Gamma[m+1]\delta\tfrac{h(\sigma)}{[g(\sigma)]^{m+1}},
\end{aligned} \tag{3}$$

where $\mathcal{R}(\sigma)=\mathcal{R}(h(\sigma),g(\sigma))$ and

$$\mathcal{J}\{t^m v^{(n)}(t)\}=(-1)^m(h(\sigma)/g'(\sigma))\frac{d}{d\sigma}\left[\underbrace{(1/g'(\sigma))(\frac{d}{d\sigma}\cdots(1/g'(\sigma))(\frac{d}{d\sigma}}_{m\ times}(\frac{1}{h(\sigma)}\mathcal{J}\{v^{(n)}(t)\}))\ldots)\right]$$

with $h(\sigma), g(\sigma)$ and $v(t)$ are differentiable $(g'(\sigma)\neq 0)$, see [1].

Now, we find the transform of equation (3) for some integral transforms:

- Sawi transform $(h(\sigma)=\frac{1}{\sigma^2}, g(\sigma)=\frac{1}{\sigma})$ gives

$$\alpha\sigma^2\mathcal{R}''(\sigma)+\sigma(2\alpha+\beta)\mathcal{R}'(\sigma)+\mathcal{R}(\sigma)(\beta+\gamma)-\delta\,\Gamma[m+1]\sigma^{m-1}=0.$$

- Elzaki transform $(h(\sigma)=\sigma, g(\sigma)=\frac{1}{\sigma})$ gives

$$\alpha\sigma^2\mathcal{R}''(\sigma)+\sigma(\beta-4\alpha)\mathcal{R}'(\sigma)+\mathcal{R}(\sigma)(6\alpha-2\beta+\gamma)-\delta\,\Gamma[m+1]\sigma^{m+1}=0.$$

- Sumudu transform $(h(\sigma)=\frac{1}{\sigma}, g(\sigma)=\frac{1}{\sigma})$ gives

$$\alpha\sigma^2\mathcal{R}''(\sigma)+\beta\sigma\mathcal{R}'(\sigma)+\gamma\mathcal{R}(\sigma)-\delta\,\Gamma[m+1]\sigma^m=0.$$

- Laplace transform $(h(\sigma) = 1, g(\sigma) = \sigma)$ gives

$$\alpha\sigma^2\mathcal{R}''(\sigma) + \sigma(4\alpha - \beta)\mathcal{R}'(\sigma) + \mathcal{R}(\sigma)(2\alpha - \beta + \gamma) - \frac{\delta\,\Gamma[m+1]}{\sigma^{m+1}} = 0.$$

- Pourreza transform $(h(\sigma) = \sigma, g(\sigma) = \sigma^2)$ gives

$$\alpha\sigma^2\mathcal{R}''(\sigma) + \sigma(5\alpha - 2\beta)\mathcal{R}'(\sigma) + \mathcal{R}(\sigma)(3\alpha - 2\beta + 4\gamma) - 4\frac{\delta\,\Gamma[m+1]}{\sigma^{2m+1}} = 0.$$

It is clear that if the coefficients of $\mathcal{R}'(\sigma)$ and $\mathcal{R}(\sigma)$ are equal to zero in Elzaki, Pourreza and Laplace transforms, then we obtain a simple second-order differential equation that can be solved easily. For example, let $\alpha = 1, \beta = 4, \gamma = 2$, and $\delta = 12$, then the best choice is to apply the Laplace transform. We obtain $\mathcal{R}''(\sigma) = \frac{24}{\sigma^5}$, the solution of this equation is $v(t) = t^2$ (exact solution).

Hence, the Jafari transform is a helping tool for choosing the best integral transform for solving a certain ordinary differential equation. In Sections 10 and 11, we applied the Jafari transform for solving a SODEs with constant coefficients as a start to the topic, and a generalization to the integral transforms used for solving a SODEs [19,40,41]. Similarly, the Jafari transform advantages, presented in this section, can be exploited in solving several different SODEs.

9. Inverse Jafari Transform

The function $v(t)$ is called the inverse Jafari transform of the function $\mathcal{R}(h(\sigma), g(\sigma))$ if it verifies $\mathcal{J}\{v(t)\} = \mathcal{R}(h(\sigma), g(\sigma))$. Hence, we can write $v(t) = \mathcal{J}^{-1}[\mathcal{R}(h(\sigma), g(\sigma))]$. Inverse Jafari transforms for some important basic functions are given in Table 1. The Linearity of the inverse Jafari transform can be shown as follows.

If $\mathcal{J}^{-1}[\mathcal{R}_1(h(\sigma), g(\sigma))] = v_1(t)$ and $\mathcal{J}^{-1}[\mathcal{R}_2(h(\sigma), g(\sigma))] = v_2(t)$, then

$$\begin{aligned}
&\mathcal{J}^{-1}[\alpha\mathcal{R}_1(h(\sigma), g(\sigma)) + \beta R_2(h(\sigma), g(\sigma))] \\
&= \alpha\mathcal{J}^{-1}[\mathcal{R}_1(h(\sigma), g(\sigma))] + \beta\mathcal{J}^{-1}[\mathcal{R}_2(h(\sigma), g(\sigma))] \\
&\Rightarrow \mathcal{J}^{-1}[\alpha\mathcal{R}_1(h(\sigma), g(\sigma)) + \beta\mathcal{R}_2(h(\sigma), g(\sigma))] = \alpha v_1(t) + \beta v_2(t).
\end{aligned}$$

All the previous properties and theorems of Jafari transform and inverse Jafari transform are helping tools for solving a system of ordinary differential equations (SODEs). Further, we use the Cramer rule for solving the algebraic system of equations produced by applying the Jafari transform on the SODEs. Then, we apply the inverse Jafari transform to find the final solution of the SODEs.

10. Jafari Transform for First Order SODEs

Consider the following SODEs

$$\left.\begin{array}{c}
\psi_1{}'(t) = r_{11}\psi_1(t) + r_{12}\psi_2(t) + \cdots + r_{1k}\psi_k(t) + s_1(t) \\
\psi_2{}'(t) = r_{21}\psi_1(t) + r_{22}\psi_2(t) + \cdots + r_{2k}\psi_k(t) + s_2(t) \\
\vdots \\
\psi_k{}'(t) = r_{k1}\psi_1(t) + r_{k2}\psi_2(t) + \cdots + r_{kk}\psi_k(t) + s_k(t)
\end{array}\right\}, \tag{4}$$

with initial conditions

$$\psi_1(0) = c_1, \psi_2(0) = c_2, \ldots, \psi_k(0) = c_k. \tag{5}$$

By the matrix notation, the system (4) with (5) can be expressed as follows

$$\psi'(t) = R\psi(t) + S(t) \text{ with } \psi(0) = C, \tag{6}$$

where

$$\psi'(t)=\begin{bmatrix}\psi_1'(t)\\ \psi_2'(t)\\ \vdots\\ \psi_k'(t)\end{bmatrix}, R=\begin{bmatrix}r_{11} & r_{12} & \dots & r_{1k}\\ r_{21} & r_{22} & \dots & r_{2k}\\ \vdots & \vdots & \vdots & \vdots\\ r_{k1} & r_{k2} & \dots & r_{kk}\end{bmatrix}, \psi(t)=\begin{bmatrix}\psi_1(t)\\ \psi_2(t)\\ \vdots\\ \psi_k(t)\end{bmatrix}, S(t)=\begin{bmatrix}s_1(t)\\ s_2(t)\\ \vdots\\ s_k(t)\end{bmatrix}, \psi(0)=\begin{bmatrix}\psi_1(0)\\ \psi_2(0)\\ \vdots\\ \psi_k(0)\end{bmatrix},$$
$$C=\begin{bmatrix}c_1\\ c_2\\ \vdots\\ c_k\end{bmatrix}.$$

Now, by applying the Jafari transform on (4) and taking into consideration the initial conditions (5), we obtain

$$\left.\begin{array}{c}(g(\sigma)-r_{11})\mathcal{J}\{\psi_1(t)\}-r_{12}\mathcal{J}\{\psi_2(t)\}-\dots-r_{1k}\mathcal{J}\{\psi_k(t)\}=\mathcal{J}\{s_1(t)\}+c_1h(\sigma)\\ -r_{21}\mathcal{J}\{\psi_1(t)\}+(g(\sigma)-r_{22})\mathcal{J}\{\psi_2(t)\}-\dots-r_{2k}\mathcal{J}\{\psi_k(t)\}=\mathcal{J}\{s_2(t)\}+c_2h(\sigma)\\ \vdots\\ -r_{k1}\mathcal{J}\{\psi_1(t)\}-r_{k2}\mathcal{J}\{\psi_2(t)\}-\dots+(g(\sigma)-r_{kk})\mathcal{J}\{\psi_k(t)\}=\mathcal{J}\{s_k(t)\}+c_kh(\sigma)\end{array}\right\}, \tag{7}$$

where

$$\begin{array}{l}\mathcal{J}\{\psi_1'(t)\}=g(\sigma)\mathcal{J}\{\psi_1(t)\}-h(\sigma)\psi_1(0)=g(\sigma)\mathcal{J}\{\psi_1(t)\}-h(\sigma)c_1,\\ \mathcal{J}\{\psi_2'(t)\}=g(\sigma)\mathcal{J}\{\psi_2(t)\}-h(\sigma)\psi_2(0)=g(\sigma)\mathcal{J}\{\psi_2(t)\}-h(\sigma)c_2,\\ \mathcal{J}\{\psi_3'(t)\}=g(\sigma)\mathcal{J}\{\psi_3(t)\}-h(\sigma)\psi_3(0)=g(\sigma)\mathcal{J}\{\psi_3(t)\}-h(\sigma)c_3.\end{array}$$

Let

$$\Delta=\begin{vmatrix}(g(\sigma)-r_{11}) & -r_{12} & \cdots & -r_{1k}\\ -r_{21} & (g(\sigma)-r_{22}) & \cdots & -r_{2k}\\ \vdots & \vdots & \vdots & \vdots\\ -r_{k1} & -r_{k2} & \cdots & (g(\sigma)-r_{kk})\end{vmatrix}\neq 0,$$
$$\Delta_1=\begin{vmatrix}\mathcal{J}\{s_1(t)\}+c_1h(\sigma) & -r_{12} & \cdots & -r_{1k}\\ \mathcal{J}\{s_2(t)\}+c_2h(\sigma) & (g(\sigma)-r_{22}) & \cdots & -r_{2k}\\ \vdots & \vdots & \vdots & \vdots\\ \mathcal{J}\{s_k(t)\}+c_kh(\sigma) & -r_{k2} & \cdots & (g(\sigma)-r_{kk})\end{vmatrix},$$
$$\Delta_2=\begin{vmatrix}(g(\sigma)-r_{11}) & \mathcal{J}\{s_1(t)\}+c_1h(\sigma) & \cdots & -r_{1k}\\ -r_{21} & \mathcal{J}\{s_2(t)\}+c_2h(\sigma) & \cdots & -r_{2k}\\ \vdots & \vdots & \vdots & \vdots\\ -r_{k1} & \mathcal{J}\{s_k(t)\}+c_kh(\sigma) & \cdots & (g(\sigma)-r_{kk})\end{vmatrix},$$
$$\Delta_k=\begin{vmatrix}(g(\sigma)-r_{11}) & -r_{12} & \cdots & \mathcal{J}\{s_1(t)\}+c_1h(\sigma)\\ -r_{21} & (g(\sigma)-r_{22}) & \cdots & \mathcal{J}\{s_2(t)\}+c_2h(\sigma)\\ \vdots & \vdots & \vdots & \vdots\\ -r_{k1} & -r_{k2} & \cdots & \mathcal{J}\{s_k(t)\}+c_kh(\sigma)\end{vmatrix}.$$

By applying the Cramer rule, the solution of the system (7) is given by

$$\mathcal{J}\{\psi_1(t)\}=\frac{\Delta_1}{\Delta}, \mathcal{J}\{\psi_2(t)\}=\frac{\Delta_2}{\Delta}, \dots, \mathcal{J}\{\psi_k(t)\}=\frac{\Delta_k}{\Delta}.$$

Hence,

$$\psi_1(t)=\mathcal{J}^{-1}\{\frac{\Delta_1}{\Delta}\}, \psi_2(t)=\mathcal{J}^{-1}\{\frac{\Delta_2}{\Delta}\}, \dots, \psi_k(t)=\mathcal{J}^{-1}\{\frac{\Delta_k}{\Delta}\}.$$

Note that the system introduced in this section is general. This means that the solution depends on the values of r_{ij}, c_α, and $s_\alpha(t); \alpha, i, j \in \{1, 2, \ldots, k\}$. If these values are known, then we can find Δ and $\Delta_i, i \in \{1, 2, \ldots, k\}$. Hence, we can find $\psi_1(t) = \mathcal{J}^{-1}\{\frac{\Delta_1}{\Delta}\}$, $\psi_2(t) = \mathcal{J}^{-1}\{\frac{\Delta_2}{\Delta}\}, \ldots, \psi_k(t) = \mathcal{J}^{-1}\{\frac{\Delta_k}{\Delta}\}$ and complete the solution.

11. Application

Integral transformations play a predominant role in medical science, mathematics, chemical engineering, physics, radar, signal processing, fluid mechanics, and theory of elasticity. The mathematical model describing the cell population dynamics in the colonic crypt and colorectal cancer [44] can be represented by the following SODEs:

$$\left.\begin{array}{l} \psi_1'(t) = (\lambda_3 - \lambda_1 - \lambda_2)\psi_1(t), \\ \psi_2'(t) = (\lambda_6 - \lambda_4 - \lambda_5)\psi_2(t) + \lambda_2\psi_1(t), \\ \psi_3'(t) = \lambda_4\psi_2(t) - \mu\psi_3(t). \end{array}\right\} \tag{8}$$

with

$$\psi_1(0) = c_1, \psi_2(0) = c_2, \psi_3(0) = c_3. \tag{9}$$

Table 2 shows the natural explanation of the parameters of the model (8).

Table 2. The natural explanation of the parameters.

Parameter	Meaning
$\psi_1(t)$	number of stem cells
$\psi_2(t)$	number of semi-differentiated cells
$\psi_3(t)$	number of differentiated cells
λ_1	the cell death rates in stem cells
λ_2	the number of stem cells that become semi-differentiated
λ_3	the cell renewal rates of stem cells
λ_4	the cell death rates in semi-differentiated cells
λ_5	the number of the semi-differentiated cells that become differentiated cells
λ_6	the cell renewal rates of semi-differentiated cells
μ	the rate of differentiated cells that are removed from the crypt

In this section, we solve this model by the Jafari transformation.

Put $\omega = \lambda_3 - \lambda_1 - \lambda_2, \ell = \lambda_6 - \lambda_4 - \lambda_5$, then the model (8) with (9) can be written as follows

$$\left.\begin{array}{l} \psi_1'(t) = \omega\psi_1(t), \\ \psi_2'(t) = \ell\psi_2(t) + \lambda_2\psi_1(t), \\ \psi_3'(t) = \lambda_4\psi_2(t) - \mu\psi_3(t), \end{array}\right\} \tag{10}$$

with

$$\psi_1(0) = c_1, \psi_2(0) = c_2, \psi_3(0) = c_3. \tag{11}$$

By the matrix notation, the system (8) with (9) can be expressed as follows

$$\psi'(t) = R\psi(t) + S(t), \psi(0) = C, \tag{12}$$

where

$$\psi'(t)=\begin{bmatrix}\psi_1'(t)\\ \psi_2'(t)\\ \psi_3'(t)\end{bmatrix}, R=\begin{bmatrix}\omega & 0 & 0\\ \lambda_2 & \ell & 0\\ 0 & \lambda_4 & -\mu\end{bmatrix}, \psi(t)=\begin{bmatrix}\psi_1(t)\\ \psi_2(t)\\ \psi_3(t)\end{bmatrix}, S(t)=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}, \psi(0)=\begin{bmatrix}\psi_1(0)\\ \psi_2(0)\\ \psi_3(0)\end{bmatrix}, C=\begin{bmatrix}c_1\\ c_2\\ c_3\end{bmatrix}.$$

Now, by applying the Jafari transform on (10) and taking into consideration the initial conditions (11), we obtain

$$\left.\begin{array}{l}(g(\sigma)-\omega)\mathcal{J}\{\psi_1(t)\}=c_1h(\sigma),\\ -\lambda_2\mathcal{J}\{\psi_1(t)\}+(g(\sigma)-\ell)\mathcal{J}\{\psi_2(t)\}=c_2h(\sigma),\\ -\lambda_4\mathcal{J}\{\psi_2(t)\}+(g(\sigma)+\mu)\mathcal{J}\{\psi_3(t)\}=c_3h(\sigma),\end{array}\right\} \tag{13}$$

Let

$$\Delta=\begin{vmatrix}g(\sigma)-\omega & 0 & 0\\ -\lambda_2 & g(\sigma)-\ell & 0\\ 0 & -\lambda_4 & g(\sigma)+\mu\end{vmatrix}=(g(\sigma)-\omega)(g(\sigma)-\ell)(g(\sigma)+\mu),$$

$$\Delta_1=\begin{vmatrix}c_1h(\sigma) & 0 & 0\\ c_2h(\sigma) & g(\sigma)-\ell & 0\\ c_3h(\sigma) & -\lambda_4 & g(\sigma)+\mu\end{vmatrix}=c_1h(\sigma)(g(\sigma)-\ell)(g(\sigma)+\mu),$$

$$\Delta_2=\begin{vmatrix}g(\sigma)-\omega & c_1h(\sigma) & 0\\ -\lambda_2 & c_2h(\sigma) & 0\\ 0 & c_3h(\sigma) & g(\sigma)+\mu\end{vmatrix}=(g(\sigma)+\mu)[c_2h(\sigma)(g(\sigma)-\omega)+\lambda_2c_1h(\sigma)],$$

$$\Delta_3=\begin{vmatrix}g(\sigma)-\omega & 0 & c_1h(\sigma)\\ -\lambda_2 & g(\sigma)-\ell & c_2h(\sigma)\\ 0 & -\lambda_4 & c_3h(\sigma)\end{vmatrix}=(g(\sigma)-\omega)[c_3h(\sigma)(g(\sigma)-\ell)+\lambda_4c_2h(\sigma)]+c_1\lambda_2\lambda_4h(\sigma).$$

By applying the Cramer rule, the solution of the system (13) is given by

$$\begin{aligned}
&\mathcal{J}\{\psi_1(t)\}=\tfrac{\Delta_1}{\Delta}=\tfrac{c_1h(\sigma)(g(\sigma)-\ell)(g(\sigma)+\mu)}{(g(\sigma)-\omega)(g(\sigma)-\ell)(g(\sigma)+\mu)}=c_1\tfrac{h(\sigma)}{g(\sigma)-\omega},\\
&\mathcal{J}\{\psi_2(t)\}=\tfrac{\Delta_2}{\Delta}=\tfrac{(g(\sigma)+\mu)[c_2h(\sigma)(g(\sigma)-\omega)+\lambda_2c_1h(\sigma)]}{(g(\sigma)-\omega)(g(\sigma)-\ell)(g(\sigma)+\mu)}=\tfrac{c_2h(\sigma)}{(g(\sigma)-\ell)}+\tfrac{\lambda_2c_1h(\sigma)}{(g(\sigma)-\omega)(g(\sigma)-\ell)}\\
&=c_2\tfrac{h(\sigma)}{g(\sigma)-\ell}-\tfrac{\lambda_2c_1}{(\ell-\omega)}\tfrac{h(\sigma)}{g(\sigma)-\omega}+\tfrac{\lambda_2c_1}{(\ell-\omega)}\tfrac{h(\sigma)}{g(\sigma)-\ell},\\
&J\{\psi_3(t)\}=\tfrac{\Delta_3}{\Delta}=\tfrac{(g(\sigma)-\omega)[c_3h(\sigma)(g(\sigma)-\ell)+\lambda_4c_2h(\sigma)]+c_1\lambda_2\lambda_4h(\sigma)}{(g(\sigma)-\omega)(g(\sigma)-\ell)(g(\sigma)+\mu)}\\
&=\tfrac{c_3h(\sigma)}{(g(\sigma)+\mu)}+\tfrac{\lambda_4c_2h(\sigma)}{(g(\sigma)-\ell)(g(\sigma)+\mu)}+\tfrac{c_1\lambda_2\lambda_4h(\sigma)}{(g(\sigma)-\omega)(g(\sigma)-\ell)(g(\sigma)+\mu)}\\
&=c_3\tfrac{h(\sigma)}{g(\sigma)+\mu}+\tfrac{\lambda_4c_2}{(\ell+\mu)}\tfrac{h(\sigma)}{g(\sigma)-\ell}-\tfrac{\lambda_4c_2}{(\ell+\mu)}\tfrac{h(\sigma)}{g(\sigma)+\mu}\\
&+\left(\tfrac{\lambda_2\lambda_4c_1}{\mu(\ell+\omega+\mu)+\ell\omega}\right)\left(\left(\tfrac{\mu+\ell}{\omega-\ell}\right)\tfrac{h(\sigma)}{g(\sigma)-\omega}-\left(\tfrac{\omega+\mu}{\ell+\omega}\right)\tfrac{h(\sigma)}{g(\sigma)-\ell}+\tfrac{h(\sigma)}{g(\sigma)+\mu}\right),
\end{aligned}$$

hence,

$$\begin{aligned}
&\psi_1(t)=\mathcal{J}^{-1}\left\{c_1\tfrac{h(\sigma)}{g(\sigma)-\omega}\right\}=c_1e^{\omega t},\ \ \omega=\lambda_3-\lambda_1-\lambda_2,\\
&\psi_2(t)=\mathcal{J}^{-1}\left\{c_2\tfrac{h(\sigma)}{g(\sigma)-\ell}-\tfrac{\lambda_2c_1}{(\ell-\omega)}\tfrac{h(\sigma)}{g(\sigma)-\omega}+\tfrac{\lambda_2c_1}{(\ell-\omega)}\tfrac{h(\sigma)}{g(\sigma)-\ell}\right\},\\
&\psi_2(t)=\left(c_2+\tfrac{\lambda_2c_1}{(\ell-\omega)}\right)e^{\ell t}-\tfrac{\lambda_2c_1}{(\ell-\omega)}e^{\omega t},\ \omega=\lambda_3-\lambda_1-\lambda_2,\ \ell=\lambda_6-\lambda_4-\lambda_5,\\
&\psi_3(t)=\mathcal{J}^{-1}\{c_3\tfrac{h(\sigma)}{g(\sigma)+\mu}+\tfrac{\lambda_4c_2}{(\ell+\mu)}\tfrac{h(\sigma)}{g(\sigma)-\ell}-\tfrac{\lambda_4c_2}{(\ell+\mu)}\tfrac{h(\sigma)}{g(\sigma)+\mu}\\
&+\left(\tfrac{\lambda_2\lambda_4c_1}{\mu(\ell+\omega+\mu)+\ell\omega}\right)\left(\left(\tfrac{\mu+\ell}{\omega-\ell}\right)\tfrac{h(\sigma)}{g(\sigma)-\omega}-\left(\tfrac{\omega+\mu}{\ell+\omega}\right)\tfrac{h(\sigma)}{g(\sigma)-\ell}+\tfrac{h(\sigma)}{g(\sigma)+\mu}\right)\},\\
&\psi_3(t)=\left(c_3-\tfrac{\lambda_4c_2}{(\ell+\mu)}+\left(\tfrac{\lambda_2\lambda_4c_1}{\mu(\ell+\omega+\mu)+\ell\omega}\right)\right)e^{-\mu t}\\
&+\left(\tfrac{\lambda_4c_2}{(\ell+\mu)}-\left(\tfrac{\lambda_2\lambda_4c_1}{\mu(\ell+\omega+\mu)+\ell\omega}\right)\left(\tfrac{\omega+\mu}{\ell+\omega}\right)\right)e^{\ell t}+\left(\tfrac{\lambda_2\lambda_4c_1}{\mu(\ell+\omega+\mu)+\ell\omega}\right)\left(\tfrac{\mu+\ell}{\omega-\ell}\right)e^{\omega t},\\
&\omega=\lambda_3-\lambda_1-\lambda_2,\ \ell=\lambda_6-\lambda_4-\lambda_5.
\end{aligned}$$

12. Conclusions

In this paper, we have applied a general integral transform called Jafari transform for solving SODEs. The Jafari transform in solving SODEs does not need a large computational work as the previous integral transforms. We have studied and proved some valuable properties and theories of this transform that have not been studied before. There is a mathematical model that describes the cell population dynamics in the colonic crypt and colorectal cancer. We have applied the Jafari transform for solving this model and illustrating the efficiency of the Jafari transform. In future work, we will handle a system with variable coefficients.

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