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On Geometric Properties of a Certain Analytic Function with Negative Coefficients

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Abstract: Various function theorists have successfully defined and investigated different kinds of analytic functions. The applications of such functions have played significant roles in geometry function theory as a field of complex analysis. In this work, therefore, a certain subclass of univalent analytic functions of the form $f(z) = z - \sum_{m=2}^{t} \frac{[\omega(2+\beta)+c\gamma-\sigma]C_m}{[m\sigma - c\omega(2+\beta)+c\gamma]K^n} z^m - \sum_{k=t+1}^{\infty} a_k z^k$ is defined using a generalized differential operator. Furthermore, some geometric properties for the class were established.

Keywords: analytic functions; univalent function; coefficient estimates; fixed coefficients; extreme points

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1. Introduction and Preliminaries

Let $\mathbb{U}$ be the unit disk, that is, $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$, A be the class of functions analytic in $\mathbb{U}$ satisfying the conditions $f(0) = 0$ and $f'(0) = 1$ and of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \tag{1}$$

We denote T the subclass of A analytic in $\mathbb{U}$ of the form

$$f(z) = z - \sum_{k=2}^{\infty} a_k z^k, \ \ a_k \geq 0. \tag{2}$$

Differential operator is one of the tools used in geometric functions theory. Various authors have used different operators in literature. See [1–7] for instance. Differential operator $D^{n,\lambda}_{\alpha,\beta,\mu_1,\mu_2}$ defined as

$$D^{n,\lambda}_{\alpha,\beta,\mu_1,\mu_2} f(z) = z + \sum_{k=2}^{\infty} \left(\frac{a + (\alpha-\beta)(\lambda+\mu_2-\mu_1)(k-1)+b}{a+b} \right)^n a_k z^k, \tag{3}$$

where $a, b \geq 0$, $a+b \neq 0$, $\alpha > \beta \geq 0$, $\lambda > \mu_2 \leq \mu_1$ and $n \in \mathbb{N}_0$ was used to define a certain class of univalent functions. See [2,6].

In this work, we set

$$K = \left(\frac{a + (\alpha-\beta)(\lambda+\mu_2-\mu_1)(k-1)+b}{a+b} \right) \tag{4}$$

in (3) above.

Lemma 1 ([6]). *Let the function $f \in A$. Then $Q^{n,\lambda,\omega}_{\alpha,\beta,\mu,\sigma}(\gamma, c)$ if and only if*

$$\sum_{k=2}^{\infty}[k\sigma - c\omega(2+\beta) + c\gamma]\left(\frac{a + (\alpha-\beta)(\lambda+\mu_2-\mu_1)(k-1)+b}{a+b}\right)^n a_k \leq \omega(2+\beta) + c\gamma - \sigma. \tag{5}$$

See [6] for the proof.

Silverman in [8] was the first to pave way for the study of functions with negative coefficients of the form (2), after which various forms of such functions have been opened up by many researchers in the field of geometric functions theory. Rather than fixing the negative coefficients from the second coefficients in (2), Owa in [9] considered fixing more coefficients, which motivated the work of Aouf and Darwish in [10] and gave birth to the investigation of univalent functions $f(z)$ with fixed finitely many negative coefficients and the behaviors of such kinds of functions. In [4–7,11–18], for instance, various classes of univalent functions with finitely many fixed coefficients were investigated.

Motivated by the work of Oluwayemi and Faisal in [6], the following class of functions $Q^{n,\lambda,\omega}_{\alpha,\beta,\mu,\sigma}(\gamma, c, C_m) \subset Q^{n,\lambda,\omega}_{\alpha,\beta,\mu,\sigma}(\gamma, c)$ is introduced.

Definition 1. *Let $f \in T$ be defined by (2). Then, $f(z)$ is in the class $Q^{n,\lambda,\omega}_{\alpha,\beta,\mu,\sigma}(\gamma, c, C_m)$ if it is of the form*

$$f(z) = z - \sum_{m=2}^{t}\frac{[\omega(2+\beta)+c\gamma-\sigma]C_m}{[m\sigma - c\omega(2+\beta)+c\gamma]K^n}z^m - \sum_{k=t+1}^{\infty}a_k z^k \tag{6}$$

where

$$C_m = \frac{[m\sigma - c\omega(2+\beta)+c\gamma]K^n}{[\omega(2+\beta)+c\gamma-\sigma]}a_m, \quad \textit{and} \quad a_k = \frac{[\omega(2+\beta)+c\gamma-\sigma](1-\sum_{m=2}^{t}C_m)}{[k\sigma - c\omega(2+\beta)+c\gamma]K^n}. \tag{7}$$

Note that: $\sigma \geq 1,\ \ 0 < \gamma \leq 1,\ \ 1 < \omega \leq \frac{1}{2}$, *and* $c\gamma - \sigma \geq 0$.

2. Main Results

Theorem 1. *Let the function $f \in T$. Then $Q^{n,\lambda,\omega}_{\alpha,\beta,\mu,\sigma}(\gamma, c, C_m)$ if*

$$\sum_{k=2}^{\infty}\frac{[k\sigma - c\omega(2+\beta)+c\gamma]K^n}{[\omega(2+\beta)+c\gamma-\sigma]}a_k \leq 1 - \sum_{m=2}^{t}C_m \tag{8}$$

Proof. Let $f \in Q^{n,\lambda,\omega}_{\alpha,\beta,\mu,\sigma}(\gamma, c, C_m)$. From (7),

$$a_m = \frac{[\omega(2+\beta)+c\gamma-\sigma]C_m}{[m\sigma - c\omega(2+\beta)+c\gamma]K^n}. \tag{9}$$

Then, $f \in Q^{n,\lambda,\omega}_{\alpha,\beta,\mu,\sigma}(\gamma, c, C_m) \subset Q^{n,\lambda,\omega}_{\alpha,\beta,\mu,\sigma}(\gamma, c)$ if and only if

$$\sum_{m=2}^{t}\frac{[m\sigma - c\omega(2+\beta)+c\gamma]K^n}{[\omega(2+\beta)+c\gamma-\sigma]}a_m + \sum_{k=t+1}^{\infty}\frac{[k\sigma - c\omega(2+\beta)+c\gamma]K^n}{[\omega(2+\beta)+c\gamma-\sigma]}a_k \leq 1$$

which also implies from (7) that,

$$\sum_{k=t+1}^{\infty}\frac{[k\sigma - c\omega(2+\beta)+c\gamma]K^n}{[\omega(2+\beta)+c\gamma-\sigma]}a_k \leq 1 - \sum_{m=2}^{t}C_m$$

which completes the proof. □

Corollary 1. *Let $f \in Q^{n,\lambda,\omega}_{\alpha,\beta,\mu,\sigma}(\gamma, c, C_m)$ for $k \geq t+1$. Then, we have that*

$$a_k \leq \frac{[\omega(2+\beta) + c\gamma - \sigma](1 - \sum_{m=2}^{t} C_m)}{[k\sigma - c\omega(2+\beta) + c\gamma]K^n}.$$

The best possible result is of the function

$$f(z) = z - \sum_{m=2}^{t} \frac{[\omega(2+\beta) + c\gamma - \sigma]}{[m\sigma - c\omega(2+\beta) + c\gamma]K^n} z^m - \sum_{k=t+1}^{\infty} \frac{[\omega(2+\beta) + c\gamma - \sigma](1 - \sum_{m=2}^{t} C_m)}{[k\sigma - c\omega(2+\beta) + c\gamma]K^n} z^k. \tag{10}$$

Corollary 2. *Let $f(z)$ be defined by (6). Then $f \in Q^{n,\lambda,\frac{1}{2}}_{\alpha,\beta,\mu,\sigma}(\gamma, c, C_m)$ for and $k \geq t+1$, we have that*

$$a_k \leq \frac{[(1+\frac{\beta}{2}) + c\gamma - \sigma](1 - \sum_{m=2}^{t} C_m)}{[k\sigma - c\omega(2+\beta) + c\gamma]K^n} \tag{11}$$

with equality only for the functions $f(z)$ of the form

$$f(z) = z - \sum_{m=2}^{t} \frac{[(1+\frac{\beta}{2}) + c\gamma - \sigma]}{[m\sigma - c\omega(2+\beta) + c\gamma]K^n} z^m - \sum_{k=t+1}^{\infty} \frac{[(1+\frac{\beta}{2}) + c\gamma - \sigma](1 - \sum_{m=2}^{t} C_m)}{[k\sigma - c\omega(2+\beta) + c\gamma]K^n} z^k.$$

Corollary 3. *Let $f(z)$ be defined by (6). Then $f \in Q^{n,\lambda,\omega}_{\alpha,0,\mu,\sigma}(\gamma, c, C_m)$ for and $k \geq t+1$, we have that*

$$a_k \leq \frac{(2\omega + c\gamma - \sigma)(1 - \sum_{m=2}^{t} C_m)}{[k\sigma - c\omega(2+\beta) + c\gamma]K^n}$$

with equality only for the functions $f(z)$ of the form

$$f(z) = z - \sum_{m=2}^{t} \frac{(2\omega + c\gamma - \sigma)}{[m\sigma - c\omega(2+\beta) + c\gamma]K^n} z^m - \sum_{k=t+1}^{\infty} \frac{(2\omega + c\gamma - \sigma)(1 - \sum_{m=2}^{t} C_m)}{[k\sigma - c\omega(2+\beta) + c\gamma]K^n} z^k.$$

Corollary 4. *Let $f(z)$ be defined by (6). Then $f \in Q^{n,\lambda,\omega}_{\alpha,0,\mu,1}(1, 1, C_m)$ for and $k \geq t+1$, we have that*

$$a_k \leq \frac{2\omega(1 - \sum_{m=2}^{t} C_m)}{[k - 2\omega + 1]K^n}$$

with equality only for the functions $f(z)$ of the form

$$f(z) = z - \sum_{m=2}^{t} \frac{2\omega}{[m - 2\omega + 1]K^n} z^m - \sum_{k=t+1}^{\infty} \frac{2\omega(1 - \sum_{m=2}^{t} C_m)}{[k - 2\omega + 1]K^n} z^k.$$

Corollary 5. *Let $f(z)$ be defined by (6). Then $f \in Q^{n,\lambda,\frac{1}{2}}_{\alpha,0,\mu,1}(1, 1, C_m)$ for and $k \geq t+1$, we have that*

$$a_k \leq \frac{1 - \sum_{m=2}^{t} C_m}{(k-1)K^n}$$

with equality only for the functions $f(z)$ of the form

$$f(z) = z - \sum_{m=2}^{t} \frac{1 - \sum_{m=2}^{t}}{(m-1)K^n} z^m - \sum_{k=t+1}^{\infty} \frac{1 - \sum_{m=2}^{t} C_m}{(k-1)K^n} z^k.$$

Theorem 2. *Let $j \in \mathbb{N}$ and $f_1(z), \ldots f_j(z)$ be defined by*

$$f_j(z) = z - \sum_{m=2}^{t} \frac{[\omega(2+\beta) + c\gamma - \sigma]C_m}{[m\sigma - c\omega(2+\beta) + c\gamma]K^n} z^m - \sum_{k=t+1}^{\infty} a_{k,j} z^k \tag{12}$$

belong to the class $Q^{n,\lambda,\omega}_{\alpha,\beta,\mu,\sigma}(\gamma, c, C_m)$. *Then,*

$$G(z) = \sum_{i=2}^{j} \varsigma_i f_i \ \textit{ and } \ \sum_{i=2}^{j} \varsigma_i = 1, \ 0 \leq \sum_{m=2}^{t} C_m \leq 1, \ 0 \leq C_m \leq 1$$

also belongs to the class $Q^{n,\lambda,\omega}_{\alpha,\beta,\mu,\sigma}(\gamma, c, C_m)$.

Proof. Let $f_j \in Q^{n,\lambda,\omega}_{\alpha,\beta,\mu,\sigma}(\gamma, c, C_m)$. It follows from Theorem 1 that

$$\sum_{k=2}^{\infty} \frac{[k\sigma - c\omega(2+\beta) + c\gamma]K^n}{[\omega(2+\beta) + c\gamma - \sigma]} a_{k,j} \leq 1 - \sum_{m=2}^{t} C_m$$

for every $i = 1, \dots j$. So that

$$G(z) = \sum_{i=2}^{j} \varsigma_i f_i = z - \sum_{m=2}^{t} \frac{[\omega(2+\beta) + c\gamma - \sigma]C_m}{[m\sigma - c\omega(2+\beta) + c\gamma]K^n} z^m - \sum_{k=t+1}^{\infty} \left(\sum_{i=2}^{j} \varsigma_i a_{k,j} \right) z^k.$$

Thus,

$$\sum_{k=t+1}^{\infty} \frac{[k\sigma - c\omega(2+\beta) + c\gamma]K^n}{\omega(2+\beta) + c\gamma - \sigma} \left(\sum_{i=2}^{j} \varsigma_i a_{k,j} \right)$$

$$\sum_{i=2}^{j} \sum_{k=t+1}^{\infty} \left(\frac{[k\sigma - c\omega(2+\beta) + c\gamma]K^n}{\omega(2+\beta) + c\gamma - \sigma} \right) \varsigma_i$$

$$< \sum_{i=2}^{j} \left(1 - \sum_{m=2}^{t} C_m \right) \varsigma_i = 1 - \sum_{m=2}^{t} C_m.$$

□

Theorem 3. *Let*

$$f_t(z) = z - \sum_{m=2}^{t} \frac{[\omega(2+\beta) + c\gamma - \sigma]C_m}{[m\sigma - c\omega(2+\beta) + c\gamma]K^n} z^m \tag{13}$$

and for $k \geq t+1$

$$f_k(z) = z - \sum_{m=2}^{t} \frac{[\omega(2+\beta) + c\gamma - \sigma]C_m}{[m\sigma - c\omega(2+\beta) + c\gamma]K^n} z^m - \sum_{k=t+1}^{\infty} \frac{[\omega(2+\beta) + c\gamma - \sigma](1 - \sum_{m=2}^{t} C_m)}{[k\sigma - c\omega(2+\beta) + c\gamma]K^n} z^k. \tag{14}$$

Then the function $f(z) \in Q^{n,\lambda,\omega}_{\alpha,\beta,\mu,\sigma}(\gamma, c, C_m)$ *if and only if it can be expressed in the form* $f(z) = \sum_{k=t}^{\infty} \lambda_k f_k(z)$, *where* $\lambda_k \geq 0$, $(k \geq t)$ *and* $\sum_{k=t}^{\infty} \lambda_k = 1$.

Proof. Let

$$f(z) = \sum_{k=t+1}^{\infty} \lambda_k f_k(z) + \lambda_t f_t(z)$$

$$= \lambda_t z - \lambda_t \sum_{m=2}^{t} \frac{[\omega(2+\beta) + c\gamma - \sigma]C_m}{[m\sigma - c\omega(2+\beta) + c\gamma]K^n} z^m + \sum_{k=t+1}^{\infty} \lambda_k z$$

$$- \sum_{k=t+1}^{\infty} \lambda_k \left(\sum_{m=2}^{t} \frac{[\omega(2+\beta) + c\gamma - \sigma]C_m}{[m\sigma - c\omega(2+\beta) + c\gamma]K^n} z^m \right)$$

$$- \sum_{k=t+1}^{\infty} \lambda_k \left(\frac{[\omega(2+\beta) + c\gamma - \sigma](1 - \sum_{m=2}^{t} C_m)}{[k\sigma - c\omega(2+\beta) + c\gamma]K^n} z^k \right)$$

$$= \left(\lambda_t + \sum_{k=t+1}^{\infty} \lambda_k\right) z - \left(\lambda_t + \sum_{k=t+1}^{\infty} \lambda_k\right) \sum_{m=2}^{t} \frac{[\omega(2+\beta) + c\gamma - \sigma] C_m}{[m\sigma - c\omega(2+\beta) + c\gamma] K^n} z^m$$

$$- \sum_{k=t+1}^{\infty} \frac{[\omega(2+\beta) + c\gamma - \sigma](1 - \sum_{m=2}^{t} C_m)}{[k\sigma - c\omega(2+\beta) + c\gamma] K^n} \lambda_k z^k$$

$$= z - \sum_{m=2}^{t} \frac{[\omega(2+\beta) + c\gamma - \sigma] C_m}{[m\sigma - c\omega(2+\beta) + c\gamma] K^n} z^m - \sum_{k=t+1}^{\infty} \frac{[\omega(2+\beta) + c\gamma - \sigma](1 - \sum_{m=2}^{t} C_m)}{[k\sigma - c\omega(2+\beta) + c\gamma] K^n} \lambda_k z^k.$$

We can further write that

$$\sum_{k=t+1}^{\infty} \frac{[k\sigma - c\omega(2+\beta) + c\gamma][\omega(2+\beta) + c\gamma - \sigma](1 - \sum_{m=2}^{t} C_m)}{[\omega(2+\beta) + c\gamma - \sigma][k\sigma - c\omega(2+\beta) + c\gamma] K^n} K^n \lambda_k$$

$$= (1 - \sum_{m=2}^{t} C_m) \sum_{k=t+1}^{\infty} \lambda_k = (1 - \sum_{m=2}^{t} C_m)(1 - \lambda_k) < 1 - \sum_{m=2}^{t} C_m.$$

Therefore $f(z) \in Q_{\alpha,\beta,\mu,\sigma}^{n,\lambda,\omega}(\gamma, c, C_m)$.

Conversely, suppose $f(z) \in Q_{\alpha,\beta,\mu,\sigma}^{n,\lambda,\omega}(\gamma, c, C_m)$. From Definition 1 and (6),

$$f(z) = z - \sum_{m=2}^{t} \frac{[\omega(2+\beta) + c\gamma - \sigma] C_m}{[m\sigma - c\omega(2+\beta) + c\gamma] K^n} z^m - \sum_{k=t+1}^{\infty} a_k z^k.$$

Set

$$\lambda_k = \frac{[k\sigma - c\omega(2+\beta) + c\gamma] K^n}{[\omega(2+\beta) + c\gamma - \sigma](1 - \sum_{m=2}^{t} C_m)} a_k$$

then $\lambda_k \geq 0$ and for $\lambda_t = 1 - \sum_{k=t+1}^{\infty} \lambda_k$; we have that

$$f(z) = z - \sum_{m=2}^{t} \frac{[\omega(2+\beta) + c\gamma - \sigma] C_m}{[m\sigma - c\omega(2+\beta) + c\gamma] K^n} z^m - \sum_{k=t+1}^{\infty} \frac{[\omega(2+\beta) + c\gamma - \sigma](1 - \sum_{m=2}^{t} C_m)}{[k\sigma - c\omega(2+\beta) + c\gamma] K^n} \lambda_k z^k$$

$$= f_t(z) - \sum_{k=t+1}^{\infty} \left(z - \sum_{m=2}^{t} \frac{[\omega(2+\beta) + c\gamma - \sigma] C_m}{[m\sigma - c\omega(2+\beta) + c\gamma] K^n} z^m - f_k(z)\right) \lambda_k$$

$$= f_t(z) - \sum_{k=t+1}^{\infty} (f_t(z) - f_k(z)) \lambda_k$$

$$= \left(1 - \sum_{k=t+1}^{\infty} \lambda_k\right) f_t(z) + \sum_{k=t+1}^{\infty} \lambda_k f_k(z) = \sum_{k=t+1}^{\infty} \lambda_k \lambda_k f_k(z)$$

□

Integral Operator

We now consider the effect of the Alexander operator, defined as

$$I(f) = \int_0^z \frac{f(t)}{t} dt \tag{15}$$

for the functions in the class S on the class $Q_{\alpha,\beta,\mu,\sigma}^{n,\lambda,\omega}(\gamma, c, C_m)$ through the following theorem.

Theorem 4. *Let $f(z)$, defined by (6), belong to the class $Q_{\alpha,\beta,\mu,\sigma}^{n,\lambda,\omega}(\gamma, c, C_m)$. Then, $I(f)$ is also in the class $Q_{\alpha,\beta,\mu,\sigma}^{n,\lambda,\omega}(\gamma, c, C_m)$.*

Proof. Assume $f(z) \in Q^{n,\lambda,\omega}_{\alpha,\beta,\mu,\sigma}(\gamma, c, C_m)$

$$I(f) = z - \sum_{m=2}^{t} \frac{[\omega(2+\beta) + c\gamma - \sigma]C_m}{[m\sigma - c\omega(2+\beta) + c\gamma]K^n} z^m - \sum_{k=t+1}^{\infty} \frac{a_k}{k} z^k. \tag{16}$$

Now

$$\sum_{k=t+1}^{\infty} [k\sigma - c\omega(2+\beta) + c\gamma]K^n \frac{a_k}{k} \leq \frac{1}{k+1}[k\sigma - c\omega(2+\beta) + c\gamma]K^n a_k$$

$$\leq \frac{1}{k+1}\left(1 - \sum_{m=2}^{t} C_m\right) = \frac{1}{k+1} - \sum_{m=2}^{t} \frac{C_m}{k+1} < 1 - \sum_{m=2}^{t} \frac{C_m}{m}$$

which implies that $I(f) \in Q^{n,\lambda,\omega}_{\alpha,\beta,\mu,\sigma}(\gamma, c, C_m)$. □

Remark 1 ([18])**.** *The operator maps the class of starlike functions onto the class of convex functions. The class of functions studied in [19] consists of the convex function with* $\alpha = 1$.

3. Conclusions

The class of functions considered in the work add to the existing knowledge in the investigation of properties of univalent functions with negative coefficients. Furthermore, the class of functions (6) reduces to (2) with $\omega = 0$ and $c\gamma = \sigma$.

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