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A New Perspective on the Exact Solutions of the Local Fractional Modified Benjamin–Bona–Mahony Equation on Cantor Sets

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Abstract: A new local fractional modified Benjamin–Bona–Mahony equation is proposed within the local fractional derivative in this study for the first time. By defining some elementary functions via the Mittag–Leffler function (MLF) on the Cantor sets (CSs), a set of nonlinear local fractional ordinary differential equations (NLFODEs) is constructed. Then, a fast algorithm namely Yang's special function method is employed to find the non-differentiable (ND) exact solutions. By this method, we can extract abundant exact solutions in just one step. Finally, the obtained solutions on the CS are outlined in the form of the 3-D plot. The whole calculation process clearly shows that Yang's special function method is simple and effective, and can be applied to investigate the exact ND solutions of the other local fractional PDEs.

Keywords: local fractional derivative; Mittag–Leffler function; Yang's special function method; cantor sets

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1. Introduction

As is known to all, many complex phenomena occurring in nature involving in optics [1–5], vibration [6,7], social and economic [8], thermal science [9,10], and others [11–13] can be modeled by the partial differential equations (PDEs). In recent years, the fractional derivative has been adopted to PDEs to describe many phenomena arising in scientific and engineering fields, such as physics [14–18], biology [19–21], chemistry [22–24], mechanics [25–27], communication engineering [28–31], and so on [32,33]. Finding the exact solution of the fractional partial differential equation is helpful to further understand and analyze the dynamic behavior of the fractional partial differential equation. Compared with the mathematical model with an integer derivative, the fractional derivative mathematical model can more accurately describe the complex phenomena. Recently, the local fractional derivative (LFD) has attracted wide attention in various fields and some outstanding research results have emerged. In [34], the q-homotopy analysis transform method is applied to study the local fractional Poisson equation. In [35], the local fractional Fokker Planck equation is proposed and the reduced differential transform method and local fractional series expansion method are considered. In [36], the factorization technique is derived to investigate some local fractional PDEs. In [37], the Sumudu transform method, alongside the Adomian decomposition method, is used to employ the local fractional PDEs. In [38], the Mittag–Leffler function-based method is adopted to find the non-differentiable exact solutions of the $(2+1)$-dimensional local fractional breaking soliton equation. In [39], the local fractional variational iteration method is presented to investigate the local fractional heat conduction equation. In [40], the extended rational fractal sine–cosine method is used and six sets of the exact solutions are obtained. In [41], the Local fractional Fourier series method is utilized to study the wave equations. Many other studies can be seen in [42–46]. On the inspiration of the latest research results about the LFD, we present a new local fractional modified Benjamin–Bona–Mahony equation (LFMBBME) below:

$$\frac{\partial^{\vartheta}\aleph_{\vartheta}}{\partial t^{\vartheta}}+\frac{\partial^{\vartheta}\aleph_{\vartheta}}{\partial x^{\vartheta}}+k\aleph_{\vartheta}^{2}\frac{\partial^{\vartheta}\aleph_{\vartheta}}{\partial x^{\vartheta}}+\frac{\partial^{2\vartheta}}{x^{2\vartheta}}\left(\frac{\partial^{\vartheta}\aleph_{\vartheta}}{\partial t^{\vartheta}}\right)=0, \tag{1}$$

where $\vartheta(0<\vartheta\leq 1)$ is the fractional order, $\frac{\partial^{\vartheta}\aleph_{\vartheta}}{\partial t^{\vartheta}}$ and $\frac{\partial^{\vartheta}\aleph_{\vartheta}}{\partial x^{\vartheta}}$ are the local fractional derivatives. The definitions are presented in Section 2. In this work, we aim to investigate the exact ND solutions of the LFMBBME via a fast algorithm known as Yang's special function method, which can avoid the complicated calculation process and obtain abundant exact solutions in one step. The ideas within work are expected to open up some new horizons in the study of local fractional PDEs. The rest of this article is structured as follows. In Section 2, the properties of the LFD and some special functions are presented. In Section 3, a set of nonlinear local fractional ODEs is constructed. In Section 4, Yang's special function method is used to find the exact ND solutions, and the behaviors of the solutions on the CS are presented. Finally, a conclusion is reached in Section 5.

2. Basic Theory

In this section, some basic theory that is used to study the problem is presented.

Definition 1. *The LFD of $\Xi(x)$ with order ϑ is defined as [47]:*

$$\frac{d^{\vartheta}\Xi(x)}{dx^{\vartheta}}|x=x_0=\lim_{x\to x_0}\frac{\Delta^{\vartheta}[\Xi(x)-\Xi(x_0)]}{(x-x_0)^{\vartheta}}, \tag{2}$$

where $\Delta^{\vartheta}[\Xi(x)-\Xi(x_0)]\cong\Gamma(1+\vartheta)[\Xi(x)-\Xi(x_0)]$ with Euler's gamma function.

$$\Gamma(1+\vartheta)=:\int_0^{\infty}x^{\vartheta-1}\exp(-x)\mathrm{d}x.$$

For the LFD, there is the following rule chain [47]:

$$\frac{d^{k\vartheta}\Xi(x)}{dx^{k\vartheta}}=\overbrace{\frac{d^{\vartheta}}{dx^{\vartheta}}\cdots\frac{d^{\vartheta}}{dx^{\vartheta}}}^{k\quad times}\Xi(x).$$

Definition 2. *The local fractional integral (LFI) of $\Xi(x)$ with the fractional order ϑ $(0<\vartheta\leq 1)$ is defined by [47]:*

$${}_a\mathrm{I}_b^{\vartheta}\Xi(x)=\frac{1}{\Gamma(1+\vartheta)}\int_a^b\Xi(x)(dx)^{\vartheta}=\frac{1}{\Gamma(1+\vartheta)}\lim_{\Delta x_k\to 0}\sum_{k=0}^{N-1}\Xi(x_k)(\Delta x_k)^{\vartheta} \tag{3}$$

Here, $\Delta x_k=x_{k+1}-x_k$ and $x_0=a<x_1<\ldots<x_{N-1}<x_N=b$.

Property 1. *The properties of the LFD are listed as follows [47]:*

$$(1)\ \frac{d^{\vartheta}}{dt^{\vartheta}}[p(t)\pm q(t)]=\frac{d^{\vartheta}}{dt^{\vartheta}}p(t)\pm\frac{d^{\vartheta}}{dt^{\vartheta}}q(t), \tag{4}$$

$$(2)\ \frac{d^{\vartheta}}{dt^{\vartheta}}[p(t)q(t)]=q(t)\frac{d^{\vartheta}}{dt^{\vartheta}}p(t)+p(t)\frac{d^{\vartheta}}{dt^{\vartheta}}q(t), \tag{5}$$

$$(3)\ \frac{d^{\vartheta}}{dt^{\vartheta}}[p(t)/q(t)]=\frac{\left[q(t)\frac{d^{\vartheta}}{dt^{\vartheta}}p(t)-p(t)\frac{d^{\vartheta}}{dt^{\vartheta}}q(t)\right]}{q(t)^2}, \tag{6}$$

Definition 3. *The MLF on the CS with fractional order ϑ is defined as [47]:*

$$\mathrm{MI}_{\vartheta}(\wp^{\gamma}) = \sum_{\Im=0}^{\infty} \frac{\wp^{\Im\vartheta}}{\Gamma(1+\Im\vartheta)}. \tag{7}$$

Definition 4. *Based on the MLF, we can derive four special functions, namely the SE function, the CH function, the SE function, and the CS function, as [47]:*

$$\mathrm{SH}_{\vartheta}\left(\wp^{\vartheta}\right) = \frac{2}{\mathrm{MI}_{\vartheta}(\wp^{\vartheta}) + \mathrm{MI}_{\vartheta}(-\wp^{\vartheta})}, \tag{8}$$

$$\mathrm{CH}_{\vartheta}\left(\wp^{\vartheta}\right) = \frac{2}{\mathrm{MI}_{\vartheta}(\wp^{\vartheta}) - \mathrm{MI}_{\vartheta}(-\wp^{\vartheta})}, \tag{9}$$

$$\mathrm{SE}_{\vartheta}\left(\wp^{\vartheta}\right) = \frac{2}{\mathrm{MI}_{\vartheta}(i^{\vartheta}\wp^{\vartheta}) + \mathrm{MI}_{\vartheta}(-i^{\vartheta}\wp^{\vartheta})}, \tag{10}$$

$$\mathrm{CS}_{\vartheta}\left(\wp^{\vartheta}\right) = \frac{2i^{\vartheta}}{\mathrm{MI}_{\vartheta}(i^{\vartheta}\wp^{\vartheta}) - \mathrm{MI}_{\vartheta}(-i^{\vartheta}\wp^{\vartheta})}. \tag{11}$$

The behaviors of the four special functions on the CS using $\vartheta = \ln 2/\ln 3$ are displayed in Figure 1.

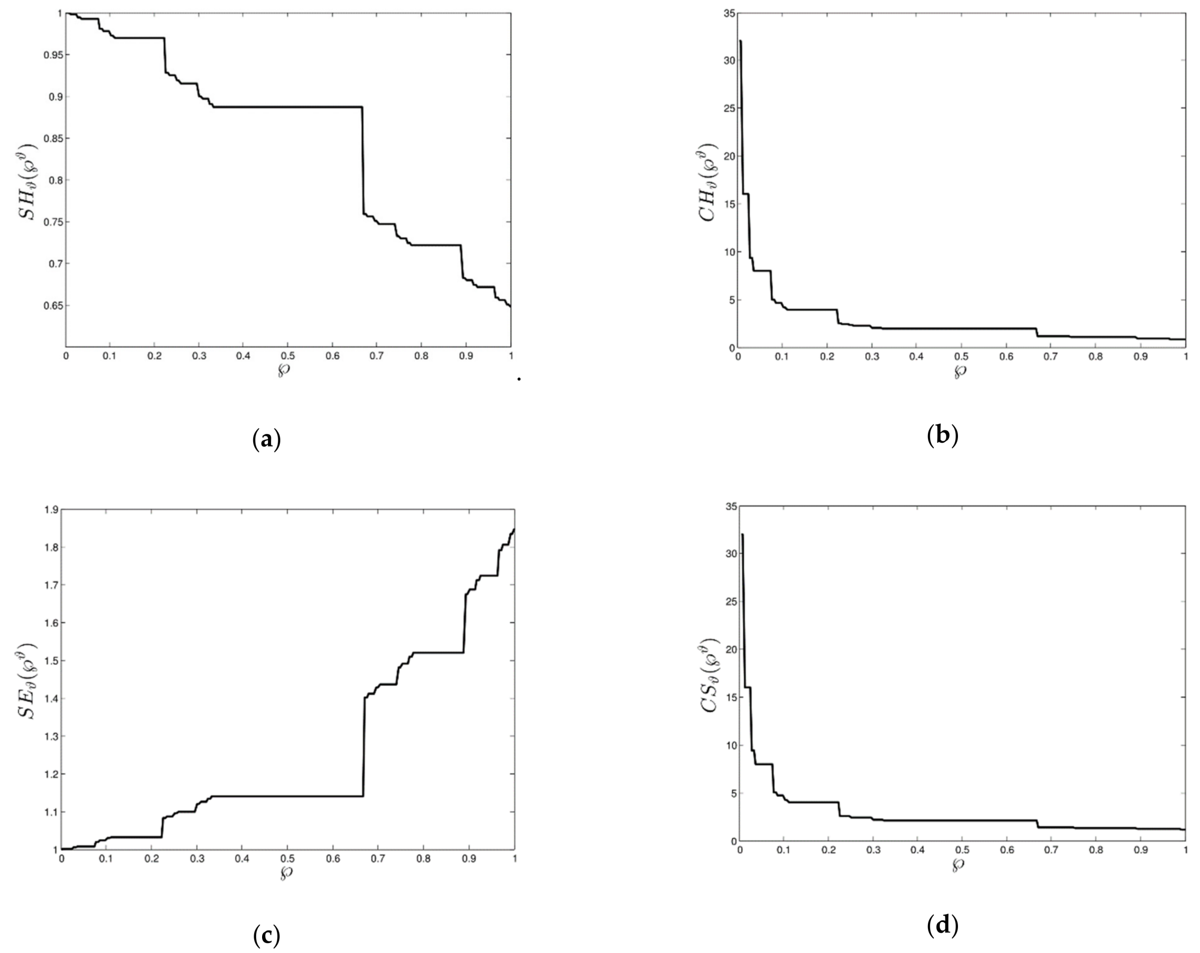

Figure 1. The outline of the special functions on the CS: (**a**) for the SE function, (**b**) for the CH function, (**c**) for the SE function, (**d**) and for the CS function.

Property 2. *The properties of the MLF are given as [47]:*

$$(1)\ D^{(\vartheta)}\mathrm{MI}_\vartheta\left(\Delta\xi^\vartheta\right) = \Delta\mathrm{MI}_\vartheta\left(\xi^\vartheta\right), \tag{12}$$

$$(2)\ \mathrm{MI}_\vartheta\left(\xi^\vartheta\right)\mathrm{MI}_\vartheta\left(\zeta^\vartheta\right) = \mathrm{MI}_\vartheta\left(\xi^\vartheta + \zeta^\vartheta\right), \tag{13}$$

$$(3)\ \mathrm{MI}_\vartheta\left(\xi^\vartheta\right)\mathrm{MI}_\vartheta\left(-\zeta^\vartheta\right) = \mathrm{MI}_\vartheta\left(\xi^\vartheta - \zeta^\vartheta\right) \tag{14}$$

$$(4)\ \mathrm{MI}_\vartheta\left(\xi^\vartheta\right)\mathrm{MI}_\vartheta\left(i^\vartheta\zeta^\vartheta\right) = \mathrm{MI}_\vartheta\left(\xi^\vartheta + i^\vartheta\zeta^\vartheta\right) \tag{15}$$

$$(5)\ \mathrm{MI}_\vartheta\left(i^\vartheta\xi^\vartheta\right)\mathrm{MI}_\vartheta\left(i^\vartheta\zeta^\vartheta\right) = \mathrm{MI}_\vartheta\left(i^\vartheta\xi^\vartheta + i^\vartheta\zeta^\vartheta\right) \tag{16}$$

3. Construct of the NLFODEs

In the view of Equation (8), we define the following NLFODE [48]:

$$\varphi_\vartheta\left(\wp^\vartheta\right) = \chi_1\mathrm{SH}_\vartheta\left(\chi_2\wp^\vartheta\right), \tag{17}$$

Taking the LFD of Equation (17), we have:

$$\begin{aligned} D^{(\vartheta)}\varphi_\vartheta\left(\wp^\vartheta\right) &= D^{(\vartheta)}\left[\chi_1\mathrm{SH}_\vartheta\left(\chi_2\wp^\vartheta\right)\right] = D^{(\vartheta)}\left[\frac{2\chi_1}{\mathrm{MI}_\vartheta\left(\chi_2\wp^\vartheta\right)+\mathrm{MI}_\vartheta\left(-\chi_2\wp^\vartheta\right)}\right] \\ &= -\frac{2\chi_1\chi_2\left[\mathrm{MI}_\vartheta\left(\chi_2\wp^\vartheta\right)-\mathrm{MI}_\vartheta\left(-\chi_2\wp^\vartheta\right)\right]}{\left[\mathrm{MI}_\vartheta\left(\chi_2\wp^\vartheta\right)+\mathrm{MI}_\vartheta\left(-\chi_2\wp^\vartheta\right)\right]^2} \end{aligned} \tag{18}$$

which gives:

$$\begin{aligned} &\left[D^{(\vartheta)}\varphi\left(\wp^\vartheta\right)\right]^2 \\ &= \left\{-\frac{2\chi_1\chi_2\left[\mathrm{MI}_\vartheta\left(\chi_2\wp^\vartheta\right)-\mathrm{MI}_\vartheta\left(-\chi_2\wp^\vartheta\right)\right]}{\left[\mathrm{MI}_\vartheta\left(\chi_2\wp^\vartheta\right)+\mathrm{MI}_\vartheta\left(-\chi_2\wp^\vartheta\right)\right]^2}\right\}^2 = 4\chi_1^2\chi_2^2\frac{\left[\mathrm{MI}_\vartheta\left(2\chi_2\wp^\vartheta\right)+\mathrm{MI}_\vartheta\left(-2\chi_2\wp^\vartheta\right)-2\right]}{\left[\mathrm{MI}_\vartheta\left(\chi_2\wp^\vartheta\right)+\mathrm{MI}_\vartheta\left(-\chi_2\wp^\vartheta\right)\right]^4} \\ &= 4\chi_1^2\chi_2^2\frac{\left[\mathrm{MI}_\vartheta\left(\chi_2\wp^\vartheta\right)+\mathrm{MI}_\vartheta\left(-\chi_2\wp^\vartheta\right)\right]^2-4}{\left[\mathrm{MI}_\vartheta\left(\chi_2\wp^\vartheta\right)+\mathrm{MI}_\vartheta\left(-\chi_2\wp^\vartheta\right)\right]^4} = \chi_1^2\chi_2^2\left(\frac{4}{\left[\mathrm{MI}_\vartheta\left(\chi_2\wp^\vartheta\right)+\mathrm{MI}_\vartheta\left(-\chi_2\wp^\vartheta\right)\right]^2}, -, \frac{16}{\left[\mathrm{MI}_\vartheta\left(\chi_2\wp^\vartheta\right)+\mathrm{MI}_\vartheta\left(-\chi_2\wp^\vartheta\right)\right]^4}\right) \\ &= \chi_2^2\chi_1^2\left[\mathrm{SH}_\vartheta^2\left(\wp^\vartheta\right) - \mathrm{SH}_\vartheta^4\left(\wp^\vartheta\right)\right] = \chi_2^2\varphi_\vartheta^2\left(\wp^\vartheta\right)\left[1 - \frac{1}{\chi_1^2}\varphi_\vartheta^2\left(\wp^\vartheta\right)\right] \\ &= \chi_2^2\varphi_\vartheta^2\left(\wp^\vartheta\right) - \frac{\chi_2^2}{\chi_1^2}\varphi_\vartheta^4\left(\wp^\vartheta\right) \end{aligned} \tag{19}$$

Based on Equation (9), we can construct the following NLFODE [48]:

$$\varphi_\vartheta\left(\wp^\vartheta\right) = \chi_1\mathrm{CH}_\vartheta\left(\chi_2\wp^\vartheta\right), \tag{20}$$

Taking the LFD of the above equation as:

$$\begin{aligned} D^{(\vartheta)}\varphi_\vartheta\left(\wp^\vartheta\right) &= D^{(\vartheta)}\left[\chi_1\mathrm{CH}_\vartheta\left(\chi_2\wp^\vartheta\right)\right] = D^{(\vartheta)}\left[\frac{2\chi_1}{\mathrm{MI}_\vartheta\left(\chi_2\wp^\vartheta\right)-\mathrm{MI}_\vartheta\left(-\chi_2\wp^\vartheta\right)}\right] \\ &= -\frac{2\chi_1\chi_2\left[\mathrm{MI}_\vartheta\left(\chi_2\wp^\vartheta\right)+\mathrm{MI}_\vartheta\left(-\chi_2\wp^\vartheta\right)\right]}{\left[\mathrm{MI}_\vartheta\left(\chi_2\wp^\vartheta\right)-\mathrm{MI}_\vartheta\left(-\chi_2\wp^\vartheta\right)\right]^2} \end{aligned} \tag{21}$$

Then, we have:

$$
\begin{aligned}
&\left[D^{(\vartheta)}\varphi\left(\wp^{\vartheta}\right)\right]^{2} \\
&= \left\{-\frac{2\chi_1\chi_2\left[\mathrm{MI}_{\vartheta}\left(\chi_2\wp^{\vartheta}\right)+\mathrm{MI}_{\vartheta}\left(-\chi_2\wp^{\vartheta}\right)\right]}{\left[\mathrm{MI}_{\vartheta}\left(\chi_2\wp^{\vartheta}\right)-\mathrm{MI}_{\vartheta}\left(-\chi_2\wp^{\vartheta}\right)\right]^2}\right\}^2 = 4\chi_1^2\chi_2^2\frac{\left[\mathrm{MI}_{\vartheta}\left(2\chi_2\wp^{\vartheta}\right)+\mathrm{MI}_{\vartheta}\left(-2\chi_2\wp^{\vartheta}\right)+2\right]}{\left[\mathrm{MI}_{\vartheta}\left(\chi_2\wp^{\vartheta}\right)-\mathrm{MI}_{\vartheta}\left(-\chi_2\wp^{\vartheta}\right)\right]^4} \\
&= 4\chi_1^2\chi_2^2\frac{\left[\mathrm{MI}_{\vartheta}\left(\chi_2\wp^{\vartheta}\right)-\mathrm{MI}_{\vartheta}\left(-\chi_2\wp^{\vartheta}\right)\right]^2+4}{\left[\mathrm{MI}_{\vartheta}\left(\chi_2\wp^{\vartheta}\right)-\mathrm{MI}_{\vartheta}\left(-\chi_2\wp^{\vartheta}\right)\right]^4} = \chi_1^2\chi_2^2\left(\begin{array}{l}\frac{4}{\left[\mathrm{MI}_{\vartheta}\left(\chi_2\wp^{\vartheta}\right)-\mathrm{MI}_{\vartheta}\left(-\chi_2\wp^{\vartheta}\right)\right]^2} \\ +\frac{16}{\left[\mathrm{MI}_{\vartheta}\left(\chi_2\wp^{\vartheta}\right)-\mathrm{MI}_{\vartheta}\left(-\chi_2\wp^{\vartheta}\right)\right]^4}\end{array}\right) \\
&= \chi_2^2\chi_1^2\left[\mathrm{CH}_{\vartheta}^2\left(\wp^{\vartheta}\right)+\mathrm{CH}_{\vartheta}^4\left(\wp^{\vartheta}\right)\right] \\
&= \chi_2^2\varphi_{\vartheta}^2\left(\wp^{\vartheta}\right)+\frac{\chi_2^2}{\chi_1^2}\varphi_{\vartheta}^4\left(\wp^{\vartheta}\right)
\end{aligned}
\tag{22}
$$

We can also consider the following NLFODE [48]:

$$
\varphi_{\vartheta}\left(\wp^{\vartheta}\right) = \chi_1\mathrm{SE}_{\vartheta}\left(\chi_2\wp^{\vartheta}\right),
\tag{23}
$$

Similarly, its LFD is given by:

$$
\begin{aligned}
D^{(\vartheta)}\varphi_{\vartheta}\left(\wp^{\vartheta}\right) = D^{(\vartheta)}\left[\chi_1\mathrm{SE}_{\vartheta}\left(\chi_2\wp^{\vartheta}\right)\right] = D^{(\vartheta)}\left[\frac{2\chi_1}{\mathrm{MI}_{\vartheta}\left(\chi_2 i^{\vartheta}\wp^{\vartheta}\right)+\mathrm{MI}_{\vartheta}\left(-\chi_2 i^{\vartheta}\wp^{\vartheta}\right)}\right] \\
= -\frac{2\chi_1\chi_2\left[i^{\vartheta}\mathrm{MI}_{\vartheta}\left(\chi_2 i^{\vartheta}\wp^{\vartheta}\right)-i^{\vartheta}\mathrm{MI}_{\vartheta}\left(-\chi_2 i^{\vartheta}\wp^{\vartheta}\right)\right]}{\left[\mathrm{MI}_{\vartheta}\left(\chi_2 i^{\vartheta}\wp^{\vartheta}\right)+\mathrm{MI}_{\vartheta}\left(-\chi_2 i^{\vartheta}\wp^{\vartheta}\right)\right]^2}
\end{aligned}
\tag{24}
$$

Such that

$$
\begin{aligned}
&\left[D^{(\vartheta)}\varphi_{\vartheta}\left(\wp^{\vartheta}\right)\right]^{2} \\
&= \left\{-\frac{2\chi_1\chi_2\left[i^{\vartheta}\mathrm{MI}_{\vartheta}\left(\chi_2 i^{\vartheta}\wp^{\vartheta}\right)-i^{\vartheta}\mathrm{MI}_{\vartheta}\left(-\chi_2 i^{\vartheta}\wp^{\vartheta}\right)\right]}{\left[\mathrm{MI}_{\vartheta}\left(\chi_2 i^{\vartheta}\wp^{\vartheta}\right)+\mathrm{MI}_{\vartheta}\left(-\chi_2 i^{\vartheta}\wp^{\vartheta}\right)\right]^2}\right\}^2 = -4\chi_1^2\chi_2^2\frac{\left[\mathrm{MI}_{\vartheta}\left(2\chi_2 i^{\vartheta}\wp^{\vartheta}\right)+\mathrm{MI}_{\vartheta}\left(-2\chi_2 i^{\vartheta}\wp^{\vartheta}\right)-2\right]}{\left[\mathrm{MI}_{\vartheta}\left(\chi_2 i^{\vartheta}\wp^{\vartheta}\right)+\mathrm{MI}_{\vartheta}\left(-\chi_2 i^{\vartheta}\wp^{\vartheta}\right)\right]^4} \\
&= -4\chi_1^2\chi_2^2\frac{\left[\mathrm{MI}_{\vartheta}\left(\chi_2 i^{\vartheta}\wp^{\vartheta}\right)+\mathrm{MI}_{\vartheta}\left(-\chi_2 i^{\vartheta}\wp^{\vartheta}\right)\right]^2-4}{\left[\mathrm{MI}_{\vartheta}\left(\chi_2 i^{\vartheta}\wp^{\vartheta}\right)+\mathrm{MI}_{\vartheta}\left(-\chi_2 i^{\vartheta}\wp^{\vartheta}\right)\right]^4} = \chi_1^2\chi_2^2\left(\begin{array}{l}-\frac{4}{\left[\mathrm{MI}_{\vartheta}\left(\chi_2 i^{\vartheta}\wp^{\vartheta}\right)+\mathrm{MI}_{\vartheta}\left(-\chi_2 i^{\vartheta}\wp^{\vartheta}\right)\right]^2} \\ +\frac{16}{\left[\mathrm{MI}_{\vartheta}\left(\chi_2 i^{\vartheta}\wp^{\vartheta}\right)+\mathrm{MI}_{\vartheta}\left(-\chi_2 i^{\vartheta}\wp^{\vartheta}\right)\right]^4}\end{array}\right) \\
&= \chi_1^2\chi_2^2\left[-\mathrm{SE}_{\vartheta}^2\left(\wp^{\vartheta}\right)+\mathrm{SE}_{\vartheta}^4\left(\wp^{\vartheta}\right)\right] = \chi_2^2\varphi_{\vartheta}^2\left(\wp^{\vartheta}\right)\left[-1+\frac{1}{\chi_1^2}\varphi_{\vartheta}^2\left(\wp^{\vartheta}\right)\right] \\
&= -\chi_2^2\varphi_{\vartheta}^2\left(\wp^{\vartheta}\right)+\frac{\chi_2^2}{\chi_1^2}\varphi_{\vartheta}^4\left(\wp^{\vartheta}\right)
\end{aligned}
\tag{25}
$$

In the light of Equation (11), we construct another NLFODE as [48]:

$$
\varphi_{\vartheta}\left(\wp^{\vartheta}\right) = \chi_1\mathrm{CS}_{\vartheta}\left(\chi_2\wp^{\vartheta}\right),
\tag{26}
$$

Applying the LFD for Equation (26) as:

$$
\begin{aligned}
D^{(\vartheta)}\varphi_{\vartheta}\left(\wp^{\vartheta}\right) = D^{(\vartheta)}\left[\chi_1\mathrm{CS}_{\vartheta}\left(\chi_2\wp^{\vartheta}\right)\right] = D^{(\vartheta)}\left[\frac{2\chi_1 i^{\vartheta}}{\mathrm{MI}_{\vartheta}\left(\chi_2 i^{\vartheta}\wp^{\vartheta}\right)-\mathrm{MI}_{\vartheta}\left(-\chi_2 i^{\vartheta}\wp^{\vartheta}\right)}\right] \\
= \frac{2\chi_1\chi_2\left[\mathrm{MI}_{\vartheta}\left(\chi_2 i^{\vartheta}\wp^{\vartheta}\right)+\mathrm{MI}_{\vartheta}\left(-\chi_2 i^{\vartheta}\wp^{\vartheta}\right)\right]}{\left[\mathrm{MI}_{\vartheta}\left(\chi_2 i^{\vartheta}\wp^{\vartheta}\right)-\mathrm{MI}_{\vartheta}\left(-\chi_2 i^{\vartheta}\wp^{\vartheta}\right)\right]^2}
\end{aligned}
\tag{27}
$$

Thus, we have:

$$\begin{aligned}
&\left[D^{(\vartheta)}\varphi\left(\wp^{\vartheta}\right)\right]^{2}\\
&=\left\{\frac{2\chi_1\chi_2\left[\mathrm{MI}_{\vartheta}\left(\chi_2 i^{\vartheta}\wp^{\vartheta}\right)+\mathrm{MI}_{\vartheta}\left(-\chi_2 i^{\vartheta}\wp^{\vartheta}\right)\right]}{\left[\mathrm{MI}_{\vartheta}\left(\chi_2 i^{\vartheta}\wp^{\vartheta}\right)-\mathrm{MI}_{\vartheta}\left(-\chi_2 i^{\vartheta}\wp^{\vartheta}\right)\right]^{2}}\right\}^{2}=4\chi_1^2\chi_2^2\frac{\left[\mathrm{MI}_{\vartheta}\left(2\chi_2 i^{\vartheta}\wp^{\vartheta}\right)+\mathrm{MI}_{\vartheta}\left(-2\chi_2 i^{\vartheta}\wp^{\vartheta}\right)+2\right]}{\left[\mathrm{MI}_{\vartheta}\left(\chi_2 i^{\vartheta}\wp^{\vartheta}\right)-\mathrm{MI}_{\vartheta}\left(-\chi_2 i^{\vartheta}\wp^{\vartheta}\right)\right]^{4}}\\
&=4\chi_1^2\chi_2^2\frac{\left[\mathrm{MI}_{\vartheta}\left(\chi_2 i^{\vartheta}\wp^{\vartheta}\right)-\mathrm{MI}_{\vartheta}\left(-\chi_2 i^{\vartheta}\wp^{\vartheta}\right)\right]^{2}+4}{\left[\mathrm{MI}_{\vartheta}\left(\chi_2 i^{\vartheta}\wp^{\vartheta}\right)-\mathrm{MI}_{\vartheta}\left(-\chi_2 i^{\vartheta}\wp^{\vartheta}\right)\right]^{4}}=\chi_1^2\chi_2^2\left(\begin{array}{c}-\frac{-4}{\left[\mathrm{MI}_{\vartheta}\left(\chi_2 i^{\vartheta}\wp^{\vartheta}\right)-\mathrm{MI}_{\vartheta}\left(-\chi_2 i^{\vartheta}\wp^{\vartheta}\right)\right]^{2}}\\+\frac{16}{\left[\mathrm{MI}_{\vartheta}\left(\chi_2 i^{\vartheta}\wp^{\vartheta}\right)-\mathrm{MI}_{\vartheta}\left(-\chi_2 i^{\vartheta}\wp^{\vartheta}\right)\right]^{4}}\end{array}\right)\\
&=\chi_1^2\chi_2^2\left[-\mathrm{CS}_{\vartheta}^{2}\left(\wp^{\vartheta}\right)+\mathrm{CS}_{\vartheta}^{4}\left(\wp^{\vartheta}\right)\right]=\chi_2^2\varphi_{\vartheta}^{2}\left(\wp^{\vartheta}\right)\left[-1+\frac{1}{\chi_1^2}\varphi_{\vartheta}^{2}\left(\wp^{\vartheta}\right)\right]\\
&=-\chi_2^2\varphi_{\vartheta}^{2}\left(\wp^{\vartheta}\right)+\frac{\chi_2^2}{\chi_1^2}\varphi_{\vartheta}^{4}\left(\wp^{\vartheta}\right)
\end{aligned}\tag{28}$$

From Equations (19), (22), (25), and (28), we can conclude the general NLFODE as the following form by introducing two parameters p and q:

$$\left[D^{(\vartheta)}\varphi_{\vartheta}\left(\wp^{\vartheta}\right)\right]^{2}=p\chi_2^2\varphi_{\vartheta}^{2}\left(\wp^{\vartheta}\right)+q\frac{\chi_2^2}{\chi_1^2}\varphi_{\vartheta}^{4}\left(\wp^{\vartheta}\right),\tag{29}$$

Obviously, its exact ND solutions are given as:

$$\varphi_{\vartheta}\left(\wp^{\vartheta}\right)=\begin{cases}\chi_1\mathrm{SH}_{\vartheta}\left(\chi_2\wp^{\vartheta}\right),\ for\ p=1,q=-1\\ \chi_1\mathrm{CH}_{\vartheta}\left(\chi_2\wp^{\vartheta}\right),\ for\ p=1,q=1\\ \chi_1\mathrm{SE}_{\vartheta}\left(\chi_2\wp^{\vartheta}\right),\ for\ p=-1,q=1\\ \chi_1\mathrm{CS}_{\vartheta}\left(\chi_2\wp^{\vartheta}\right),\ for\ p=-1,q=1\end{cases}.\tag{30}$$

4. Yang's Special Function Method

In this section, Yang's special function method will be adopted to search for the exact ND solutions. For this goal, the following ND transformation is considered [49–51]:

$$\aleph_{\vartheta}\left(x^{\vartheta},t^{\vartheta}\right)=\aleph_{\vartheta}\left(\wp^{\vartheta}\right),\ \wp^{\vartheta}=\rho^{\vartheta}x^{\vartheta}-\varpi^{\vartheta}t^{\vartheta},\tag{31}$$

Additionally, there is:

$$\lim_{\vartheta\to1}\wp^{\vartheta}=\rho x-\varpi t,\tag{32}$$

Putting Equation (31) into Equation (1) gives:

$$\frac{\partial^{\vartheta}\aleph}{\partial t^{\vartheta}}=-\varpi^{\vartheta}\frac{d^{\vartheta}\aleph}{d\wp^{\vartheta}},\tag{33}$$

$$\frac{\partial^{\vartheta}\aleph_{\vartheta}}{\partial x^{\vartheta}}=\rho^{\vartheta}\frac{d^{\vartheta}\aleph_{\vartheta}}{d\wp^{\vartheta}},\tag{34}$$

$$\frac{\partial^{2\vartheta}}{x^{2\vartheta}}\left(\frac{\partial^{\vartheta}\aleph_{\vartheta}}{\partial t^{\vartheta}}\right)=-\rho^{2\vartheta}\varpi^{\vartheta}\frac{d^{3\vartheta}\aleph_{\vartheta}}{d\zeta^{3\vartheta}},\tag{35}$$

Taking them into Equation (1) yields:

$$\left(\rho^{\vartheta}-\varpi^{\vartheta}\right)\frac{d^{\vartheta}\aleph}{d\wp^{\vartheta}}+k\rho^{\vartheta}\aleph^{2}\frac{d^{\vartheta}\aleph}{d\wp^{\vartheta}}-\rho^{2\vartheta}\varpi^{\gamma}\frac{d^{3\vartheta}\aleph}{d\wp^{3\vartheta}}=0,\tag{36}$$

where $\rho^{\vartheta}-\varpi^{\vartheta}\neq0$.

Applying the LFI to Equation (36) and ignoring the integral constant yields:

$$\left(\rho^{\vartheta}-\varpi^{\vartheta}\right)\aleph_{\vartheta}+\frac{1}{3}k\rho^{\vartheta}\aleph_{\vartheta}^{3}-\rho^{2\vartheta}\varpi^{\gamma}\frac{d^{2\vartheta}\aleph_{\vartheta}}{d\wp^{2\vartheta}}=0,\tag{37}$$

By multiplying both sides of above equation by $\frac{d^{\vartheta}\aleph_{\vartheta}}{d\wp^{\vartheta}}$, we have:

$$\left(\rho^{\vartheta}-\varpi^{\vartheta}\right)\aleph_{\vartheta}\frac{d^{\vartheta}\aleph_{\vartheta}}{d\wp^{\vartheta}}+\frac{1}{3}k\rho^{\vartheta}\aleph_{\vartheta}^{3}\frac{d^{\vartheta}\aleph_{\vartheta}}{d\wp^{\vartheta}}-\rho^{2\vartheta}\varpi^{\gamma}\frac{d^{2\vartheta}\aleph_{\vartheta}}{d\wp^{2\vartheta}}\frac{d^{\vartheta}\aleph_{\vartheta}}{d\wp^{\vartheta}}=0, \tag{38}$$

Taking the LFI of the above equation leads to:

$$\frac{1}{2}\left(\rho^{\vartheta}-\varpi^{\vartheta}\right)\aleph_{\vartheta}^{2}+\frac{1}{12}k\rho^{\vartheta}\aleph_{\vartheta}^{4}-\frac{1}{2}\rho^{2\vartheta}\varpi^{\gamma}\left(\frac{d^{\vartheta}\aleph_{\vartheta}}{d\wp^{\vartheta}}\right)^{2}=\Delta, \tag{39}$$

Here, Δ is the integral constant. Letting Δ to be zero, we have:

$$\frac{1}{2}\left(\rho^{\vartheta}-\varpi^{\vartheta}\right)\aleph_{\vartheta}^{2}+\frac{1}{12}k\rho^{\vartheta}\aleph_{\vartheta}^{4}-\frac{1}{2}\rho^{2\vartheta}\varpi^{\gamma}\left(\frac{d^{\vartheta}\aleph_{\vartheta}}{d\wp^{\vartheta}}\right)^{2}=0. \tag{40}$$

Such that:

$$\left(\frac{d^{\vartheta}\aleph_{\vartheta}}{d\wp^{\vartheta}}\right)^{2}=\frac{\rho^{\vartheta}-\varpi^{\vartheta}}{\rho^{2\vartheta}\varpi^{\vartheta}}\aleph_{\vartheta}^{2}+\frac{k}{6\rho^{\vartheta}\varpi^{\vartheta}}\aleph_{\vartheta}^{4}. \tag{41}$$

By comparing Equation (41) and Equation (29), we have:
Set 1: For $p=1$, $q=-1$, there is:

$$\frac{\rho^{\vartheta}-\varpi^{\vartheta}}{\rho^{2\vartheta}\varpi^{\vartheta}}=\chi_{2}^{2}, \tag{42}$$

$$\frac{k}{6\rho^{\vartheta}\varpi^{\vartheta}}=-\frac{\chi_{2}^{2}}{\chi_{1}^{2}}, \tag{43}$$

According to Equations (42) and (43), we have:

$$\chi_{1}=\sqrt{\frac{6\left(\varpi^{\vartheta}-\rho^{\vartheta}\right)}{k\rho^{\vartheta}}},\ \chi_{2}=\sqrt{\frac{\rho^{\vartheta}-\varpi^{\vartheta}}{\rho^{2\vartheta}\varpi^{\vartheta}}}, \tag{44}$$

Thus, we can obtain the exact solution of Equation (1) as:

$$\aleph_{\vartheta}(x,t)=\sqrt{\frac{6\left(\varpi^{\vartheta}-\rho^{\vartheta}\right)}{k\rho^{\vartheta}}}\mathrm{SH}_{\vartheta}\left(\sqrt{\frac{\rho^{\vartheta}-\varpi^{\vartheta}}{\rho^{2\vartheta}\varpi^{\vartheta}}}\left(\rho^{\vartheta}x^{\vartheta}-\varpi^{\vartheta}t^{\vartheta}\right)\right). \tag{45}$$

For $\varpi^{\vartheta}=1$, $\rho^{\vartheta}=2$, $k=-1$, we display the profile of the exact ND solution given by Equation (45) on the CS in Figure 2. Here, the t and x are both selected on the CS range 0 to 1, and the fractional order is used as $\vartheta=\ln 2/\ln 3$. It can be found that the value of the solution is between 1 and 1.8. In addition, the figure is the blocky structure which conforms to the CS characteristics.

Set 2: For $p=1$, $q=1$, there is:

$$\frac{\rho^{\vartheta}-\varpi^{\vartheta}}{\rho^{2\vartheta}\varpi^{\vartheta}}=\chi_{2}^{2}, \tag{46}$$

$$\frac{k}{6\rho^{\vartheta}\varpi^{\vartheta}}=\frac{\chi_{2}^{2}}{\chi_{1}^{2}}, \tag{47}$$

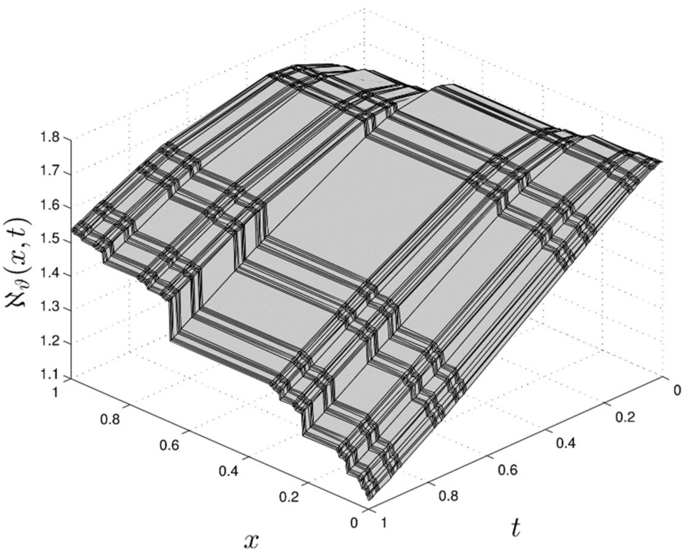

Figure 2. The profile of Equation (45) on CS with $\varpi^\vartheta = 1$, $\rho^\vartheta = 2$, and $k = -1$ for $\vartheta = \ln 2/\ln 3$.

We have:

$$\chi_1 = \sqrt{\frac{6(\rho^\vartheta - \varpi^\vartheta)}{k\rho^\vartheta}},\ \chi_2 = \sqrt{\frac{\rho^\vartheta - \varpi^\vartheta}{\rho^{2\vartheta}\varpi^\vartheta}}, \tag{48}$$

Then, the second exact ND solution of Equation (1) is attained as:

$$\aleph_\vartheta(x,t) = \sqrt{\frac{6(\rho^\vartheta - \varpi^\vartheta)}{k\rho^\vartheta}}\mathrm{CH}_\vartheta\left(\sqrt{\frac{\rho^\vartheta - \varpi^\vartheta}{\rho^{2\vartheta}\varpi^\vartheta}}\left(\rho^\vartheta x^\vartheta - \varpi^\vartheta t^\vartheta\right)\right). \tag{49}$$

For using $\varpi^\vartheta = 1$, $\rho^\vartheta = 2$, $k = 1$, we display the solution Equation (49) on the CS for $\vartheta = \ln 2/\ln 3$ in Figure 3. The values of t and x are all selected on the CS from 0 to 1. It can be found that the profile of Equation (49) is the blocky structure, and when the coordinate (x, t) is close to (0, 0), the value of the solution increases rapidly.

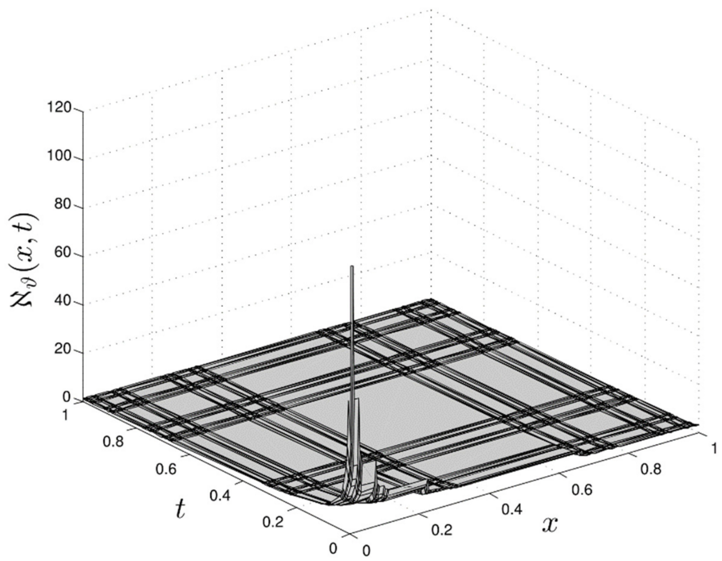

Figure 3. The profile of Equation (49) on CS with $\varpi^\vartheta = 1$, $\rho^\vartheta = 2$, $k = 1$ for $\vartheta = \ln 2/\ln 3$.

Set 3: For $p = -1$, $q = 1$, there is:

$$\frac{\rho^{\vartheta} - \varpi^{\vartheta}}{\rho^{2\vartheta}\varpi^{\vartheta}} = -\chi_2^2, \tag{50}$$

$$\frac{k}{6\rho^{\vartheta}\varpi^{\vartheta}} = \frac{\chi_2^2}{\chi_1^2}, \tag{51}$$

There is:

$$\chi_1 = \sqrt{\frac{6(\varpi^{\vartheta} - \rho^{\vartheta})}{k\rho^{\vartheta}}}, \ \chi_2 = \sqrt{\frac{\varpi^{\vartheta} - \rho^{\vartheta}}{\rho^{2\vartheta}\varpi^{\vartheta}}}, \tag{52}$$

Correspondingly, we can find the exact ND solutions of Equation (1) as:

$$\aleph_{\vartheta}(x,t) = \sqrt{\frac{6(\varpi^{\vartheta} - \rho^{\vartheta})}{k\rho^{\vartheta}}} \mathrm{SE}_{\vartheta}\left(\sqrt{\frac{\varpi^{\vartheta} - \rho^{\vartheta}}{\rho^{2\vartheta}\varpi^{\vartheta}}}\left(\rho^{\vartheta}x^{\vartheta} - \varpi^{\vartheta}t^{\vartheta}\right)\right), \tag{53}$$

By choosing the parameters as $\varpi^{\vartheta} = 2$, $\rho^{\vartheta} = 1$, $k = 1$, we display the outline of Equation (53) on CS for $\vartheta = \ln 2 / \ln 3$ in Figure 4. Here, the outline of the solution is also the blocky structure which corresponds to the characteristics of the CS. Additionally, the value of the solution increases rapidly when (x, t) is close to (1, 0).

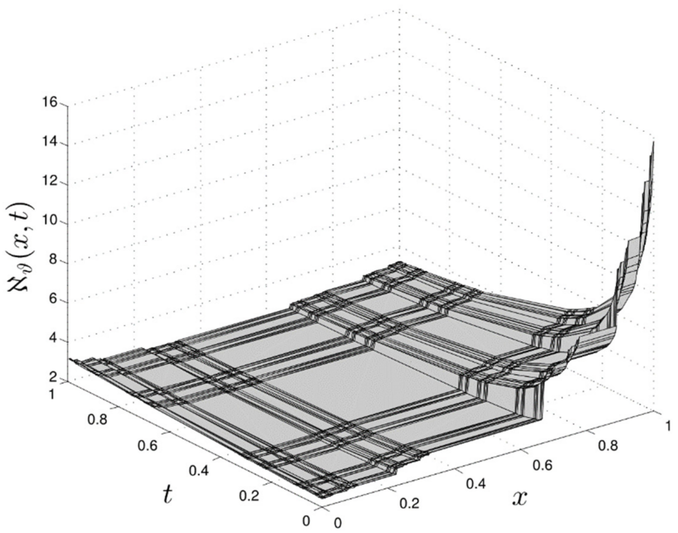

Figure 4. The profile of Equation (53) on CS with $\varpi^{\vartheta} = 2$, $\rho^{\vartheta} = 1$, $k = 1$ for $\vartheta = \ln 2 / \ln 3$.

Set 4: For $p = -1$, $q = 1$, there is:

$$\frac{\rho^{\vartheta} - \varpi^{\vartheta}}{\rho^{2\vartheta}\varpi^{\vartheta}} = -\chi_2^2, \tag{54}$$

$$\frac{k}{6\rho^{\vartheta}\varpi^{\vartheta}} = \frac{\chi_2^2}{\chi_1^2}, \tag{55}$$

There is:

$$\chi_1 = \sqrt{\frac{6(\varpi^{\vartheta} - \rho^{\vartheta})}{k\rho^{\vartheta}}}, \ \chi_2 = \sqrt{\frac{\varpi^{\vartheta} - \rho^{\vartheta}}{\rho^{2\vartheta}\varpi^{\vartheta}}}, \tag{56}$$

Correspondingly, we can find the exact ND solutions of Equation (1) as:

$$\aleph_\vartheta(x,t) = \sqrt{\frac{6(\varpi^\vartheta - \rho^\vartheta)}{k\rho^\vartheta}} \mathrm{CS}_\vartheta\left(\sqrt{\frac{\varpi^\vartheta - \rho^\vartheta}{\rho^{2\vartheta}\varpi^\vartheta}}\left(\rho^\vartheta x^\vartheta - \varpi^\vartheta t^\vartheta\right)\right). \tag{57}$$

For using $\varpi^\vartheta = -1$, $\rho^\vartheta = 1$, $k = -1$, we display the profile of the Equation (57) solution on the CS with $\vartheta = \ln 2/\ln 3$ in Figure 5. Similar to Equation (49), the blocky structure increases rapidly when (x, t) is close to (0, 0).

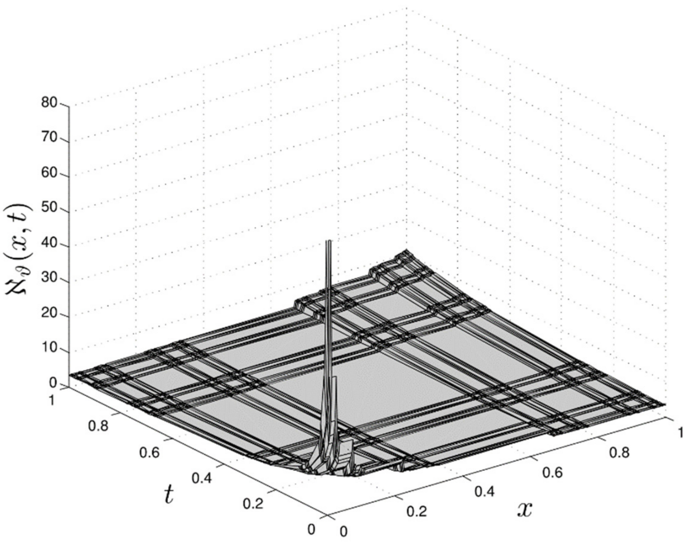

Figure 5. The profile of Equation (57) on CS with $\varpi^\vartheta = -1$, $\rho^\vartheta = 1$, $k = -1$ for $\vartheta = \ln 2/\ln 3$.

It should be noted that the correctness of the exact obtained ND solutions provided by Equations (45), (49), (53), and (57) are verified by substituting them into Equation (41).

5. Conclusions

This paper proposes a new local fractional modified Benjamin–Bona–Mahony equation based on the local fractional derivative. A group of nonlinear local fractional ordinary differential equations is constructed by defining some elementary functions via the Mittag–Leffler function on the Cantor set. A simple and effective approach, called Yang's special function method, is suggested for the first time to solve this problem. By using this method, we can obtain four different exact solutions in just one step. Furthermore, the obtained solutions on the Cantor set are outlined in the form of a 3-D plot. It is revealed that the one-step method is effective and can be utilized to study the other local fractional PDEs.

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