fractal and fractional

Article

Adaptive Fuzzy Fault-Tolerant Control of Uncertain Fractional-Order Nonlinear Systems with Sensor and Actuator Faults

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Abstract: In this work, an adaptive fuzzy backstepping fault-tolerant control (FTC) issue is tackled for uncertain fractional-order (FO) nonlinear systems with sensor and actuator faults. A fuzzy logic system is exploited to manage unknown nonlinearity. In addition, a novel FO nonlinear filter-based dynamic surface control (DSC) method is constructed, effectively avoiding the inherent complexity explosion problem in the backstepping recursive process, and in the light of the construction of auxiliary functions, compensating the coupling term introduced by faults. On account of certain assumptions, the stability criterion of the FO Lyapunov function is applied to guarantee the stability of the closed-loop system. Finally, the simulation example verifies the validity of the presented control strategy.

Keywords: fractional-order nonlinear systems; fault-tolerant control; nonlinear filter; dynamic surface control; adaptive fuzzy control

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1. Introduction

The fractional order system is an extension of the integer-order system in traditional control theory. It can describe non-rigid dynamic systems more accurately, and has been widely used in bioengineering, thermoelectric systems, electronic power systems, battery management systems and so on. In recent years, with the wide application of fractional-order systems, fractional-order control theory has been developed rapidly and made some progress. To go a step further, a vast array of fascinating results has been gained for controller design of fractional order systems [1–4]. However, the above works are only applied to fractional order systems in which the system mode is linear or the nonlinearities are known. Thus, these schemes cannot be applied to fractional-order systems with unknown nonlinearities. Generally speaking, due to changes in the internal or external environment of the system, nonlinear and uncertainty often exist simultaneously in the control research of nonlinear systems, which makes the control design and stability analysis of the FO nonlinear systems extremely difficult. Therefore, some scholars have studied the control design of fractional nonlinear systems and made some progress [5–8]. In [5], a fractional order fast terminal sliding mode control method is proposed under the assumption that the nonlinear function is bounded. In [6,7], state feedback control methods based on the indirect Lyapunov method and direct Lyapunov method are proposed for Lipschitz-type nonlinear systems and multi-agent nonlinear systems, respectively. Further, for linear parameterized fractional-order nonlinear systems, an adaptive control method based on fractional-order parameter updating law is presented in [8]. The control design method proposed in [5–8] requires that the nonlinear system must meet the matching condition, that is, the nonlinearity of the system must appear in the same equation as the controller, and the control scheme obtained is established under the theoretical framework of feedback linearization. In order to solve the control problem that the controlled object is a strict feedback nonlinear system (a nonlinear system that meets the non-matching conditions), with the help of backstepping control technology and based on direct Lyapunov function method, fractional adaptive backward step recursive state feedback and output feedback control methods are presented in [9,10], respectively. Based on the indirect Lyapunov function method, the authors in [11,12] proposed adaptive backstepping recurrent state feedback and output feedback control methods, respectively.

At present, although some research achievements have been made in the control problems of frit-order nonlinear systems, the above research methods all require that the model of the controlled object is known or the nonlinear function can be parameterized. Obviously, a great majority of the works of fractional-order adaptive control assume that uncertainties existing in the system can be linearly parameterized, which is unrealistic in practical control design. Therefore, it is a challenging task to solve the controller design problem for FO nonlinear systems with unknown nonlinear functions. To handle such an issue, since fuzzy logic systems or neural networks have good approximation and learning ability to unknown nonlinear dynamics, nonlinear intelligent control schemes based on fuzzy logic systems or neural networks are often developed, and are widely used in uncertain fractional-order nonlinear systems, and many fruitful research results are achieved [13–18]. The main representative achievements are as follows: in [13], a fuzzy adaptive backstepping recursive state feedback control scheme is proposed for the nonlinear system with uncertain fractional order strict feedback with single input and single output. In the design process, fuzzy logic system is used to model the unknown nonlinear system, and the parameters are updated online by designing the fractional order adaptive law. Based on finite-time sliding mode control theory, a fractional-order finite-time fuzzy adaptive state feedback sliding mode control method is proposed in [14,15] for single-input single-output uncertain fractional-order strict feedback nonlinear systems. For uncertain switching fractional-order nonlinear systems, based on the common Lyapunov function method, a fractional-order switching neural network adaptive state feedback control scheme is proposed in [16]. In [17,18], for uncertain fractional nonlinear multi-agent systems, the robust consistency tracking problem in general undirected topology and directed topology is studied by using neural network to model the controlled object. Although fractional nonlinear systems have made some achievements in the design of adaptive control and fuzzy/neural network control, the above achievements are all under the premise of normal operation of the system actuator or sensor. When actuator or sensor failure occurs in the system, the above method can not meet the dual requirements of control performance and stability analysis. If the fault can not be identified and compensated in time, the control accuracy is bound to decline, and even affect the normal operation of the system.

In practical engineering, there usually exists an inescapable fact that system components (e.g., actuators, sensors, etc.) may abruptly encounter an unexpected occurrence during the operating process, which impairs system performance, disrupts system stability, and even potentially triggers a fatal disaster [19,20]. To assure reliability and security, the area of exploring faults has drawn intensive concern. Regarding this, the FTC strategy was presented, aiming at handling various faults while ensuring system stability and performance, and recently fruitful developments have been reported on actuator faults [21–27]. Focusing on linear systems, several FTC approaches were designed to tackle partial loss of effectiveness and locked faults [21,22], further extended in distinct varieties to nonlinear cases [23–27]. Nevertheless, collating the aforementioned works, it is obvious that they only concentrate on the integer-order nonlinear case. Actually, compared to traditional integer-order case, FO nonlinear systems have broader prospects due to their unique characteristics, e.g., memory and inheritance, and have received the favor of numerous scholars. Certainly, the achievements on actuator faults in FO nonlinear systems will not be absent [28–32]. Through the sliding mode control technique, the FTC issue of triangular

FO nonlinear systems with actuator faults was addressed [28], where the multiple control inputs are translated into the special mode in view of actuator fault characteristics, while the nonlinear dynamics considered in [28] must meet the matching condition. Further, the authors in [29] extend the previous results to control nonlinear systems with actuator faults that the non-matching condition is not satisfied. On the basis of adaptive control technique, actuator faults coefficients are effectively estimated in [30], where the fault compensation relies on the introduction of multiple parameter adaptive laws, which inevitably increases the complexity of the controller. To erase such a limitation, the Nussbaum gain technique is introduced in [31] to handle the lumped uncertainties of multiple faulty actuators and unknown control gains; however, the catch is that high-frequency oscillations may occur. It should be emphasized that the above control methods are only suitable for the limited number of faults in the actuator, and the fault mode remains unchanged. Aiming at this issue, a special auxiliary function is employed in [32] to compensate the effect of intermittent actuator faults. Unfortunately, this method relies too much on the ability of approximation technique, which may reduce the performance. Although the research on intelligent fault-tolerant control for uncertain fractional nonlinear systems has achieved some achievements, the works [28–32] are limited to the study of actuator fault compensation control, and the fault-tolerant control of fractional nonlinear systems with sensor faults is not reported. It is vital to be aware that the sensor is more prone to faults than the actuator, and the case sensor encounter unexpected occurrence may be even worse since misleading information from faulty sensors causes the entire system to be embroiled in risks. Note that in actuator faults compensation control design, the actuator's output u^F is dependent on the output of controller u, where u is available but u^F is unknown. Meanwhile, in the sensor faults case, the measurement values χ_i^F depend on unknown system state variables χ_i. It is obvious that the previous actuator faults compensation schemes are invalid for sensor faults compensation control problem. Thus, it is still a tremendous challenge to develop a sensor FTC strategy, not to mention the case with both actuator and sensor faults.

Meanwhile, with the repeated differentiation of the virtual control function and the continuous increase of the system dimension in the control process, the computational load of the backstepping control design is obviously terrible. This phenomenon is known as computational expansion. For the sake of maintaining system stability and reducing the computational burden existed in traditional backstepping recursive methods, the inverse control design idea based on filter technology was presented for the first time in [33]. Similar ideas were also explored in [34,35]. However, the results obtained based on this technique in [33–35] only referred to the integer-order nonlinear case. Compared to the integer-order case, the parameters in FO case have more degrees of freedom, resulting in the inapplicability of Newton–Leibniz formula and the derivative rules in integer-order case, which hence makes it exceedingly thorny to employ integer-order control method directly to FO. In [36,37], authors drew on the aforementioned ideas to explore FO, and made momentous progresses via linear filter, while neglecting the compensation of the error boundary, which lead to the increase of the error boundary. In order to better compensate the boundary errors, which were caused by the introduction of the filter, the authors of [38,39] proposed the design idea of nonlinear filtering by introducing auxiliary functions, and, combined with the traditional backstepping control, they effectively avoided the computational expansion and the impact of the boundary error. But the common feature of these works is that the actuators and sensors in the FO systems studied are under normal conditions. In the process of research, we find that when both actuators and sensors fail, it would bring about the emergence of coupling terms. Unfortunately, these existing nonlinear filtering-based strategies cannot deal with the coupling terms caused by the coexistence of actuator and sensor faults.

Enlightened by the above motivations, we focus on the adaptive fuzzy FTC problem for FO nonlinear systems with both actuator and sensor faults. The main contributions of this paper are divided into two points:

(1) This paper addresses the FTC issue of FO nonlinear systems with simultaneous actuator faults and sensor faults. It should be mentioned that unlike this paper, the authors in [28–32] considered the adaptive fault-tolerant control issue for fractional-order nonlinear systems with actuator faults. However, in the actual control system, the sensor is more prone to failure than the actuator, and the performance of the system is also heavily dependent on the output signal of the sensor, so even if the sensor undergoes a small fault, the feedback control and stability of the closed-loop system will be greatly affected. It is obvious that the previous actuator faults compensation schemes are invalid for sensor faults compensation control problem, not to mention the case with both actuator and sensor faults. This makes the research of this work more difficult and challenging.
(2) A nonlinear filtering-based DSC strategy is established by introducing auxiliary functions, which not only effectively solves the issue of computational burden existing in traditional FO nonlinear strict feedback systems, but also improves the control performance in contrast to the traditional linear filters-based DSC results [36,37]. Different from nonlinear filtering results [38,39], this paper compensates the effects of lumped uncertainties caused by actuator faults and sensor faults by designing a quadratic Lyapunov function which includes the lower bound of actuator and sensor faults coefficients.
(3) The proposed fault compensation mechanism can erase the limitation condition that the unknown functions dependent on state variable χ_i must satisfy the monotonically increasing property, by use of the characteristics of fuzzy basis functions.

The paper is organized as follows. The preliminaries and problem formulation are presented in Section 2. In Section 3, nonlinear filter-based adaptive controller design and stability analysis are addressed. Effectiveness of the proposed scheme has been demonstrated via simulation study in Section 4. The conclusion is drawn in Section 5.

Notation. In this paper, some specific notations are employed. C represents a complex set; N is a set of natural numbers; $|\cdot|$ is the absolute value of real number; $\|\cdot\|$ denotes the Euclidean norm of a vector or the corresponding induced norm of a matrix; R^i denotes i-dimensional Euclidean space.

2. Preliminaries and Problem Formulation

2.1. Preliminaries

To facilitate the subsequent strategy, several preliminaries are presented.

Definition 1 ([40])**.** *A continuous function $\alpha : [0, \alpha) \to [0, \infty)$, $\alpha > 0$ is said to belong to class-K if it is strictly increasing and $\alpha(0) = 0$.*

Definition 2 ([41])**.** *Let $F(t)$ be a class-K function, then its FO integral satisfies*

$$ {}_{t_0}I_t^{\omega} F(t) = \frac{1}{\Gamma(\omega)} \int_{t_0}^{t} \frac{F(\mathcal{T})}{(t-\mathcal{T})^{1-\omega}} d\mathcal{T} \tag{1} $$

with $\omega \in \mathbb{R}^+$, ${}_{t_0}I_t^{\omega}$ being the fractional–integral of order ω subject to initial time t_0, the Gamma function $\Gamma(\cdot)$ is denoted as $\Gamma(\hbar) = \int_0^{\infty} e^{-\mathcal{T}} \mathcal{T}^{\hbar-1} d\mathcal{T}$, where $\hbar \in C$. A key property for Gamma function is that $\Gamma(\Im + 1) = \Im\Gamma(\Im)$, $\Gamma(-\Im) = \infty$, $\Im \in N_0 = \{\Im | \Im \geq 0, \Im \in N\}$.

Definition 3 ([41])**.** *The Caputo FO derivative of $F(t)$ satisfies*

$$ \begin{aligned} {}_{t_0}^{C}D_t^{\omega} F(t) &= {}_{t_0}D_t^{l-\omega} \tfrac{d^l}{dt^l} F(t) \\ &= \tfrac{1}{\Gamma(l-\omega)} \int_{t_0}^{t} \tfrac{F^{(l)}(\mathcal{T})}{(t-\mathcal{T})^{\omega-l+1}} d\mathcal{T} \end{aligned} \tag{2} $$

where $l - 1 < \omega < l \in \mathbb{R}^+$.

Lemma 1 ([42]). *Considering a smooth function $\xi(l) \in \mathbb{R}^n$, then $\forall l \geq l_0$*

$$\frac{1}{2}{}_0^C D_t^{\omega}\left[\xi^T(l)P\xi(l)\right] \leq \xi^T(l)P{}_0^C D_t^{\omega}\xi(l) \tag{3}$$

where $0 < \omega < 1$ and constant matrix $P \in \mathbb{R}^{n\times n}$ is positive-definite.

Lemma 2 ([41]). *Denote $\omega \in \mathbb{R}^+$ satisfying $0 < \omega < 2$, $\nu \in C$ and $\bar{\gamma} \in \mathbb{R}$ satisfying $\frac{\pi\omega}{2} < \bar{\gamma} < \min\{\pi, \pi\omega\}$, then for any integer $h \geq 1$,*

$$E_{\omega,\nu}(\iota) = -\sum_{k=1}^{\hbar} \frac{\iota^{-k}}{\Gamma(\nu - \omega k)} + o\left(|\iota|^{-1-h}\right) \tag{4}$$

with $\bar{\gamma} \leq |\arg(\iota)| \leq \pi$ and $|\iota| \to \infty$. Here, $E_{\omega,\nu}(\iota)$ represents the Mittag–Leffler function defined as:

$$E_{\omega,\nu}(\iota) = \sum_{k=0}^{\infty} \frac{\iota^k}{\Gamma(\omega k+\nu)}$$

Lemma 3 ([41]). *Consider the real numbers ν, $\bar{\gamma}$, ω and ι defined in Lemma 2. Then, the following inequality holds*

$$E_{\omega,\nu}(\iota) \leq \frac{\lambda}{1+|\iota|} \tag{5}$$

where $\lambda > 0$.

Lemma 4 ([43]). *Consider FO nonlinear system ${}_0^C\mathrm{D}_t^{\omega}\zeta(t) = \psi(t,\zeta(t))$ with $\psi(t,\zeta(t))$ being Lipschitz continuous, whose equilibrium point is $\zeta = 0$. For class-K functions h_j ($j = 1,2,3$), and function $V(t,\zeta(t))$, if*

$$h_1(\|\zeta(t)\|) \leq V \leq h_2(\|\zeta(t)\|) \tag{6}$$

$${}_0^C\mathrm{D}_t^{\omega} V \leq -h_3(\|\zeta(t)\|) \tag{7}$$

hold, asymptotical stability of the system ${}_0^C\mathrm{D}_t^{\omega}\zeta(t) = \psi(\zeta,t)$ is achieved.

Lemma 5 ([44]). *Denote $\psi(\jmath)$ be a continous function on a compact set $\bar{\Xi}$. Then, $\forall\kappa > 0$, a FLS $\bar{\Theta}^{*T}\bar{\Psi}(\jmath)$ satisfies*

$$\sup_{\chi\in\bar{\Xi}}\left|\psi(\jmath) - \bar{\Theta}^{*T}\bar{\Psi}(\jmath)\right| \leq \kappa \tag{8}$$

where $\bar{\Psi}(\jmath) = [\bar{\Psi}_1(\jmath), \bar{\Psi}_2(\jmath), \ldots, \bar{\Psi}_\imath(\jmath)]^T$ denotes the fuzzy basis function satisfying the fact that $0 < \bar{\Psi}^T(\jmath)\bar{\Psi}(\jmath) \leq 1$, while $\bar{\Theta}^* = \left[\bar{\Theta}_1^*, \ldots, \bar{\Theta}_\imath^*\right]^T$ denotes the optimal weight, $\imath$ being the maximum rule number, and κ being regarded as a bounded fuzzy approximation error.

2.2. Problem Formulation

Consider a class of FO nonlinear systems with sensor and actuator faults in following form:

$$\begin{cases} {}_0^C\mathrm{D}_t^{\omega}\chi_i = \chi_{i+1} + \psi_i(\bar{\chi}_i) & 1 \leq i \leq n-1 \\ {}_0^C\mathrm{D}_t^{\omega}\chi_n = u^{\mathcal{F}} + \psi_n(\bar{\chi}_n) \\ y = \chi_1 \end{cases} \tag{9}$$

with $\bar{\chi}_i = [\chi_1, \ldots, \chi_i]^T \in \mathbb{R}^i$ ($i = 1,\ldots,n$) being the system states, $u^{\mathcal{F}}$ and $y \in \mathbb{R}$ being the output of actuator and system, $\omega \in (0,1)$, and smooth functions ψ_s ($s = 1,\ldots,n$) being unknown.

Remark 1. *System (9) concerned in this article is more universal than results [28–32], since in this work each subsystem involves a sensor fault and the last subsystem contains an actuator and sensor fault. The coupling term induced by sensor fault is quite distinct from the traditional coupling term, which determines that this work is more challenging. In the sequel, we will present a novel FO nonlinear filter to tackle it.*

This work considers potential faults that occurred on actuator and sensors simultaneously, whose definitions are presented as follows

Definition 4 ([32,45])**.** *The actuator fault is*

$$u^{\mathcal{F}}(t_u) = \breve{\rho}_0 u(t_u) + \breve{\tau}_0(t_u) \tag{10}$$

with $\breve{\rho}_0 \in (0,1]$, $\breve{\tau}_0(t_u)$ being unknown but bounded, t_u denotes actuator fault occurrence time.

The s-th sensor fault is

$$\chi_s^{\mathcal{F}}(t_s) = \breve{\rho}_s \chi_s(t_s) + \breve{\tau}_s(t_s) \tag{11}$$

with $s = 1, \ldots, n$, $\breve{\rho}_s \in (0,1]$, $\breve{\tau}_s(t)$ denotes bias faults that is unknown but bounded, $t_s, s = 1, \ldots, n$ denote sensor faults occurrence time.

The fault $u^{\mathcal{F}}(t)$ and $\chi_s^{\mathcal{F}}(t)$ ($s = 1, \ldots, n$) can be divided into 4 cases, as described in Table 1, where $0 \le \underline{\breve{\rho}}_m < \breve{\rho}_m < \bar{\breve{\rho}}_m \le 1$ ($m = 0, s$).

Table 1. Fault Model.

	$\underline{\breve{\rho}}_0$ & $\underline{\breve{\rho}}_s$	$\bar{\breve{\rho}}_0$ & $\bar{\breve{\rho}}_s$	$\breve{\tau}_0$ & $\breve{\tau}_s$
Fault-free	1	1	0
Partial loss of effectiveness fault	>0	<1	0
Bias	1	1	$\neq 0$
Total loss of effectiveness (TLOE) fault	0	0	$\neq 0$

The block diagram of the fractional-order nonlinear systems with actuator and sensor faults is provided in Figure 1.

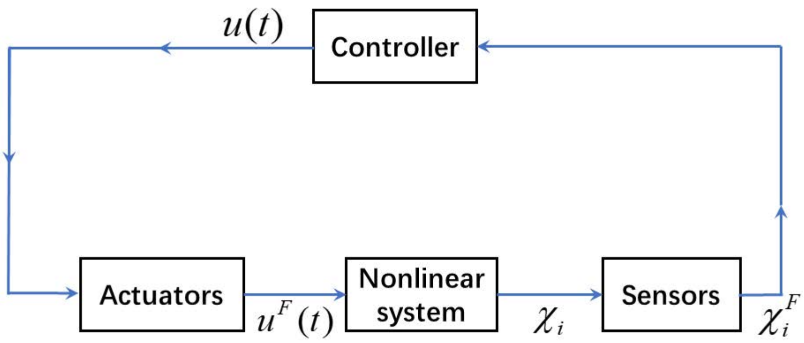

Figure 1. The block diagram of the faulty nonlinear systems.

Remark 2. *From Figure 1, it is obvious that the function of the sensor is to transmit the signal from the system to the controller, while the transmitted signal will be biased due to the existence of sensor failure. However, sensors are detection devices that can feel the information being measured, thus the measurement values χ_i^F after sensor faults are measurable. In fact, since that the fault coefficients $\breve{\rho}_i$ and $\breve{\tau}_i$ are unknown, the states χ_i become unknown, which makes control design difficult and challenging to some extent.*

Control Objective: Construct a FO nonlinear filter-based adaptive controller for system (9), to guarantee: (1) all the closed-loop signals are bounded; (2) the system output y is forced to track the reference tracjectory y_d.

To this end, some assumptions and lemmas are needed.

Assumption 1 ([32]). *$\breve{\tau}_0$ and $\breve{\tau}_s$ are limited by unknown constants $\bar{\breve{\tau}}_0$ and $\bar{\breve{\tau}}_s$, i.e., $|\breve{\tau}_0| \leq \bar{\breve{\tau}}_0$ and $|\breve{\tau}_s| \leq \bar{\breve{\tau}}_s$.*

Assumption 2 ([46–48]). *The desired signal y_d and ${}_0^C D_t^{\omega} y_d$ are smooth and bounded.*

Lemma 6 ([49]). *For any variable $\wp$, the following property satisfies:*

$$|\wp| - \wp \tanh\left(\frac{\wp}{\delta}\right) \leq \varsigma\delta \tag{12}$$

where $\delta > 0$ and $\varsigma \approx 0.2785$.

3. Nonlinear Filter-Based Adaptive Controller Design and Stability Analysis

3.1. Novel Nonlinear Filter Design

To avoid the issue of explosion of complexity, a valid nonlinear filter is constructed

$${}_0^C D_t^{\omega} s_\ell = -\frac{\varpi_\ell}{\varrho_\ell} - \hat{M}_\ell \tanh(\frac{\hat{M}_\ell \varpi_\ell}{\bar{\delta}_\ell}) - \hat{N}_\ell \xi_\ell \tanh(\frac{\xi_\ell \varpi_\ell}{\bar{\delta}_\ell}) \tag{13}$$

with

$$s_\ell(0) = \alpha_\ell(0), \ell = 1, 2, \cdots, n-1$$

where $\varrho_\ell > 0$, $\bar{\delta}_\ell > 0$, filter s_ℓ is utilized in place of the intermediate controller α_ℓ, while $\varpi_{i-1} = s_{i-1} - \alpha_{i-1}$ represents the ℓ-th boundary layer error. $\hat{M}_\ell$ and $\hat{N}_\ell$ are emplyed to estimate M_ℓ and N_ℓ with M_k, N_k ($k = 1, \ldots, n-2$) and N_{n-1} being the unknown upper bounds of ${}_0^C D_t^{\omega} \alpha_k$, $\breve{\rho}_k$ and $\breve{\rho}_n \breve{\rho}_0$, respectively.

Remark 3. *In contrast to the classical strategy utilizing linear filter* [36]*, the merit of presented method lies in that the constructed nonlinear filter is reflected in the introduction of $-\hat{M}_j \tanh(\frac{\hat{M}_j \varpi_j}{\bar{\delta}_j})$ and $-\hat{N}_j \xi_j \tanh(\frac{\xi_j \varpi_j}{\bar{\delta}_j})$, in which the former is employed to counterbalance the upper bound of the derivative of the virtual control law, while the latter is employed to expunge the coupling effect emerged from the actuator and sensor faults.*

3.2. Adaptive Controller Design

In this work, the adaptive backstepping FTC scheme will be proposed in accordance with the changes of coordinates:

$$\xi_1 = y^{\mathcal{F}} - y_d \tag{14}$$

$$\xi_i = \chi_i^{\mathcal{F}} - s_{i-1}, \ \ i = 2, \ldots, n \tag{15}$$

$$\varpi_{i-1} = s_{i-1} - \alpha_{i-1} \tag{16}$$

with $y^{\mathcal{F}} = \chi_1^{\mathcal{F}}$ being a measurement value, ξ_i being an error surface, s_{i-1} being obtained according to an FO filter on α_{i-1} and ϖ_{i-1} being the FO filter output error designed in the sequel.

For the sake of simplicity, we will omit state dependence.

Step 1: In view of (14) and (15), ${}_0^C D_t^{\omega} \xi_1$ is introduced as:

$${}_0^C \mathrm{D}_t^{\omega} \xi_1 = \breve{\rho}_1(\chi_2 + \psi_1) + {}_0^C \mathrm{D}_t^{\omega} \breve{\tau}_1 - {}_0^C \mathrm{D}_t^{\omega} y_d \tag{17}$$

Owing to the unknown property of function ψ_1, fuzzy logic system is utilized to model ψ_1 as

$$\psi_1 = \theta_1^T \varphi_1(\chi_1) + \kappa_1 \tag{18}$$

where $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$ with θ_1 being the ideal weight, and κ_1 being bounded by $|\kappa_1| < \kappa_1^*$.

Define the Lyapunov function candidate

$$V_1 = \frac{1}{2}\xi_1{}^2 + \frac{1}{2r_1}\tilde{\vartheta}_1^2 + \frac{\underline{\breve{\rho}}_1}{2q_1}\tilde{\epsilon}_1^2 \tag{19}$$

where $r_1 > 0$ and $q_1 > 0$ are design constants, $\underline{\breve{\rho}}_1 = \inf_{t \geq 0} \breve{\rho}_1$, $\tilde{\vartheta}_1 = \vartheta_1 - \hat{\vartheta}_1$ and $\tilde{\epsilon}_1 = \epsilon_1 - \hat{\epsilon}_1$, and $\hat{\epsilon}_1$ and $\hat{\vartheta}_1$ denote the estimations of $\epsilon_1 = \frac{1}{\underline{\breve{\rho}}_1}$ and $\vartheta_1 = \sup_{t \geq 0}\|\Theta_1(t)\|$, respectively, with $\Theta_1 = \left[\breve{\rho}_1^2\|\theta_1\|^2, \breve{\rho}_1^2, {}_0^C\mathrm{D}_t^\omega \breve{\tau}_1\right]$.

The derivative of V_1 along with (17)–(19) and Lemma 1 is

$$\begin{aligned}
{}_0^C\mathrm{D}_t^\omega V_1 &\leq \xi_1 {}_0^C\mathrm{D}_t^\omega \xi_1 - \tfrac{1}{r_1}\tilde{\theta}_1 {}_0^C\mathrm{D}_t^\omega \hat{\theta}_1 - \tfrac{\underline{\breve{\rho}}_1}{q_1}\tilde{\epsilon}_1 {}_0^C\mathrm{D}_t^\omega \hat{\epsilon}_1 \\
&\leq \xi_1\Big[\underline{\breve{\rho}}_1\big(\chi_2 + \theta_1^T\varphi_1(\chi_1) + \kappa_1\big) + {}_0^C\mathrm{D}_t^\omega \breve{\tau}_1 \\
&\quad - {}_0^C\mathrm{D}_t^\omega y_d\Big] - \tfrac{1}{r_1}\tilde{\theta}_1 {}_0^C\mathrm{D}_t^\omega \hat{\theta}_1 - \tfrac{\underline{\breve{\rho}}_1}{q_1}\tilde{\epsilon}_1 {}_0^C\mathrm{D}_t^\omega \hat{\epsilon}_1
\end{aligned} \tag{20}$$

In the light of Young's Inequality, one can compute

$$\xi_1\breve{\rho}_1\theta_1^T\varphi_1(\chi_1) \leq \frac{\xi_1^2\breve{\rho}_1^2\|\theta_1\|^2}{2\varphi_1^T(\chi_1^{\mathcal{F}})\varphi_1(\chi_1^{\mathcal{F}})} + \frac{1}{2} \tag{21}$$

$$\xi_1\breve{\rho}_1\kappa_1 \leq \frac{1}{2}\xi_1^2\breve{\rho}_1^2 + \frac{1}{2}\kappa_1^{*2} \tag{22}$$

Define $\eta_1 = \frac{\xi_1\phi_1^T\phi_1}{\sqrt{\xi_1^2\phi_1^T\phi_1 + \delta_1^2}}$ with $\phi_1 = \left[\frac{\xi_1}{2\varphi_1^T(\chi_1^{\mathcal{F}})\varphi_1(\chi_1^{\mathcal{F}})}, \frac{1}{2}\xi_1, 1\right]^T$, then, on the basis of Proposition 1 in [50], it yields

$$\xi_1\Theta_1{}^T\phi_1 \leq \xi_1\tilde{\vartheta}_1\eta_1 + \xi_1\hat{\vartheta}_1\eta_1 + \delta_1\vartheta_1 \tag{23}$$

where $\delta_1 > 0$ is a constant. In accordance with (15), one can deduce

$$\xi_1\breve{\rho}_1\chi_2 = \xi_1\breve{\rho}_1\xi_2 + \xi_1\breve{\rho}_1\alpha_1 + \xi_1\bar{\alpha}_1 - \xi_1\bar{\alpha}_1 \tag{24}$$

where $\bar{\alpha}_1$ will be given later. Using (21)–(24), ${}_0^C\mathrm{D}_t^\omega V_1$ becomes

$$\begin{aligned}
{}_0^C\mathrm{D}_t^\omega V_1 &\leq \xi_1\big(\breve{\rho}_1(\xi_2 + \varpi_1) + \breve{\rho}_1\alpha_1 - \bar{\alpha}_1 + \hat{\vartheta}_1\eta_1 \\
&\quad - {}_0^C\mathrm{D}_t^\omega y_d\big) + \tfrac{1}{r_1}\tilde{\vartheta}_1\big(r_1\xi_1\eta_1 - {}_0^C\mathrm{D}_t^\omega \hat{\vartheta}_1\big) \\
&\quad + \xi_1\bar{\alpha}_1 + \delta_1\vartheta_1 + \tfrac{1}{2}\kappa_1^{*2} + \tfrac{1}{2} - \tfrac{\underline{\breve{\rho}}_1}{q_1}\tilde{\epsilon}_1 {}_0^C\mathrm{D}_t^\omega \hat{\epsilon}_1
\end{aligned} \tag{25}$$

The virtual control law α_1 can be given as

$$\alpha_1 = -\hat{\epsilon}_1\bar{\alpha}_1 \tanh\left(\frac{\hat{\epsilon}_1\bar{\alpha}_1\xi_1}{\delta_1}\right) \tag{26}$$

with $\bar{\alpha}_1$ being defined as

$$\bar{\alpha}_1 = c_1\xi_1 + \hat{\vartheta}_1\eta_1 - {}_0^C\mathrm{D}_t^\omega y_d \tag{27}$$

where $c_1 > 0$ is a constant. In terms of Lemma 6, one can calculate

$$-\xi_1\hat{\epsilon}_1\bar{\alpha}_1 \tanh\left(\frac{\hat{\epsilon}_1\bar{\alpha}_1\xi_1}{\delta_1}\right) \leq -\hat{\epsilon}_1\bar{\alpha}_1\xi_1 + \varsigma\delta_1 \tag{28}$$

Then (25) satisfies

$$
\begin{aligned}
{}_0^C\mathrm{D}_t^\omega V_1 \le& -c_1\xi_1^2 + \xi_1\breve{\rho}_1(\xi_2+\varpi_1) + \bar{\alpha}_1\xi_1 \\
&-\breve{\rho}_1\hat{\epsilon}_1\bar{\alpha}_1\xi_1 + \tfrac{1}{r_1}\tilde{\vartheta}_1\big(r_1\xi_1\eta_1 - {}_0^C\mathrm{D}_t^\omega\hat{\vartheta}_1\big) \\
&-\tfrac{\underline{\breve{\rho}}_1}{q_1}\tilde{\epsilon}_1{}_0^C\mathrm{D}_t^\omega\hat{\epsilon}_1 + \delta_1\vartheta_1 + \underline{\breve{\rho}}_1\varsigma\delta_1 + \tfrac{1}{2}\kappa_1^{*2} + \tfrac{1}{2} \\
\le& -c_1\xi_1^2 + \mu_1 + \tfrac{1}{r_1}\tilde{\vartheta}_1\big(r_1\xi_1\eta_1 - {}_0^C\mathrm{D}_t^\omega\hat{\vartheta}_1\big) + \xi_1 \\
&\times\breve{\rho}_1(\xi_2+\varpi_1) + \tfrac{\underline{\breve{\rho}}_1}{q_1}\tilde{\epsilon}_1\big(q_1\xi_1\bar{\alpha}_1 - {}_0^C\mathrm{D}_t^\omega\hat{\epsilon}_1\big)
\end{aligned} \tag{29}
$$

with $\mu_1 = \delta_1\vartheta_1 + \underline{\breve{\rho}}_1\varsigma\delta_1 + \frac{1}{2}\kappa_1^{*2} + \frac{1}{2} > 0$. Construct the adaptive law ${}_0^C\mathrm{D}_t^\omega\hat{\vartheta}_1$ and ${}_0^C\mathrm{D}_t^\omega\hat{\epsilon}_1$ as

$$
{}_0^C\mathrm{D}_t^\omega\hat{\vartheta}_1 = r_1\xi_1\eta_1 - \sigma_1\hat{\vartheta}_1 \tag{30}
$$

$$
{}_0^C\mathrm{D}_t^\omega\hat{\epsilon}_1 = q_1\xi_1\bar{\alpha}_1 - \bar{\sigma}_1\hat{\epsilon}_1 \tag{31}
$$

where $\sigma_1 > 0$ and $\bar{\sigma}_1 > 0$ are design constants. Substituting (30) and (31) into (29) produces

$$
\begin{aligned}
{}_0^C\mathrm{D}_t^\omega V_1 \le& -c_1\xi_1^2 + \xi_1\breve{\rho}_1(\xi_2+\varpi_1) \\
&-\tfrac{\sigma_1}{r_1}\tilde{\vartheta}_1\hat{\vartheta}_1 - \tfrac{\underline{\breve{\rho}}_1\bar{\sigma}_1}{q_1}\tilde{\epsilon}_1\hat{\epsilon}_1 + \mu_1
\end{aligned} \tag{32}
$$

Step i ($i = 2, \dots, n-1$): In line with (15), ${}_0^C D_t^\omega \xi_i$ is computed as

$$
\begin{aligned}
{}_0^C\mathrm{D}_t^\omega\xi_i =& {}_0^C\mathrm{D}_t^\omega\chi_i - {}_0^C\mathrm{D}_t^\omega s_{i-1} \\
=& \breve{\rho}_i(\chi_{i+1}+\psi_i) + {}_0^C\mathrm{D}_t^\omega\breve{\tau}_i + \tfrac{\varpi_{i-1}}{\varrho_{i-1}} \\
&+ \hat{M}_{i-1}\tanh\Big(\tfrac{\hat{M}_{i-1}\varpi_{i-1}}{\delta_{i-1}}\Big) \\
&+ \hat{N}_{i-1}\xi_{i-1}\tanh\Big(\tfrac{\xi_{i-1}\varpi_{i-1}}{\delta_{i-1}}\Big)
\end{aligned} \tag{33}
$$

Similarly, due to the unknown property of function ψ_i, fuzzy logic system is utilized to model ψ_i as

$$
\psi_i = \theta_i^T\varphi_i(\bar{\chi}_i) + \kappa_i \tag{34}
$$

with θ_i being the ideal weight, and κ_i being bounded by $|\kappa_i| < \kappa_i^*$.

Define the Lyapunov function candidate

$$
\begin{aligned}
V_i =& V_{i-1} + \tfrac{1}{2}\xi_i^2 + \tfrac{1}{2r_i}\tilde{\vartheta}_i^2 + \tfrac{\underline{\breve{\rho}}_i}{2q_i}\tilde{\epsilon}_i^2 \\
&+ \tfrac{1}{2}\varpi_{i-1}^2 + \tfrac{1}{2\beta_{i-1}}\tilde{M}_{i-1}^2 + \tfrac{1}{2\pi_{i-1}}\tilde{N}_{i-1}^2
\end{aligned} \tag{35}
$$

where $r_i > 0$, $q_i > 0$, β_{i-1} and π_{i-1} are design constants, $\underline{\breve{\rho}}_i = \inf_{t\ge 0}\breve{\rho}_i$, $\tilde{\vartheta}_i = \vartheta_i - \hat{\vartheta}_i$ and $\tilde{\epsilon}_i = \epsilon_i - \hat{\epsilon}_i$, and $\hat{\epsilon}_i$ and $\hat{\vartheta}_i$ denote the estimations of $\epsilon_i = \frac{1}{\underline{\breve{\rho}}_i}$ and $\vartheta_i = \sup_{t\ge 0}\|\Theta_i(t)\|$, respectively, with $\Theta_i = \Big[\breve{\rho}_{i-1}, \breve{\rho}_i^2\|\theta_i\|^2, \breve{\rho}_i^2, {}_0^C\mathrm{D}_t^\omega\breve{\tau}_i\Big]$.

In view of Lemma 1 and (33)–(35), ${}_0^C\mathrm{D}_t^\omega V_i$ is

$$
\begin{aligned}
{}_0^C\mathrm{D}_t^\omega V_i \le& {}_0^C\mathrm{D}_t^\omega V_{i-1} + \xi_i\big(\breve{\rho}_i\big(\chi_{i+1} + \theta_i^T\varphi_i(\bar{\chi}_i) + \kappa_i\big) \\
&+ {}_0^C\mathrm{D}_t^\omega\breve{\tau}_i + \tfrac{\varpi_{i-1}}{\varrho_{i-1}} + \hat{M}_{i-1}\tanh\Big(\tfrac{\hat{M}_{i-1}\varpi_{i-1}}{\delta_{i-1}}\Big) \\
&+\hat{N}_{i-1}\xi_{i-1}\tanh\Big(\tfrac{\xi_{i-1}\varpi_{i-1}}{\delta_{i-1}}\Big)\Big) - \tfrac{1}{r_i}\tilde{\vartheta}_i{}_0^C\mathrm{D}_t^\omega\hat{\vartheta}_i \\
&- \varpi_{i-1}\Big(\tfrac{\varpi_{i-1}}{\varrho_{i-1}} + \hat{M}_{i-1}\tanh\Big(\tfrac{\hat{M}_{i-1}\varpi_{i-1}}{\delta_{i-1}}\Big) \\
&+\hat{N}_{i-1}\xi_{i-1}\tanh\Big(\tfrac{\xi_{i-1}\varpi_{i-1}}{\delta_{i-1}}\Big) + {}_0^C\mathrm{D}_t^\omega\alpha_{i-1}\Big) \\
&- \tfrac{\underline{\breve{\rho}}_i}{q_i}\tilde{\epsilon}_i{}_0^C\mathrm{D}_t^\omega\hat{\epsilon}_i - \tfrac{1}{\beta_{i-1}}\tilde{M}_{i-1}{}_0^C\mathrm{D}_t^\omega\hat{M}_{i-1} \\
&- \tfrac{1}{\pi_{i-1}}\tilde{N}_{i-1}{}_0^C\mathrm{D}_t^\omega\hat{N}_{i-1}
\end{aligned} \tag{36}
$$

In a similar way to the first step, one can obtain

$$\xi_i \breve{\rho}_i \theta_i^T \varphi_i(\bar{\chi}_i) \leq \frac{\xi_i^2 \breve{\rho}_i^2 \|\theta_i\|^2}{2\varphi_i^T(\bar{\chi}_i^{\mathcal{F}})\varphi_i(\bar{\chi}_i^{\mathcal{F}})} + \frac{1}{2} \tag{37}$$

$$\xi_i \breve{\rho}_i \kappa_i \leq \frac{1}{2}\xi_i^2 \breve{\rho}_i^2 + \frac{1}{2}\kappa_i^{*2} \tag{38}$$

Denote $\eta_i = \frac{\xi_i \phi_i^T \phi_i}{\sqrt{\xi_i^2 \phi_i^T \phi_i + \delta_i^2}}$ with $\phi_i = \left[\xi_{i-1}, \frac{\xi_i}{2\varphi_i^T(\bar{\chi}_i^{\mathcal{F}})\varphi_i(\bar{\chi}_i^{\mathcal{F}})}, \frac{1}{2}\xi_i, 1\right]^T$, then, it is easy to obtain

$$\xi_i {\Theta_i}^T \phi_i \leq \xi_i \tilde{\vartheta}_i \eta_i + \xi_i \hat{\vartheta}_i \eta_i + \delta_i \vartheta_i \tag{39}$$

where $\delta_i > 0$ is a constant. Thus, one can describe

$$\xi_i \breve{\rho}_i \chi_{i+1} = \xi_i \breve{\rho}_i \xi_{i+1} + \xi_i \breve{\rho}_i \alpha_i + \xi_i \bar{\alpha}_i - \xi_1 \bar{\alpha}_i \tag{40}$$

with $\bar{\alpha}_i$ being given later. Then, some transformations are established as

$$\begin{aligned} -\varpi_{i-1}{}_0^C\mathrm{D}_t^{\omega}\alpha_{i-1} &\leq |\varpi_{i-1}|\tilde{M}_{i-1} + |\varpi_{i-1}|\hat{M}_{i-1} \\ &\leq \varsigma\bar{\delta}_{i-1} + |\varpi_{i-1}|\tilde{M}_{i-1} \\ &\quad + \varpi_{i-1}\hat{M}_{i-1}\tanh\left(\frac{\hat{M}_{i-1}\varpi_{i-1}}{\bar{\delta}_{i-1}}\right) \end{aligned} \tag{41}$$

$$\begin{aligned} &\xi_{i-1}\breve{\rho}_{i-1}\varpi_{i-1} - N_{i-1}\xi_{i-1}\varpi_{i-1}\tanh\left(\frac{\xi_{i-1}\varpi_{i-1}}{\bar{\delta}_{i-1}}\right) \\ &\leq N_{i-1}|\xi_{i-1}||\varpi_{i-1}| - N_{i-1}\xi_{i-1}\varpi_{i-1}\tanh\left(\frac{\xi_{i-1}\varpi_{i-1}}{\bar{\delta}_{i-1}}\right) \\ &\leq \varsigma N_{i-1}\bar{\delta}_{i-1} \end{aligned} \tag{42}$$

Using (37)–(42), ${}_0^C\mathrm{D}_t^{\omega} V_i$ is rewritten as

$$\begin{aligned} {}_0^C\mathrm{D}_t^{\omega} V_i \leq &-\sum_{k=1}^{i}\left(c_k\xi_k^2 + \frac{\sigma_k}{r_k}\tilde{\vartheta}_k\hat{\vartheta}_k + \frac{\underline{\rho}_k\bar{\sigma}_k}{q_k}\tilde{\epsilon}_k\hat{\epsilon}_k\right) + \delta_i\vartheta_i \\ &+ \frac{1}{2}\kappa_i^{*2} + \frac{1}{2} - \frac{\varpi_{i-1}^2}{\varrho_{i-1}} + \xi_i(\breve{\rho}_i(\xi_{i+1} + \varpi_i) - \bar{\alpha}_i \\ &+ \bar{\alpha}_i + \breve{\rho}_i\alpha_i + \frac{\varpi_{i-1}}{\varrho_{i-1}} + \hat{M}_{i-1}\tanh\left(\frac{\hat{M}_{i-1}\varpi_{i-1}}{\bar{\delta}_{i-1}}\right) \\ &+ \hat{N}_{i-1}\xi_{i-1}\tanh\left(\frac{\xi_{i-1}\varpi_{i-1}}{\bar{\delta}_{i-1}}\right)\Big) - \frac{\underline{\rho}_i}{q_i}\tilde{\epsilon}_i{}_0^C\mathrm{D}_t^{\omega}\hat{\epsilon}_i \\ &+ \frac{1}{\beta_{i-1}}\tilde{M}_{i-1}\left(\beta_{i-1}|\varpi_{i-1}| - {}_0^C\mathrm{D}_t^{\omega}\hat{M}_{i-1}\right) \\ &+ \frac{1}{\pi_{i-1}}\tilde{N}_{i-1}\Big(\pi_{i-1}\xi_{i-1}\varpi_{i-1}\tanh\left(\frac{\xi_{i-1}\varpi_{i-1}}{\bar{\delta}_{i-1}}\right) \\ &- {}_0^C\mathrm{D}_t^{\omega}\hat{N}_{i-1}\Big) - \frac{1}{r_i}\tilde{\vartheta}_i\left(r_i\xi_i\eta_i - {}_0^C\mathrm{D}_t^{\omega}\hat{\vartheta}_i\right) \end{aligned} \tag{43}$$

where $\mu_{i-1} = \mu_{i-2} + \delta_{i-1}\vartheta_{i-1} + \underline{\breve{\rho}}_{i-1}\varsigma\delta_{i-1} + \varsigma\bar{\delta}_{i-2}(1 + N_{i-2}) + \frac{1}{2}\kappa_{i-1}^{*2} + \frac{1}{2} > 0$.

The virtual control law α_i is given as

$$\alpha_i = -\hat{\epsilon}_i\bar{\alpha}_i\tanh\left(\frac{\hat{\epsilon}_i\bar{\alpha}_i\xi_i}{\delta_i}\right) \tag{44}$$

with $\bar{\alpha}_i$ being defined as

$$\begin{aligned} \bar{\alpha}_i = &c_i\xi_i + \hat{\vartheta}_i\eta_i + \hat{M}_{i-1}\tanh\left(\frac{\hat{M}_{i-1}\varpi_{i-1}}{\bar{\delta}_{i-1}}\right) \\ &+ \frac{\varpi_{i-1}}{\varrho_{i-1}} + \hat{N}_{i-1}\xi_{i-1}\tanh\left(\frac{\xi_{i-1}\varpi_{i-1}}{\bar{\delta}_{i-1}}\right) \end{aligned} \tag{45}$$

where $c_i > 0$ is a constant. According to Lemma 6, one can calculate

$$-\xi_i\hat{\epsilon}_i\bar{\alpha}_i \tanh\left(\frac{\hat{\epsilon}_i\bar{\alpha}_i\xi_i}{\delta_i}\right) \le -\hat{\epsilon}_i\bar{\alpha}_i\xi_i + \varsigma\delta_i \tag{46}$$

Then, (46) satisfies

$$\begin{aligned}
{}_0^C\mathrm{D}_t^\omega V_i \le & -\sum_{k=1}^{i}\left(c_k\xi_k^2 + \tfrac{\sigma_k}{r_k}\tilde{\vartheta}_k\hat{\vartheta}_k + \tfrac{\underline{\breve{\rho}}_k\bar{\sigma}_k}{q_k}\tilde{\epsilon}_k\hat{\epsilon}_k\right) - \tfrac{\varpi_{i-1}^2}{\varrho_{i-1}} \\
& + \tfrac{\underline{\breve{\rho}}_i}{q_i}\tilde{\epsilon}_i\left(q_i\bar{\alpha}_i\xi_i - {}_0^C\mathrm{D}_t^\omega\hat{\epsilon}_i\right) + \xi_i\breve{\rho}_i(\xi_i + \varpi_{i-1}) \\
& + \mu_i + \tfrac{1}{\beta_{i-1}}\tilde{M}_{i-1}\left(\beta_{i-1}|\varpi_{i-1}| - {}_0^C\mathrm{D}_t^\omega\hat{M}_{i-1}\right) \\
& + \tfrac{1}{\pi_{i-1}}\tilde{N}_{i-1}\Big(\pi_{i-1}\xi_{i-1}\varpi_{i-1}\tanh\left(\tfrac{\xi_{i-1}\varpi_{i-1}}{\delta_{i-1}}\right) \\
& - {}_0^C\mathrm{D}_t^\omega\hat{N}_{i-1}\Big) - \tfrac{1}{r_i}\tilde{\vartheta}_i\left(r_i\xi_i\eta_i - {}_0^C\mathrm{D}_t^\omega\hat{\vartheta}_i\right)
\end{aligned} \tag{47}$$

where $\mu_i = \mu_{i-1} + \delta_i\vartheta_i + \underline{\breve{\rho}}_i\varsigma\delta_i + \varsigma\bar{\delta}_{i-1}(1 + N_{i-1}) + \frac{1}{2}\kappa_i^{*2} + \frac{1}{2} > 0$.

Construct the adaptive laws ${}_0^C\mathrm{D}_t^\omega\hat{\vartheta}_i$, ${}_0^C\mathrm{D}_t^\omega\hat{\epsilon}_i$, ${}_0^C\mathrm{D}_t^\omega\hat{M}_{i-1}$ and ${}_0^C\mathrm{D}_t^\omega\hat{N}_{i-1}$ as

$${}_0^C\mathrm{D}_t^\omega\hat{\vartheta}_i = r_i\xi_i\eta_i - \sigma_i\hat{\vartheta}_i \tag{48}$$

$${}_0^C\mathrm{D}_t^\omega\hat{\epsilon}_i = q_i\xi_i\bar{\alpha}_i - \bar{\sigma}_i\hat{\epsilon}_i \tag{49}$$

$${}_0^C\mathrm{D}_t^\omega\hat{M}_{i-1} = \beta_{i-1}|\varpi_{i-1}| - \bar{\beta}_{i-1}\hat{M}_{i-1} \tag{50}$$

$${}_0^C\mathrm{D}_t^\omega\hat{N}_{i-1} = \pi_{i-1}\xi_{i-1}\varpi_{i-1}\tanh\left(\frac{\xi_{i-1}\varpi_{i-1}}{\delta_{i-1}}\right) - \bar{\pi}_{i-1}\hat{N}_{i-1} \tag{51}$$

with $\sigma_i > 0$, $\bar{\sigma}_i > 0$, $\bar{\beta}_{i-1} > 0$ and $\bar{\pi}_{i-1} > 0$ being design constants. Invoking (48)–(51) produces

$$\begin{aligned}
{}_0^C\mathrm{D}_t^\omega V_i \le & -\sum_{k=1}^{i}\left(c_k\xi_k^2 + \tfrac{\sigma_k}{r_k}\tilde{\vartheta}_k\hat{\vartheta}_k + \tfrac{\underline{\breve{\rho}}_k\bar{\sigma}_k}{q_k}\tilde{\epsilon}_k\hat{\epsilon}_k\right) \\
& -\sum_{j=1}^{i-1}\left(\tfrac{\varpi_j^2}{\varrho_j} + \tfrac{\bar{\beta}_j}{\beta_j}\tilde{M}_j\hat{M}_j + \tfrac{\bar{\pi}_j}{\pi_j}\tilde{N}_j\hat{N}_j\right) \\
& + \xi_i\breve{\rho}_i(\xi_i + \varpi_{i-1}) + \mu_i
\end{aligned} \tag{52}$$

Step n: The last phase is devoted to construct the actual controller. On account of (15), ${}_0^C D_t^\omega\xi_n$ satisfies

$$\begin{aligned}
{}_0^C\mathrm{D}_t^\omega\xi_n &= {}_0^C\mathrm{D}_t^\omega\chi_n - {}_0^C\mathrm{D}_t^\omega s_{n-1} \\
&= \breve{\rho}_n(\breve{\rho}_0 v + \breve{\tau}_0 + \psi_n) + {}_0^C\mathrm{D}_t^\omega\breve{\tau}_n \\
&\quad + \tfrac{\varpi_{n-1}}{\varrho_{n-1}} + \hat{M}_{n-1}\tanh\left(\tfrac{\hat{M}_{n-1}\varpi_{n-1}}{\delta_{n-1}}\right) \\
&\quad + \hat{N}_{n-1}\xi_{n-1}\tanh\left(\tfrac{\xi_{n-1}\varpi_{n-1}}{\delta_{n-1}}\right)
\end{aligned} \tag{53}$$

Define the Lyapunov function

$$\begin{aligned}
V_n = & V_{n-1} + \tfrac{1}{2}\xi_n^2 + \tfrac{1}{2r_n}\tilde{\vartheta}_n^2 + \tfrac{\underline{\breve{\rho}}_n}{2q_n}\tilde{\epsilon}_n^2 \\
& + \tfrac{1}{2}\varpi_{n-1}^2 + \tfrac{1}{2\beta_{n-1}}\tilde{M}_{n-1}^2 + \tfrac{1}{2\pi_{n-1}}\tilde{N}_{n-1}^2
\end{aligned} \tag{54}$$

where $r_n > 0$, $q_n > 0$, β_{n-1} and π_{n-1} are design constants, $\underline{\breve{\rho}}_n = \inf_{t\ge 0}(\breve{\rho}_n\breve{\rho}_0)$, $\tilde{\vartheta}_n = \vartheta_n - \hat{\vartheta}_n$ and $\tilde{\epsilon}_n = \epsilon_n - \hat{\epsilon}_n$. Note that $\hat{\epsilon}_n$ and $\hat{\vartheta}_n$ are the estimates of ϵ_n and ϑ_n, respectively, where $\epsilon_n = \frac{1}{\underline{\breve{\rho}}_n}$, $\vartheta_n = \sup_{t\ge 0}\|\Theta_n(t)\|$ with $\Theta_n = \left[\breve{\rho}_{n-1}, \breve{\rho}_n^2\|\theta_n\|^2, \breve{\rho}_n^2, \breve{\tau}_0\breve{\rho}_n, {}_0^C\mathrm{D}_t^\omega\breve{\tau}_n\right]$.

The actual controller v is given as

$$v = -\hat{\epsilon}_n\bar{\alpha}_n\tanh\left(\frac{\hat{\epsilon}_n\bar{\alpha}_n\xi_n}{\delta_n}\right) \tag{55}$$

with $\bar{\alpha}_n$ being defined as

$$\begin{aligned}\bar{\alpha}_n = c_n\xi_n + \hat{\vartheta}_n\eta_n + \hat{M}_{n-1}\tanh\left(\frac{\hat{M}_{n-1}\varpi_{n-1}}{\bar{\delta}_{n-1}}\right)\\ + \frac{\varpi_{n-1}}{\varrho_{n-1}} + \hat{N}_{n-1}\xi_{n-1}\tanh\left(\frac{\xi_{n-1}\varpi_{n-1}}{\bar{\delta}_{n-1}}\right)\end{aligned} \tag{56}$$

where $c_n > 0$ is a constant, $\eta_n = \frac{\xi_n\phi_n^T\phi_n}{\sqrt{\xi_n^2\phi_n^T\phi_n+\delta_n^2}}$ $(\delta_n > 0)$, $\phi_n = \left[\xi_{n-1}, \frac{\xi_n}{2\varphi_n^T(\bar{\chi}_n^F)\varphi_n(\bar{\chi}_n^F)}, \frac{1}{2}\xi_n, 1, 1\right]^T$.

Construct ${}_0^C\mathrm{D}_t^\omega\hat{\vartheta}_n$, ${}_0^C\mathrm{D}_t^\omega\hat{\epsilon}_n$, ${}_0^C\mathrm{D}_t^\omega\hat{M}_{n-1}$ and ${}_0^C\mathrm{D}_t^\omega\hat{N}_{n-1}$ as

$${}_0^C\mathrm{D}_t^\omega\hat{\vartheta}_n = r_n\xi_n\eta_n - \sigma_n\hat{\vartheta}_n \tag{57}$$

$${}_0^C\mathrm{D}_t^\omega\hat{\epsilon}_n = q_n\xi_n\bar{\alpha}_n - \bar{\sigma}_n\hat{\epsilon}_n \tag{58}$$

$${}_0^C\mathrm{D}_t^\omega\hat{M}_{n-1} = \beta_{n-1}|\varpi_{n-1}| - \bar{\beta}_{n-1}\hat{M}_{n-1} \tag{59}$$

$$\begin{aligned}{}_0^C\mathrm{D}_t^\omega\hat{N}_{n-1} = \pi_{n-1}\xi_{n-1}\varpi_{n-1}\tanh\left(\frac{\xi_{n-1}\varpi_{n-1}}{\bar{\delta}_{n-1}}\right)\\ - \bar{\pi}_{n-1}\hat{N}_{n-1}\end{aligned} \tag{60}$$

where $\sigma_n > 0$, $\bar{\sigma}_n > 0$, $\bar{\beta}_{n-1} > 0$ and $\bar{\pi}_{n-1} > 0$ are design constants. Invoking (55)–(60) gives

$$\begin{aligned}{}_0^C\mathrm{D}_t^\omega V_n \le -\sum_{k=1}^{n}\left(c_k\xi_k^2 + \frac{\sigma_k}{r_k}\tilde{\vartheta}_k\hat{\theta}_k + \frac{\underline{\breve{\rho}}_k\bar{\sigma}_k}{q_k}\tilde{\epsilon}_k\hat{\epsilon}_k\right) + \mu_n\\ -\sum_{j=1}^{n-1}\frac{\varpi_j{}^2}{\varrho_j} - \sum_{j=1}^{n-1}\left(\frac{\bar{\beta}_j}{\beta_j}\tilde{M}_j\hat{M}_j + \frac{\bar{\pi}_j}{\pi_j}\tilde{N}_j\hat{N}_j\right)\end{aligned} \tag{61}$$

with $\mu_n = \mu_{n-1} + \delta_n\vartheta_n + \underline{\breve{\rho}}_n\varsigma\delta_n + \varsigma\bar{\delta}_{n-1}(1 + N_{n-1}) + \frac{1}{2}\kappa_n^{*2} + \frac{1}{2} > 0$.

3.3. Stability Analysis

The presented control scheme achieves the following result.

Theorem 1. *Consider an uncertain FO nonlinear system with actuator and sensor faults (9) subject to Assumptions 1–3. Through the virtual control laws (26) and (44), the actual control law (55) and the adaptive laws (30) and (31), (48)–(51) and (57)–(60), it is ensured that all the closed-loop signals are bounded, and the output y follows the desired signal y_d well.*

Proof. Denote $\Xi = \Xi_1 \cup \Xi_2 \cdots \cup \Xi_n = \{V_n(t) \le q\}$, hence a constant $M_i > 0$ exists such that $\left|{}_0^C D_t^\omega\alpha_i\right| \le M_i$, on compact set Ξ. □

Then, the partial terms in (61) are rearranged as

$$-\frac{\sigma_k}{r_k}\tilde{\vartheta}_k\hat{\vartheta}_k \le -\frac{\sigma_k}{2r_k}\tilde{\vartheta}_k^2 + \frac{\sigma_k}{2r_k}\vartheta_k^2 \tag{62}$$

$$-\frac{\underline{\breve{\rho}}_k\bar{\sigma}_k}{q_k}\tilde{\xi}_k\hat{\xi}_k \le -\frac{\underline{\breve{\rho}}_k\bar{\sigma}_k}{2q_k}\tilde{\xi}_k^2 + \frac{\underline{\breve{\rho}}_k\bar{\sigma}_k}{2q_k}\xi_k^2 \tag{63}$$

$$-\frac{\bar{\beta}_j}{\beta_j}\tilde{M}_j\hat{M}_j \le -\frac{\bar{\beta}_j}{2\beta_j}\tilde{M}_j^2 + \frac{\bar{\beta}_j}{2\beta_j}M_j^2 \tag{64}$$

$$-\frac{\bar{\pi}_j}{\pi_j}\tilde{N}_j\hat{N}_j \le -\frac{\bar{\pi}_j}{2\pi_j}\tilde{N}_j^2 + \frac{\bar{\pi}_j}{2\pi_j}N_j^2 \tag{65}$$

Substituting (62)–(65) into (61) results in

$$\begin{aligned} {}_0^C\mathrm{D}_t^\omega V_n \le & -\sum_{k=1}^{n}\left(c_k\xi_k^2+\tfrac{\sigma_k}{2r_k}\tilde{\vartheta}_k^2+\tfrac{\breve{\rho}_k\bar{\sigma}_k}{2q_k}\tilde{\epsilon}_k^2\right)+\bar{\mu}_n \\ & -\sum_{j=1}^{n-1}\tfrac{\varpi_j{}^2}{\varrho_j}-\sum_{j=1}^{n-1}\left(\tfrac{\bar{\beta}_j}{2\beta_j}\tilde{M}_j^2+\tfrac{\bar{\pi}_j}{2\pi_j}\tilde{N}_j^2\right) \end{aligned} \tag{66}$$

with $\bar{\mu}_n=\mu_n+\sum_{k=1}^{n}\left(\tfrac{\sigma_k}{2r_k}\vartheta_k^2+\tfrac{\breve{\rho}_k\bar{\sigma}_k}{2q_k}\zeta_k^2\right)+\sum_{j=1}^{n-1}\left(\tfrac{\bar{\beta}_j}{2\beta_j}M_j^2+\tfrac{\bar{\pi}_j}{2\pi_j}N_j^2\right)$.

Denote $\gamma=\min\{2c_s,\sigma_s,\underline{\breve{\rho}}_s\bar{\sigma}_s,\bar{\beta}_\ell,\bar{\pi}_\ell,s=1,\ldots,n;\ell=1,\ldots,n-1\}$. Then, there must exist a parameter $p(t)>0$ satisfying

$$ {}_0^C\mathrm{D}_t^\omega V_n+p(t)=-\gamma V_n+\bar{\mu}_n \tag{67}$$

Taking Laplace transform on (67) becomes

$$\begin{aligned} V_n(s) &= \tfrac{s^{\omega-1}}{s^\omega+\gamma}V_n(0)+\tfrac{\bar{\mu}_n}{s(s^\omega+\gamma)}-\tfrac{p(s)}{s^\omega+\gamma} \\ &= \tfrac{s^{\omega-1}}{s^\omega+\gamma}V_n(0)+\tfrac{s^{\omega-(\omega+1)}}{s^\omega+\gamma}\bar{\mu}_n-\tfrac{p(s)}{s^\omega+\gamma} \end{aligned} \tag{68}$$

which indicates

$$\begin{aligned} V_n(t)= & E_{\omega,1}(-\gamma t^\omega)V_n(0)+t^\omega E_{\omega,1+\omega}(-\gamma t^\omega)\bar{\mu}_n \\ & -p(t)*t^{-1}E_{\omega,0}(-\gamma t^\omega) \end{aligned} \tag{69}$$

where $*$ denotes the convolution operator. Using Lemma 2 induces

$$t^{-1}E_{\omega,0}(-ct^\omega)=\frac{dE_\omega(-ct^\omega)}{dt}>0 \tag{70}$$

Thus, the last term in (69), i.e., $-p(t)*t^{-1}E_{\omega,0}(-\gamma t^\omega)$ is negative.
Since $\arg(-ct^\omega)=-\pi$, $|-ct^\omega|\ge 0$ and $0<\omega<2$, in the light of Lemma 3,

$$|E_{\omega,1}(-\gamma t^\omega)|\le\frac{\lambda}{\gamma t^\omega+1} \tag{71}$$

Then, one can know via Assumption 3 that

$$\lim_{t\to\infty}E_{\omega,1}(-\gamma t^\omega)V_n(0)=0 \tag{72}$$

Consequently, one can find a time instant $t_1>0$, for $\zeta_1>0$ and $\forall t>t_1$, it is guaranteed that

$$E_{\omega,1}(-\gamma t^\omega)V_n(0)\le\zeta_1 \tag{73}$$

Denote integer $h=1$, using Lemma 1 gives

$$E_{\omega,1+\omega}(-\gamma t^\omega)=\frac{1}{\Gamma(1)\gamma t^\omega}+o\left(\frac{1}{|\gamma t^\omega|^2}\right) \tag{74}$$

which induces

$$t^\omega E_{\omega,1+\omega}(-\gamma t^\omega)\bar{\mu}_n=\frac{\bar{\mu}_n}{\gamma}+t^\omega\mu o\left(\frac{1}{|\gamma t^\omega|^2}\right) \tag{75}$$

On the basis of the definition of infinitesimal amount, it is clear that $t^{\omega}\mu o\left(\frac{1}{|\gamma t^{\omega}|^2}\right) \leq \zeta_2$, $\forall t > t_2$, with $\zeta_2 > 0$ and $t_2 > 0$ being a time instant. Hence, (75) is rearranged as

$$t^{\omega} E_{\omega,1+\alpha}(-\gamma t^{\omega})\bar{\mu}_n \leq \zeta_2 + \zeta_3 \tag{76}$$

where $\zeta_3 > 0$. Subsequently, one can conclude

$$V_n(t) \leq \zeta_1 + \zeta_2 + \zeta_3 \tag{77}$$

Thus, the boundedness of all the closed-loop signals is guaranteed, and the tracking error approaches a small neighborhood of the original $|\xi_1| \leq \sqrt{2(\zeta_1 + \zeta_2 + \zeta_3)}$, for all $t > \max\{t_1, t_2\}$.

4. Simulation Study

A Chua–Hartley system [51] is considered to demonstrate the reliability of the presented FTC approach:

$$\begin{cases} {}_0^C D_t^{\omega} \chi_1 = \chi_2 + \frac{10}{7}(\chi_1 - \chi_1^3) \\ {}_0^C D_t^{\omega} \chi_2 = \chi_3 + 10\chi_1 - \chi_2 \\ {}_0^C D_t^{\omega} \chi_3 = -\frac{100}{7}\chi_2 + u^{\mathcal{F}} \\ y = \chi_1, \end{cases} \tag{78}$$

where $\omega = 0.98$. When $u = 0$, system (78) shows rich dynamical behavior, which is depicted in Figure 2.

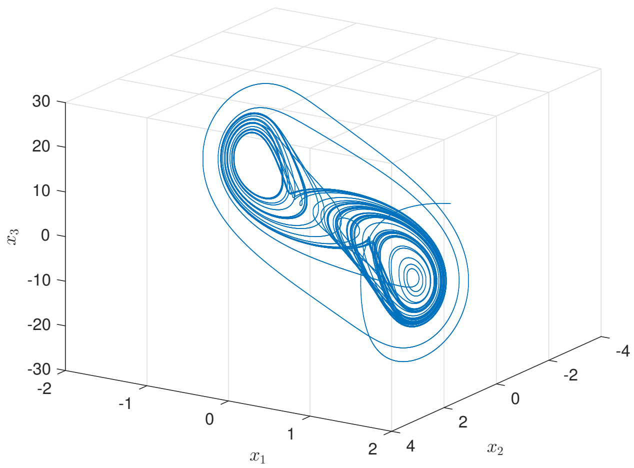

Figure 2. Dynamical behavior of the fractional-order Chua-Hartley system.

The sensor and actuator faults are selected as follows:

$$\chi_s^{\mathcal{F}}(t_s) = \breve{\rho}_s \chi_s(t_s) + \breve{\tau}_s(t_s), \;\; s = 1,2,3 \tag{79}$$

$$u^{\mathcal{F}}(t_u) = \breve{\rho}_0 u(t_u) + \breve{\tau}_0(t_u) \tag{80}$$

with $t_s = 10$, $t_u = 20$, $\breve{\rho}_1 = 0.5$, $\breve{\tau}_1 = 0.3\cos(t)$, $\breve{\rho}_2 = 0.5$, $\breve{\tau}_2 = 0.3\sin(t)$, $\breve{\rho}_3 = 0.9$, $\breve{\tau}_3 = 0.01\sin(t)$, $\breve{\rho}_0 = 0.8$, $\breve{\tau}_0 = 0.05\sin(t)$.

Three fuzzy logic systems are employed to estimate the unknown nonlinearities existed in system (78). The fuzzy membership functions are chosen as: $\varphi_{F_h^j}(x_h) = \exp\left(-\frac{x_h+3-j}{4}\right)$, $h = 1,2,3$, $j = 1,2,\ldots,5$.

In order to explore the influence of controller gains on system tracking error, two cases are considered:

CASE 1: Controller gains are chosen as: $c_1 = 100, c_2 = 15, c_3 = 20$.

In this case, the design parameters in a fault-tolerant controller, the adaptive laws parameters and fractional-order nonlinear filters are selected as: $q_1 = q_2 = 0.01$, $q_3 = 0.1$, $\bar{\sigma}_1 = 1$, $\bar{\sigma}_2 = \bar{\sigma}_3 = 3$, $r_1 = r_3 = 0.001$, $r_2 = 0.03$, $\sigma_1 = 1$, $\sigma_2 = 0.3$, $\sigma_3 = 0.8$, $\beta_1 = \beta_2 = 0.1$, $\bar{\beta}_1 = \bar{\beta}_2 = 1$, $\pi_1 = 0.01$, $\pi_2 = 0.1$, $\bar{\pi}_1 = \bar{\pi}_2 = 1$, $\frac{1}{\varrho_0} = 50$, $\frac{1}{\varrho_1} = 20$, $\bar{\delta}_k = 0.1$ $(k = 1,2,3)$.

The initial conditions are: $\chi_1(0) = 0.2$, $\chi_2(0) = 3$, $\chi_3(0) = 3$, $\hat{\vartheta}_1(0) = 0.2$, $\hat{\vartheta}_2(0) = \hat{\vartheta}_3(0) = 0.1$, $\hat{\epsilon}_1(0) = 0.2$, $\hat{\epsilon}_2(0) = \hat{\epsilon}_3(0) = 0.1$, $\hat{M}_1(0) = 0$, $\hat{M}_2(0) = 0.1$, $\hat{N}_1(0) = 0$ and $\hat{N}_2(0) = 0.1$, the others are zeros. The desired trajectory is chosen as $y_d = 1.4\sin(2t)$.

The simulation results are shown in Figures 3–7, the trajectories of y, $y^{\mathcal{F}}$ and y_d are showed in Figure 3; the trajectories of states χ_k $(k = 1,2,3)$ are depicted in Figure 4; the trajectories of $\hat{\vartheta}_k$ and $\hat{\epsilon}_k$ $(k = 1,2,3)$ are showed in Figure 5; the trajectories of $\hat{M}_j$ and $\hat{N}_j$ $(j = 0,1)$ are depicted in Figure 6; the control input is shown in Figure 7.

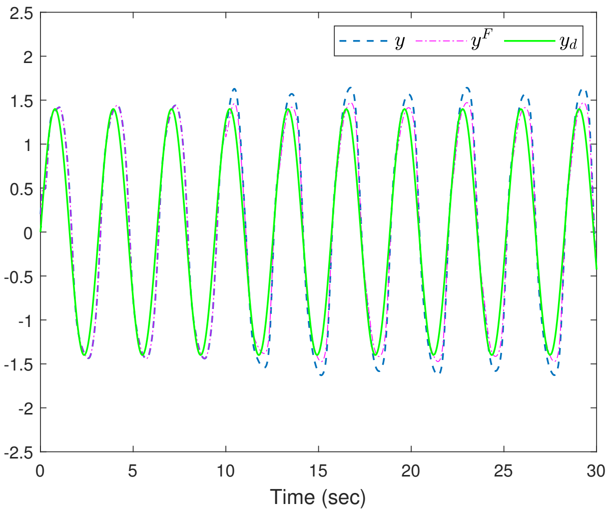

Figure 3. The trajectories of y, $y^{\mathcal{F}}$ and y_d.

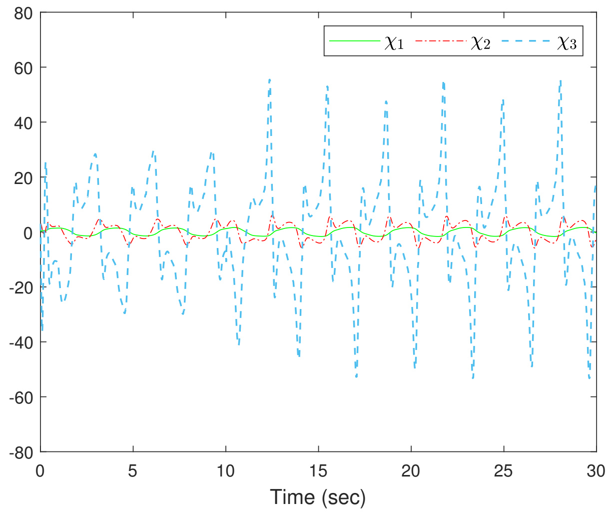

Figure 4. The system states χ_k $(k = 1,2,3)$.

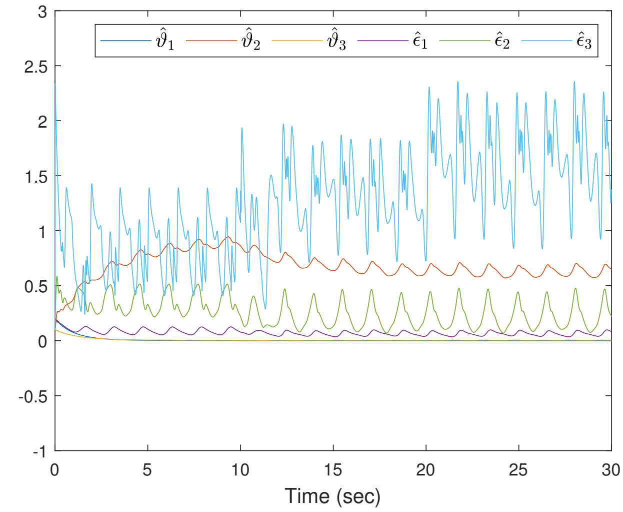

Figure 5. The parameters $\hat{\vartheta}_k$ and $\hat{\epsilon}_k$ $(k = 1, 2, 3)$.

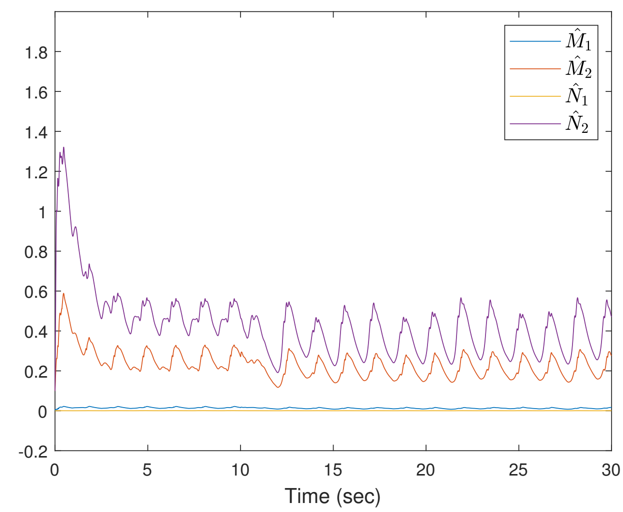

Figure 6. The parameters $\hat{M}_j$ and $\hat{N}_j$ $(j = 1, 2)$.

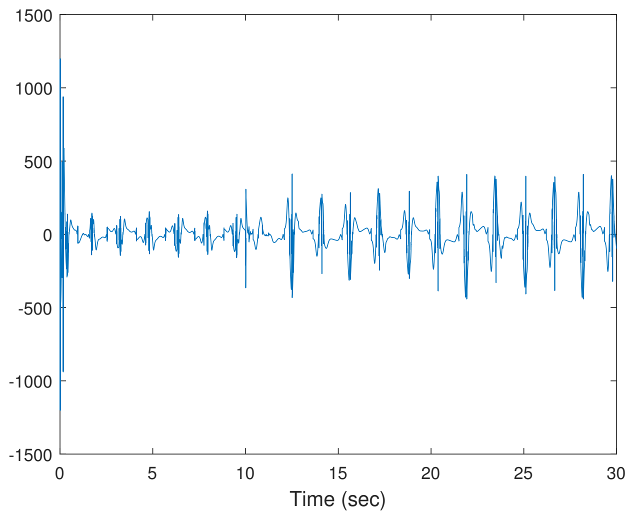

Figure 7. The control input u.

CASE 2: Controller gains are chosen as: $c_1 = 20, c_2 = 10, c_3 = 10$.

In this case, the design parameters in the fault-tolerant controller, the adaptive laws parameters and fractional-order nonlinear filters and initial conditions are same as CASE 1.

The contrastive simulation results for the trajectories of y, $y^{\mathcal{F}}$, y_d and the control input are depicted in Figures 8 and 9, respectively.

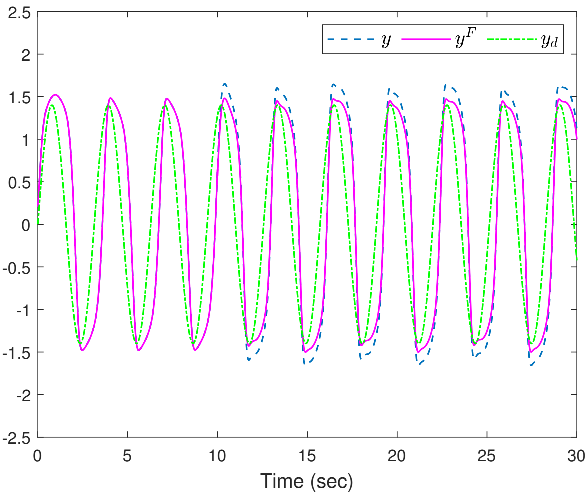

Figure 8. The trajectories of y, $y^{\mathcal{F}}$ and y_d in CASE 2.

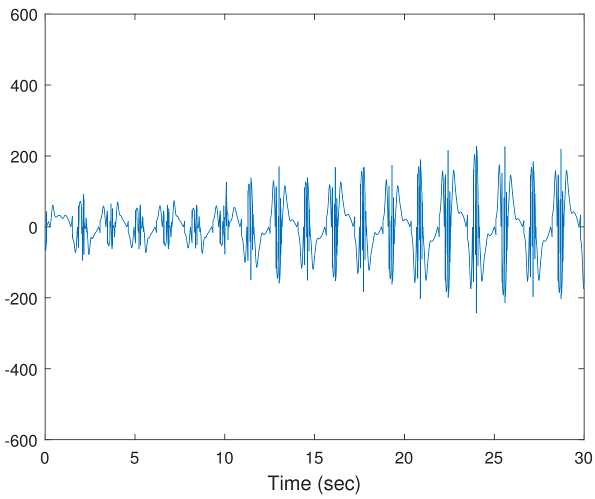

Figure 9. The control input u in CASE 2.

From the aforementioned simulation results, it is obvious that the following conclusions are proved.

(i) It can be seen from Figure 3 that the good tracking performance is guaranteed even if the actuator and sensor faults exist in the system. Figures 4–7 show that the presented control strategy ensures the boundedness of the closed-loop signals.

(ii) By comparing Figures 3 and 8, it is clear that Figure 3 has better tracing performance. By comparing Figures 7 and 9, it is easy to see that the control input in Figure 9 is smaller.

In summary, if the controller gain is increased, the tracking performance will be better. On the contrary, if you reduce controller gain, tracking performance deteriorates. Therefore, in practical applications, it is necessary to trade-off the transient performance and control action by selecting the design parameters suitably.

To better show the advantages of this paper in dealing with simultaneous actuator and sensor faults, we apply the existing adaptive control scheme for nonlinear systems with actuator and sensor faults using the fuzzy approximation criterion [45] to the Chua-Hartley system (78). Under the similar design parameters and initial conditions of CASE 1, the simulations results are shown in Figures 10 and 11. It is clear that under the premise of similar tracking effect, the control input of our proposed control method is much smaller than the one in [45].

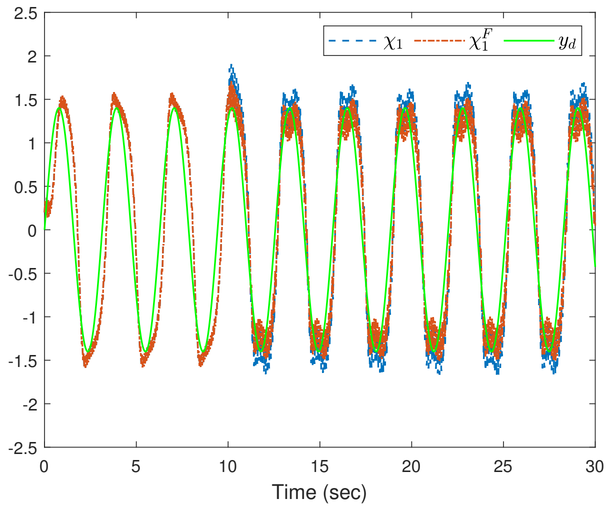

Figure 10. The trajectories of y, $y^{\mathcal{F}}$ and y_d using the strategy in [45].

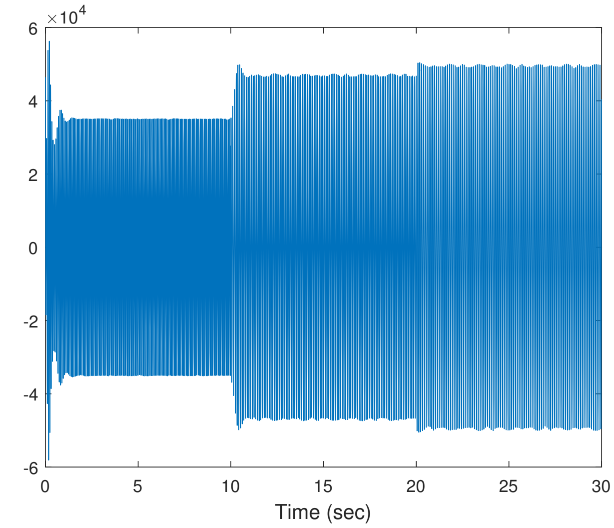

Figure 11. The control input u using the strategy in [45].

Remark 4. *By comparing Figures 7 and 11, it is easy to see that the results in [45] rely too much on the ability of the approximation technique, which has to improve the tracking performance by only increasing the control gains constantly. In comparison to [45], the control design method proposed in this paper has higher design freedom, and shows better robustness and transient performance.*

5. Conclusions

This work has addressed the adaptive fuzzy FTC issue for uncertain FO nonlinear systems with sensor and actuator faults. The fuzzy logic syatem has been exploited to manage unknown nonlinearity. On account of the constructed FO nonlinear filters, a DSC strategy has been developed. In line with the stability criterion of FO Lyapunov function, the system stability has been achieved. Exploring state constraints while maintaining a similar philosophy of the presented scheme is an interesting challenge for future study.

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