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Article

Chance or Chaos? Fractal Geometry Aimed to Inspect the Nature of Bitcoin

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Abstract: The aim of this paper is to analyse Bitcoin in order to shed some light on its nature and behaviour. We select 9 cryptocurrencies that account for almost 75% of total market capitalisation and compare their evolution with that of a wide variety of traditional assets: commodities with spot and future contracts, treasury bonds, stock indices, and growth and value stocks. Fractal geometry will be applied to carry out a careful statistical analysis of the performance of Bitcoin returns. As a main conclusion, we have detected a high degree of persistence in its prices, which decreases the efficiency but increases its predictability. Moreover, we observe that the underlying technology influences price dynamics, with fully decentralised cryptocurrencies being the only ones to exhibit self-similarity features at any time scale.

Keywords: Bitcoin; cryptocurrencies; fractal geometry; hurst exponent; long-term memory; efficient market

1. Introduction

Within the context of great uncertainty regarding the regulation of Bitcoin (BTC), with a wide variety of legislative solutions ranging from prohibition to full incorporation into economies, the key question to characterize its nature and behaviour has spread beyond the financial press. Indeed, there are multitudes of scientific papers with the aim of providing grounded arguments for its classification as money [1], a technology-based product [2], or a safe-haven investment [3].

Paradoxically, a deep understanding of the behaviour of BTC and similar assets is still pending, as the academic literature has reached no common consensus, for instance, about the market efficiency of cryptocurrencies (CC hereafter). In fact, on one side, the pioneering works by Urquhart [4] and Bariviera [5] conclude that BTC is mostly inefficient, but evolving towards random dynamics. On the contrary, there are sources claiming that BTC moves similarly most of the time, but there is no agreement between those who advocate that the mainstream characterists are efficiency [6–8], long-term memory [9–12], or anti-persistency [13], no matter the frequency or time frame considered.

In this unsettled framework, the overall goal of this paper is to shed some light on the nature of BTC in order to foresee whether its prices move randomly or in a chaotic but predictable way, which would be a starting point to allow forecasting opportunities. Additionally, as claimed by Grobys in the recent paper [10], the economy has lately been suffering from widespread disruptions caused by COVID-19, as well as the current Russian–Ukrainian conflict, leading to the crucial need to re-inspect the market efficiency of both traditional and digital assets.

More precisely, the author in [10] focuses exclusively on BTC (with a sample of 512 weekly observations) and its direct comparison to S&P 500 as a paradigm of a long-established market, after which he encourages future more detailed reseach including other CC. We take up this suggestion by considering 9 different digital coins which accummulate

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almost 75% of the traded volume, and our sample takes observations in much higher frequencies (daily, 1 h and each 15 min), as this will better reflect the fast trading in digital exchanges. Moreover, we do not restrict to a single classical asset for comparison, but consider a range of 13 traditional assets of a different nature: gold, crude oil, wheat, silver, 10-year US bonds, two indices and 6 stocks, including value and growth companies.

Here is a quick overview of our analysis:

- To identify factors detecting differences in the dynamics of CC in the markets, we have chosen CC representing diverse consensus protocols.
- For each of the 21 different assets, as already mentioned, we have three time series corresponding to daily and two different intraday frecuencies. Moreover, we will split each sample in two different non-overlapping subperiods, which we will analyze in parallel to the full sample. The goal is to judge whether varying the time frame or frequency would lead to different indicators or whether the data present scale invariance properties.
- After construction of the corresponding series of logarithmic returns and the mandatory descriptive statistics (including a normality test), we conduct a careful analysis of the precise characteristics of our samples, in order to choose an ad hoc method to address the question about their efficiency.
  - First, we tested the potential presence of multifractality features in our series (see Section 6.1). Our outcomes agree with those by Bariviera in [14], who analyzes 84 CC to conclude that the most capitalized ones (like our selection of nine CC) display monofractal patterns.
  - Then, as stationarity is a crucial feature to decide the technique, we perform the classical Augmented Dickey–Fuller (ADF) unit root test. Moreover, to double check its findings, we have implemented a second generation unit root test (KSS) which tackles the possible failure of ADF in the presence of non linearities. The outcomes (see Sections 6.2 and 6.3) confirm that our series are stationary at 99% confidence.
- After these a priori tests to check that the assumptions of monofractality and stationarity are fulfilled by our data, we are enabled to apply the R/S method, which goes back to Mandelbrot's original development of fractal geometry [15,16], to compute the fractal dimension (by means of the Hurst exponent), boosted with suitable inference tests to validate our outcomes (see Section 5.3 for a detailed account of the methodology).
- Apart from applying the technique to the different time frequencies and periods to assess self-similarity properties, to avoid that our estimates include spurious effects due to the presence of autocorrelations in the time series, we generate randomly shuffled copies of them for which we re-calculate the Hurst exponent.
- Finally, we study how CC are correlated between them and with a selection of 13 already established asset classes.

As highlights of our findings, for all CC with purely decentralized technologies (that is, without intermediaries or audit control), we detect clear signs of inefficiency. Furthermore, the computation of the fractal dimension under different time frames and scales certainly reveals that these CC present long-term memory and self-similarity properties. Hence, a technical analysis of past data is well-founded but shifting the scale is helpless to obtain finer predictions. This information will be helpful to develop forecasting algorithms in future research.

We also confirm that these properties are shared neither by centralized CC nor by assets like gold, silver, crude, wheat or treasury bonds, which are quite poorly correlated to BTC that is in turn tightly interrelated to most actors of the *cryptosphere*. This disables any narrative or marketing strategy to declare BTC or new gold and its often claimed diversifying power.

Our results upgrade the previous literature in several directions: we pioneer in disclosing that it is the underlying technology, instead of the capitalization, that is the crucial element influencing CC prices. Accordingly, it is interesting to assess in subsequent research whether the change of protocol in Ethereum (which is the second most capitalized CC) from 15 September 2022 modifies its efficiency, self-similarity patterns and trends in comparison to BTC.

We also open a new research line that challenges the widely accepted impermeability of the CC market to the macroeconomic environment. In fact, we detect a major paradigm shift (driven possibly by rising inflation and interest rates) in CC interrelations, and provide feedback on external products by zooming in on the last trimester of 2022 and 2023 data.

If we focus on BTC, the present paper is very much in the line of [10], since we share the conclusions that BTC is persistent with behaviour that does not change over time. Methodologically, the author in [10] also applies the R/S method enhanced by several robustness tests to confirm its current validity, as opposed to the large amount of replication failures in finance studies (which is well recorded in [17]). The latter even suggests to treat with caution and re-examine conclusions derived from standard statistical models (see further discussion in Section 5.2). In short, several authors share our concern about the rigorous application of the techniques, by a careful check of the hypotheses before using a procedure and a later confirmation of the statistical significance of the computations; However, both steps are often naively ignored in the financial literature, as pointed out in [18].

This paper is organized as follows: in Section 2, we introduce the basic notions concerning CC consensus protocols, the memory/efficiency character of time series and the fractality that will be used extensively throughout the paper; whereas, Section 3 includes a detailed survey of the literature to stress that the debate about the nature of BTC is a vivid area of current research that has not yet reached any consensus and deserves further attention, and contradictory results in previous papers could be a statistical artefact of inaccurate techniques. Later on, in Section 4, we provide descriptive statistics of our datasets explaining the sources and criteria of selection of the different assets. Then, we explain the details of how we perfom the R/S method (cf. Section 5) and thoroughly discuss the outcomes and the corresponding implications in Section 6. Finally, we include a summary of main conclusions (Section 7) and, for completion, three appendices with additional tables on the overview of the techniques used in the previous literature and calculations of the Hurst exponent, as well as full correlation matrices.

2. Background Material: Basic Facts and Definitions

2.1. Blockchain and Its Different Consensus Algorithms

The paradigm change brought by BTC represents an evolution towards decentralised networks, allowing peer-to-peer interactions (with no need of intermediaries or central authorities that audit the operations) and a secure exchange, not only of information, but also of value. The technology that enables the latter is known as blockchain; roughly speaking, it uses cryptographic techniques to run a shared and secure digital register to record transactions, like a ledger.

Different consensus protocols/algorithms have been developed to validate and secure operations: Proof-of-Work (PoW), where different users (*miners*) compete to solve a mathematical problem that requires high computational cost; or Proof-of-Stake (PoS), which selects validators randomly, giving a higher probability to those who deposit a larger amount of CC as a guarantee.

We will not go into technical details of other variants, like Nominated PoS (NPoS), PoS Authority (PoSA), Delegated PoS (DPoS) and Ripple Protocol Consensus Algorithm (RPCA). Let us just point out that some of them are controlled by very few nodes as in the traditional transaction systems.

In short, the consensus method provides each CC with distinct features in terms of energy efficiency, security and scalability; thus, we wonder the following:

Does the technology of CC influence their behaviour in the markets?

Accordingly, we will focus on the comparative study of CC with different consensus mechanisms (see second table in Section 4.1). Up to our knowledge, it is the first time that the underlying technology is used in the literature as a distinctive characteristic.

2.2. Efficiency versus Persistence and Fractality

An essential yet unresolved matter is the predictability of the CC behaviour, a notion that challenges the Efficient Market Hypothesis (EMH), which is one of the key cornerstones for modeling financial data [19]. A market is efficient if prices follow a random Brownian motion, reflecting all available information; thus, a market is said to be efficient if past data cannot be exploited to predict future returns.

More precisely, three characteristics determine this motion: independence (i.e., prices have no memory and their dynamics are fully random), stationarity (the magnitude of changes does not vary with time) and normality, which implies that extreme events occur with very low probability. The latter cannot explain the sudden and sharp movements in financial markets. Illustratively, from 1916 to 2003, the Dow Jones index had 48 days with a swing bigger than 7%, but under a normal distribution this should occur 1 day every 300,000 years [20].

In short, the evidence from real data leads us to look for alternatives to EMH that allow us to predict these abrupt changes, which also happen in waves and are not isolated nor orderly, thus showing the chaotic fluctuation of prices. This is precisely what led B. Mandelbrot to use chaos theory as an inspiration to create fractal geometry in the 1970s, which seeks to quantify complex patterns in nature.

Indeed, fractal objects work like chaotic systems where instantaneous shifts can have significant effects in the long term. Roughly speaking, two main features define a fractal: it has a fractional (non-integer) dimension, and it is scale invariant or self-similar, i.e., it presents copies of itself as it is zoomed in (like a snowflake). For a time series, this means that its basic features are kept if we consider time subperiods or alter the data frequency of the sample.

When prices evolve inefficiently (then predictably), there are two types of memory: antipersistent or mean reverting if an increase is followed by a fall and, conversely, drawing an oscillatory path around the mean; and persistent or long-term memory, that is, after a rise/drop comes another move with the same trend. In practice, the higher the persistence is, the more difficult it is for the values to return to their predetermined target in the event of a fall or exogenous shock.

3. Literature Review

This section pretends to highlight that, despite the expansive literature about this topic, there is still no agreement (in fact, one finds clear contradictions from one paper to another) concerning the nature of BTC in terms of efficiency and self-similar dynamics. Clarifying this by means of a meticulous application of the methods to avoid misleading conclusions is a key issue and any new contribution is welcome, as argued in [10], because of the raising popularity of CC as investment tools, the high amounts of money traded and the crucial time for policy-makers due to the pressing need to decide how to regulate this reality.

Let us first stress that, despite this heterogeneous landscape and apart from the coincidence with [10], our conclusions about persistent behaviour, no matter the time period, are also in line with new approaches to the problem by means of wavelet analysis [21], or deep learning techniques [22]. However, both of them restrict their interest to daily returns of BTC as an isolated object. In contrast, we expand our view to other CC in relation to a wide variety of traditional assets and consider not only daily returns, but also higher intraday frequencies, in order to grasp the extremely quick movements of digitally traded markets.

3.1. Under Which Label Do We Classify Bitcoin?

The right description of BTC is a desirable goal to grasp its potential role in the market for risk management and portfolio diversification. While its design has similarities with gold (mining, decentralization, not government-backed and globally traded 24/7) and currencies (medium of exchange), if BTC were a real unit of account or a store of value, it would not exhibit the high volatility characterized by bubbles and crashes (cf. [23]). Recall that volatility/fluctuation are typically characterized in terms of the standard deviation for the sampling time period.

In this spirit, it more closely resembles a highly speculative asset than a typical commodity or currency (see [24]), or at least belongs to a category in between the latter two (cf. [25]). Moreover, ref. [26] shows that BTC has its own risk-return features and is uncorrelated with traditional assets. Because of this, refs. [27,28] conclude that it cannot play the role of a safe-haven from an econometric perspective.

In fact, its evolution does not respond to monetary policy news, but it reacts to events related to CC, having significant correlations with them [29,30]. Analogously, ref. [31] claims that the isolation of BTC from the global financial system implies that it is not an actual source of economic instability.

However, the dependence between CC and other assets may change over time. Indeed, ref. [32] checks that the gold-BTC correlation reached a maximum during the peak of COVID-19, dropping to almost zero in July 2021. Because of this, it is not completely hopeless to include CC in a portfolio. Even more, ref. [25] argues that BTC is helpful for risk-averse investors in anticipation of negative shocks, as its reactions to market sentiment are quicker.

3.2. Efficient Market Hypothesis (EMH) in the "Cryptosphere"

Despite the extensive analysis about EMH's applicability to the BTC market (see [33] and Table A1 for a survey), there is no agreement on whether the periods of efficiency alternate either with mean reverting dynamics (as claimed in [4,34]) or with long-term memory trends (stated by [5,35,36]).

More precisely, ref. [37] unveils a decreasing trend in the predictability, confirmed in [38] by checking a reduction in the price reaction time to unexpected events. This is further supported in [39], as no pattern in returns can be discovered away from price clustering. But, there is no common narrative to explain the varying efficiency over time: according to [40], inefficiency is higher during price rises; ref. [7] found out that liquidity (volatility) has a significant positive (negative) effect on the efficiency, which can be enhanced by introducing BTC futures [41].

Despite the almost exclusive restriction to BTC, some authors compare it with other CC. In this spirit, ref. [42] concludes that BTC is the least predictable, which is reinforced by [14], with notable efficiency in lower volume quantiles and anti-persistance in higher ones. However, refs. [43,44] also find evidence of the reverse arguments, showing that less capitalized coins are more efficient than BTC, while the latter presents long-term memory (see also [45]).

Finally, efficiency can be altered during exceptional circumstances, such as the COVID-19 pandemic, which introduced significant regime changes in crypto and traditional markets (see [46]). Surprisingly enough, BTC is more resilient to efficiency decreases than other financial assets (cf. [47]).

3.3. Unraveling Complexity by Means of Fractal Geometry

Refs. [48,49] claim that the efficiency of CC varies across frequencies, as there is heterogeneous memory behaviour against the self-similarity required for fractal objects. This view is shared by [7], which checked that the higher the frequency, the lower the pricing efficiency. In the same spirit, refs. [44,50] conclude that the regime of persistence depends on the time scale and the period considered.

On the contrary, ref. [51] reports similar memory patterns, no matter the time frequencies, implying a self-similar process. This is confirmed in [52] via a big data-driven study joint with statistical testing, providing evidence of dominant fractal traits at all high frequency rates for BTC prices.

Later on, ref. [14] clearly reveals that one cannot apply a common model to address the whole CC landscape. Indeed, highly capitalized coins display roughly unifractal processes, and thus can be described via fractional Brownian motion; however, cryptoassets with fewer liquidities exhibit strong multifractality, and more sophisticated models would be required for capturing their complex dynamics.

As pointed out in [53], multifractal features are not exclusive of CC; in fact, they are similar to that of stock markets, but differ from regular coins. Additionally, ref. [54] checked that BTC shows higher fractality than gold, which reopens the debate about its nature also from the outlook of fractal geometry.

4. Data and Descriptive Statistics

4.1. Types of Assets: Selection Criteria and Sources

We have worked with a dataset (obtained from Binance) containing the prices of BTC and eight other CC (see Table 1) over the time period from 20 August 2020 to 24 February 2023, considering opening values every 15 min ($N = 88{,}060$ observations), 1 h ($N = 22{,}019$) and 1 day ($N = 919$). The aim is to contrast daily dynamics with intraday or high frequency movements in the view of seeking self-similarity features.

Table 1. Description of cryptoassets and traditional commodities.

Asset	Ticker	Market	Initial Release/Public Offering/Description
Bitcoin	BTC	Binance	9 January 2009
Ethereum	ETH	Binance	30 July 2015
Binance Coin	BNB	Binance	3 July 2017
Ripple	XRP	Binance	2 June 2012
Cardano	ADA	Binance	27 September 2017
Polygon	MATIC	Binance	24 April 2019
Solana	SOL	Binance	24 March 2020
Tron	TRX	Binance	25 July 2018
Polkadot	DOT	Binance	26 May 2020
Amazon	AMZN	NASDAQ	15 May 1997
Tesla	TSLA	NASDAQ	29 June 2010
Netflix	NFLX	NASDAQ	23 May 2002
Procter & Gamble	PG	NYSE	13 January 1978
Johnson & Johnson	JNJ	NYSE	24 September 1944
The Coca-Cola Co.	KO	NYSE	September 1919
Silver	XAGUSD	LBMA	Spot price per ounce in US dollars
Gold	GCG23	COMEX	100 troy ounce FC due in February 2023
Crude oil	CLK23	NYMEX	West Texas Intermediate FC due May 2023
Wheat	ZWK23	NYMEX	FC expiring in May 2023
US Treasury bonds	ZNM23	CBOT	10-year FC due in June 2023
Nasdaq	CCMP	NASDAQ	1971
EuroStoxx	SX5E	EUROSTOXX	26 February 1998

In order to compare with other assets traded on traditional financial markets, time series have been selected from Bloomberg with daily values of:

- ○ Three commodities with future contracts (gold, crude oil and wheat);
- ○ One spot commodity (silver);
- ○ A 10-year US Treasury bond future contract (FC);
- ○ Two stock market indices (Nasdaq and EuroStoxx);
- ○ Three growth stocks (Tesla (Austin, TX, USA), Netflix (Los Gatos, CA, USA) and Amazon (Seattle, WA, USA)) and three value stocks (The Coca-Cola Co. (Atlanta, GA, USA), Procter & Gamble (Cincinnati, OH, USA), and Johnson & Johnson (New Brunswick, NJ, USA)).

We conjecture that growth stocks (i.e., those with a 5-year average sales growth over 15%) will perform similar to cryptoassets having an actual positive correlation; while it is expected that when CC go up, value stocks (that is, the ones with price-to-sales ratio < 1) will go down.

On the other hand, to assess whether CC perform as a safe haven asset, commodities have been chosen, as well as low volatility assets as treasury bonds. Moreover, to compare the cryptoeconomy to the traditional financial market, we include stock indices, which act as a thermometer of market movements.

Regarding the selection of CC, we applied the Pareto principle to their capitalisation, according to which 80% of the results are due to 20% of the variables involved. Although the 9 selected CC "only" accumulate approximately 74% of the total market capitalisation (see Table 2), we have ruled out CC with volume percentages below 0.5%, which we consider to be unrepresentative. Let us stress that we have chosen archetypes of five different consensus protocols to enable our aim of comparing CC on the basis of their underlying technology.

Table 2. Description of the 9 selected CC (from www.binance.com, 24 May 2023).

Ticker	Protocol	Capitalization (Million $)	Percentage	Cumulative
BTC	PoW	517,890	45.51%	45.51%
ETH	PoW/PoS	218,230	19.18%	64.69%
BNB	PoSA	48,060	4.22%	68.91%
XRP	RPCA	23,590	2.07%	70.98%
ADA	PoS	12,760	1.12%	72.10%
MATIC	PoS	10,080	0.89%	72.99%
SOL	PoS	7690	0.68%	73.66%
TRX	DPoS	6960	0.61%	74.28%
DOT	Nominated PoS	6300	0.55%	74.83%

As can be observed in Figure 1, BTC and ETH account for almost 65% of the total volume, while the 9 chosen CC together have a total capitalisation of more than USD 851 billion, which will provide us a global view of the market by accounting for almost 75% of the total capitalisation estimated at USD 1138 trillion, according to Investing (www.investing.com), as of 24 May 2023.

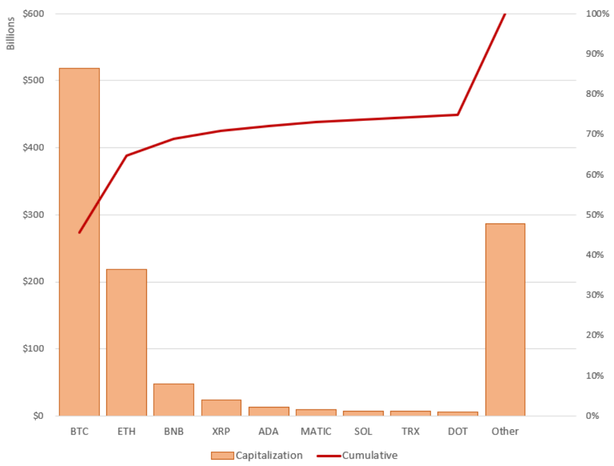

Figure 1. Pareto diagram of the cryptocurrency market according to capitalization.

4.2. Descriptive Statistical Analysis of Data

Let us first point out that daily opening values have been taken. In addition, to facilitate the direct comparison, for the descriptive statistics (see Table 3) and the computation of correlations (cf. Section 6.7) only weekday values are included, even though CC are still trading on weekends.

Table 3. Descriptive statistics and normality test (Jarque Bera) for daily returns from 20 August 2020 to 24 February 2023. *** means significant results at 0.1% level of significance.

Asset	Mean	Median	Std.	Max.	Min.	Skew.	Kurt.	J. Bera
BTC	0.0401	−0.0190	3.8451	17.8448	−16.7093	−0.2419	5.9025	376.16 ***
ETH	0.1428	0.2390	5.1954	23.3707	−32.4864	−0.4250	7.5343	288.07 ***
BNB	0.2217	0.1630	5.6390	29.5648	−41.6751	−0.2158	11.5924	7311.6 ***
XRP	−0.1597	0.1424	6.6802	36.6201	−53.8523	−0.6736	15.8684	3289.8 ***
ADA	−0.0909	−0.1909	5.9398	28.7239	−31.1317	0.1538	6.5777	435.14 ***
MATIC	0.4301	−0.0706	8.1566	48.7557	−41.0080	1.0209	9.6473	1639.1 ***
SOL	0.0081	−0.2235	7.9623	38.0494	−54.9008	−0.5784	9.8461	665.94 ***
TRX	0.0556	0.1906	5.5029	34.3355	−38.8245	−0.2373	11.5803	1096.2 ***
DOT	−0.0957	−0.2126	6.5311	28.0615	−48.3208	−0.3569	10.0600	271.22 ***
AMZN	−0.1167	0.0016	2.5189	10.4044	−15.1499	−0.3680	6.4681	296.59 ***
TSLA	0.0707	0.1384	4.3051	14.4446	−17.0308	−0.1375	4.3593	47.421 ***
NFLX	−0.1004	−0.0627	3.0387	12.0963	−30.6729	−2.4777	28.1416	15228 ***
PG	0.0034	0.0768	1.1469	3.6368	−7.5586	−0.8748	7.1694	498.61 ***
JNJ	−0.0043	0.0270	1.0430	5.5584	−3.7804	0.1567	4.6431	66.278 ***
KO	0.0419	0.0869	1.1405	5.5815	−7.0610	−0.2622	6.9038	333.44 ***
Silver	−0.0544	−0.0637	1.8350	7.9983	−8.5103	−0.1167	5.6815	154.02 ***
Gold	−0.0143	0.0429	0.9210	2.8140	−4.8252	−0.5177	4.9956	134.18 ***
Crude	0.0892	0.2208	2.5703	11.6753	−12.3624	−0.1827	5.1919	181.26 ***
Wheat	0.0630	−0.1375	2.2508	17.5554	−9.6304	0.8261	9.6193	1179.1 ***
Bonds	−0.0303	−0.0036	0.4281	1.7628	−1.4895	0.0768	4.3337	46.587 ***
Nasdaq	−0.0005	0.1616	1.6119	6.8863	−7.0825	−0.3765	4.4745	22.102 ***
EuroSt.	0.0310	0.1008	1.0992	5.6815	−5.3614	−0.2227	5.9040	276.7 ***

To analyse the normality of the time series, one typically converts them from price series $P = \{P_t : t \in \mathbb{N}\}$ to return series $R = \{R_t : t \in \mathbb{N}\}$. In our case, we have calculated a total of 608 returns for each series on a logarithmic scale, i.e., we work with the sample $R = \{R_t : t = 1, \ldots, 608\}$, where

$$R_t := \log \frac{P_t}{P_{t-1}} = \log P_t - \log P_{t-1}. \tag{1}$$

Table 3 reports the descriptive statistics for the full sample period from 2 August 2020 to 24 February 2023, which leads us to the following preliminary conclusions:

- CC have a much higher volatility than other assets; in particular, BTC's volatility (3.85) is much higher than gold's volatility (0.92). As a result, this is the first indication that BTC does not behave as a safe haven asset.
- For a normal distribution, the kurtosis is around three and the skewness is near zero. The values of the latter do not seem conclusive, as most of them are close to zero; however, the kurtosis seems to point out that neither CC nor traditional stock market players follow a normal pattern.
- The normality test (Jarque Bera) provides strongly significant evidence that none of the series under study follows Gaussian moves, which confirms Mandelbrot's criticism [20] to classical economic theories arguing that the normality assumption does not properly capture price evolution.

Additionally, we split our sample into two subperiods (see Table 4) in order to test whether the efficiency varied over time. A first inspection shows that descriptive statistics are quite stable for BTC, while other assets present sign shifts in the mean (TRX, NFLX) or kurtosis (silver). This provides a first hint that self-similarity features may not be shared for all items under review. Let us also point out that the significativity level for the normality test rises to 5% during the second subperiod for wheat, oil, gold, silver, Nasdaq, US bonds, TSLA and JNJ, which may foresee a change in performance.

Table 4. Extract from the descriptive statistics of the two subsample periods.

Sample Period	N	Mean	SD	Skew.	Kurt.
BTC					
20 August 2020–1 July 2022	681	0.0395	4.1134	−0.1986	5.2739
1 July 2022–24 February 2023	239	0.0358	2.9172	−0.5396	8.9691
TRX					
20 August 2020–1 July 2022	681	0.1229	6.2265	−0.2374	9.4431
1 July 2022–24 February 2023	239	−0.1431	−0.6032	−0.5396	5.9418
NFLX					
20 August 2020–1 July 2022	470	−0.2597	3.0527	−3.4819	34.9563
1 July 2022–24 February 2023	164	0.36498	2.9482	0.7096	3.9657
Silver					
20 August 2020–1 July 2022	485	−0.0710	1.7950	−0.4791	6.0914
1 July 2022–24 February 2023	169	−0.0199	0.7124	0.7096	4.5917

Finally, let us mention that we do not display descriptive statistics for intraday frequencies of CC for brevity, since there are not remarkable differences with Table 3, apart from the higher number of observations.

5. Methodology: R/S Analysis Enhanced by a Test of Significance

5.1. Hurst Exponent: Origin, Definition and Interpretation

A fractal is defined by the scale invariance and chaotic nature, whose complexity is measured by the fractal dimension D. As the graphs of returns are bumpy/peaky curves in the plane, D should be between 1 (dim. of a smooth curve) and 2 (dim. of the plane); then, we can write $D = 2 - H$, with $0 < H < 1$.

For time series, one typically computes the value of H, known as Hurst exponent, which was introduced to study the Nile overflows [55,56]. Economists use H to assess the efficiency of the market; thus, it is considered efficient if $H = 0.5$, and inefficient otherwise (see more details in Table 5).

Table 5. Interpretation of the values of the Hurst exponent.

	$H = 0.5$	$0 < H < 0.5$	$0.5 < H < 1$
Motion	Random Brownian	Fractional Brownian	Fractional Brownian
Persistence	None (independent)	Anti-persistent (mean-reverting)	Persistent
Memory/Correlation	None	Short term	Long term
Efficiency	Efficient	Inefficient	Inefficient

5.2. Justification of the Chosen Methodology: R/S versus DFA

Hereafter, we apply the rescaled range (R/S) method (cf. [16,57,58]) to obtain H without imposing independence nor a normality of the returns (as other methods more widely used in the literature do). The only restriction is to work with stationary series, but we will check that this is the case (see Section 6.2, where we perform a traditional ADF test, and Section 6.3, which confirms the results of such a test by applying a second generation unit root test which accounts for the presence of non-linearities in the time series).

One of the additional advantages of the R/S analysis is that it is robust in the sense that can detect non-periodic cycles even if they are longer than the sample period, as well as long-term correlations. For instance, ref. [59] applied it to conclude that many natural phenomena are not independent random processes. We choose this instead of the modified R/S statistic, as [60] evidenced that the latter produces a strong bias towards accepting the null hypothesis of data independence.

Apart from a solid pure mathematical theory (that is, the fractal geometry developed by Mandelbrot) on the basis of the R/S method, the belief that it is an outdated technique, displaced by new algorithms, is dismissed by its relevance in recent studies such as [10], which also warns about standard statistical models based on the assumption of finite variances, as they may produce misleading answers.

In contrast to the deep mathematical base of the R/S technique, the probabilistic foundations of Detrended Fluctuation Analysis (DFA, and its subsequent modifications) are still unclear [61] and a current topic of research, despite being one of the most popular algorithms to handle non-stationarities. Further advances in this direction of providing a theoretical justification for DFA is regarded in [61] as a key step towards the assessment of statistical estimation errors.

In fact, the criticism towards the efficacy of DFA in addressing nonstationarities is not new: Bryce and Sprague [62] already pointed out that its validity is overemphasized by several reasons: it introduces uncontrollable biases, its computational demands surpass those of the R/S analysis; and it fails to grant an effective and comprehensive protection against non-stationarities. In this spirit, ref. [63] highlights that the nonlinear filtering characteristics of DFA's detrending process may induce instabilities in estimating the scaling exponent, leading to estimation errors when computing H.

In short, while methodological enhancements and the theoretical backup for DFA are developed, we will double check the stationarity of our series, and we opted for the R/S method, which has very recently been applied in similar settings by [10]. Let us finally point out that the search of new methods to beat DFA is an active topic of research, including Bayesian methods [64], wavelet analysis [21] or deep learning models [22].

5.3. Details of the Rescaled Range Algorithm (R/S Method)

We have divided the calculation procedure into the steps described below:

(1) Take a series of returns $R = \{R_1, \cdots, R_N\}$, computed as in (1).

(2) Split the full series R into d sub-series of length $n = \frac{N}{d}$:

- 1st sub-series: $\{R_1^1, \cdots, R_n^1\}$,
- 2nd sub-series: $\{R_1^2, \cdots, R_n^2\}$, and so on, up to the
- dth sub-series: $\{R_1^d, \cdots, R_n^d\}$.

In short, we consider a pack $\{R_i^m\}_{i=1,\ldots,n;m=1,\ldots,d}$ with all sub-series. This step is conducted with $d = 1, 2, 3, \ldots, \frac{N}{n}$ for different values of $6 < n < N$.

(3) For each sub-series R_i^m, compute the mean E_m and the standard deviation S_m (which will be functions of n).

(4) Determine the series of distances to the mean Z_i^m by means of

$$Z_i^m = R_i^m - E_m,$$

and create the cumulative time series Y_i^m for each sub-series $m = 1, 2, \ldots, d$:

$$Y_i^m = Z_1^m + Z_2^m + \cdots Z_i^m = \sum_{j=1}^{i} Z_j^m.$$

(5) Find the range $\boldsymbol{R}_m$ of each cumulative subset, for all m:

$$\boldsymbol{R}_m(n) = \max\{Y_1^m, Y_2^m, \ldots, Y_n^m\} - \min\{Y_1^m, Y_2^m, \ldots, Y_n^m\}.$$

Then, divide this by the corresponding standard deviation S_m, that is, compute $\boldsymbol{R}_m/S_m$. Thus, one obtains a dimensionless measure that depends on n and allows for comparing the relative variability of sets of different sizes.

(6) Obtain the rescaled range statistic $(\boldsymbol{R}/S)_n$ by averaging for all sub-series:

$$(\boldsymbol{R}/S)_n = \frac{1}{d}\sum_{m=1}^{d} \frac{\boldsymbol{R}_m}{S_m}(n)$$

Note that the different partition sizes (d) from **(2)** lead to the set of values:

$$\left\{(\boldsymbol{R}/S)_N,\ (\boldsymbol{R}/S)_{\frac{N}{2}},\ (\boldsymbol{R}/S)_{\frac{N}{4}},\ \cdots,\ (\boldsymbol{R}/S)_6\right\} \;=\; \{(\boldsymbol{R}/S)_n\}_{n=N,\frac{N}{2},\frac{N}{3},\ldots}$$

(7) Computation of $\boldsymbol{H}$. With an analogy of Hurst's ideas, assume that the variability of the data follows a potential law of the form:

$$(\boldsymbol{R}/S)_n = c \cdot n^H,$$

and now take logarithms on both sides to reach the linear relation

$$\log(\boldsymbol{R}/S)_n = \log c + H \log n.$$

We can plot $(\log n, \log(\boldsymbol{R}/S)_n)$ for $n \in \left\{N, \frac{N}{2}, \frac{N}{3}, \ldots\right\}$, and obtain the corresponding regression line (see Figure 2), whose slope is the desired H.

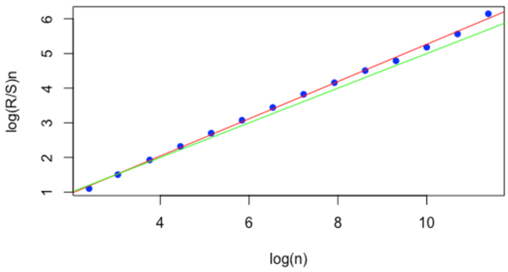

Figure 2. Graphic representation of the Hurst exponent. The slope of the red line represents the Hurst exponent, while the green line has a slope of 0.5. Blue dots represent the values of $(\log n, \log(\boldsymbol{R}/S)_n)$.

Many papers in the literature stop here, which is a naive approach, as pointed out in [18], since one needs an extra statistical test to judge whether H computed for the sample is significatively different from the value 0.5 characterizing an independent process. This is why we have developed our own R script to top up the R/S method by adding a t-test for the slope of the line (see Figure 3). Let us stress that the code is just a trivial modification of the standard routine included in the `pracma` R library to additionally perform an inference test on the slope of the linear regression used to estimate H. Moreover, all our results for H coincide up to the second decimal place with the R outputs for corrected H provided by the instruction `hurstexp`.

Additionally, to avoid estimates of H being misleading or including spurious effects due to the presence of autocorrelations in our time series, we generate randomly shuffled copies of our series (applying a `sample` function in R multiple times to each series, independently) and re-compute the Hurst exponent for them.

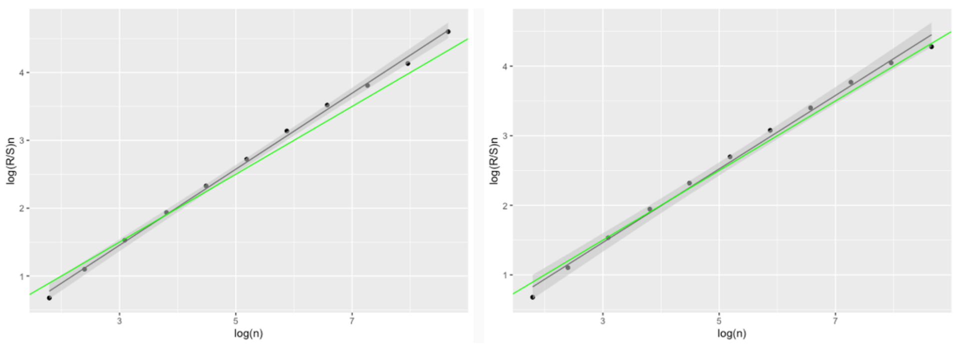

Figure 3. Graphical example of estimation of Hurst exponent (slope of the black line) and its 99% confidence interval (shadow region) for a long-term memory time series (**left**) and a random process (**right**). The green lines have slope of exactly 0.5.

6. Results and Discussion

First, let us point out that, in general, we work with much tighter significance levels (0.1% and 1%) than those normally found in finance literature, where typical results use levels of 5% and even 10%.

6.1. Test for Monofractality via the Mass Exponent

Our study takes, as a starting point, the outcomes by Bariviera in [14], who analyzed a sample of 84 CC and concluded that the largest coins (those in the first quartile of the traded volume) seem to move according to monofractal processes. Although we only work with the most capitalized CC, here we check by ourselves whether the claims in [14] are valid for our concrete time series.

Let us briefly recall that if a stochastic process follows a monofractal pattern, the scaling function or mass exponent $\tau(q) := q\,H(q) - 1$ as a function of an integer q should be close to a straight line, where $H(q)$ denotes the generalized Hurst exponent introduced by [65]. In turn, the presence of concavities/curvature in this function is a sign of multifractality.

Accordingly, we need to estimate $\tau(q)$ and perform a linear regression to test the plausibility of its linearity. With this goal, we use the MFDFA package developed by [66] in the R language. Our outcomes are totally in line with the aforementioned results from [14], as all the regressions we performed show that the function $\tau(q)$ admits a good approximation by a linear function of q, no matter the time frequency of picking the samples. As all our results are very similar, as an illustration, Figure 4 shows the linear plots for the BTC, which monopolises 45% of the total capitalization as compared with ADA, which accounts for less than 2% of the traded volume (see Table 2).

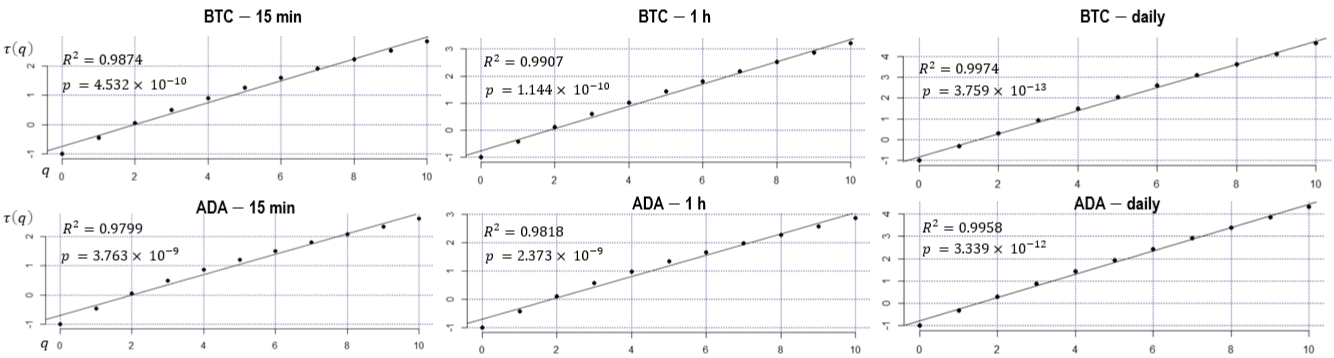

Figure 4. The function $\tau(q)$ versus q and the corresponding linear regression, showing the corresponding adjusted R^2 and p-value.

These upshots are also stable for the three different periods under consideration, as exemplified by Figure 5.

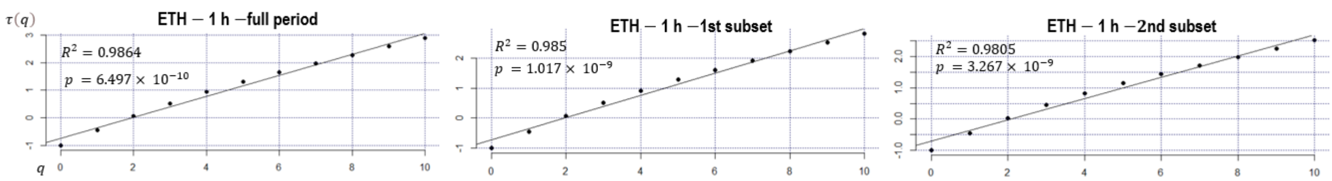

Figure 5. Linear regression for $\tau(q)$ across the different time periods under consideration for ETH.

6.2. Test for Stationarity as Prerequisite for R/S Analysis

Despite this step often being skipped, in order to be rigorous, one needs to check that the assumptions of a method are fulfilled before applying it. In this case, the assumption is that all time series are stationary. For the CC, the corresponding test ensures, at more than 99% confidence, that this is the case (see Table 6) at any scale in the full interval (these results are not altered for the subperiods).

Table 6. Results of the Augmented Dickey–Fuller (ADF) test for stationarity for cryptoassets with different frequencies for the period of 20 August 2020–24 February 2023. ** concludes that all series are stationary at less than 1% level of significance.

	15 min (Lag Order = 44)	1 h (Lag = 28)	Daily (Lag = 9)
ADA	−44.340 **	−29.644 **	−8.2618 **
BNB	−42.009 **	−29.384 **	−7.5603 **
BTC	−43.426 **	−28.566 **	−8.7709 **
DOT	−43.915 **	−29.196 **	−9.8369 **
ETH	−43.757 **	−28.549 **	−9.1639 **
MATIC	−43.159 **	−29.456 **	−9.3751 **
SOL	−44.326 **	−29.367 **	−8.3510 **
TRX	−43.360 **	−30.270 **	−9.0032 **
XRP	−43.736 **	−27.587 **	−8.9680 **

Concerning the remaining assets, stationarity is guaranteed at the same level of significance for daily data in all the three periods under study, as can be checked in Table 7.

Table 7. Results of the Augmented Dickey-Fuller (ADF) test for stationarity for traditional assets with daily values for the three different periods. ** concludes that all series are stationary at less than 1% level of significance.

	Full Period (Lag Order = 8)	1st Subset (Lag = 7)	2nd Subset (Lag = 5)
Gold	−8.7680 **	−8.7733 **	−5.2928 **
Silver	−8.8266 **	−8.1462 **	−5.6915 **
Nasdaq	−8.0335 **	−8.5723 **	−5.1938 **
Eurotoxx	−7.8613 **	−8.1981 **	−5.1350 **
Oil	−10.0500 **	−9.6601 **	−6.7996 **
US bonds	−8.1775 **	−7.6914 **	−4.9403 **
Wheat	−6.8168 **	−5.5172 **	−4.8443 **
AMZN	−8.1451 **	−8.2698 **	−5.1547 **
NFLX	−7.8880 **	−7.6475 **	−5.2018 **

Let us stress that, regardless of the stationary character of all of the return series, a quick visual inspection (see Figure 6) already provides us with an early warning about the different complexity levels in the graphs of CC versus traditional assets (even though the curves for CC are drawn with average daily values to smooth out irregularities). The tool to quantify this and grasp finer disparities within CC, which are not displayed at first sight by the graphs, is the fractal dimension or, equivalently, the Hurst exponent that we compute in the sequel.

6.3. Second Generation Stationarity Test to Account for Nonlinearities

Although the conclusions of the previous subsection pointed clearly towards the stationary character of all time series under consideration, we can still perform an additional double-check to assess whether the presence of non-linearities could alter the outcomes of conventional unit root tests, like ADF, which may have a limited power in distinguishing non-linear equilibrium adjustments.

To detect non-stationary features hidden behind non-linear effects, we apply the non-linear KSS unit root test designed by Kapetanios, Shin and Snell [67]. The outcomes ensure at more than 99.9% confidence that the series of CC under consideration are stationary (see Table 8) at any scale in the full interval (these results are similar for the subsamples).

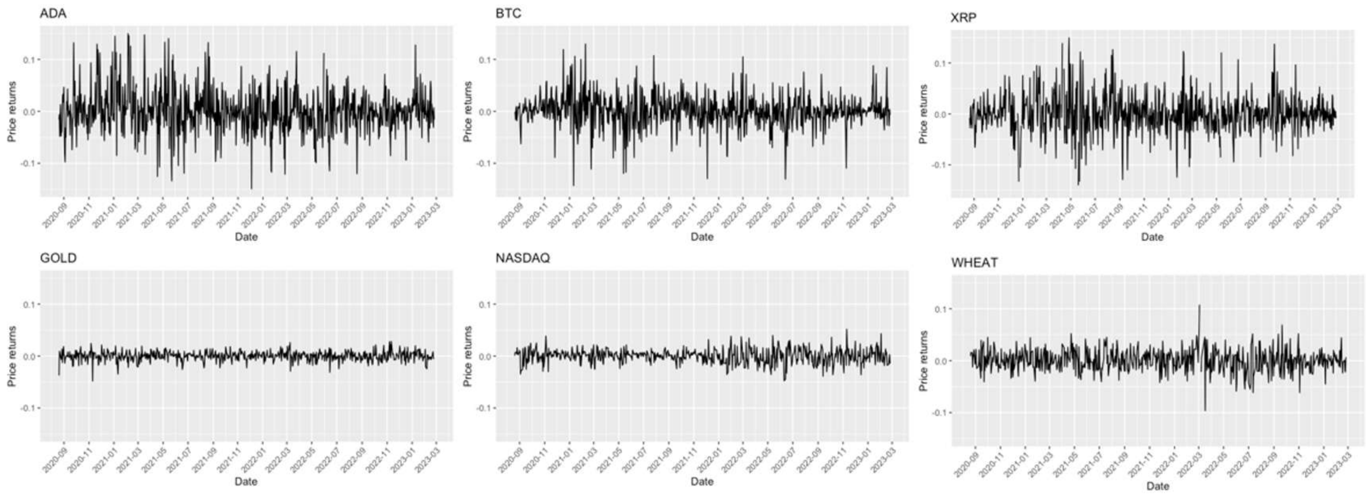

Figure 6. Comparison of daily return graphs for several assets during the whole period.

Table 8. Results of the KSS unit root test for cryptoassets with different frequencies for the period of 20 August 2020–24 February 2023. *** concludes that all series are stationary at <0.1% level of significance.

	15 min (Lag Order = 44)	**1 h (Lag = 28)**	**Daily (Lag = 9)**
ADA	−8.9580 ***	−13.2326 ***	−7.1024 ***
BNB	−6.2327 ***	−12.4460 ***	−7.5603 ***
BTC	−14.0094 ***	−3.5715 ***	−7.7599 ***
DOT	−17.9212 ***	−5.7037 ***	−3.9958 ***
ETH	−10.6192 ***	−10.3832 ***	−4.2671 ***
MATIC	−16.3656 ***	−9.7870 ***	−6.0347 ***
SOL	−4.8659 ***	−11.9307 ***	−3.8921 ***
TRX	−10.1991 ***	−11.8249 ***	−5.0051 ***
XRP	−20.3144 ***	−5.0958 ***	−5.0386 ***

The table above displays results for raw data, but similar outputs are obtained for detrended series; the AIC method was chosen to find the optimal lag. The outputs for traditional assets are also in the same line as those displayed for CC, and hence support the use of the R/S method, as no signs of non-stationarity are detected.

6.4. Memory and Efficiency in Cryptocurrencies and Traditional Assets

We start by analysing the Hurst index (see Table 9) of the time series for the full temporal frame with daily opening data for CC, commodities, stock indices and the US bonds. At a first glance, we conclude that all CC (except XRP) have long-term memory, and thus it would make sense to carry out a technical analysis to predict their future behaviour. As a corollary, CC do not fulfil the EMH of the statistical independence of prices.

The same is true for most of the remaining assets; however, our data do not provide enough evidence to discard the fact that silver or gold move randomly. Hence, they "forget" the past. Nasdaq and two of the stocks that compose it (AMZN and NFLX) also have aleatory patterns; in contrast, Eurostoxx has a long-term memory and, paradoxically, from an efficiency viewpoint it more closely resembles the BTC than the other index, which includes more disruptive companies.

Table 9. Hurst exponent between 20 August 2020 and 24 February 2023 with daily data. The error is estimated by displaying the p-value, as well as a 99% confidence interval. For the shuffled series, *** and ** mean that the null hypothesis of efficiency is rejected at $<0.1\%$ and 1% level of significance, respectively. No asterisk reveals that there is not enough evidence to support that the series does not follow a Gaussian path.

Name	N	Hurst	p-Value	99%-CI	H-Shuffled
BTC	919	0.64169	8.56×10^{-5}	(0.58542, 0.69796)	0.61997 ***
ETH	919	0.63337	2.28×10^{-5}	(0.59135, 0.67540)	0.60187 ***
BNB	919	0.65587	6.20×10^{-5}	(0.59739, 0.71435)	0.62742 ***
XRP	919	0.56646	0.01623	(0.49199, 0.64093)	0.55066
ADA	919	0.65739	7.57×10^{-4}	(0.56451, 0.75027)	0.55847 **
MATIC	919	0.68630	1.48×10^{-5}	(0.63179, 0.74082)	0.63666 ***
SOL	919	0.67028	5.53×10^{-4}	(0.57552, 0.76505)	0.65067 ***
TRX	919	0.59418	2.41×10^{-3}	(0.52457, 0.66378)	0.60926 **
DOT	919	0.63734	3.81×10^{-4}	(0.56599, 0.70869)	0.56258 **
Silver	653	0.50182	0.97286	(0.29685, 0.70678)	0.53399
Gold	652	0.54179	0.23721	(0.41627, 0.66731)	0.56801
Crude	633	0.59764	3.29×10^{-4}	(0.55248, 0.64279)	0.56763 **
Wheat	608	0.58424	5.67×10^{-3}	(0.51091, 0.65757)	0.58761 **
Bonds	633	0.54451	2.41×10^{-3}	(0.51276, 0.57625)	0.58484 **
Nasdaq	633	0.53580	0.32121	(0.40463, 0.66697)	0.55920
EuroSt.	650	0.58617	3.96×10^{-4}	(0.54471, 0.62762)	0.59152 ***
AMZN	633	0.45961	0.28361	(0.32397, 0.59524)	0.51260
NFLX	633	0.62543	0.03299	(0.45225, 0.79861)	0.53410

Notice that the aleatory behaviour of XRP already provides a hint that the consensus methods may influence the efficiency of CC. Indeed, XRP cannot be regarded as decentralised, since most of its nodes are controlled by a single company. This is because its aim is not to replace banks as intermediaries; it is to improve their effectiveness, as a substitute for the banking SWIFT.

Next, to discern whether TRX and BNB, of which neither are fully decentralised, also display random patterns, we will perform a more refined fractal analysis, considering different time periods and/or modifying the time frequencies. Actually, when varying the latter and taking opening values every 15 min, 1 h or day, the outcomes in Table 10 reveal that BTC's movements are non-efficient at any scale, which also exhibits long-term memory. In contrast, XRP and TRX operate mostly in an efficient way.

But, to observe traits of aleatoriness in BNB, we still need to reduce the time arc (or zoom in) to the second subperiod (see Table 11). As before, although using daily splits TRX exhibits long-term memory, if we take high frequency data, it behaves following a random motion, which is an early clue of the lack of self-similarity.

Regarding the other six CC, whose algorithms are completely decentralised, they show persistent memory in the full period (Table A2), and in both sub-intervals (Tables A3 and A4). Moreover, this remains invariant at any time scale; thus, we can claim that they have a consistent inefficient behaviour.

Table 10. Hurst exponent: BTC, XRP, TRX and BNB for the full period but different time frequencies: 15 min (N = 88,060 observations), 1 h (N = 22,019) and daily (N = 919). For the shuffled series, *** and ** mean that the hypothesis of efficiency is rejected at <0.1% and 1% level of significance, respectively. No asterisk says that there is not enough evidence to ensure that the data is non-random.

	Hurst	p-Value	99%-CI	H-Shuffled
BTC				
15 min	0.53736	1.12×10^{-4}	(0.51707, 0.55766)	0.55294 ***
1 h	0.55226	9.92×10^{-5}	(0.52562, 0.57889)	0.55334 ***
Daily	0.64169	8.56×10^{-5}	(0.58542, 0.69796)	0.61997 ***
XRP				
15 min	0.51401	0.06970	(0.49252, 0.53550)	0.52702
1 h	0.52551	0.01164	(0.49926, 0.55177)	0.51974
Daily	0.56646	0.01623	(0.49199, 0.64093)	0.55066
TRX				
15 min	0.51740	0.04995	(0.49301, 0.54179)	0.52523
1 h	0.52192	0.04746	(0.49117, 0.55268)	0.52843
Daily	0.59418	2.41×10^{-3}	(0.52457, 0.66378)	0.60926 **
BNB				
15 min	0.55127	1.11×10^{-6}	(0.53386, 0.56868)	0.54502 ***
1 h	0.56468	1.30×10^{-5}	(0.53876, 0.59061)	0.54211 ***
Daily	0.65587	6.20×10^{-5}	(0.59739, 0.71435)	0.62742 ***

Table 11. Hurst exponent: XRP, TRX and BNB for the second subperiod (1 July 2022–24 February 2023) and different time frequencies: 15 min (N = 22,869 observations), 1 h (N = 5718) and daily (N = 239).

	Hurst	p-Value	99%-CI
XRP			
15 min	0.51450	0.14754	(0.48522, 0.54377)
1 hora	0.51419	0.28596	(0.47254, 0.55585)
Daily	0.56816	0.13156	(0.40223, 0.73408)
TRX			
15 min	0.51719	0.07521	(0.48975, 0.54463)
1 h	0.52535	0.04041	(0.49053, 0.56017)
Daily	0.62667	1.35×10^{-3}	(0.55335, 0.69999)
BNB			
15 min	0.53628	1.04×10^{-3}	(0.51107, 0.56149)
1 h	0.53901	9.70×10^{-3}	(0.50823, 0.57778)
Daily	0.62968	0.04020	(0.43023, 0.82913)

6.5. Fractal Features: Does the Graph Change When Zooming in?

Next, we inspect whether the return graphs of BTC and other assets exhibit self-similarity features, that is, whether they look the same on different time scales. For this to hold, the efficiency character must not vary at different subintervals or if we change the frequency to take the data.

Unlike BTC, which moves persistently at any frequency (cf. Table 10), silver and Eurostoxx (Table 12) change the pattern as we reduce the partition. Specifically, for daily data, the silver moves randomly, but it is mean-reverting for high frequencies. Due to this shift, we deduce that it does not behave as a fractal.

Moreover, one can also observe that traditional assets display a change in the memory character for different periods (see Table 13). Therefore, none of them can be regarded as a fractal object. Let us further mention that confidence intervals are not displayed for the simplicity of the table, as p-values over 0.01 already denote that the result is not significative enough to reject the null hypothesis of randomness. As an illustration of this, the 99%-CI of gold in the second subperiod turns out to be $(0.42126, 0.91643)$, meaning that any value in that wide range (including 0.5) has the same probabilities as the slope of the linear regression used to compute H. Again, as 0.5 belongs to this interval, despite the high $H = 0.67$, our data do not provide enough evidence to reject the hypothesis of random dynamics.

Table 12. H index: silver and Eurostoxx between 20 August 2020 and 24 February 2023 and different frequencies.

	N	Hurst	p-Value	99%-CI
Silver				
15 min.	60,320	0.42234	3.82×10^{-5}	(0.38585, 0.45883)
1 h	15,463	0.42702	4.03×10^{-4}	(0.38251, 0.47153)
Daily	653	0.50182	0.97286	(0.29685, 0.70678)
Eurostoxx				
15 min.	23,376	0.45768	1.43×10^{-3}	(0.42730, 0.48807)
1 h	5849	0.48318	0.06186	(0.45753, 0.50883)
Daily	650	0.58617	3.96×10^{-4}	(0.54472, 0.62763)

Table 13. Hurst exponent: traditional assets for different time periods and daily frequency. With colors, we mark random and long-term memory behaviour.

	20 August 2020–24 February 2023		20 August 2020–1 July 2022		1 July 2022–24 February 2023	
	Hurst	p-Value	Hurst	p-Value	Hurst	p-Value
Silver	0.5018	0.97286	0.5746	0.04957	0.6340	2.81×10^{-3}
Gold	0.5418	0.23721	0.5880	3.82×10^{-3}	0.6688	0.02832
Crude	0.5976	3.29×10^{-4}	0.5561	0.03144	0.4498	0.38206
Wheat	0.5842	5.67×10^{-3}	0.5861	5.36×10^{-3}	0.5647	0.23887
US bonds	0.5445	2.41×10^{-3}	0.5631	0.02122	0.6406	3.43×10^{-3}
Nasdaq	0.5358	0.32121	0.5835	0.03252	0.6475	0.01860
Eurostoxx	0.5862	3.96×10^{-4}	0.6135	1.19×10^{-3}	0.6768	0.01810

In contrast, the scale-invariance of BTC is shared by the five CC whose consensus protocols are fully decentralised (see Figure 7). This holds both in the complete study period and in the two sub-intervals considered, thus confirming the fractal nature of CC that operates peer-to-peer. Furthermore, this corroborates our guess that the underlying consensus protocol is the key to determine the efficiency (instead of the capitalization, as claimed in previous literature, since BNB and XRP are third and fourth in volume, respectively; recall Table 2).

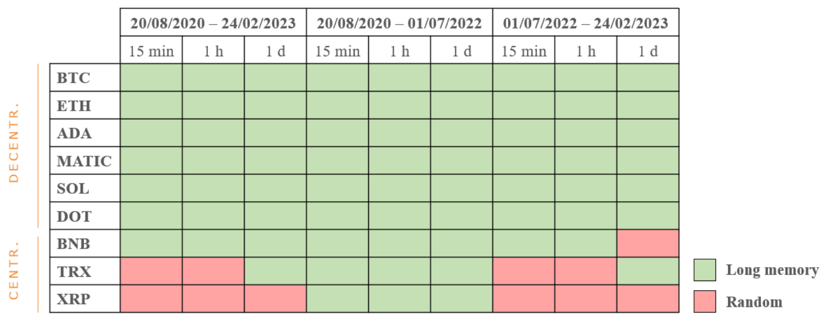

Figure 7. Illustration of the self-similarity of CC for different time frames and frequencies (made with the outcomes from Tables A2–A4). Coloured to show 0.1% significance.

Additionally, we analysed how the BTC's Hurst values vary as a function of time from March 2018 to February 2023 with daily opening data. To address this question and calculate $H(t)$, a 150-value rolling window was used. The outcomes (see Figure 8) show that BTC presents long-term memory most of the time, with H values between 0.55 and 0.75, except at specific moments such as November 2019, when sharp oscillations within the anti-persistence zone occur.

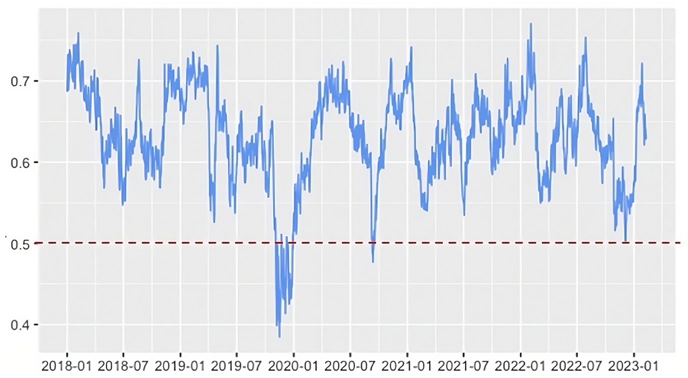

Figure 8. Evolution of the Hurst exponent of BTC over time. Blue line indicates the Hurst values estimated, and the dashed line is the randomness barrier that separates the areas of long memory (over) and anti-persistance (below).

Notice that in the last trimester of 2022, the dynamics seem quite random, which could be influenced by the macroeconomomic situation tensioned by high inflation and rising interest rates.

6.6. A Striking Turnabout: Efficiency May Happen (under a Change of Variable)

We were puzzled by the claim in [6] that if, instead of working directly with the BTC returns defined via the (1), we take the values R_t^{17}, then the resulting series becomes more efficient, except for the fact that their tests do not provide enough evidence to reject independence. As the authors restrict to BTC and do not compute the Hurst exponent, we decided to update their study and broaden it to include the variety of CC and traditional assets we are dealing with.

A quick glance at Figure 9 compared to Figure 7 already reveals that this simple transformation distorts the conclusions about efficiency, and unlike [6], we also learn that, for most time frames and scales, the new values behave independently. Furthermore, if we relax the significance requirements to 1%, most of the table will be coloured randomly.

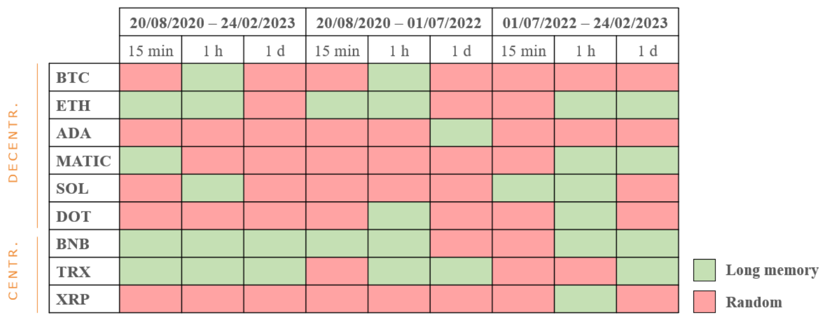

Figure 9. Illustration of the self-similarity of CC for different time frames and frequencies (with the values of the return series raised to 17). Coloured to show 0.1% significance.

Nevertheless, our outcomes also reveal that the reply to the change of variable and shift to higher efficiency is not consistent, as TRX moves oppositely towards the long-term memory character. Additionally, we lose information, since the new outcomes do not evidence any coherent common trend or difference according to neither technology nor liquidity.

Interestingly, traditional assets are more resilient to changes in the efficiency character in a coherent way (compare Tables 13 and 14), which provides further support from an alternative perspective to our conclusion about the different nature of CC versus traditional assets.

Table 14. Hurst exponent: traditional assets for various periods and daily frequency (with return series raised to 17). Colors mark random and long-term memory behaviour.

	20 August 2020–24 February 2023		20 August 2020–1 July 2022		1 July 2022–24 February 2023	
	Hurst	***p*-Value**	**Hurst**	***p*-Value**	**Hurst**	***p*-Value**
Silver	0.5213	0.0627	0.5255	0.0434	0.5654	0.0264
Gold	0.5291	0.0971	0.5273	0.0205	0.6534	2.68×10^{-3}
Crude	0.4845	0.345	0.4847	0.4560	0.6208	8.55×10^{-5}
Wheat	0.5241	3.76×10^{-3}	0.5329	7.32×10^{-3}	0.5542	0.0205
US bonds	0.5600	7.94×10^{-4}	0.5913	0.0140	0.5586	0.0709
Nasdaq	0.5203	0.2530	0.5814	9.32×10^{-5}	0.5603	0.0102
Eurostoxx	0.5928	3.86×10^{-4}	0.5977	4.49×10^{-4}	0.5571	0.0154

For the purposes of the present paper, we leave this subsection as a matter of curiosity that deserves further and closer inspection, as we disagree with the premature statement in [6] that any odd power will produce similar results. Indeed, very recent research [68] proves that for a return series with $R_t > 0$ for all t, the fractional dimension keeps the invariant under the change R_t^q for every q.

It is actually an interesting future research line to find an optimal change of variable that turns any time series into a random Brownian motion, as this will enable us to use the plethora of statistical techniques based on the normal distribution, and drive conclusions after undoing the transformation. However, despite some numerical experiments being helpful, a rigorous approach to this problem should wait for extra advances in fractal geometry, as it is an open problem to find a concrete formula of how the dimension would vary.

6.7. Interdependence of Bitcoin and Other Assets

To determine whether a diversified portfolio can be built with CC alone or whether it should be combined with other types of products, we obtain the Pearson correlations between the different assets in the whole period and in both sub-intervals. Let us point out that all time stamps have been converted to Central European Time (CET) to facilitate the direct comparison of values from different assets. In view of the outcomes (see Table 15), the second option can occur, as BTC has very strong positive correlations with all CC except MATIC.

We also confirm our previous guess that growth stocks (TSLA, NFLX and AMZN, and hence the Nasdaq index) have similar dynamics as BTC. However, Bitcoin has a weak linear relation with value stocks, hence the mix of these with BTC has no diversification power, and it is a clever strategy to combine Bitcoin with, e.g., NFLX, with whom strong negative correlations pop up (see Table A8). Moreover, we conclude that BTC is far from acting like the more conservative safe haven products for investors, as no correlation with them arose.

If we now restrict to the interrelations within the cryptosphere, all CC are very tightly correlated as expected (see Tables A5 and A6 for the full matrices of the full period and the first subinterval, respectively). Nevertheless, it is quite surprising that, if we focus on the last subperiod, the completely unprecedented event of negative correlations between them emerges (cf. Table A7).

If we move from the whole time frame to the earlier subset, most of the values match up to the first decimal, except some shifts in the interdependences of gold and silver with growth stocks (see Tables A8 and A9). But, if we look at the last subinterval, unexpectedly the landscape changes quite radically. Certainly, BTC loses correlation strength with growth stocks (cf. Table A10); in parallel, slight negative correlations start to emerge with assets such as JNJ or silver.

Table 15. Correlation of daily returns (for weekdays) between BTC and other assets: period from 20 August 2020 to 24 February 2023, sub-intervals from 20 August 2020 to 1 July 2022 and from 1 July 2022 to 24 February 2023. *** (**, *) denotes significance at 0.1% (1%, 5%) significance level.

	Full Period	1st Subperiod	2nd Subperiod
BNB	0.7318 ***	0.8217 ***	0.5129 ***
TRX	0.7089 ***	0.7785 ***	0.9059 ***
SOL	0.6606 ***	0.6259 ***	0.6932 ***
MATIC	0.5037 ***	0.5779 ***	0.4791 ***
ETH	0.8422 ***	0.8342 ***	0.8396 ***
DOT	0.9527 ***	0.9380 ***	0.8011 ***
ADA	0.8275 ***	0.7851 ***	0.7125 ***
XRP	0.8050 ***	0.7656 ***	0.0441
TSLA	0.6240 ***	0.6812 ***	0.4635 ***
NFLX	0.5564 ***	0.3544 ***	−0.006
AMZN	0.5540 ***	0.2980 ***	0.5396 ***
KO	−0.1100 **	0.2099 ***	0.0053
PG	0.1017 *	0.0924 *	−0.1615 *
JNJ	0.2074 ***	0.4573 ***	−0.5148 ***
Crude	0.1331 ***	0.3493 ***	0.3249 ***
Wheat	0.1005 *	0.2084 ***	−0.0784
US bonds	0.2886 ***	−0.2597 ***	0.4490 ***
Silver	0.3457 ***	0.012	−0.1935 *
Gold	−0.1281 **	−0.5741 ***	0.2076 **
Nasdaq	0.8458 ***	0.7984 ***	0.6902 ***
EuroStoxx	0.6749 ***	0.8005 ***	0.1553

7. Conclusions

Our main goal was to decide whether the evolution of BTC prices is random (as assumed by the EMH) or follows chaotic (but more predictable) patterns, opening the door to forecasting opportunities (see, e.g., new prediction models in [69]). Via the Hurst values (H), we confirm that the BTC dynamics are not independent, but have a long-term memory, even if we change the time interval or the frequency of data collection. The latter indicates that the graphs of BTC returns are self similar, which is an essential feature to confirm their nature as fractals.

On the contrary, when changing the scale, we obtain efficiency shifts for traditional assets, as well as the CC whose consensus protocols have centralised features (BNB, TRX and XRP). To our knowledge, we pioneer in stressing that it is the underlying technology (intead of the liquidity) that the key to determine differences between the performance of CC within the market.

As a corollary, the fractality of BTC, characterised by chaotic events occurring in waves rather than isolatedly, contradicts every conservative investor's desire for it to be a store of value. This invalidates that BTC could be considered a safe haven, while not holding the narrative that it is digital gold.

From our correlational study, BTC generally exhibits the same trends as other CC and growth stocks. In practice, this implies that it is not possible to build a risk-controlled portfolio with these types of assets alone. At the same time, there is no tight correlation of BTC with conservative products; consequently, it is also very difficult to exploit BTC as a diversification tool.

Complementarily, if we focus on the later subperiod, the correlations of all assets become disrupted. It will be interesting for future work to analyse whether this paradigm shift is an isolated event or is sustained over time. A plausible justification within the actual context could come from the influence of a sharp rise in interest rates, combined with a global economy strained by inflation.

To recap, we detect a high degree of persistence in BTC prices. This may be due to the lack of investor confidence in this market, as it is unstructured and lacks oversight by the authorities. With more legislative certainty, participation will increase, which may lead to a rise in efficiency (or decrease in predictability) that would stabilise the *cryptoeconomy*.

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Data Availability Statement: Data will be available upon request.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A. Table Providing a Literature Overview

Table A1. Overview of literature about efficiency/persistence.

Ref.	Coin	Period	Method	Conclusions
[4]	BTC	1 August 2010–31 July 2016	standard tests, R/S statistic	strong anti-persistance, but moving to efficiency in 2nd half
[39]	BTC	1 May 2012–30 April 2017	regression, clustering kurtosis test	price clustering at round numbers, no significant pattern otherwise
[37]	BTC, LTC, XRP	2013–2017	R/S analysis and fractional integration	decreasing trend in inefficiency
[6]	BTC	1August 2010–31 July 2016	8 different tests (no Hurst)	after raise to 17th-power, returns are efficient
[38]	75 CC	31 August 2015–31 August 2018	3 delay measures	average price delay decreases during the last 3 years
[5]	BTC	18 August 2011–15 February 2017	dynamic Hurst , detrended fluctuation analysis (DFA)	long term memory, becoming more efficient over time. Volatility is persistent for the whole period.
[44]	BTC, ETH, intraday	1 June 2013–23 June 2018 (from 1 June 2016, ETH)	generalized Hurst, asymmetric multifractal DFA (A-MF-DFA)	ETH more efficient, inefficiency more accentuated when the market moves downwards
[34]	BTC	30 June 2013–30 June 2017	DFA over sliding windows	periods of efficiency alternating with anti-persistence (at day, hour and second frequencies)
[40]	BTC-USD, BTC-CYN	18 July 2010–31 July 2017, 1 February 2014–31 July 2017	Efficiency Index	inefficient, apart from periods of "cooling off" after price surges
[42]	73 CC	31 August 2015–30 November 2017	different variance ratio tests, R/S statistic	CC become less predictable / inefficient as liquidity increases.
[36]	224 CC	varying start- until 31 July 2017	Ljung–Box (L–B) Q-test for autocorrelation,and normality tests	all show long memory, leverage, stochastic volatility and heavy tailedness
[8]	BTC	18 July 2010 - 16 June 2017	"various long-range dependence estimators"	efficient with some exception to the period of April–August 2013 and August–November 2016.
[13]	BTC, high frequency	1 April 2017–30 November 2017	Wavelet transform modulus maxima, MF-DFA	anti-persistent, being the main drivers high kurtosis and fat tails
[9]	BTC, USD, gold, MSCI	18 July 2010–31 September 2017	MF-DFA	long-memory in all four markets: BTC is the most inefficient and MSCI intex the least one
[12]	BTC	1 December 2110–30 November 2017	generalized Hurst	long-memory, does not become efficient over time
[18]	BTC, ETH, LTC, XRP, XMR, ETC	July 2016–March 2019	Auto-Regressive Integrated Moving Average (ARIMA), fractional (ARFIMA)	except for BTC, the other CC are mean reverting, showing a lower predictability.
[11]	31 CC	17 August 2017–16 January 2019	panel unit root tests	cross-sectional dependence among the CC. Confirms inefficiency in highly-traded CC
[45]	BTC, LTC, ETH, XRP	29 April 2013–1 February 2018 8 August 2015–2 February 2018	ICSS algorithms tests	inefficiency of all the considered markets, with the exception of ETH
[29]	7 biggest size CC	1 January 2019–20 January 2021	fractal connectivity matrix, ARFIMA	using high-frequency returns, the null hypothesis of long memory is rejected for all series
[7]	BTC-USD, BTC-EUR	1 January 2013–5 March 2018	permutation entropy	BTC-USD market is slightly more efficient . Higher the frequency, lower the efficiency.
[49]	8 major CC	25 August 2015–13 March 2018	Log-periodogram, ARFIMA-FIGARCH models	market (in)efficiency and the intensity of volatility persistence is sensitive to time-scales
[10]	BTC	28 July 2013–14 May 2023	R/S analysis (weekly data)	persistent, character does not change over time
[47]	BTC, gold, S&P 500, USD	2 January 2018–29 May 2020	"entropy via RCMFE method, efficiency index"	COVID-19 leads to efficiency decreases in all four markets. Resiliency of BTC market efficiency and disjoint change with other markets

Appendix B. Complete Tables with Hurst Exponents

Table A2. Hurst exponent: cryptocurrencies between 20 August 2020 and 24 February 2023.

	Hurst	*p*-Value		Hurst	*p*-Value		Hurst	*p*-Value
BTC			XRP			SOL		
15 min	0.5374	1.12×10^{-4}	15 min	0.5140	0.06970	15 min	0.5480	1.45×10^{-4}
1 h	0.5523	9.92×10^{-5}	1 h	0.5219	0.04746	1 h	0.5632	2.39×10^{-4}
Daily	0.6417	8.56×10^{-5}	Daily	0.5942	2.41×10^{-3}	Daily	0.6703	5.53×10^{-4}
ETH			ADA			TRX		
15 min	0.5375	2.87×10^{-6}	15 min	0.5356	1.52×10^{-3}	15 min	0.5174	0.04995
1 h	0.5525	5.99×10^{-7}	1 h	0.5540	8.30×10^{-4}	1 h	0.5219	0.04746
Daily	0.6334	2.28×10^{-5}	Daily	0.6574	7.57×10^{-4}	Daily	0.5942	2.41×10^{-3}
BNB			MATIC			DOT		
15 min	0.5513	1.11×10^{-6}	15 min	0.5539	3.77×10^{-5}	15 min	0.5276	8.51×10^{-4}
1 h	0.5647	1.30×10^{-5}	1 h	0.5767	3.56×10^{-5}	1 h	0.5413	5.00×10^{-4}
Daily	0.6559	6.20×10^{-5}	Daily	0.6863	1.48×10^{-5}	Daily	0.6373	3.81×10^{-4}

Table A3. Hurst exponent: cryptocurrencies between 20 August 2020 and 1 July 2022.

	Hurst	*p*-Value		Hurst	*p*-Value		Hurst	*p*-Value
BTC			XRP			SOL		
15 min	0.5462	1.62×10^{-6}	15 min	0.5271	5.88×10^{-4}	15 min	0.5627	5.13×10^{-6}
1 h	0.5651	6.70×10^{-7}	1 h	0.5446	3.75×10^{-5}	1 h	0.5819	9.38×10^{-6}
Daily	0.6670	1.20×10^{-5}	Daily	0.5820	3.52×10^{-3}	Daily	0.7275	5.69×10^{-5}
ETH			ADA			TRX		
15 min	0.5450	1.72×10^{-5}	15 min	0.5411	2.04×10^{-4}	15 min	0.5310	6.40×10^{-4}
1 h	0.5637	9.66×10^{-6}	1 h	0.5630	8.95×10^{-5}	1 h	0.5384	8.33×10^{-4}
Daily	0.6685	1.43×10^{-4}	Daily	0.6733	8.42×10^{-4}	Daily	0.5949	1.42×10^{-3}
BNB			MATIC			DOT		
15 min	0.5569	1.20×10^{-6}	15 min	0.5505	6.43×10^{-4}	15 min	0.5352	6.60×10^{-6}
1 h	0.5736	5.70×10^{-6}	1 h	0.5752	4.11×10^{-4}	1 h	0.5528	2.58×10^{-6}
Daily	0.6622	4.05×10^{-4}	Daily	0.6918	2.85×10^{-3}	Daily	0.6398	3.11×10^{-4}

Table A4. Hurst exponent: cryptocurrencies between 1 July 2022 and 24 February 2023.

	Hurst	*p*-Value		Hurst	*p*-Value		Hurst	*p*-Value
BTC			XRP			SOL		
15 min	0.5406	1.36×10^{-4}	15 min	0.5145	0.14754	15 min	0.5604	1.03×10^{-7}
1 h	0.5508	2.39×10^{-4}	1 h	0.5141	0.28596	1 h	0.5854	2.83×10^{-7}
Daily	0.6227	4.60×10^{-3}	Daily	0.5682	0.13156	Daily	0.6767	1.85×10^{-4}
ETH			ADA			TRX		
15 min	0.5491	1.41×10^{-7}	15 min	0.5376	1.46×10^{-5}	15 min	0.5172	0.07521
1 h	0.5641	6.68×10^{-7}	1 h	0.5531	1.56×10^{-5}	1 h	0.5254	0.04041
Daily	0.6645	9.69×10^{-4}	Daily	0.6832	3.98×10^{-5}	Daily	0.6267	1.35×10^{-3}
BNB			MATIC			DOT		
15 min	0.5363	1.04×10^{-3}	15 min	0.5377	4.22×10^{-4}	15 min	0.5322	1.54×10^{-3}
1 h	0.5390	9.70×10^{-3}	1 h	0.5506	5.50×10^{-4}	1 h	0.5463	2.41×10^{-3}
Daily	0.6297	0.04020	Daily	0.6468	2.50×10^{-3}	Daily	0.6829	2.74×10^{-3}

Appendix C. Correlation Matrices for Cryptoassets and BTC versus Traditional Assets

Hereafter *** (**, *) denotes significance at 0.1% (1%, 5%) significance level.

Table A5. Correlation between cryptoassets between 20 August 2020 & 24 February 2023.

	BTC	BNB	TRX	SOL	MATIC	ETH	DOT	ADA	XRP
BTC	1								
BNB	0.7318 ***	1							
TRX	0.7089 ***	0.8866 ***	1						
SOL	0.6606 ***	0.7489 ***	0.5968 ***	1					
MATIC	0.5037 ***	0.8222 ***	0.5711 ***	0.7780 ***	1				
ETH	0.8422 ***	0.9057 ***	0.7742 ***	0.8825 ***	0.8378 ***	1			
DOT	0.9527 ***	0.7042 ***	0.7252 ***	0.6598 ***	0.4609 ***	0.8008 ***	1		
ADA	0.8275 ***	0.7456 ***	0.7379 ***	0.7017 ***	0.6312 ***	0.8524 ***	0.8423 ***	1	
XRP	0.8050 ***	0.7997 ***	0.8387 ***	0.6221 ***	0.5788 ***	0.8152 ***	0.8233 ***	0.8548 ***	1

Table A6. Correlation between CC for the first subperiod between 20 August 2020 and 1 July 2022.

	BTC	BNB	TRX	SOL	MATIC	ETH	DOT	ADA	XRP
BTC	1								
BNB	0.8217 ***	1							
TRX	0.7785 ***	0.8917 ***	1						
SOL	0.6259 ***	0.7686 ***	0.5974 ***	1					
MATIC	0.5779 ***	0.8250 ***	0.5768 ***	0.8175 ***	1				
ETH	0.8342 ***	0.9392 ***	0.7925 ***	0.8741 ***	0.8827 ***	1			
DOT	0.9380 ***	0.7820 ***	0.7933 ***	0.6235 ***	0.5258 ***	0.7821 ***	1		
ADA	0.7851 ***	0.7998 ***	0.7793 ***	0.6684 ***	0.6965 ***	0.8371 ***	0.8029 ***	1	
XRP	0.7656 ***	0.8538 ***	0.8927 ***	0.5820 ***	0.6313 ***	0.7979 ***	0.7868 ***	0.8290 ***	1

Table A7. Correlation between CC for the second subperiod between 1 July 2022 and 24 February 2023.

	BTC	BNB	TRX	SOL	MATIC	ETH	DOT	ADA	XRP
BTC	1								
BNB	0.5929 ***	1							
TRX	0.9059 ***	0.2921 ***	1						
SOL	0.6932 ***	0.1368	0.8407 ***	1					
MATIC	0.4791 ***	0.7286 ***	0.2211 **	−0.1586 *	1				
ETH	0.8396 ***	0.7048 ***	0.6931 ***	0.5532 ***	0.5588 ***	1			
DOT	0.8011 ***	0.3992 ***	0.8477 ***	0.8809 ***	0.1309	0.7540 ***	1		
ADA	0.7125 ***	0.2197 ***	0.8101 ***	0.9460 ***	−0.0652	0.6323 ***	0.9413 ***	1	
XRP	0.044 ***	0.3717 ***	−0.002 ***	0.090 ***	0.1914 *	0.0062	−0.0515	−0.0496	1

Table A8. Correlation between CC and traditional assets (20 August 2020–24 February 2023).

	BTC	TSLA	NFLX	AMZN	KO	PG	JNJ	OIL	WHEAT	ZNM23	SILVER	GOLD	CCMP	SX5E
BTC	1													
TSLA	0.6240 ***	1												
NFLX	0.5564 ***	0.1255 **	1											
AMZN	0.5540 ***	0.3264 ***	0.8124 ***	1										
KO	−0.1100 **	0.2883 ***	−0.7136 ***	−0.5929 ***	1									
PG	0.1017 *	0.4000 ***	−0.1337 ***	−0.0511	0.6250 ***	1								
JNJ	0.2074 ***	0.3453 ***	−0.4702 ***	−0.3805 ***	0.7655 ***	0.4230 ***	1							
OIL	0.1331 ***	0.4739 ***	−0.6360 ***	−0.4502 ***	0.8346 ***	0.4178 ***	0.7680 ***	1						
WHEAT	0.1005 *	0.4453 ***	−0.5995 ***	−0.3983 ***	0.7225 ***	0.4370 ***	0.6566 ***	0.8853 ***	1					
ZNM23	0.2886 ***	−0.0723	0.8077 ***	0.8527 ***	−0.7906 ***	−0.1962 ***	−0.6048 ***	−0.7307 ***	−0.6281 ***	1				
SILVER	0.3457 ***	−0.1752 ***	0.6281 ***	0.5899 ***	−0.5643 ***	−0.1753 ***	−0.2711 ***	−0.5135 ***	−0.4070 ***	0.7052 ***	1			
GOLD	−0.1281 **	−0.1695 ***	0.1380 ***	0.1976 ***	−0.0755	0.3312 ***	−0.1653 ***	−0.1631 ***	0.018	0.3385 ***	0.5770 ***	1		
CCMP	0.8458 ***	0.6579 ***	0.7100 ***	0.7734 ***	−0.1598 ***	0.2298 ***	0.1021 *	−0.0114	−0.068	0.4794 ***	0.3704 ***	0.0016	1	
SX5E	0.6749 ***	0.4388 ***	0.3532 ***	0.1908 ***	0.2798 ***	0.3701 ***	0.4401 ***	0.2734 ***	0.1016 *	−0.0676	0.1415 ***	−0.075	0.7065 ***	1

Table A9. Correlation between CC and traditional assets (20 August 2020–1 July 2022).

	BTC	TSLA	NFLX	AMZN	KO	PG	JNJ	OIL	WHEAT	ZNM23	SILVER	GOLD	CCMP	SX5E
BTC	1													
TSLA	0.6812 ***	1												
NFLX	0.3544 ***	0.0853	1											
AMZN	0.2980 ***	0.0984 *	0.8644 ***	1										
KO	0.2099 **	0.5322 ***	−0.6331 ***	−0.4738 ***	1									
PG	0.0924 *	0.5644 ***	−0.2808 ***	−0.1528 **	0.7504 ***	1								
JNJ	0.4573 ***	0.5669 ***	−0.4379 ***	−0.2835 ***	0.7824 ***	0.4341 ***	1							
OIL	0.3493 ***	0.5853 ***	−0.6032 ***	−0.5135 ***	0.8831 ***	0.5302 ***	0.8046 ***	1						
WHEAT	0.2084 ***	0.5141 ***	−0.6787 ***	−0.5854 ***	0.8468 ***	0.5623 ***	0.7137 ***	0.9100 ***	1					
ZNM23	−0.2597 ***	−0.5172 ***	0.7155 ***	0.6348 ***	−0.8814 ***	−0.5346 ***	−0.7664 ***	−0.9465 ***	−0.8981 ***	1				
SILVER	0.012	−0.2938 ***	0.3056 ***	0.4494 ***	−0.4883 ***	−0.5377 ***	−0.2296 ***	−0.4530 ***	−0.4421 ***	0.5016 ***	1			
GOLD	−0.5741 ***	−0.1756 ***	−0.3667 ***	−0.1677 ***	−0.1346 **	0.2643 ***	−0.1043 *	0.008	0.2110 ***	−0.033	0.2536 ***	1		
CCMP	0.7984 ***	0.7068 ***	0.5825 ***	0.6223 ***	0.1693 ***	0.2419 ***	0.3897 ***	0.2023 ***	0.0303	−0.041	−0.007	−0.4723	1	
SX5E	0.8005 ***	0.6782 ***	0.3206 ***	0.3571 ***	0.4124 ***	0.3419 ***	0.5725 ***	0.4249 ***	0.1995 ***	−0.2916 ***	−0.1215 **	−0.5028 ***	0.9041 ***	1

Table A10. Correlation between CC and traditional assets (1 July 2022–24 February 2023).

	BTC	TSLA	NFLX	AMZN	KO	PG	JNJ	OIL	WHEAT	ZNM23	SILVER	GOLD	CCMP	SX5E
BTC	1													
TSLA	0.4625 ***	1												
NFLX	−0.0057	−0.6776 ***	1											
AMZN	0.5396 ***	0.9061 ***	−0.6243 ***	1										
KO	0.0053	0.0187	−0.0855	0.0868	1									
PG	−0.1615 *	−0.3325 ***	0.1678 *	−0.2334 **	0.8862 ***	1								
JNJ	−0.5115 ***	−0.3869 ***	−0.0964	−0.3739 ***	0.5586 ***	0.6238 ***	1							
OIL	0.3249 ***	0.5943 ***	−0.7818 ***	0.5898 ***	0.1343	−0.1611 *	0.0887	1						
WHEAT	−0.0784	0.4506 ***	−0.5450 ***	0.3125 ***	−0.6074 ***	−0.7372 ***	−0.2562 **	0.4156 ***	1					
ZNM23	0.4490 ***	0.4700 ***	−0.4555 ***	0.5972 ***	0.6617 ***	0.4577 ***	0.1813 *	0.5487 ***	−0.2809 ***	1				
SILVER	−0.1935 *	−0.8158 ***	0.7776 ***	−0.6900 ***	0.2269 **	0.5357 ***	−0.2835 ***	−0.6520 ***	−0.6288 ***	−0.1823 *	1			
GOLD	0.20761 **	−0.5669 ***	0.6723 ***	−0.3722 ***	0.3678 ***	0.5710 ***	0.0768	−0.4172 ***	−0.7638 ***	0.2084 **	0.8354 ***	1		
CCMP	0.6902 ***	0.6555 ***	−0.1618 *	0.7480 ***	0.4621 ***	0.2024 *	−0.2785 ***	0.3319 ***	−0.2431 **	0.7113 ***	−0.2581 **	0.1508	1	
SX5E	0.1553	−0.5850 ***	0.8704 ***	−0.5044 ***	0.2686 ***	0.4704 ***	0.0049	−0.6055 ***	−0.7780 ***	−0.0851	0.7946 **	0.8497 ***	0.1278	1

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