Investigation of the Relationship between Three- and One-Dimensional Infiltration Using a Mini Disc Infiltrometer †

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Abstract: Disc infiltrometer experiments were conducted in the laboratory on two disturbed soils, a loam and a silty clay loam soil, in order to investigate the relationship between three- and one-dimensional infiltration using the proposed equation of Smettem et al. A mini disc infiltrometer of a radius of 45 mm with suction ranged from −5 mm to −70 mm was used. Three- and one-dimensional infiltration tests were performed on repacked cores by applying pressure heads −70, −40 and −10 mm for loam soil, and −30 and −10 mm for silty clay loam soil. Analysis of the results showed that the difference between the three- and one-dimensional infiltration is linear with time confirming the equation of Smettem et al. [1]. Also, this difference is used to calculate the value of an additional infiltration parameter.

Keywords: infiltration; soil sorptivity; mini disc infiltrometer

1. Introduction

Knowledge of both soil water infiltration characteristics, saturated hydraulic conductivity and soil sorptivity, of the upper soil layers is essential for the modeling of field infiltration process for hydrological applications and agricultural water management.

Tension disc infiltrometers, among other experimental apparatus used in situ for determining these two infiltration characteristics, has been extensively used in last decades [1–5]. Tension disc infiltrometers allow measurements of infiltration with a constant and small negative pressure head, h_0, at the soil surface and has been extensively used to measure the near-saturated hydraulic conductivity, K_0, and sorptivity S_0 [1–5]. Disc infiltrometers have the advantage that are portable and use small volumes of water making them suitable for spatial variability studies [6–8].

Various methodologies have been proposed to determine K_0 and S_0 from three-dimensional infiltration data from circular source at soil surface using tension disc infiltrometer. Among these, some are based on steady-state flow data and others on transient flow data [2,4,9–14].

In case of steady-state flow from circular source with constant negative pressure head at soil surface, the analytical solution of Wooding [15], is used. Wooding's equation is applied by assuming that the soil is homogenous and isotropic and the initial water content is uniform. Hydraulic conductivity near saturation can be determined by steady-state infiltration experiments either using multiple-discs with the same pressure head imposed at the surface, or by single-disc infiltration experiments with multiple pressure heads [16]. Wooding's approach uses a two-parametric model for determining the hydraulic conductivity [17]. However, it is quite difficult to obtain reliable values

of sorptivity using infiltration data by disc infiltrometer since there is a difficult to determine the required short time where gravity ignored [12].

Due to uncertainties about the time at which a steady-state infiltration regime is attained, together with the fact that useful information is lost by ignoring the transient stage, several researchers use a transient three-dimensional infiltration equation for disc infiltrometers [18]. Although, several expressions for transient infiltration [3,4,12], have in common the following two-term cumulative infiltration equation analogous to Philip [19],

$$I_{3D} = C_1 \sqrt{t} + C_2 t ,$$

(1)

where I is the cumulative infiltration (L) and the subscript 3D refers to three-dimensional infiltration process. The coefficients C_1 (L T-0.5) and C_2 (L T-1) differ among the expressions used [18].

Haverkamp et al. [4], related C_1 and C_2 to S_0 and K_0 for short and medium times as follows:

$$C_1 = S_0,$$

(2)

$$C_2 = \frac{2-\beta}{3} K_0 + \frac{\gamma S_0^2}{r(\theta_0 - \theta_n)},$$

(3)

where the subscript 0 refers to values on the supply boundary where the pressure head is h_0 (L), n refers to the initial conditions, r is the disc radius (L), β is a shape factor lying between 0 and 1, and γ is a proportionality coefficient. The coefficient γ was originally set equal to $\sqrt{0.3}$ by [1], and later was revised to be $0.6 < \gamma < 0.8$ by [4]. Consequently, the Equation (1) takes the following form:

$$I_{3D} = S_0 \sqrt{t} + \frac{2-\beta}{3} K_0 t + \frac{\gamma S_0^2}{r(\theta_0 - \theta_n)} t ,$$

(4)

The first term of the right-hand side in Equation (4) represents vertical capillary flow and dominates the infiltration during the initial stage. The second term represents gravity-driven vertical flow and the third term represents lateral capillary flow.

Smettem et al. [1], showed that the additional term accounting for the side effects due the axisymetric flow geometry is linear in time:

$$I_{3D} - I_{1D} = \frac{\gamma S_0^2}{r(\theta_0 - \theta_n)} t ,$$

(5)

where the subscript 1D refers to one-dimensional infiltration process.

In order to apply Equation (5) an estimation of $C_1 = S_0$ and γ is required.

However, Vandervaere et al. [5], had reported that the calculation of C_1 and C_2 parameters by direct nonlinear fitting of Equation (1) on experimental data can provide fitted values for C_1 and C_2, even if these values have no physical meaning, e.g., negative C_2 value. So, negative K_0 is a physical impossibility indicating a problem with the data set. To overcome this problem, various linear fitting techniques of Equation (1) have been proposed to detect possible inadequacy of Equation (1), as well as scattering of data points.

Smiles and Knight [20], proposed linearizing of Equation (1) by dividing both sides by $\sqrt{t}$, giving the equation:

$$\frac{I_{3D}}{\sqrt{t}} = C_1 + C_2 \sqrt{t} ,$$

(6)

and then plotting $I_{3D}/\sqrt{t}$ as a function of $\sqrt{t}$. Thus, it is easy to determine $C_1 = S_0$ as the intercept and C_2 as the slope of the fitted line on experimental data. Also, S_0 could be estimated from cumulative one-dimensional infiltration data versus square root of time, at short times, where this relationship is linear with slope S_0.

The main purpose of this study is to investigate the difference between three- and one-dimensional infiltration on two disturbed soils, a loam and a silty clay loam soil, using a mini disc infiltrometer and to compare this difference with the proposed equation of Smettem et al. [1].

2. Materials and Methods

A mini disc infiltrometer [21], with a radius of 22.5 mm was used to perform three-dimensional infiltration experiments on repacked soils into a cylinder of 300 mm diameter and 300 mm length and the disc was placed in the center of the box to allow fully unconfined three-dimensional flow. On the other hand, one-dimensional infiltration experiments were performed on repacked soils into a cylinder of 45 mm diameter and 500 mm length and the disc was placed on the top surface of the soils.

The soils examined were a Loam (L) and a Silty Clay Loam (SiCL). The soil texture and the bulk density, ϱ_φ, of soils are presented in Table 1. The disturbed soil samples used had been air dried before the beginning of the experiments and consequently the initial soil water content was very low ($\theta_n = 0.04$ cm^3 cm^{-3} for both soils). The pressure heads applied, during the three- and one-dimensional infiltration tests, were −70, −40 and −10 mm for the L soil, and −30 and −10 mm for the SiCL soil. After cessation of each infiltration experiment, soil samples were obtained to determination soil water content.

Table 1. Soil particle size distribution and bulk density, ϱ_φ, of the soils examined.

Soil Type	Clay (%)	Silt (%)	Sand (%)	ϱ_φ (g.cm^{-3})
L	20.0	38	42.0	1.17
SiCL	36.5	52	11.5	1.23

The duration of the experiments ranged from 600 to 900 s. The sorptivity S_0 value for each pressure head h$_0$ was obtained by plotting one-dimensional infiltration data I_{1D} as a function of $\sqrt{t}$, for short times up to 60 s where the $I_{1D}\left(\sqrt{t}\right)$ relationship is linear.

3. Results and Discussion

In Figure 1, three- and one-dimensional infiltration relationships are depicted for L and SiCL soils for each pressure head used.
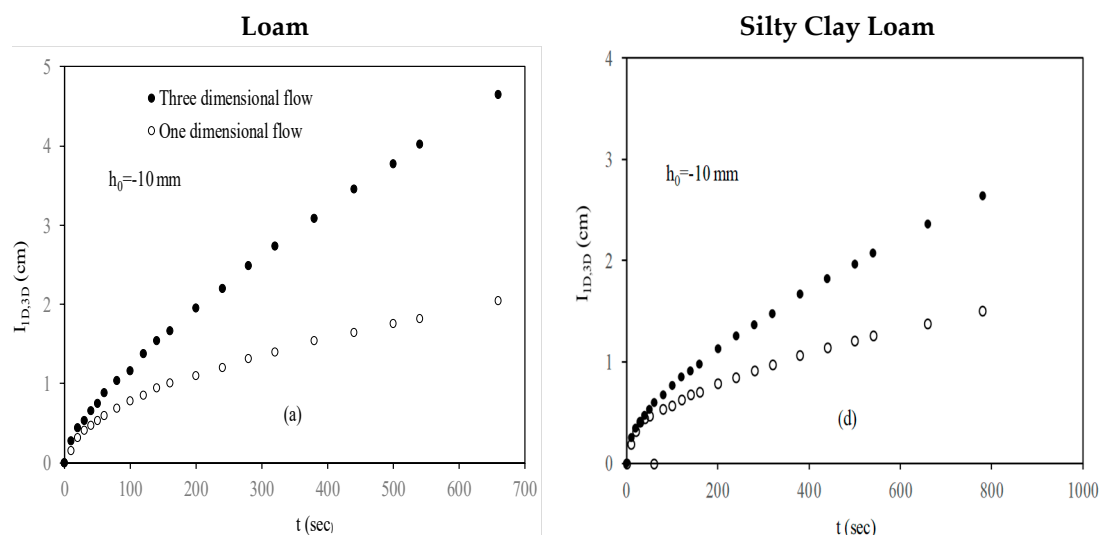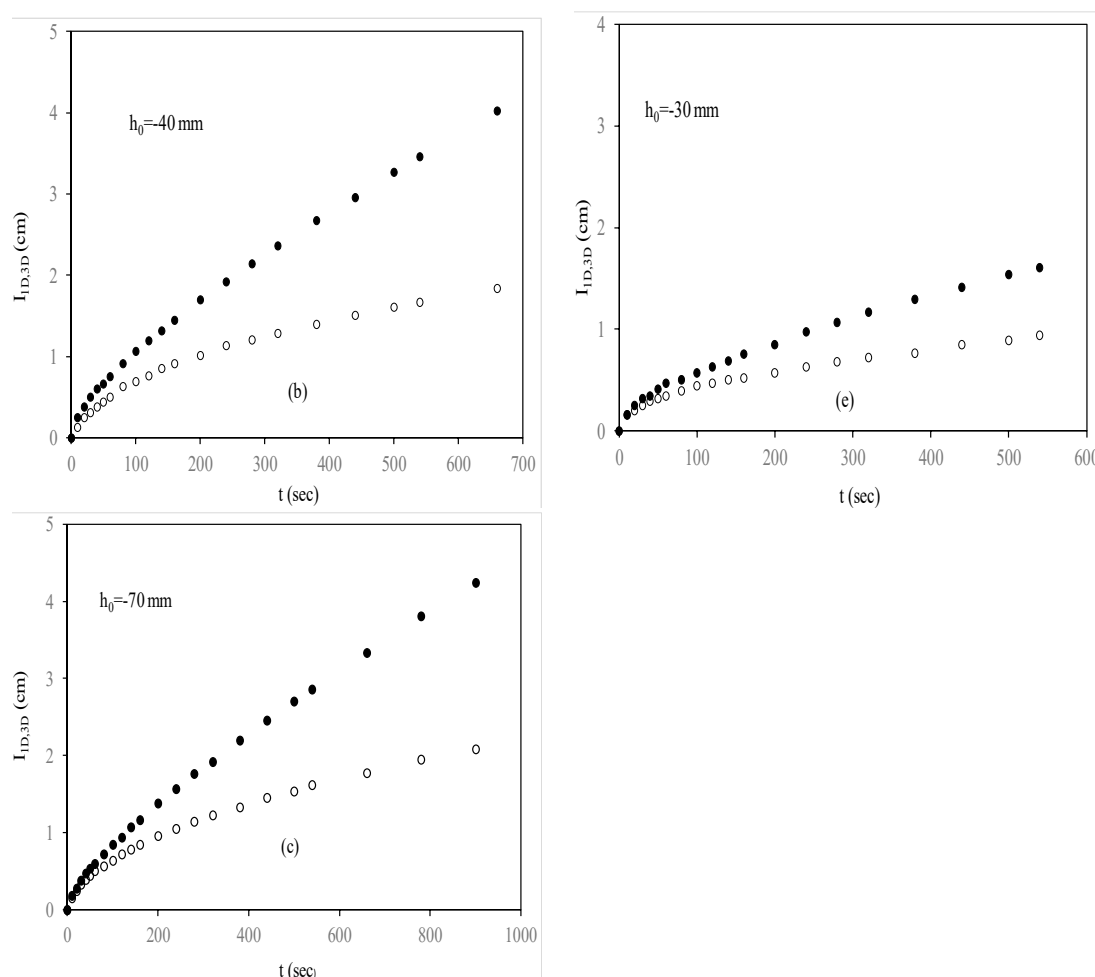
Figure 1. Representation of three- and one-dimensional experimental infiltration data with time, $I_{1D,3D}(t)$, for various pressure heads: (**a**) $h_0 = -10$ mm; (**b**) $h_0 = -40$ mm; (**c**) $h_0 = -70$ mm for Loam soil, and (**d**) $h_0 = -10$ mm; (**e**) $h_0 = -30$ mm for Silty Clay Loam soil.

At short times ($t \rightarrow 0$), the sorptivity S_0 can be calculated by the equation $I_{3D} = I_{1D} = S_0 \sqrt{t}$. However, Smettem [22], and Minasny and McBratney [23], showed that the classical approach to calculate sorptivity from a disk infiltrometer experiment (three-dimensional infiltration flow) may give erroneous result (overestimated). For this reason, S_0 value, for each pressure head, was obtained from the slope of the linear relationship of the one-dimensional infiltration data versus square root of time, $I_{1D}\left(\sqrt{t}\right)$, (data not shown) for short times where the effect of the gravity is negligible (Table 2).

Table 2. Parameters of one-dimensional infiltration experiment for loam (L) and silty clay loam (SiCL) soil and the coefficient of determination R^2 between the experimental infiltration data $I_{1D}\left(\sqrt{t}\right)$ and fitted curve for S_0 estimation.

Soil Type	Pressure Head, h_0 (mm)	Water Content, θ_0 (cm³cm⁻³)	Soil Sorptivity, S_0 (cm s⁻⁰·⁵)	R^2
L	−10	0.385	0.072	0.970
	−40	0.369	0.063	0.958
	−70	0.362	0.058	0.986
SiCL	−10	0.464	0.061	0.990
	−30	0.446	0.046	0.990

The one-dimensional infiltration data were used in contrast to the three-dimensional infiltration data, since the corresponding time where the $I_{3D}\left(\sqrt{t}\right)$ function is linear, is much shorter and therefore is uncertain in estimation of S_0 [4]. The choice of the appropriate time interval for calculating S_0 in the

one-dimensional infiltration experiments, where the soils used are fine-textured and disturbed, is relatively easy since no sand layer is needed to be used on the surface of each porous medium to ensure hydraulic contact between the infiltrometer and the soil. The presence of this layer can be affect on the first stages of infiltration making difficult the estimation of the appropriate time interval for calculating S_0 [5,24]. The value of S_0 at each pressure head can also be calculated by linearization methodologies of the Equation (1) using one- or three-dimensional infiltration data [5,20,22]. The applied methodology, in this study, for estimation of S_0 value for each pressure head compared with the aforementioned linearization method gave similar results. Perroux and White [25], after conducting disc infiltrometer experiments, showed that the time for estimating S_0 depends on the soil type and ranged from 6 to 2450 s with an average value of 60 s.

In Figure 2, the fitted curves of the linear relationship $\dfrac{I}{\sqrt{t}} = C_1 + C_2\sqrt{t}$ on the three- and one-dimensional infiltration data using pressure head $h_0 = -10$ mm, for the Loam soil, are presented. Similar results obtained using the other pressure heads for both soils (data not shown).
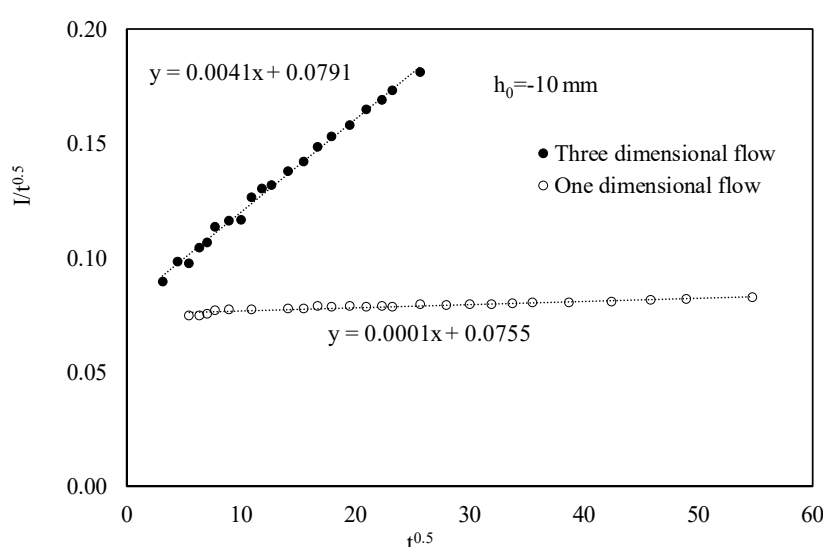

Figure 2. Linear representation of three- and one-dimensional experimental infiltration data and fitted linear lines for pressure head $h_0 = -10$mm, for Loam soil.

As shown in Figure 2, both fitting functions (i.e., on three- and one-dimensional infiltration data) have almost the same intercept value $C_1 = S_0$ [13]. The intercept values for all pressure heads tested in the two soils were very close to S_0 values obtained from the slope of the linear relationship of the one-dimentional infiltration data versus square root of time, $I_{1D}\left(\sqrt{t}\right)$, and are presented in Table 2. Vandervaere et al. [13], report that a difference up to 15% between S_0 values obtained from three- and one-dimensional infiltration is acceptable due to possible different soil packing.

In Figure 3, the difference between three- and one-dimensional infiltration (I_{3D}–I_{1D}) as a function of time t, for each pressure head tested in the two soils, is presented. As shown the difference I_{3D}-I_{1D} is linear in time t for all cases studied, with a coefficient of determination $R^2 > 0.99$, as predicted by the Equation (5) of Smettem et al. [1], and independent of gravity. It also appears that the slope of this linear function decreases by reducing the pressure head. This practically means that the reduction of S_0 is greater than the difference of soil water content θ_0–θ_n.
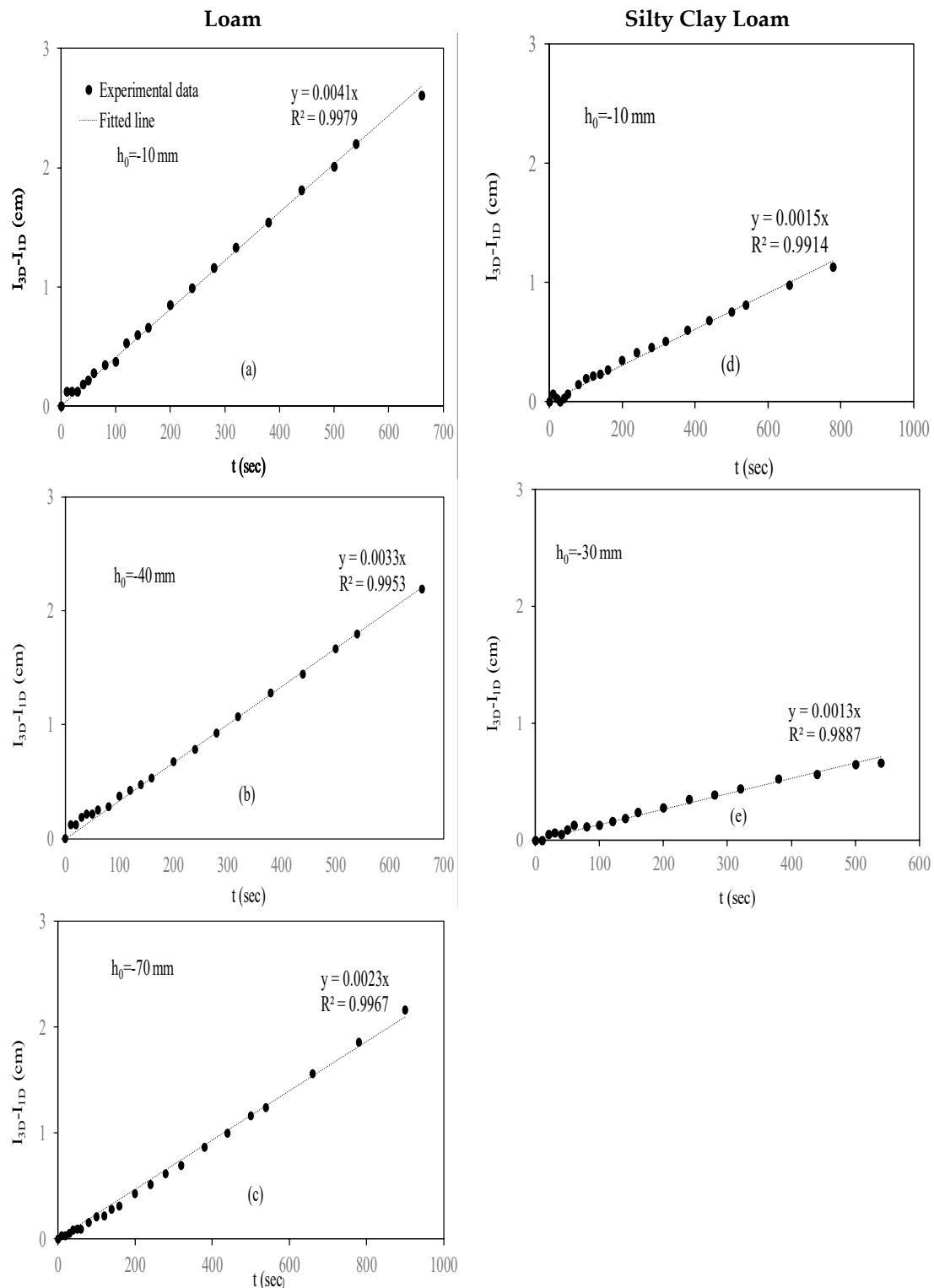
Figure 3. The difference between three- and one-dimensional infiltration flow over time, for pressure heads: (**a**) $h_0 = -10$ mm; (**b**) $h_0 = -40$ mm; (**c**) $h_0 = -70$ mm for Loam soil, and (**d**) $h_0 = -10$ mm; (**e**) $h_0 = -30$ mm for Silty Clay Loam soil. The fitted line is from Equation (5).

The comparison between the experimental relationship I_{3D}–I_{1D}(t) and the predicted one by the Equation (5) showed a very good agreement for the proportionality constant $\gamma = \sqrt{0.3}$, for both porous media used. This value of γ parameter is given in the model of Smettem et al. [1]. However, the value of $\gamma = 0.75$ was suggested by [1], as a result of the comparison between experimental and analytical results in the case of a sandy loam soil. This difference of γ values was attributed to the use of sharp wetting front approximation in developing the analytical solution. Haverkamp et al. [4],

reported that in normal working conditions are reasonably bounded by 0.6–0.8. In addition, Smettem et al. [22], using a double disc tension infiltrometer in three- and one-dimensional infiltration experiments on a sandy loam soil with pressure head $h_0 = -30$ mm, found a value of $\gamma = 0.726$.

The difference between the experimental value of γ of our study and those ones are referred in the works of [4], and [22], can be attributed to various factors such as soil type, initial and boundary conditions, as well as radius of infiltrometer.

4. Conclusions

The difference between three- and one-dimensional infiltration flow (I_{3D}–I_{1D}) is proportional to time as predicted by the equation of Smettem et al. [1]. However, the coefficient γ is different from that calculated from other experiments on different soils using different infiltrometers. The calculation of γ value, in combination with the value of S_0, as calculated by various methodologies, can be used to calculate the contribution of gravitational flow during three-dimensional infiltration.

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