## NOMENCLATURE

| A | constant appearing in the analytical solution (dim.) |
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| A* | constant appearing in the analytical solution (dim.) |
| $\mathrm{A}_{\mathrm{ij}}$ | interphase area for i-j phases ( $\mathrm{m}^{2}$ ) |
| $\mathrm{a}_{\mathrm{v}}$ | microscale effective area ( $\mathrm{m}^{2} \mathrm{~m}^{-3}$ ) |
| $\mathrm{a}_{\mathrm{v}}$ | macroscale effective area ( $\mathrm{m}^{2} \mathrm{~m}^{-3}$ ) |
| $\mathrm{c}_{\mathrm{ji}}$ | i -ion concentration in j-phase ( $\mathrm{mol} \mathrm{m}^{-3}$ ) |
| $\widetilde{c}_{j i}$ | deviation of j-ion concentration in i-phase ( $\mathrm{mol} \mathrm{m}^{-3}$ ) |
| $\left\langle\mathrm{c}_{j i}\right\rangle^{\mathrm{i}}$ | intrinsic phase average i-ion concentration in j -phase ( $\mathrm{mol} \mathrm{m}^{-3}$ ) |
| $\left\langle\mathrm{c}_{\mathrm{i}}\right\rangle^{*}$ | one-equation model phase average i-ion concentration ( $\mathrm{mol} \mathrm{m}^{-3}$ ) |
| $\mathrm{d}_{\mathrm{p}}$ | pore diameter (m) |
| $D_{i}$ | bulk solution diffusion coefficient ( $\mathrm{m}^{2} \mathrm{~s}^{-1}$ ) |
| $\underline{\underline{D}}{ }_{i}$ | one-equation model total effective diffusivity tensor ( $\mathrm{m}^{2} \mathrm{~s}^{-1}$ ) |
| $\underline{\underline{D}}_{i, d i f f}$ | global effective diffusivity tensor ( $\mathrm{m}^{2} \mathrm{~s}^{-1}$ ) |
| $\underline{\underline{D}}_{i, d i s p}$ | dispersion effective diffusivity tensor ( $\mathrm{m}^{2} \mathrm{~s}^{-1}$ ) |
| $\underline{\underline{D}}_{i, e f f}$ | microscale effective diffusivity tensor ( $\mathrm{m}^{2} \mathrm{~s}^{-1}$ ) |
| $\underline{\underline{D}}_{\text {ii }}$ | diagonal effective diffusivity tensor, $\mathrm{i}=\mathrm{\kappa}, \gamma\left(\mathrm{~m}^{2} \mathrm{~s}^{-1}\right)$ |
| $\underline{\underline{D}}_{\text {ij }}$ | cross-term effective diffusivity tensor, $\mathrm{i}, \mathrm{j}=\kappa, \gamma$ and $\mathrm{i} \neq \mathrm{j}\left(\mathrm{m}^{2} \mathrm{~s}^{-1}\right)$ |
| $\underline{\underline{D}}_{i, \text { eff }}$ | i-ion microscale effective diffusivity tensor ( $\mathrm{m}^{2} \mathrm{~s}^{-1}$ ) |
| $\underline{\underline{D}}{ }_{i j}$ | effective diffusivity tensor for the two-equation model ( $\mathrm{m}^{2} \mathrm{~s}^{-1}$ ) |
| $f$ | microscale closure vector fields (m) |
| $f_{j i}$ | macroscale closure vector fields, $\mathrm{j}=\kappa, \gamma(\mathrm{m})$ |
| F | Faraday constant ( $\mathrm{C} \mathrm{mol}^{-1}$ ) |
| $g$ | microscale closure vector fields (m) |
| $g_{i}$ | macroscale closure vector fields $\mathrm{s}, \mathrm{i}=\kappa, \gamma$ (m) |
| $g_{j i}$ | macroscale closure vector fields, $\mathrm{j}=\kappa, \gamma(\mathrm{m})$ |
| $\underline{h_{i}}$ | macroscale closure vector fields (m) |
| $\underline{h}_{j i}$ | macroscale closure vector fields, $\mathrm{j}=\mathrm{\kappa}, \gamma(\mathrm{~m})$ |

unit tensor (dim.)
closure vector fields, $\mathrm{i}=\kappa, \gamma(\mathrm{m})$
interfacial transfer coefficient for potential $\left(\mathrm{m} \mathrm{s}^{-1}\right)$
interfacial transfer coefficient for i-ion concentration ( $\mathrm{m} \mathrm{s}^{-1}$ )
characteristic length in i-phase (m)
mixed length scale combining interfacial and mass transport (m)
electrode length scale (m)
unit normal vector from the i into the j -phase (dim.)
ion flux ( $\mathrm{mol} \mathrm{m}-{ }^{-2} \mathrm{~s}^{-1}$ )
Peclet number (dim.)
radial cylindrical coordinate (m)
Chang's unit cell particle dimension (m)
Chang's unit cell dimension (m)
gas constant ( $\mathrm{J} \mathrm{K}^{-1} \mathrm{~mol}^{-1}$ )
radius of the microscale representative elementary volume (m)
radius of the macroscale representative elementary volume (m)
macroscale closure vector fields (m)
microscale interfacial dimensionless mass transport coefficient (dim.)
macroscale interfacial dimensionless mass transport coefficient (dim.)
macroscale closure scalar fields (dim.)
time (s)
macroscale closure vector fields, $\mathrm{j}=\kappa, \gamma($ dim.;)
microscale effective mobility tensor $\left(\mathrm{m}^{2} \mathrm{~V}^{-1} \mathrm{~s}^{-1}\right)$
macroscale total effective mobility tensor $\left(\mathrm{m}^{2} \mathrm{~V}^{-1} \mathrm{~s}^{-1}\right)$
two-equation model effective mobility tensors, $\mathrm{i}, \mathrm{j}=\kappa, \gamma\left(\mathrm{m}^{2} \mathrm{~V}^{-1} \mathrm{~s}^{-1}\right)$
macroscale closure vector fields, $\mathrm{j}=\kappa, \gamma(\mathrm{m})$
i-phase volume in REV ( $\mathrm{m}^{3}$ )
microscale REV (m ${ }^{3}$ )
macroscale REV ( $\mathrm{m}^{3}$ )
convective velocity ( $\mathrm{m} \mathrm{s}^{-1}$ )

| $\underline{\tilde{v}}_{s}$ | convective velocity deviation ( $\mathrm{m} \mathrm{s}^{-1}$ ) |
| :---: | :---: |
| $\left\langle\underline{v}_{s}\right\rangle^{\gamma}$ | intrinsic phase average velocity ( $\mathrm{m} \mathrm{s}^{-1}$ ) |
| $\mathrm{V}_{T}$ | thermal voltage (V) |
| $\underline{w}_{j i}$ | macroscale closure vector fields, $\mathrm{j}=\kappa, \gamma(\mathrm{m})$ |
| $\mathrm{Z}_{\mathrm{i}}$ | ionic charge number (dim.) |
| Greek letters |  |
| $\delta$ | Mass diffusivity ratio, ( $\left.\varepsilon_{a} \mathrm{D}_{\text {к,eff }} / \mathrm{D}_{\gamma}, \mathrm{dim}.\right)$ |
| $\mathcal{E}_{i}$ | i-phase volume fraction (dim.) |
| $\langle\varepsilon\rangle$ | factor defined in eq. (25) (dim.) |
| $\phi$ | electrostatic potential (V) |
| $\psi_{i}$ | dimensionless electrostatic potential in i-phase (dim.) |
| $\widetilde{\psi}_{i}$ | potential deviation in i-phase (V) |
| $\left\langle\psi_{i}\right\rangle^{\mathrm{i}}$ | intrinsic phase average potential in i-phase (dim.) |
| $\langle\psi\rangle^{*}$ | one-equation model phase average potential (dim.) |
| $\eta$ | phase permittivity ( $\mathrm{F} \mathrm{m}^{-1}$ ) |
| $\underline{\underline{\eta}}{ }^{*}$ | one-equation model effective permittivity tensor ( $\mathrm{F} \mathrm{m}^{-1}$ ) |
| $\underline{\eta}_{e f f}$ | microscale effective permittivity tensor ( $\mathrm{F} \mathrm{m}^{-1}$ ) |
| $\eta_{=i j}$ | two-equation model effective permittivity tensors; $\mathrm{i}, \mathrm{j}=\kappa, \gamma\left(\mathrm{F} \mathrm{m}^{-1}\right)$ |
| $\lambda_{D}$ | Debye length (m) |
| $\theta$ | azimuthal cylindrical coordinate (m) |
| V | nabla operator ( $\mathrm{m}^{-1}$ ) |
| $\sigma$ | charge density ( $\mathrm{C} \mathrm{m}^{-2}$ ) |
| $\langle\sigma\rangle_{i j}$ | charge density area average; $\mathrm{i}, \mathrm{j}=\kappa, \gamma ; \mathrm{i} \neq \mathrm{j}\left(\mathrm{C} \mathrm{m}^{-2}\right)$ |
| $\xi_{j i}$ | closure variables in eqns. (54) and (55), $\mathrm{j}=\kappa, \gamma\left(\mathrm{mol} \mathrm{m}^{-3}\right)$ |
| $\zeta_{j i}$ | closure variables in eqns. (46) and (47), $\mathrm{j}=\kappa, \gamma\left(\mathrm{mol} \mathrm{m}^{-3}\right)$ |
| Subscripts |  |
| $\alpha$ | $\alpha$-phase |
| $\beta$ | $\beta$-phase |
| $\gamma$ | $\gamma$-phase |
| $\kappa$ | $\kappa$-phase |

