NOMENCLATURE

А	constant appearing in the analytical solution (dim.)
A^*	constant appearing in the analytical solution (dim.)
A _{ij}	interphase area for i-j phases (m ²)
a _v	microscale effective area $(m^2 m^{-3})$
a _{vM}	macroscale effective area ($m^2 m^{-3}$)
c _{ji}	i-ion concentration in j-phase (mol m ⁻³)
c _{ji}	deviation of j-ion concentration in i-phase (mol m ⁻³)
$\langle c_{ji} \rangle^{i}$	intrinsic phase average i-ion concentration in j-phase (mol m ⁻³)
$\langle c_i \rangle^*$	one-equation model phase average i-ion concentration (mol m ⁻³)
d _p	pore diameter (m)
D_i	bulk solution diffusion coefficient (m ² s ⁻¹)
$\underline{\underline{D}}_{i}^{*}$	one-equation model total effective diffusivity tensor (m ^{2} s ⁻¹)
$\underline{\underline{D}}_{i,diff}$	global effective diffusivity tensor ($m^2 s^{-1}$)
$\underline{\underline{D}}_{\underline{i},disp}$	dispersion effective diffusivity tensor $(m^2 s^{-1})$
$\underline{\underline{D}}_{i,eff}$	microscale effective diffusivity tensor $(m^2 s^{-1})$
D_ii	diagonal effective diffusivity tensor, $i=\kappa$, γ (m ² s ⁻¹)
$\underline{\underline{D}}_{ij}$	cross-term effective diffusivity tensor, i, j = κ , γ and i \neq j (m ² s ⁻¹)
$\underline{\underline{D}}_{i,e\!f\!f}$	i-ion microscale effective diffusivity tensor (m 2 s $^{-1}$)
$\underline{\underline{D}}_{ij}$	effective diffusivity tensor for the two-equation model ($m^2 s^{-1}$)
£	microscale closure vector fields (m)
Ĺji	macroscale closure vector fields, $j = \kappa, \gamma$ (m)
F	Faraday constant (C mol ⁻¹)
g	microscale closure vector fields (m)
\underline{g}_i	macroscale closure vector fields s, $i=\kappa, \gamma$ (m)
Q ji	macroscale closure vector fields, $j=\kappa, \gamma$ (m)
\underline{h}_i	macroscale closure vector fields (m)
<u>h</u> ji	macroscale closure vector fields, $j=\kappa, \gamma(m)$

Ī	unit tensor (dim.)
<u>Ĵ</u> i	closure vector fields, $i=\kappa$, γ (m)
k	interfacial transfer coefficient for potential(m s ⁻¹)
k _i	interfacial transfer coefficient for i-ion concentration (m s ⁻¹)
li	characteristic length in i-phase (m)
l_{κ}	mixed length scale combining interfacial and mass transport (m)
L _e	electrode length scale (m)
<u>n</u> _{ij}	unit normal vector from the i into the j-phase (dim.)
N _i	ion flux (mol $m^{-2} s^{-1}$)
Pe	Peclet number (dim.)
r	radial cylindrical coordinate (m)
\mathbf{r}_1	Chang's unit cell particle dimension (m)
r ₂	Chang's unit cell dimension (m)
R	gas constant (J K ⁻¹ mol ⁻¹)
R _m	radius of the microscale representative elementary volume (m)
R _M	radius of the macroscale representative elementary volume (m)
\underline{h}_i	macroscale closure vector fields (m)
Sh	microscale interfacial dimensionless mass transport coefficient (dim.)
Sh_M	macroscale interfacial dimensionless mass transport coefficient (dim.)
Si	macroscale closure scalar fields (dim.)
t	time (s)
<u>t</u> ji	macroscale closure vector fields, $j=\kappa, \gamma$ (dim.;)
$\underbrace{U}_{=i,\textit{eff}}$	microscale effective mobility tensor (m ² V ⁻¹ s ⁻¹)
$\underline{\underline{U}}_{i}^{*}$	macroscale total effective mobility tensor $(m^2 V^{-1} s^{-1})$
Uij	two-equation model effective mobility tensors, i, j = κ , γ (m ² V ⁻¹ s ⁻¹)
<u>V</u> ji	macroscale closure vector fields, $j=\kappa, \gamma(m)$
Vi	i-phase volume in REV (m ³)
V _m	microscale REV (m ³)
V_{M}	macroscale REV (m ³)
$\underline{\nu}_s$	convective velocity (m s ⁻¹)

$\frac{\widetilde{\mathcal{V}}_{s}}{s}$	convective velocity deviation (m s ⁻¹)
$\left< \underline{\nu}_s \right>^{\gamma}$	intrinsic phase average velocity (m s ⁻¹)
V _T	thermal voltage (V)
\underline{W}_{ji}	macroscale closure vector fields, $j=\kappa, \gamma(m)$
Zi	ionic charge number (dim.)
Greek letters	
δ ϵ_i	Mass diffusivity ratio, ($\varepsilon_a D_{\kappa, eff}/D_{\gamma}$, dim.) i-phase volume fraction (dim.)
$\langle \mathcal{E} \rangle$	factor defined in eq. (25) (dim.)
$egin{array}{lll} \phi & & \ \psi_i & \ \widetilde{\psi}_i & \ \end{array}$	electrostatic potential (V) dimensionless electrostatic potential in i-phase (dim.) potential deviation in i-phase (V)
$\langle \psi_i angle^{ m i}$	intrinsic phase average potential in i-phase (dim.)
$\langle\psi angle^{*}$	one-equation model phase average potential (dim.)
η	phase permittivity (F m ⁻¹)
$\underline{\underline{\eta}} *$	one-equation model effective permittivity tensor (F m ⁻¹)
$\eta_{=e\!f\!f}$	microscale effective permittivity tensor (F m ⁻¹)
$\eta_{=ij}$	two-equation model effective permittivity tensors; i, j = κ , γ (F m ⁻¹)
λ_D	Debye length (m)
θ	azimuthal cylindrical coordinate (m)
$\frac{\nabla}{\sigma}$	nabla operator (m ⁻¹) charge density (C m ⁻²)
$\langle \sigma angle_{_{ij}}$	charge density area average; i, j =κ, γ; i≠j (C m ⁻²)
ξ_{ji}	closure variables in eqns. (54) and (55), $j = \kappa, \gamma \pmod{m^{-3}}$
ζ_{ji}	closure variables in eqns. (46) and (47), $j = \kappa, \gamma \pmod{m^{-3}}$

Subscripts	
α	

α	α-phase
β	β-phase
γ	γ-phase
к	к-phase